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ANALYSIS OF PRODUCTION AND LOCATION DECISIONS BY MEANS OF MULTI-CRITERIA ANALYSIS

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During the last few years economists and operations researchers have paid much attention to multi-criteria analysis as a tool in modern decision-making. The basic feature of multi-criteria analysis is the fact that a wide variety of relevant decision aspects can be taken into account without a necessity to translate all these aspects in monetary terms. This article will give a brief survey of these new methods in both a quantitative and in a qualitative sense. After this survey the relevance of multi-criteria analysis for entrepreneurial decisions in the field of production and investments will be exposed. The analysis will be illustrated by means of two examples of entrepreneurial decision-problems, which have been solved by means of multi-criteria analysis.

1. INTRODUCTION

During the last few years economists and operations researchers have paid much attention to multi-dimensional optimization methods as a tool in modern decision-making. The background of this deepgoing interest for new decision analyses is the lack of operationality of traditional decision techniques. A frequently felt shortcoming of almost all these techniques is the fact that all dimensions of a decision problem have to be translated into a common denominator (like income, profit, efficiency etc.) or at least have to be made commensurate with the primary objective of a decision problem.

The awareness of a multiplicity of different objectives in decision-making and management has evoked the need of more adequate techniques which take into account the multidimensionality and heterogeneity of individual, social or entrepreneurial behaviour. The need of such adjusted methods is even more apparent due to the mutually conflicting or non-commensurable nature of many objectives. The presence of (partially) incompatible priorities can be considered as an essential characteristic of a wide variety of modern planning and decision problems.

Therefore, recently several attempts have been made to develop more adequate theories and methods which take explicitly into account the existence of multiple criteria in decision-making (see for example, Cochrane and Zeleny |1973|, Van Delft and Nijkamp |1977|, Haimes et al. |1975|, Keeney and Raiffa |1976|, Nijkamp |1977|, Nijkamp and Rietveld|1976|, Nijkamp and Spronk |1977|, Roy|1971|, Thiriez and Zionts |1976|, Wallenius |1975|, Wilhelm |1975| and Zeleny |1974, 1976|). The basic feature of these techniques is that a wide variety of relevant decision aspects is included without translating them into monetary units or any other common denominator. These multidimensional optimization methods are able to integrate also intangibles normally falling outside the realm of the traditional price and market system.

It is clear that these new approaches are extremely relevant for entrepreneurial decision-making in the sphere of production, investment, location, marketing etc. In all these cases pecuniary elements (like profitability) play an important role, but in addition several other elements are important as well like social aspects, environmental impacts of production, use of scarce natural resources, risk characteristics, labour conditions etc.).

This paper will first present a (brief) survey of these multidimensional optimization methods, based on a systematic typology of these methods (section 2). Next a plea will be made in favour of the use of goal-programming methods in the area of managerial decisions (section 3). Then a new optimization technique, based on interactive goal programming methods, will be proposed as a useful tool for managerial decision-making (section 4). Finally, the analysis will be illustrated by means of some examples from the field of production planning and location problems.

2. TYPOLOGY OF MULTIDIMENSIONAL OPTIMIZATION METHODS

Multidimensional optimization (MO-)methods are based on the presence of a set of different (conflicting or at least diverging) objectives instead of one primary objective like in the traditional single-objective optimization models. Clearly, the treatment of several non-commensurable objectives implies that a compromise has to be found between diverging priorities of one decision-maker or between diverging interests of multiple decision-makers.

The general formulation of a MO-model is:

$$\max_{\underline{x} \in R} \underline{w}\,(\underline{x}) \,, \tag{2.1}$$

where $\underline{w}$ is a vector (or profile) encompassing the various objective functions w_i $(i=1,\ldots,m)$ and $\underline{x}$ a vector with decision arguments x_j $(j=1,\ldots,n)$. R represents the feasible area for the decision arguments.

It is clear that a pure maximization of one objective w_i will prevent the remaining objectives from attaining their maxima. This conflict implies essentially a double choice problem: (a) the optimal values of the successive objective functions w_i, and (b) the optimal values of the successive arguments x_j. This choice problem can be attacked in several ways depending on the nature of the decision problem at hand. Therefore, first a typology of MO-models will be presented, based on a classification into discrete and continuous MO-models. Discrete MO-models are models in which the number of feasible alternative choices or strategies is finite; they are usually called multi-criteria models. Continuous MO-models are based on an infinite number of possible values for the decision arguments and hence for the objective functions; they are usually called multi-objective programming models. Both types of models can be further classified into quantitative and qualitative models, deterministic and stochastic models, and static and dynamic models.

Quantitative MO-models are based on information measured on a cardinal (ratio or interval) scale, whereas qualitative MO-models include ordinal or nominal information. Hence the following typology may be made:

Table 1. A classification of MO-models

	scale		accuracy		time dimension	
	quantitative	qualitative	deterministic	stochastic	static	dynamic
discrete						
continuous						

2.1. Discrete MO-models

There is a wide variety of discrete MO-models. An extensive survey of these multi-criteria analyses as well as several applications can be found in Van Delft and Nijkamp |1977|. A first step in all these methods is the construction of an impact table which reflects the outcomes of all alternative plans for all relevant decision criteria:

Table 2. An impact table

objective \ plan	1 . . . . j . . . . , n
1 . . i . . , m	w_{ij}

The elements w_{ji} reflect the values of the i^{th} criterion with respect to the j^{th} plan and can be measured in any appropriate unit.

The next step is the specification of a set of weights which reflect the relative importance attached by the decision-maker to the outcomes of each criterion. These weights, which may be linear or non-linear, reflect the priority scheme of a decision-maker. In several multi-criteria analyses these weights are not specified explicitly a priori, but can be derived from an interactive process during which the decision-maker specifies in a stepwise manner his preferences regarding the values of certain decision criteria (see section 4).

Apart from a cost-benefit analysis and a cost-effectiveness analysis the following quantitative multi-criteria methods can be distinguished:

I. Trade-off analysis

A trade-off analysis attempts to identify the best means to attain a prespecified set of goals, so that one may analyse whether one alternative plan is better than another, given the same set of goals (see Edmunds and Letey |1973|). A basic problem in the use of a trade-off analysis is the translation of the trade-offs between alternative outcomes into opportunity costs. A trade-off analysis is essentially the dual formulation of a cost-effectiveness analysis and has therefore the same shortcomings.

II. Expected value method

The expected value method assigns a set of weights to the criteria of a plan evaluation problem (see Kahne |1975|, Schimpeler and Grecco |1968| and Schlager |1968| and treats these weights as semi-probabilities, so that the expected value of the plan outcomes of each alternative can be calculated by multiplying these semi-probabilities with the plan outcomes and next by aggregating them over all criteria. This method is a rather rigid approach which does not allow for the relative discrepancies and the relative priority differences among alternatives.

