Bank Performance Benchmarking in Stochastic Environments using Log-Linear Mean-Variance Data Envelopment Analysis

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In this paper, we address the potential non-robustness of standard DEA models with respect to stochastic disturbances in the data. Disturbances such as measurement errors, external effects and outliers often occur in bank performance studies. We propose to extend the log-linear CCR DEA model by including mean-variance conditions that are consistent with rational choice behaviour for general preference structures and general disturbance distributions. The result is a technique that yields performance benchmarks that are not only dominant in deterministic settings, but also stochastically dominate evaluated DMUs by second order. The basis concept and properties of the resulting Mean Variance DEA are illustrated for a large sample of Indonesian banks.

1. Introduction

Research on production and cost structure analysis and performance evaluation and benchmarking for banks concentrates on two distinct analytical techniques. Stochastic Frontier Analysis (Berger et al., 1987; Berger and Humphrey, 1991; Pulley and Braunstein, 1992; Bauer and Hancock, 1993; Berger et al., 1993; Mester, 1993; Kaparakis et al., 1994; Lang and Welzel, 1996; Mahajan et al., 1996; Mester, 1996; Mitchell and Onvural (1996); Goldberg and A. Rai, 1996) uses econometric estimation techniques to estimate a parametric production or cost frontier. The SFA, approach is handicapped by the potential specification bias that may arise from its requirement to a priori specify the functional form of the production or cost relationships and the distributional structure of inefficient deviations from the efficient frontier.

Data Envelopment Analysis uses Linear Programming techniques to estimate a non-parametric frontier. Some published accounts of uses of DEA in bank performance evaluation are Sherman and Gold (1985), Rangan et al. (1988), Ferrier and Lovell (1990), Oral and Yolalan (1990), Vassiloglou and Giokas (1990), Giokas (1991), Leibenstein and Maital (1992), Yue (1992), Fried et al. (1993), Drake and Howcroft (1994), Golany and Storbeck (1995), Kantor and Maital (1995), Soteriou and Zenios (1995), Sherman and Ladino (1995), Miller and Noulas (1996), Soteriou and Zenios (1996), Yeh (1996) and Resti (1997). One of the main attractions of DEA is its minimal requirement for prior production assumptions. Nevertheless, its practical applicability is handicapped by its implicit distributional assumption that all input-output variables are measured accurately, and its consequent non-robustness regarding external effects, outliers and measurement error.

This limitation has been recognized in the DEA literature and a number of solutions has been proposed to deal with stochastic environments. For example, robust efficiency scores and robust reference units can be obtained by adding to conventional deterministic DEA models restrictions regarding sensitivity measures, such as the so-called 'regions of stability' (Charnes et al., 1992) and the so-called 'polyhedrons of robustness' (Zhu, 1996). Alternatively, disturbances can be incorporated by using certainty equivalence of stochastic input-output variables (Gong and Sun, 1995). In addition, noise can be filtered from DEA inefficiencies by estimating a parametric error structure, as in the so-called DEA+ approach of Gstach (1996).

Post (1997) proposed an alternative approach, Mean-Variance Data Envelopment Analysis (MV-DEA), relying on the application of mean-variance conditions derived from the stochastic dominance theory. In this paper, we illustrate the operation and the characteristics of the MV-DEA approach using a large sample of Indonesian banks. In addition, we extend the original MV-DEA model by employing a multiplicative error structure and a log-linear DEA model, so as to circumvent certain limitations of MV-DEA models employing an additive error structure and a linear DEA model.

The remainder of this paper is organised as follows. Section 2 describes the conventional deterministic DEA methodology and its inherent non-robustness in stochastic environments. Section 3 describes the stochastic MV-DEA technique. Section 4 introduces the proposed log-linear MV-DEA model. Section 5 presents an empirical application of the proposed log-linear MV-DEA model to a large sample of Indonesian banks MV-DEA. Finally, section 6 offers some concluding remarks.

2. The standard DEA methodology

In the standard CCR-DEA model (Charnes et al., 1978), the performance of a set of comparable Decision Making Units (DMUs) is measured against an empirical Production Possibility Set (PPS). In general, that PPS is defined as the smallest subset in input-output space which is consistent with both the observations and the imposed assumption of a monotone increasing and concave production function with constant-returns-to-scale properties[1].

By projecting inefficient DMUs onto the efficient frontier, efficient reference units are selected from the PPS, so as to act as performance benchmarks for inefficient DMUs. The projection path employed reflects the preference assumptions imposed. The input-oriented version of the standard CCR-DEA model selects as a reference unit, for each DMU separately, the composite unit that consumes the lowest possible fraction of that DMU's current input levels to produce at least that DMU's current output levels[2]. More formally, the reference units are identified simultaneously by solving the following linear programming problem:

$$(1) \quad \min_{\theta_k, \lambda_{kj}} \sum_{k=1}^{n} \theta_k$$

$$s.t. \sum_{j=1}^{n} \lambda_{kj} y_{rj} \geq y_{rk} \quad k=1,\ldots,n;\ r=1,\ldots,s$$

$$\sum_{j=1}^{n} \lambda_{kj} x_{ij} \leq \theta_k x_{ik} \quad k=1,\ldots,n;\ i=1,\ldots,m$$

$$\lambda_{kj} \geq 0 \quad k=1,\ldots,n;\ j=1,\ldots,n$$

Notation

θ_k = efficiency score of k-th DMU
y_{rj} = quantity of r-th output for j-th DMU
x_{ij} = quantity of i-th input for j-th DMU
λ_{kj} = proportion of j-th DMU in reference unit for k-th DMU

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[1] Although the standard CCR-DEA model implicitly makes such simple production assumptions, the DEA methodology offers considerable flexibility to incorporate alternative or additional production information. For example, the standard DEA model has been extended and refined to include variable returns-to-scale properties (see Banker et al., 1984, Deprins et al., 1984, and Petersen, 1990), non-linear input substitutability and output transformability (see Charnes et al., 1994), categorical input-output variables (see Banker and Morey, 1986) and ordinal input-output variables (see Olesen and Petersen, 1995 and Cook et al., 1996). Another method to incorporate additional production information relies on expanding the observed reference group, using a set of 'standard' DMUs, constructed by using industrial engineering standards or observed operating practice of similar organizations (see Golany and Roll, 1994).

[2] Although the standard model implicitly makes a number of rather strong preference assumptions, a number of extensions and refinements has been proposed to incorporate alternative or additional preference information. For example, judgement can be incorporated by limiting the flexibility of the model in assigning values to the input-output weights (see Dyson and Thanassoulis (1988); Thompson et al., 1986; Wong and Beasley, 1990; Charnes et al., 1990 and Thompson et al., 1995), or by combining DEA with interactive decision procedures (see Golany, 1988; Belton and Vickers, 1990, 1992, 1993 and Post and Spronk, 1996).

3. Mean-Variance Data Envelopment Analysis

Formulation (1) demonstrates that DEA reference units dominate evaluated DMUs on all input-output variables. Nevertheless, since the input-output data are assumed to be measured accurately, the dominance relationship may be disturbed by external effects, measurement error or random noise. Post (1997) proposed to incorporate stochastic input-output variables by adding to conventional deterministic DEA models the condition that reference units should necessarily have higher (respectively lower) means and lower variances than the corresponding evaluated DMUs, for all its output variables (respectively input variables):

$$(2A) \quad E[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] \geq E[y_{rk}] \quad r=1,\ldots,s;\ k=1,\ldots,n$$

$$(2B) \quad VAR[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] \leq VAR[y_{rk}] \quad r=1,\ldots,s;\ k=1,\ldots,n$$

$$(2C) \quad E[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] \leq E[x_{ik}] \quad i=1,\ldots,m;\ k=1,\ldots,n$$

$$(2D) \quad VAR[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] \leq VAR[x_{ik}] \quad i=1,\ldots,m;\ k=1,\ldots,n$$