III. Correspondence analysis

Correspondence analysis focusses on the differences between alternative plans by means of generalized principal component methods (cf. Spliid |1974|) and is essentially a technique for pattern recognition based on different criteria. The relationships between the decision criteria and the alternative plans are then examined on the basis of clustering procedures, so that the plan with a maximum correspondence to a priority profile can be identified. A drawback of this procedure is the fact that the inferences are mainly based on the statistical pattern of the impact table and less on the relative weights of the decision criteria.

IV. Permutation method

The permutation method is based on successive rank orders of alternative plans (cf. Jacquet-Lagrèze |1969| and Paelinck |1976|). This method examines the dominance relationships resulting from permutations of the successive decision criteria, as well as of the weights assigned to these criteria. In this way the most probable ranking of plans may be derived. A possible difficulty in using these methods is that in case of a less apparent dominant plan rather complicated conditions for the values of the weights may arise.

V. Entropy analysis

Entropy analysis provides a measure for the diversification of the information contained in the project table (see Van Delft and Nijkamp |1977|). By means of a diversification factor for weighted plan outcomes the most probable one is identified. A possible drawback of this analysis is the straightforward aggregation in order to arrive at a conclusion about the most probable plan.

VI. Discrepancy analysis

Discrepancy analysis is a statistical correlation technique which attempts to find a rank order of plans according to their minimum discrepancy with respect to an a priori specified optimum plan (i.e., a plan which satisfies a set of prespecified targets) (see also Nijkamp |1978|). This method should be used carefully, because it is not able to discriminate among discrepancies in plan outcomes and in weights. An alternative way is to make use of a distance metric for pairwise plan discrepancies.

VII. Concordance analysis

Concordance analysis is a widely used multi-criteria analysis, based on a pairwise comparison of plans (see Van Delft and Nijkamp |1977|). This method measures the degree at which plan outcomes and preference weights confirm or contradict the pairwise dominance relationships between alternatives. Both the differences in weights and in plan outcomes are analysed separately via a concordance and a disconcordance procedure. This method uses the available information in an appropriate and efficient manner and can be considered as one of the most satisfactory multicriteria methods, especially when the plan outcomes are related to a prespecified profile of achievement levels of criteria.

VIII. Goals-achievement method

The goals-achievement method is a technique which relates the objectives to quantative achievement levels (see Hill |1973|). Each decision criterion is assigned a index of relative importance. Then for each plan outcome an achievement index is calculated, on the basis of which an aggregate achievement index of each plan can be calculated. This procedure bears some resemblance to the first steps of a concordance analysis based on achievement levels. Clearly, a problem in all these analyses is the specification and the treatment of the set of weights, although interactive techniques may be helpful to overcome this problem (section 4)

The general feature of these multi-criteria methods is that they include a multiplicity of decision-criteria, so that they are more appropriate for modern planning and management problems. Especially the concordance method and the goals-achievement method appear to be rather successful.

2.2. Continuous MO-models

An extensive survey of continuous MO-models is contained in Nijkamp |1977|. A central concept in these types of models is the notion of a Pareto solution (non-inferior, efficient or non-dominated solution). This notion is based on the fact that the value of one objective function cannot be improved without affecting the values of other objective functions. Any feasible solution that is not dominated by other points can be regarded as a Pareto solution. In formal terms: A Pareto solution is a vector $\underline{x}^*$ for which no other feasible solution vector $\underline{x}$ does exist such that:

$$(2.2) \qquad \underline{w}(\underline{x}) \geq \underline{w}(\underline{x}^*)$$

and

$$w_i(\underline{x}) \neq w_i(\underline{x}^*) \quad \text{for at least one } i.$$

It has been proved among others by Geoffrion |1968| that a feasible solution vector is a Pareto solution, if and only if a vector of weights $\underline{\lambda}$ does exist (with $\underline{\iota}'\underline{\lambda} = 1$ and $\underline{\lambda} \geq \underline{0}$), such that this vector is the optimal solution of the following single-objective model:

$$(2.3) \qquad \max Q = \underline{\lambda}'\underline{w}(\underline{x}) \qquad \underline{x} \in R$$

Since any appropriate solution of an MO-model should always be a (non-dominated) Pareto solution, the parameter vector $\underline{\lambda}$ plays an important role in identifying a compromise solution for an MO-problem.

The following types of MO-models can be distinguished:

I. Utility models

Utility models are based on the assumption that the whole vector of relevant objectives can be translated by means of a weighing procedure into the master control of one unambiguous utility function. This assumption of explicit and known trade-offs between objectives is essential in neoclassical utility theory. This implies that (2.1) is re-specified as:

$$(2.4) \qquad \max Q\,\{\underline{w}(\underline{x})\} \qquad \underline{x} \in R \quad ,$$

where Q is the master control of a scalar-valued welfare function.

II. Penalty models

Penalty models assume the existence of an achievement or ideal vector $\underline{w}^o$, so that any discrepancy between an actual value $\underline{w}$ and an ideal value $\underline{w}^o$ is penalized by means of a penalty function. A well-known specification of a penalty function is a quadratic one (see Theil |1968| among others):

$$(2.5) \qquad \min \chi = (\underline{w} - \underline{w}^o)'\, \hat{A}\, (\underline{w} - \underline{w}^o) \qquad \underline{x} \in R \quad ,$$

where $\hat{A}$ is a diagonal matrix with coefficients a_i $(i=1,\ldots,m)$ representing the weights assigned to the i^{th} deviation.

III. Goal programming models

Goal programming models are one of the most frequently used MO-models and essentially a sub-class of the abovementioned penalty models. For all decision criteria an achievement level $\underline{w}^o$ is specified which has to be attained as closely as possible by an appropriate choice of the decision variables $\underline{x}$. A linear goal programming model can be formalized as:

$$(2.6) \quad \min \chi = \underline{\iota}' \, (\underline{w}^+ + \underline{w}^-)$$
$$x \in R$$
$$\underline{w} - \underline{w}^+ + \underline{w}^- = \underline{w}^o \quad ,$$

where $\underline{w}^+$ and $\underline{w}^-$ are the respective over- and underachievements of $\underline{w}$ with respect to $\underline{w}^o$. If necessary, priority weights may be specified to evaluate the successive deviations from the achievement levels. For a further discussion of goal programming models, see section 4.