Reference units satisfying these conditions not only dominate evaluated DMUs in an deterministic environment, but also stochastically dominate by second order[3], in case of normalized risk comparable stochastic input-output variables[4] that are measured with mutually independent disturbance terms. To operationalize these mean-variance conditions, Post (1997) proposed an additive error structure with mutually independent, zero-mean disturbances:

$$(3A) \quad y_{rj} = y_{rj}^{obs} + u_{rj} \quad r=1,\ldots,s;\ j=1,\ldots,n$$

$$E[u_{rj}]=0 \quad r=1,\ldots,s;\ j=1,\ldots,n$$

$$E[u_{rj}^2]=\sigma_r^2 \quad r=1,\ldots,s;\ j=1,\ldots,n$$

$$E[u_{rj}u_{rk}]=0 \quad r=1,\ldots,s;\ j\neq k$$

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[3] The second order stochastic dominance criterion (Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970 and Whitmore, 1970) is a decision criterion that is consistent with rational choice behaviour for all non-satiable and risk averse decision makers. For a detailed survey and analysis of the stochastic dominance literature, see Levy (1992).

[4] Normalized risk comparable random variables can be completely ordered by Rothschild-Stiglitz (1970) increasing risk, at least after adjusting for differences in their means. They constitute a general class of random variables, including all random variables that differ only by a scale and a location parameter (Meyer, 1987), such as normally distributed random variables (Tobin, 1958).

$$(3B) \quad x_{ij} = x_{ij}^{obs} + u_{ij} \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}] = 0 \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}^2] = \sigma_i^2 \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}u_{ik}] = 0 \quad i = 1,\ldots,m;\ j \neq k$$

Notation

x_{ij}^{obs} observed quantity of i-th input of j-th DMU
u_{ij} disturbance on i-th input of j-th DMU
σ_i^2 variance of disturbances on i-th input
y_{rj}^{obs} observed quantity of r-th output of j-th DMU
u_{rj} disturbance on r-th output of j-th DMU
σ_r^2 variance of disturbances on r-th ouput

In this specification, the levels of variance (σ_r^2, σ_i^2) are not restricted and, moreover, general heteroscedastic patterns between input-output variables can be accounted for. Using the proposed error structure, the mean and variance terms in (2A)-(2D) can be reformulated as:

$$(4A) \quad E[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] = \sum_{j=1}^{n} \lambda_{kj} y_{rj}^{obs} \quad r = 1,\ldots,s;\ k = 1,\ldots,n$$

$$(4B) \quad VAR[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] = \sum_{j=1}^{n} \lambda_{kj}^2 \sigma_r^2 \quad r = 1,\ldots,s;\ k = 1,\ldots,n$$

$$(4C) \quad E[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] = \sum_{j=1}^{n} \lambda_{kj} x_{ij}^{obs} \quad i = 1,\ldots,m;\ k = 1,\ldots,n$$

$$(4D) \quad VAR[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] = \sum_{j=1}^{n} \lambda_{kj}^2 \sigma_i^2 \quad i = 1,\ldots,m;\ k = 1,\ldots,n$$

Consequently, the mean-variance conditions (2A)-(2D) can be rewritten to yield:

$$(5A) \quad \sum_{j=1}^{n} \lambda_{kj} y_{rj}^{obs} \geq y_{rk}^{obs} \quad k = 1,\ldots,n;\ r = 1,\ldots,s$$

$$(5B) \quad \sum_{j=1}^{n} \lambda_{kj} x_{ij}^{obs} \leq x_{ik}^{obs} \quad k = 1,\ldots,n;\ i = 1,\ldots,m$$

$$(5C) \quad \sum_{j=1}^{n} \lambda_{jk}^2 \leq 1 \quad k = 1,\ldots,n$$

These mean-variance conditions have the attraction of being consistent with rational choice behaviour for general preference structures and general disturbance distributions. In addition, these conditions can be implemented by simply imposing additional restrictions on conventional deterministic DEA models, and, consequently, do not require complementary analysis. In addition, the mean-variance conditions preserve the attractions of the conventional DEA mathematical programming structure regarding convexity of the feasible set and feasibility and statistical consistency of solutions (see Post, 1997).