IV. Constraint models

Constraint models are models in which one of the objectives is selected as a primary objective to be maximized, while the remaining objectives are included by means of lower and upper constraints. If the first objective is taken as the primary one, the following model is obtained:

$$(2.7) \quad \max w_1(\underline{x})$$
$$\underline{x} \in R$$
$$\underline{w}^{min} \leq \underline{w} \leq \underline{w}^{max} \quad ,$$

where $\underline{w}^{min}$ and $\underline{w}^{max}$ represent vectors with lower and upper constraints on the objectives, respectively.

V. Hierarchical optimization models

Hierarchical optimization models can be regarded as more refined constraints models. The assumption of this class of models is that all objectives can be ranked according to their decreasing degree of relative priority. The optimization is carried out in a stepwise way, so that higher-ranking objective functions are maximized prior to lower-ranking objective functions. Assume that such a lexicographic ranking leads to the rank order $w_1 > w_2 > \dots > w_m$. Then a hierarchical MO-model can be formalized as:

(2.8) A. $\max w_1(\underline{x})$
$$\underline{x} \in R$$

B. $\max w_2(\underline{x})$
$$\underline{x} \in R$$
$$w_1(\underline{x}) \geq \beta_1 w_1(\underline{x}_1^o)$$

C. $\max w_3(\underline{x})$
$$\underline{x} \in R$$
$$w_1(\underline{x}) \geq \beta_1 w_1(\underline{x}_1^o)$$
$$w_2(\underline{x}) \geq \beta_2 w_2(\underline{x}_2^o), \text{ and so on}$$

where β_i $(i=1,\dots,m-1)$ is a tolerance parameter indicating the maximum deviation from the optimum $w_i(\underline{x}_i^o)$ which is considered to be allowable by the decision-maker ($\beta_i < 1$).

VI. Min-max models

Min-max models are based on the use of a pay-off matrix for conflicting objectives (see Nijkamp and Rietveld |1976|). The first step is a separate optimization of all individual functions:

$$(2.9) \quad \max w_i(\underline{x}) \qquad \text{for all } i$$
$$\underline{x} \in R$$

The optimal value of each i^{th} objective function from (2.9) will now be denoted by $w_i^o(\underline{x}^i)$ are denoted by x^i. Then the pay-off table representing the conflicts between the successive objectives can be constructed as:

Table 3. A pay off table for conflicting objectives.

	objectives			
arguments	w_1	w_2	.....	w_m
$\underline{x}^1$	$w_1^o(\underline{x}^1)$	$w_2^o(\underline{x}^1)$		
.	$w_2(\underline{x}^2)$	$w_2^o(\underline{x}^2)$		
.			.	
.			.	
$\underline{x}^m$				$w_m^o(\underline{x}^m)$

Next, the equilibrium solution of such a pay-off table may be calculated such that this solution is nearest to the set of ideal solutions represented on the main diagonal of Table 3. One of the possibilities to derive an equilibrium solution is the use of the following model (see Benayoun et al. |1973|):

$$(2.10) \quad \min \eta$$
$$\hat{B} \, \{ \underline{w}^o(\underline{x}) - \underline{w}(\underline{x}) \} \leq \eta \underline{\iota}$$
$$\underline{x} \in R \quad ,$$

where $\hat{B}$ is a diagonal matrix with elements b_i representing the weights attached to the discrepancy between the ideal value $w_i^o(\underline{x}^i)$ and the actual value $w_i(\underline{x})$. Other approaches to min-max models are contained among others in Nijkamp and Rietveld |1976|.

VII. Pareto compromise models

Pareto compromise models are based on a distance metric for the deviation between ideal solutions $w_i^o(\underline{x}^i)$ on the one hand and the set of Pareto solutions $w_i(\underline{x}^*)$ on the other hand. A compromise solution is characterized by a minimum distance between the ideal solution and one point from the set of Pareto solutions. Such a distance metric requires a normalization of the objective functions:

$$(2.11) \qquad w_i(\underline{x}) = w_i(\underline{x})/w_i^o(\underline{x}^i),$$

so that the 'ideal' value of the normalized objective function is equal to 1. Then the following (Minkowski) distance metric may be specified:

$$(2.12) \qquad \min \psi = \{\sum_{i=1}^{m} (1-w_i)^{\alpha}\}^{1/\alpha}, \quad \alpha \geq 1$$

$$w_i = w_i/w_i^o$$

$$\underline{x} \in R$$

The parameter of the Minkowski metric may be set equal, for example, to 1 (rectangular distance, 2 (Euclidean distance) or ∞. These Pareto models appear to be rather manageable tools in decision analyses.

The basic problem in the use of these multi-objective programming models is the specification of the trade-offs. Especially the approaches described in I, II and IV require a lot of prior information about trade-offs between conflicting objectives. Particularly, the methods described in III, V, VI and VII appear to be very appropriate to deal with multiple objective functions. Clearly, any specification of a utility function, a contraint or an achievement level implies a certain implicit or explicit specification of the trade-offs. The problem is of course to construct the decision problem in such a way that it is not a heavy task for a decision-maker to reveal his preferences. This will require the use of interactive techniques, so that in that case the abovementioned MO-models have to be adjusted for and extended with interactive procedures (see section 4).

This holds also true for quantitative and qualitative MO-models (see Van Delft and Nijkamp |1977|), for deterministic and stochastic models (see Nijkamp |1978|), and for static and dynamic models. For the moment, the conclusion can be drawn that there is a set of modern MO-models which are appropriate to attack decision problems with multiple objectives, diverging interests, conflicting priorities or incommensurable objectives.

3. MULTIPLE GOAL PROGRAMMING AND MANAGERIAL DECISION-MAKING

In this section we will make a plea for multiple goal programming as an important tool in managerial decision-making. Furthermore, we will mention some applications of multiple goal programming in the field of business and managerial economics reported thus far. Before concentrating on this specific technique one may wonder whether multidimentional optimization (MO-)methods have to be used anyway in the sphere of managerial decisions. Many answers can and actually have been given. In general the accent in these answers depends on the theory at hand to describe the firm and its objectives. On the one hand there are advocates of the classical theory of the firm, who consider the enterprise as an holistic entity striving for the maximization of the wealth of the firm's owner(s) (which may be interpreted in several ways - see for instance Philippatos |1973|). On the other hand, there are 'behavioral' theories of the firm (March & Simon |1958| and Cyert & March |1963| should be mentioned), in which the firm is conceived of as an 'organization' in which 'participants' cooperate to 'satisfy' the firm's objectives, consisting of the collection of the participants' objectives, thus ensuring the continued existence of the firm[1].

1) In broad lines we agree with these behavioral theories. In our opinion, the firm is an (open) system in which participants cooperate. In their organizational coherence the participants provide the firm's cabinet of instruments, which can be employed in many directions. A continuous need of compromises between the participants determines the way in which the firm's instruments are employed. In this process the participants may use both passive and active means.

Besides these two lines of thought, many other theories have been developed (see for a survey e.g. McGuire |1964|). Although several empirical studies have been carried out (see for instance Johnsen |1969| and Bilkey |1973|), many questions regarding business goals remain unsolved. However, whatever view on the firm's objectives will proof to be correct, in our view MO-methods are useful in managerial decision-making. This is clear for the case in which the firm is striving for multiple objectives, but even a single-objective (e.g. profit maximizing) firm may use these methods. For instance, in the latter case the constituents of 'profit' can be embodied in a M.O.-model, in order to create the possibility to 'translate' the profit goal into various organizational subgoals. Furthermore, M.O.-models can be used to investigate the influence of restrictions imposed by the environment of the firm on the firm's possibilities to maximize its profit function.

In our opinion, 'multiple goal programming' is one of the most valuable techniques within the class of multidimensional programming methods. However, it is not a generally accepted technique. Opponents often mention as a disadvantage that the method requires a considerable amount of prior information on the decision-maker's preferences. As will be shown in the next section this difficulty can be side-stepped when an interactive variant of the method is used. In this section we confine ourselves to mention some of the advantages of multiple goal programming. (A more detailed discussion of multiple goal programming is given by Nijkamp and Spronk |1977|).

Multiple goal programming is appropriate for decision situations in which multiple, possibly incompatible, goals fight for the use of the firm's resources. These goals are formulated as goal levels 'aspired' by the decision-maker. Weights may be assigned to the various deviational variables (both under- and overattainments of the aspired goal levels). The weighted combination is then minimized in order to satisfy the goals 'as close as possible'. Within this framework, multiple goal programming has the possibility to include 'preemptive priority factors', which may be used when one goal has to be fulfilled before even thinking of another.

Several empirical findings from decision-making practice are, in our opinion, rather convincing to demonstrate the practical usefulness of multiple goal programming. As mentioned by several writers, the method corresponds fairly well to the results of the behavioral theory of the firm. In practice, decision-makers are aiming at various goals, formulated as aspiration levels. The intensity with which the goals are strived for may vary from goal to goal - in other words - different 'weights' may be assigned to different goals[2]. The use of aspiration levels in decision-making is also reported by theorists from other fields, as for instance psychology (see for a short overview Monarchi et al |1976|). In the same way, also preemptive priorities are known in real life problems. Support for this in fact lexicographic viewpoint is provided by Fishburn |1974| and Monarchi et al |1976|. A more concrete example of the correspondence of multiple goal programming and practice is provided by Ijiri |1965|, who views multiple goal programming as an extension of break-even analysis, which is widely used in business practice.

The above plea for multiple goal programming is of a somewhat theoretical nature. Of course, the operational usefulness of multiple goal programming can only be shown in practice. Although it is a relatively 'young' method, many applications have been reported in literature. To give an idea, we have listed some of these applications, especially in the fields of business and managerial economics. Because we only want to give a general view of the potential of goal programming, we merely mention the subjects of application as reported thusfar, together with the names of the writers involved. For more detailed discussions we refer to the original articles and to the reviews of Charnes & Cooper |1975|, Lane |1970|, and Kornbluth |1973|.

In 1955 Charnes, Cooper and Ferguson presented the earliest example of goal programming. In their article |1955|, an archetype of the goal programming model is formulated in order to estimate executive compensation. Related applications concern <u>personnel recruitment</u> - Charnes, Cooper, Niehaus & Stedry |1969| - and manpower planning - Charnes, Cooper & Niehaus |1969|. Goal programming may also be

2) As shown by Lane |1970|, the correspondence of the behavioral theory and multiple goal programming is not complete, because the latter gives a specific interpretation of 'satisfying goals as close as possible' (see Lane, pp. 57-60).

used in strategic and aggregate planning, as demonstrated by Goodman |1974| and Jääskeläinen |1969|, |1972|. Traditional operational research problems, such as the location and the transportation problem, were discussed by Ignizio |1976|. In the field of marketing there is a case study of Lee & Nicely |1974|, while there are articles on sales effort allocation by Lee & Bird |1970| and on media planning by Charnes c.s. |1968|.
Management accounting and control has been attacked by Ijiri |1965|, Charnes, Cooper and Ijiri |1963| and Killough & Souders |1973|. Within this domain special attention has been paid to the decentralization problem by Ruefli |1969| and Charnes, Clower and Kortanek |1967|. Many applications can be found in finance. Salkin & Jones formulated a model for merger strategy |1972|. Sartoris and Spruill used goal programming for working capital management |1974|. Lee and Lerro developed a model for portfolio selection |1973| and together with McGinnis they worked on tax switching for commercial banks |1971|. Concluding this cataloque, we mention the subject which has been mother goal programming's darling. This subject, capital budgeting, was discussed by Bronsema & Tempelaar |1974|, Callahan |1973|, Chateau |1975|, Forsyth |1969|, Hawkins & Addams |1974|, Ignizio |1976|, Lane |1970|, Lee & Lerro |1974|, Lee & Jääskeläinen |1978| and Osteryoung |1973|. Although we do not pretend to have been complete (references not mentioned here will be gladly received), we think the above list gives a good impression of the enormous potential of goal programming.

4. MULTIPLE GOAL PROGRAMMING WITHIN AN INTERACTIVE FRAMEWORK

4.1. Multiple Goal Programming : Pro and Contra

Multiple goal programming, mainly developed by Charnes & Cooper, was one of the earliest practicable techniques in multiple criteria decision-making [3]. We believe goal programming still to be one of the stronger methods available. As claimed in the preceding section, its use of aspiration levels and preemptive priorities closely corresponds to decision-making in practice.

An important drawback of multiple goal programming is its need for fairly detailed a priori information on the decision-maker's preferences. Goal programming asks the definition of aspiration levels, the division into preemptive priority classes and the assessment of weights within these classes. We agree with those scholars advocating interactive approaches to the multiple goal problem (cf. section 2). Unfortunately, most of the usual interactive approaches lack some of the advantages of 'traditional' multiple goal programming, such as for instance the possibility to include preemptive priorities. Furthermore multiple goal programming can handle situations of satisficing behavior (see section 3) in contrast with most existing interactive methods. This situation, combined with the repeatedly shown power of the traditional approach to include piecewise linear functions (cf. Charnes & Cooper |1977|), justifies the effort to seek for an interactive variant of the traditional approach.