However, the above specification is based on the assumption of homoscedasticity of disturbances across DMUs (see 3A and 3B). This assumption may not be tenable if DMUs substantially differ in size, and may induce specification bias. For example, if variance increases with size, the above conditions discriminate against small-sized DMUs, because their relative variance is overestimated, and, consequently, the variance conditions are more restrictive for these units. To circumvent this problem, more general variance patterns can be specified. For example, Post (1997) proposed the following heteroscedastic error-structure:

$$(6A)\quad y_{rj} = y_{rj}^{obs} + u_{rj} \quad r = 1,\ldots,s;\ j = 1,\ldots,n$$
$$E[u_{rj}] = 0 \quad r = 1,\ldots,s;\ j = 1,\ldots,n$$
$$E[u_{rj}^2] = \sigma_r^2 (y_{rj}^{obs})^{\gamma_r} \quad r = 1,\ldots,s;\ j = 1,\ldots,n;\ \gamma_r \in [0,2]$$
$$E[u_{rj}u_{rk}] = 0 \quad r = 1,\ldots,s;\ k \neq j$$

$$(6B)\quad x_{ij} = x_{ij}^{obs} + u_{ij} \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$
$$E[u_{ij}] = 0 \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$
$$E[u_{ij}^2] = \sigma_i^2 (x_{ij}^{obs})^{\gamma_i} \quad i = 1,\ldots,m;\ j = 1,\ldots,n;\ \gamma_i \in [0,2]$$
$$E[u_{ij}u_{ik}] = 0 \quad i = 1,\ldots,m;\ k \neq j$$

Under this error structure, the mean and variance terms in (2A)-(2D) can be rewritten as:

$$(7A)\quad E[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] = \sum_{j=1}^{n} \lambda_{kj} y_{rj}^{obs} \quad r = 1,\ldots,s;\ k = 1,\ldots,n$$

$$(7B)\quad VAR[\sum_{j=1}^{n} \lambda_{kj} y_{rj}] = \sum_{j=1}^{n} \lambda_{kj}^2 \sigma_r^2 (y_{rj}^{obs})^{\gamma_r} \quad r = 1,\ldots,s;\ k = 1,\ldots,n$$

$$(7C)\quad E[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] = \sum_{j=1}^{n} \lambda_{kj} x_{ij}^{obs} \quad i = 1,\ldots,m;\ k = 1,\ldots,n$$

$$(7D)\quad VAR[\sum_{j=1}^{n} \lambda_{kj} x_{ij}] = \sum_{j=1}^{n} \lambda_{kj}^2 \sigma_i^2 (x_{ij}^{obs})^{\gamma_i} \quad i = 1,\ldots,m;\ k = 1,\ldots,n$$

Consequently, the mean-variance conditions (2A-2D) can be reformulated to yield:

$$(8A)\quad \sum_{j=1}^{n} \lambda_{kj}\, y_{rj}^{obs} \geq y_{rk}^{obs} \quad r = 1,\ldots,s;\ k = 1,\ldots,n$$

$$(8B)\quad \sum_{j=1}^{n} \lambda_{kj}^{2} \sigma_{r}^{2} (y_{rj}^{obs})^{\gamma_r} \leq \sigma_{r}^{2} (y_{rk}^{obs})^{\gamma_r} \quad r = 1,\ldots,s;\ k = 1,\ldots,n;\ \gamma_r \in [0,2]$$

$$(8C)\quad \sum_{j=1}^{n} \lambda_{kj}\, x_{ij}^{obs} \leq x_{ij}^{obs} \quad i = 1,\ldots,m;\ k = 1,\ldots,n$$

$$(8D)\quad \sum_{j=1}^{n} \lambda_{ij}^{2} \sigma_{i}^{2} (x_{ij}^{obs})^{\gamma_i} \leq \sigma_{i}^{2} (x_{ij}^{obs})^{\gamma_i} \quad i = 1,\ldots,m;\ k = 1,\ldots,n;\ \gamma_i \in [0,2]$$