Recently, interactive methods have become rather popular in decision analyses. These methods are based on a mutual and successive interplay between a decision-maker and an expert (or analyst).
These methods do neither require an explicit representation or specification of the decision-maker's preference function nor an explicit quantitative representation of trade-offs among conflicting objectives. Obviously, the solution of a decision problem requires the decision-maker to provide information about his priorities regarding alternative feasible states, but in normal interactive procedures only a set of achievement levels (or 'satisficing' levels) for the various objectives have to be specified in a stepwise manner. The task of the analyst is to provide all relevant information especially concerning permissible values of the criteria and about reasonable compromise solutions.

By means of interactive decision-methods a decision-maker may get more closely involved in evaluation problems, while he also obtains more insight in the trade-offs among different criteria. The feed-back process inherent in interactive decision-methods leads to a closer co-operation between decision-maker and analyst. Therefore, interactive decision-methods can be regarded as an operational application of learning theory (cf. also Atkinson et al. |1965|, Golledge |1969|, and Hilgard and Bower |1969|).

Interactive decision-methods have also been applied in the field of goal

3) An extensive survey of multiple goal programming and decision-making is given in Nijkamp and Spronk |1977|.

programming, although the number of its applications is rather limited so far.
In this section a sample of interactive goal programming methods will be mentioned (cf. also Nijkamp and Spronk |1978|).
One of the first interactive goal programming methods was proposed by Dyer |1972| for the one-sided variant of goal programming. Dyer's approach relates interactive procedures to gradient methods. Another contribution was provided by Fichefet |1974|, who links an interactive min-max model (see subsection 2.2) called STEM to the solution of goal programming problems. This is done by means of a parametric linear program and a game procedure. A rather practical procedure is contained in Monarchi et al. |1975|, although it yields the problem of a possibly large number of iterations. The same holds for a similar approach provided by Price |1976|.

4.2. Interactive Multiple Goal Programming.
In this subsection we present the general lines of a new, interactive variant of multiple goal programming (I.M.G.P.).
A more detailed description is given in a subsequent report (Nijkamp and Spronk |1978|).

I.M.G.P. is capable of including all advantages of multiple goal programming. For instance, preemptive priorities and piecewise linear functions can be handled in a straightforward way. Furthermore, the interactive process imitates practice in formulating aspiration levels, assessing priorities, seeking for a solution and readjustment of the aspiration levels.
The method needs no more a priori information on the decision-maker's preference structure than other interactive multiobjective programming models. However, all available a priori information can be incorporated within the procedure.

Step 0.
First identify the goal variables $w_i(\underline{x})$, $i=1,\dots,m$ as linear or piecewise linear functions of $\underline{x}$, the vector of instrumental variables $x_1,x_2,\dots,x_n$. We assume the $w_i(\underline{x})$ to be concave in $\underline{x}$. Then specify the feasible set R, which is assumed to be convex and within which an optimal solution must be found. When the decision-maker's preferences could be described by a preference function f (note however, that we do not make any attempt in this direction), this function should be a concave function of both $w_i(\underline{x})$, $i=1,\dots,m$ and x_i, $i=1,\dots,n$. An optimal solution is then defined by:

$$(4.1) \quad \text{Max } f = f\{w_i(\underline{x}), i=1,\dots,m\}, \text{subject to } \underline{x} \in R.$$

To simplify this brief exposition, we assume further

$$(4.2) \quad (\partial f/\partial w_i) > 0 \quad \text{for } i=1,\dots,m,$$

so that we presuppose a higher value of each of the goal variables is preferred to a lower value of (the same) goal variable[4].

Step 1.
Next maximize successively each of the m goal variables $w_i(\underline{x})$ separately and denote the maxima by w_i^* and the m corresponding combinations of the instrumental variables by $\underline{x}_i^*$, $i=1,\dots,m$.
It is not possible to find a feasible value of $w_i(\underline{x})$ which exceeds w_i^*. On the other hand, it is not necessary to accept a value of $w_i(\underline{x})$ which is lower than w_i^{min}, defined as:

$$(4.3) \quad w_i^{min} = \text{Min}_{j=1}^{m} \{ w_i(\underline{x}_j^*) \},$$

the lowest value of $w_i(\underline{x})$ resulting from the successive maximizations of the goal variables. In I.M.G.P. we define a 'solution' $\underline{S}$ as a vector of minimum values imposed on each of the goal variables. Therefore, it is clear that a final solution $\underline{S}^*$ must be found between the 'ideal' (but mostly infeasible) solution $\underline{I}$, and the 'pessimistic' solution $\underline{Q}$, which are defined respectively as:

$$(4.4) \quad \underline{I} = [w_1^*, w_2^*, \dots, w_m^*] \text{ and}$$
$$\underline{Q} = [w_1^{min}, w_2^{min}, \dots, w_m^{min}]$$

To facilitate the notation we have included the optimistic solution $\underline{I}$ and the pessimistic solution $\underline{Q}$ in the (2 x m) 'potence matrix' P.

Step 2.
For each goal variable $w_i(\underline{x})$, the decision-maker may have defined aspiration levels w_{ij}, $j=2,\dots,k_i-1$; with the following property

$$(4.5) \quad w_i^{min} < w_{i2} < w_{i3} < \dots < w_{ik_{i-1}} < w_i^*$$

---

4) In the full description of I.M.G.P. it is shown that cases in which $\partial f/\partial w_i$ is negative and cases in which f is not a monotone function of the $w_i(\underline{x})$ can also be included.

Furthermore we define

$$(4.6) \quad w_{i1} = w_i^{min} \quad \text{and}$$

$$w_{ik_i} = w_i$$

In the following steps these goal values are used in constructing trial solutions $\hat{S}_i$ which have to be evaluated by the decision-maker. Because proposed goal levels are sometimes considered as being too high, we need the auxiliary vector $\underline{\delta}$, whose elements δ_j, j=1,...,m correspond to the m goal variables. We define δ_j as the difference of the lowest level of $w_j(\underline{x})$ being rejected by the decision-maker and the highest level of $w_j(\underline{x})$ being accepted thus far. At the first stage of the procedure, no proposal have been made and consequently, no goal level has been rejected. Therefore we put $\delta_j = 0$ for j=1,...,m during the first step.

Step 3.
Define the starting solution as:

$$(4.7) \quad \underline{S}_1 = [w_{11}, w_{21}, \ldots, w_{m1}] ,$$

which is thus equal to the pessimistic solution defined in (4.4).
Present this solution together with the potence matrix P_1 to the decision-maker.

Step 4.
If the proposed solution is satisfactory for the decision-maker, one may accept it; if not, continue with step 5. Define R_i as the subset of R defined by the goal levels in $\underline{S}_i$.

Step 5.
The decision-maker then has to answer the following question: "Given the provisional solution $\underline{S}_i$, which goal variable should be improved first?" 5)

Step 6.
Let us assume that the decision-maker wants to augment the j'th goal variable. Then construct a new trial solution $\hat{\underline{S}}_{i+1}$, which differs with respect to $\underline{S}_i$ only as far as the value of the j'th goal variable is concerned (denoted by $w_j(\underline{x})_{\hat{\underline{S}}_{i+1}}$ and $w_j(\underline{x})_{\underline{S}_i}$ respectively).

5) After step 9 we discuss the case in which the decision-maker wants to raise more than one goal variable at the same time.

If $\delta_j = 0$ no proposed value of $w_j(\underline{x})$ has been rejected thus far, by which we can propose the next higher aspiration level listed in step 2. If $\delta_j > 0$, a value of $w_j(\underline{x})$ which exceeds the current solution by an amount δ_j has been rejected by the decision-maker.
In this case, define: 6)

$$(4.8) \quad w_j(\underline{x})_{\hat{\underline{S}}_{i+1}} = w_j(\underline{x})_{\underline{S}_i} + \tfrac{1}{2}.\delta_j$$

When a provisional value for $w_j(\underline{x})$ has been calculated in one of both above mentioned ways, we introduce the restriction:

$$(4.9) \quad w_j(\underline{x}) \geq w_j(\underline{x})_{\hat{\underline{S}}_{i+1}}$$

and proceed to step 7.

Step 7.
Join the restriction formulated in step 6 or in step 9 to the set of restrictions describing region R_i. Next calculate a new potence matrix, like in step 2, but subject to the new set of restrictions. Label this potence matrix $\hat{P}_{i+1}$.

Step 8.
Confront the decision-maker with $\underline{S}_i$ and $\hat{\underline{S}}_{i+1}$ on one hand and with P_i and $\hat{P}_{i+1}$ on the other hand. The shifts in the potence matrix can be viewed as a 'sacrifice' for reaching the proposed solution. If the decision-maker judges this sacrifice to be justified, accept the proposed solution by putting $\underline{S}_{i+1} = \hat{\underline{S}}_{i+1}$ and $P_{i+1} = \hat{P}_{i+1}$.
Furthermore, in the computer algorithm (see figure 4.1), put $\delta_j = \tfrac{1}{2}.\delta_j$. (which is only relevant for $\delta_j > 0$). If the decision-maker considers the sacrifice unjustified, the proposed value of $w_j(\underline{x})$ is obviously too high. Therefore, drop the constraint added in step 7 and proceed to step 9.

Step 9.
We now know that $w_j(\underline{x})_{\underline{S}_i}$ is too low and

that $w_j(\underline{x})_{\hat{\underline{S}}_{i=1}}$ is too high in the decision-maker's view. By definition, we thus may set δ_j equal to the difference between these two values. A new proposal value $\hat{\underline{S}}_{i+1}$ is then calculated 7) by defining:

6) At this moment, the decision-maker may wish to define a new aspiration level. In our opinion, it is wise to give him explicitly the opportunity to do so.

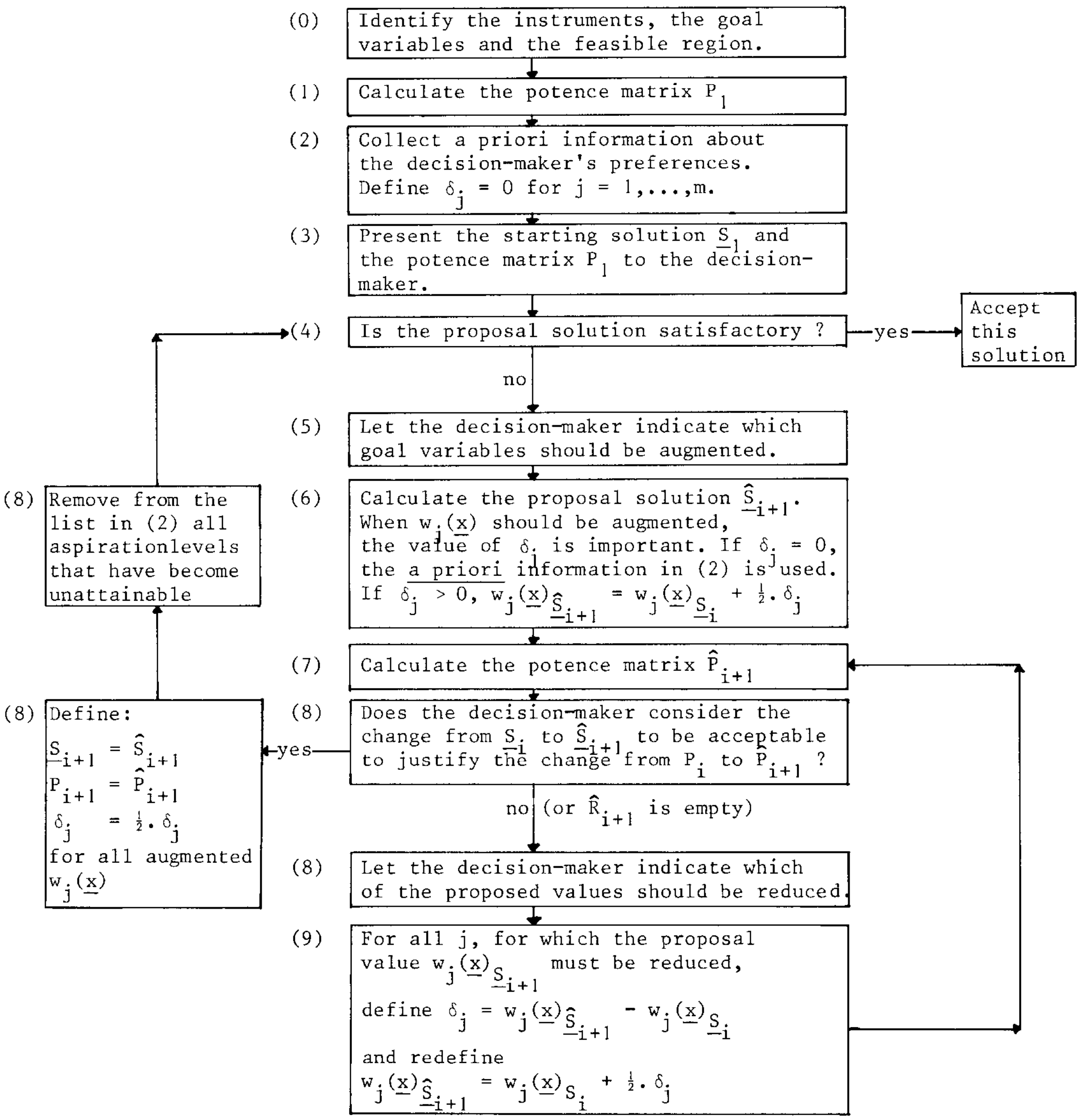
7) Also in this case the decision-maker himself may wish to define a new aspiration level.

$$(4.10) \qquad w_j(\underline{x})_{\hat{\underline{S}}_{i+1}} = w_j(\underline{x})_{\underline{S}_i} + \tfrac{1}{2}\cdot\delta_j$$

like in step 6 we add the restriction that $w_j(\underline{x})$ must equal or exceed the new proposal value and go to step 7 in order to calculate a new potence matrix $\hat{P}_{i+1}$.