Using the convexity property of the variance terms of reference units relative to evaluated units[5], this set of mean-variance conditions can be reformulated to yield the following equivalent set of mean-variance conditions:

$$(9A)\quad \sum_{j=1}^{n} \lambda_{kj}\, y_{rj}^{obs} \geq y_{rk}^{obs} \quad k = 1,\ldots,n;\ r = 1,\ldots,s$$

$$(9B)\quad \sum_{j=1}^{n} \lambda_{kj}^{2} (y_{rj}^{obs})^{2} \leq (y_{rk}^{obs})^{2} \quad k = 1,\ldots,n;\ r = 1,\ldots,s$$

$$(9C)\quad \sum_{j=1}^{n} \lambda_{kj}\, x_{ij}^{obs} \leq x_{ik}^{obs} \quad k = 1,\ldots,n;\ i = 1,\ldots,m$$

$$(9D)\quad \sum_{j=1}^{n} \lambda_{kj}^{2} (x_{ij}^{obs})^{2} \leq (x_{ik}^{obs})^{2} \quad k = 1,\ldots,n;\ i = 1,\ldots,m$$

$$(9E)\quad \sum_{j=1}^{n} \lambda_{jk}^{2} \leq 1 \quad k = 1,\ldots,n$$

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[5] The relative variance of an input variable i of a reference unit relative to an evaluated unit k can be represented as a function $f(\gamma_i) = \sum_{j=1}^{n} \lambda_{kj}^{2} (x_{ij}^{obs} / x_{ik}^{obs})^{\gamma_i}$. This relative variance function is a convex function of γ_i, because its second derivative $f''(\gamma_i) = \sum_{j=1}^{n} \lambda_{kj}^{2} (\log(x_{ij}^{obs} / x_{ik}^{obs}))^{2} (x_{ij}^{obs} / x_{ik}^{obs})^{\gamma_i}$ is nonnegative. Therefore, $f(\gamma_i) \leq \theta f(0) + (1-\theta) f(2) \quad \forall\, \theta \in [0,1]$.
Consequently, $f(0) \leq 1,\ f(2) \leq 1 \Rightarrow f(\gamma_i) \leq 1 \quad \forall\, \gamma_i \in [0,2]$.

4. The Log-Linear MV-DEA Model

Unfortunately, as is true for production, preference and distribution conditions in general, imposing the mean-variance conditions may significantly reduce the discriminating power of DEA assessments in small samples. Therefore, the potential specification bias associated with stronger distribution assumptions, such as the homoscedasticity assumption in (3A) and (3B), has to be balanced against the potential gain in discriminating power associated with more distributional structure. To circumvent this problem, we propose to replace the additive error structure in (3A) and (3B) with the following multiplicative error structure:

$$(10A) \quad y_{rj} = y_{rj}^{obs} \cdot e^{u_{rj}} \quad r = 1,\ldots,s;\ j = 1,\ldots,n$$

$$E[u_{rj}] = 0 \quad r = 1,\ldots,s;\ j = 1,\ldots,n$$

$$E[u_{rj}^2] = \sigma_r^2 \quad r = 1,\ldots,s;\ j = 1,\ldots,n$$

$$E[u_{rj}u_{rk}] = 0 \quad r = 1,\ldots,s;\ j \neq k$$

$$(10B) \quad x_{ij} = x_{ij}^{obs} \cdot e^{u_{ij}} \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}] = 0 \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}^2] = \sigma_i^2 \quad i = 1,\ldots,m;\ j = 1,\ldots,n$$

$$E[u_{ij}u_{ik}] = 0 \quad i = 1,\ldots,m;\ j \neq k$$

In our opinion, for this error structure, the assumption of homoscedasticity is more tenable, and, consequently, discrimimating power can be gained with less specification bias than under the additive error structure. Moreover, the multiplicative error structure excludes unrealistic negative input-output values. Unfortunately, the proposed multiplicative error structure cannot easily be combined with a linear DEA model, such as specification (1). Therefore, we propose to combine the multiplicative error structure with multiplicative DEA models. In contrast to the piecewise linear envelopment offered by linear DEA models, multiplicative DEA models offer a piecewise log-linear envelopment of the input-output data (Charnes et al., 1982, 1983). For example, combination of the proposed multiplicative error structure with the multiplicative CCR-DEA model yields the following log-linear MV-DEA model:

$$(11) \quad \min_{\theta_k, \lambda_{kj}} \sum_{k=1}^{n} \theta_k$$

$$s.t. \sum_{j=1}^{n} \lambda_{kj} \log(y_{rj}^{obs}) \geq \log(y_{rk}^{obs}) \quad k=1,\ldots,n;\ r=1,\ldots,s$$

$$\sum_{j=1}^{n} \lambda_{kj} \log(x_{ij}^{obs}) \leq \theta_k \log(x_{ik}^{obs}) \quad k = 1,\ldots,n;\ i = 1,\ldots,m$$

$$\sum_{j=1}^{n} \lambda_{jk}^2 \leq 1 \quad k = 1,\ldots,n$$

$$\lambda_{kj} \geq 0 \quad k=1,\ldots,n;\ j=1,\ldots,n$$

5. Empirical Application

To illustrate the operation and the characteristics of the MV-DEA approach, this section presents an empirical application of the proposed log-linear MV-DEA model to a large bank data set. The data set contains six input-output variables referring to the 1995 financial statements of 49 Indonesian banks. Following Yue (1992) and Yeh (1996), The input variables used are: (1) Total Net Loans (loan and advances plus secured loans minus loan loss reserves), (2) Interest Income, and (3) Other Income (commission fees plus other operating income). The output variables used are: (1) Customer Deposits (demand Deposits plus time deposits plus saving deposits), (2) Interest Expenses, and (3) Other Expenses (payroll expenses plus loan loss provisions plus other operating expenses). Some descriptive statistics of the data set employed are displayed in table 1.

	Total Net Loans	Interest Income	Other Income	Customer Deposits	Interest Expenses	Other expenses
Mean	3222.57	556.59	44.89	3384.73	406.90	142.93
Median	861.30	183.40	11.40	884.00	127.60	40.30
Maximum	21826.60	3990.00	447.00	22562.80	2562.00	1494.40
Minimum	148.00	48.70	1.30	161.90	35.50	7.40
Std. Dev.	5432.77	871.19	82.85	5392.75	633.83	271.45
Skewness	2.28	2.32	3.04	2.11	2.18	3.32
Kurtosis	7.14	7.81	13.18	6.48	6.77	14.97

Table1 Descriptive statistics of input-output variables of 49 Indonesian banks

The descriptive statistics illustrate the wide variation and the skewness of the size distribution. As in most reported bank studies, the Indonesian banking industry is characterized by a small number of big institutions a large number of mid-sized and small-sized institutions. This finding makes the assumption of homoscedasticity of across banks disturbances, required to increase the discriminating power of MV-DEA model by dropping conditions (4B) and (4D), unrealistic. Therefore, a MV-DEA model employing an additive error structure would either involve a large number of restrictions, and, consequently, have little discriminating power, or would involve specification bias and discriminate against small-sized banks. Therefore, we decided to uses the log-linear MV-DEA model in (7). The resulting model corresponds to the following mathematical programming problem:

$$(12) \quad \min_{\theta_k, \lambda_{kj}} \sum_{k=1}^{49} \theta_k$$

$$s.t. \sum_{j=1}^{49} \lambda_j \log(Total\ Net\ Loans_j) \geq \log(Total\ Net\ Loans_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_j \log(Interest\ Income_j) \geq \log(Interest\ Income_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_j \log(Other\ Income_j) \geq \log(Other\ Income_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_j \log(Customer\ Deposits_j) \leq \log(Customer\ Deposits_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_j \log(Interest\ Expenses_j) \leq \theta_k \log(Interest\ Expenses_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_j \log(Other\ Expenses_j) \leq \theta_k \log(Other\ Expenses_k) \quad k = 1,\dots,49$$

$$\sum_{j=1}^{49} \lambda_{kj}^2 \leq 1 \quad k = 1,\dots 49$$

$$\lambda_{kj} \geq 0 \quad k = 1,\dots,49;\ j = 1,\dots,49$$

Both this stochastic MV-DEA model and its deterministic counterpart (i.e. model 8 excluding the constraint on the sum of the squared lambda values) were solved. Appendix A displays, for each bank separately, the resulting DEA efficiency score, the sum of the lambda values, as an indicator of the size of the reference unit relative to the evaluated bank, and the sum of the squared lambda values. The latter serves as an indicator of the variance of the input-output variables of a reference unit relative to the evaluated bank, since the variance is proportional to the sum of squared lambda values under the homoscedasticity assumption. For bank1, these data are also displayed in table 2, together with the composition of the reference unit.

Model	θ_1	$\Sigma\lambda_{j1}$	$\Sigma\lambda_{j1}^2$	Reference Unit
CCR DEA	0.622	1.821	1.712	0.745 Bank20 1.076 Bank43
MV-DEA	0.635	1.726	1	0.360 Bank20 0.378 Bank41 0.840 Bank43 0.002 Bank46 0.145 Bank49

Table2 Detailed estimation output for Bank1

For 17 banks, the deterministic DEA model selects reference units with sums of squared lambda values exceeding unity, and, consequently, with higher variances than the evaluated banks. For example, for Bank1, the deterministic model selects a reference unit - an ill-diversified composite of Bank20 and Bank43 - with 71.2 percent excess variance. By contrast, the stochastic MV-DEA model selects reference units with variances lower or equal than the evaluated banks. For example, for Bank1, the stochastic MV-DEA model selects reference unit - a more-diversified, and, consequently, less risky, composite Bank20, bank41, Bank43, Bank46 and Bank49 - with 71.2 percent less variance. Imposing the constraint on the sum of squared lambda values significantly reduces the uncertainty regarding the reference units; the average reduction in the sum of the squared lambda values is 43 percent. However, it reduces the performance levels of the reference units only marginally; the average reduction in efficiency scores is 1 percent. For example, for Bank1, the DEA efficiency score decreases only by 1.3 percent. This means that additional empirical support can be found for the feasibility of the input-output levels of the reference units in the deterministic model. This empirical application demonstrates how, at least in large samples, MV-DEA can significantly reduce reference unit performance variances, while only marginally reducing reference unit performance means.

6. Concluding Remarks

Mean-Variance Data Envelopment Analysis (MV-DEA) extends the conventional deterministic DEA technique to deal with external effects, outliers and measurement error, by incorporating mean-variance conditions derived from stochastic dominance theory. These conditions are consistent with rational choice behaviour for general preference structures and general disturbance distributions, and, consequently, preserve the conservative nature of the conventional DEA methodology. In addition, the mean-variance conditions can be implemented in conventional deterministic DEA models by simply imposing additional restrictions, and, consequently, do not require complementary analysis. Moreover, MV-DEA preserves the attractions of the conventional DEA mathematical programming structure regarding convexity of the feasible set and feasibility and statistical consistency of solutions. Unfortunately, as is true for production, preference and distribution conditions in general, imposing the mean-variance conditions may significantly reduce the discriminating power of DEA assessments in small samples. To circumvent this problem, we propose to replace the additive error structure and the linear DEA model of the original linear MV-DEA model a multiplicative error structure and a multiplicative DEA model. For the resulting log-linear MV-DEA model, the assumption of homoscedasticity of disturbances across DMUs, required to improve the discriminating power of MV-DEA assessments, is more tenable than for the original model, and, consequently, can be imposed with less specification bias. Moreover, the multiplicative error structure excludes unrealistic negative input-output values.

Appendix A DEA Estimation output

Bank	Deterministic CCR DEA model			Mean-Variance CCR DEA model		
	θ_k	$\Sigma\lambda_{jk}$	$\Sigma\lambda_{jk}^2$	θ_k	$\Sigma\lambda_{jk}$	$\Sigma\lambda_{jk}^2$
Bank1	0.622	1.821	1.712	0.635	1.726	1.000
Bank2	0.695	1.838	1.823	0.735	1.662	1.000
Bank3	0.639	1.895	2.354	0.688	1.752	1.000
Bank4	0.815	1.199	0.755	u[6]	u	u
Bank5	0.759	1.003	0.381	u	u	u
Bank6	0.785	1.310	0.880	u	u	u
Bank7	0.711	1.022	0.418	u	u	u
Bank8	0.673	1.608	1.294	0.682	1.578	1.000
Bank9	0.765	1.336	0.906	u	u	u
Bank10	0.655	1.393	0.999	u	u	u
Bank11	0.690	1.884	1.801	0.705	1.782	1.000
Bank12	0.784	1.159	0.680	u	u	u
Bank13	0.794	0.973	0.409	u	u	u
Bank14	0.720	1.721	1.483	0.728	1.669	1.000
Bank15	0.696	1.768	1.569	0.705	1.705	1.000
Bank16	0.760	1.263	0.802	u	u	u
Bank17	0.779	1.057	0.415	u	u	u
Bank18	0.911	1.174	0.487	u	u	u
Bank19	0.772	1.461	1.067	0.772	1.456	1.000
Bank20	1.000	1.000	1.000	u	u	u
Bank21	0.666	1.747	1.525	0.673	1.693	1.000
Bank22	0.764	1.268	0.901	u	u	u
Bank23	0.909	0.987	0.363	u	u	u
Bank24	0.660	1.608	1.334	0.661	1.590	1.000
Bank25	0.778	1.183	0.758	u	u	u
Bank26	0.838	0.964	0.744	u	u	u
Bank27	0.791	1.188	0.669	u	u	u
Bank28	0.771	1.215	0.879	u	u	u
Bank29	0.808	1.257	0.922	u	u	u
Bank30	0.751	1.285	0.865	u	u	u
Bank31	0.727	1.589	1.390	0.737	1.538	1.000
Bank32	0.681	1.435	1.140	0.682	1.429	1.000
Bank33	0.749	1.402	1.092	0.749	1.398	1.000
Bank34	0.728	1.077	0.572	u	u	u
Bank35	0.668	1.661	1.407	0.675	1.613	1.000
Bank36	0.710	1.512	1.186	0.711	1.503	1.000
Bank37	0.692	1.177	0.814	u	u	u
Bank38	0.749	1.328	1.085	0.749	1.325	1.000
Bank39	0.724	1.391	1.030	0.725	1.390	1.000
Bank40	0.715	1.231	0.909	u	u	u
Bank41	0.929	1.218	0.785	u	u	u
Bank42	0.815	1.151	0.630	u	u	u
Bank43	1.000	1.000	1.000	u	u	u
Bank44	0.873	0.976	0.700	u	u	u
Bank45	0.840	1.050	0.890	u	u	u
Bank46	0.912	1.124	0.449	u	u	u
Bank47	0.918	1.078	0.590	u	u	u
Bank48	0.683	1.262	0.825	u	u	u
Bank49	1.000	1.000	1.000	u	u	u

[6] Unchanged

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