When the decision-maker is not able to indicate which single goal variable should be argumented, we assume he is at least capable of defining a set of goal variables whose values need to be augmented. In this case, the procedure must be modified slightly. This is shown in figure 4.1 where we give a flow chart of the procedure.

Figure 4.1 A flow chart of the extended interactive procedure



5. SOME EXAMPLES

In this section we discuss two simple examples in which I.M.G.P. is used. The first, described in subsection 5.1, is concerned with the choice of an 'optimal' production combination of two product varieties out of an infinite number of alternatives (continuous case).
In subsection 5.2 we show that I.M.G.P., with some minor modifications, can be used in making an optimal choice out of a finite number of alternatives (discrete case). The example describes a simple location problem. Of course, many other applications can be proposed (for the next future we have planned to investigate the use of I.M.G.P. in capital budgeting, both theoreticaly and empiricaly). In our opinion many problems which have been attacked by traditional goal programming can be handled by I.M.G.P. as well. To give an idea of its potential, we refer to subsection 3.1 in which we listed some applications of traditional goal programming in business and managerial economics, as reported in the literature.

5.1. An Example in Production Planning

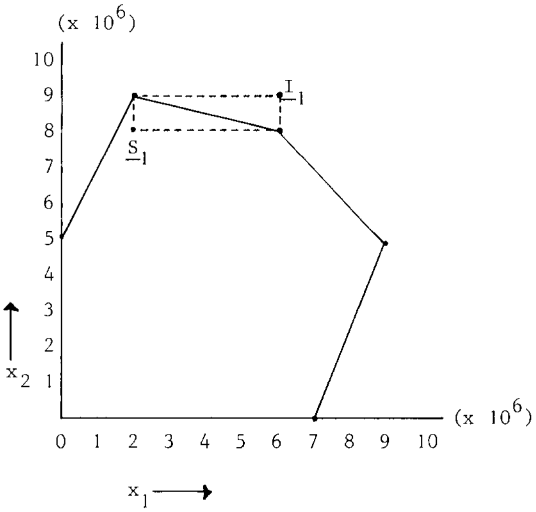
A brick factory can produce two brick varieties, but due to the limited capacity of machines, brick-kiln and drying-room and to the limited availability of skilled personnel, these products cannot be fabricated in any desired combination. We show the region of feasible production combinations in figure 5.1, where x_1 and x_2 stand for the quantity produced of variety 1 and variety 2 respectively (both in millions). For the planning period concerned management cannot define a profit function (let aside another preference function) in terms of x_1 and x_2, due to very uncertain conditions of the market and due to problems in the factory, where a recently installed machine causes many difficulties. Therefore management wants to consider both x_1 and x_2 as goal variables. We thus have:

$$(5.1) \qquad w_1(x_1,x_2) = x_1 \quad \text{and} \quad w_2(x_1,x_2) = x_2$$

Although the maximum production of variety 1 is equal to 9,000,000 it is the 'trouble machine' causing difficulties when the production of x_1 is raised over 7,000,000 units. In fact this machine runs best when approx. 6,000,000 units are produced with it. On the other hand, the factory has contracts to deliver 4,000,000 units of variety 1. Although this variety has been estimated as less profitable than variety 2, management wants to meet the contractual obligations because the customers concerned also buy a lot of variety 2 and offer a promising buying potential in the near future. Thus the preferences for $w_1(\underline{x})$ seem to be monotone non-decreasing for $w_1(\underline{x}) = x_1 \leq 6,000,000$ and monotone non-increasing for $w_1(\underline{x}) = x_1 \geq 6,000,000$. Therefore it is reasonable to consider $w_1^* = 6,000,000$ as an aspiration level for $w_1(\underline{x})$, together with $w_1(\underline{x}) = 4,000,000$ as the aspired level defined above. There are no problems at all in the production of the fairly profitable second variety. Management wants to produce as much as possible of this second variety (thus $\max \{w_2(x_1,x_2)\}$.

Figure 5.1

The feasible region R of the production combinations (the pessimistic and ideal starting solutions are indicated by $\underline{S}_1$ and $\underline{I}_1$)



When $w_2(\underline{x})$ is maximized, we have $w_2^* = 9$ and $w_1(\underline{x}) = 2$. The latter value is that of w_1^{min}, because in the subset of the feasible region R satisfying $w_2(\underline{x}) \geq 9 - \varepsilon$ $(0 \leq \varepsilon \leq 9)$, there is at least one point for which $w_1(\underline{x}) \geq 2$. Thus, no matter what is the minimal value for $w_2(\underline{x})$ required by the decision-maker, there is always a solution for which at the same time $w_1(\underline{x}) \geq 2$. By setting $w_1(\underline{x})$ equal to the most desired production volume $w_1(\underline{x}) = w_1 = 6$, the value of $w_2(\underline{x})$ becomes $w_2(\underline{x}) = 8$, which is at the same time (by similar reasoning) the value of w_2^{min} . Therefore, we must find a final solution in which $2 \leq w_1(\underline{x}) \leq 6$ and $8 \leq w_2(\underline{x}) \leq 9$, Together with the information provided by the management we thus can list (step 2) the following aspiration levels. For $w_1(\underline{x})$ the values 2, 4 and 6; for $w_2(\underline{x})$ the values 8 and 9. The first potence matrix can be written as:

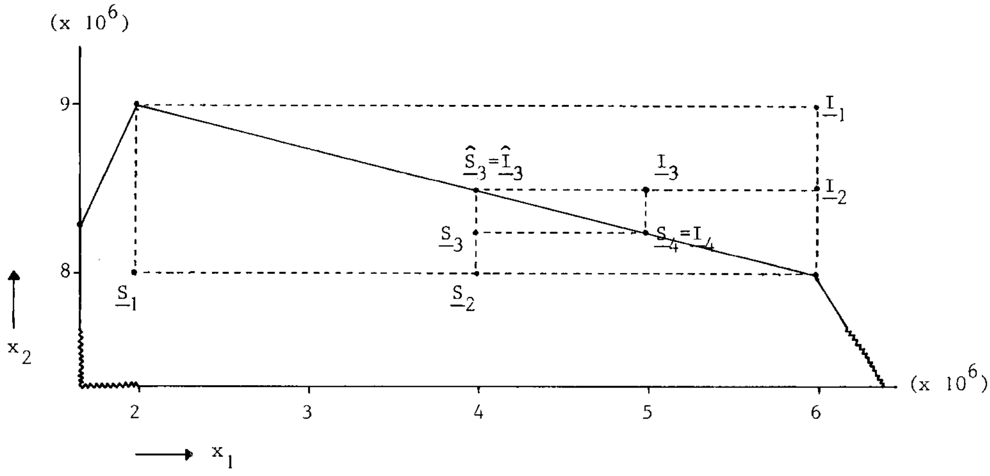
$$P_1 = \begin{bmatrix} 6 & 9 \\ 2 & 8 \end{bmatrix} \tag{5.2}$$

In the third step the first solution ($\underline{S}_1$) is set equal to the pessimistic solution (ϕ) and presented to the decision-maker, together with the potence matrix P_1. The pessimistic starting solution $\underline{S}_1$ and the ideal solution $\underline{I}_1$ are indicated in figure 5.1. This solution has to be evaluated by the decision-maker and subsequently integrated in the model. The successive steps in this hypothetical example are shown in the following table and illustrated in figure 5.2.

Table 5.1 Successive (proposal) solutions and the opinion of the decision-maker.

STEP(S)	(PROPOSAL) SOLUTION TO BE EVALUATED	(PROPOSAL) POTENCF MATRIX	STEP(S)	EVALUATION
2,3	$S_1 = [2\ ,\ 8\ ]$	$P_1 = \begin{bmatrix} 6 & 9 \\ 2 & 8 \end{bmatrix}$	4,5	$\underline{S}_1$ not satifactory, raise value $g_1(\underline{x})$
6,7	$\hat{S}_2 = [4\ ,\ 8\ ]$	$\hat{P}_2 = \begin{bmatrix} 6 & 8.5 \\ 4 & 8 \end{bmatrix}$	8	shifts in P justified, set $\underline{S}_2 = \hat{\underline{S}}_2$ and $P_2 = \hat{P}_2$
			4,5	$\underline{S}_2$ not satisfactory, raise value $g_2(\underline{x})$
6,7	$\hat{S}_3 = [4\ ,\ 8.5\ ]$	$\hat{P}_3 = \begin{bmatrix} 4 & 8.5 \\ 4 & 8.5 \end{bmatrix}$	8	shifts in P not justified, lower value $g_2(\underline{x})$
9,7	$\hat{S}_3 = [4\ ,\ 8.25]$	$\hat{P}_3 = \begin{bmatrix} 5 & 8.5 \\ 4 & 8.25 \end{bmatrix}$	8	shifts in P justified, set $\underline{S}_3 = \hat{\underline{S}}_3$ and $P_3 = \hat{P}_3$
			4,5	$\underline{S}_3$ not satisfactory, raise value of $g_1(\underline{x})$
6,7	$\hat{S}_4 = [5\ ,\ 8.25]$	$\hat{P}_4 = \begin{bmatrix} 5 & 8.25 \\ 5 & 8.25 \end{bmatrix}$	8	shifts in P justified, set $\underline{S}_4 = \hat{\underline{S}}_4$ and $P_4 = \hat{P}_4$
			4	end of procedure

Figure 5.2 All solutions from the starting solution up to and including the final solution.



5.2. A Simple Location Problem

An enterprise is planning to build a new factory for the production of storage-batteries. There are twenty candidates for the location of this new factory. Each alternative has been described in terms of its contributions to the goal variables which management considers to be relevant in this situation. These goal variables are:

w_1 = capacity of the factory the annual number of units produced (in millions). Between certain limits, management wants to have a capacity, which is as large as possible.

w_2 = costs of establishing the new factory (purchase of land and cost of construction). Of course management wants a value of this variable which is as low as possible.

w_3 = score for the quality of the facilities provided by the local government (subsidies, advice, licences). These scores are presented on an ordinal scale of increasing priority:--,-,0,+, ++. The element -- represents a strongly negative outcome for the local facilities concerned, whereas the element ++ represents a strongly positive outcome.

w_4 = score for the possibilities to attract skilled labour. (again represented on an ordinal scale with --,-,0,+,++)

w_5 = score for the quality of the transportation network to be used by the factory (again represented on an ordinal scale with --,-,0,+,++)

w_6 = estimated size of the total local market (measured in millions of units sold per year). Of course, management prefers a more voluminous market to a smaller one.

w_7 = score for the possibilities to enter the local market. (again represented on an ordinal scale with --,-,0,+,++).

The 'industrial profiles' (see Paelinck and Nijkamp |1976|) of the 20 possible locations are given in table 5.2. In contrast with the example in the preceding subsection we now have a discrete decision situation, i.e. only a limited number of alternatives are available to the decision-maker. Although this situation is in conflict with the requirement that the feasible region R should be convex, I.M.G.P. can be employed, be it with one modification. That is, w_i^{min} must now be defined as the minimum value of g_i listed for all available alternatives. (If R is convex, w_i can be defined as the minimum value of w_i listed for the respective maxima of the goal variables w_i, i=1,...,m). Under this condition, the first potence matrix can be written as:

$$(5.3) \quad P_1 = \begin{bmatrix} 30 & 20 & ++ & ++ & ++ & 50 & ++ \\ 11 & 50 & -- & -- & -- & 5 & -- \end{bmatrix}$$

and the first solution as

(5.4) $\underline{S}_1 = [11,50,--,--,--,\ 5,--]$

Assume management's first wishes are that the quality of the local governmental facilities are not too low ($w_3 \geq -$) and that there are not too many problems in attracting labour ($w_4 \geq -$). We then have:

(5.5) $\hat{\underline{S}}_2 = [11,50,\ -,\ -,--,\ 5,--]$

As can be seen in table 5.2 the profile numbers 2,8,10 and 18 do not meet these requirements. The accompanying potence matrix is

(5.6) $$\hat{P}_2 = \begin{bmatrix} 30 & 20 & ++ & ++ & ++ & 50 & ++ \\ 11 & 48 & - & - & -- & 5 & -- \end{bmatrix}$$

Table 5.2 Profiles of the location alternatives

PROFILE NUMBER	(SCORING) VALUES OF THE GOAL VARIABLES w_1	w_2	w_3	w_4	w_5	w_6	w_7	(PROPOSAL) SOLUTIONS BY WHICH PROFILES ARE REJECTED
1	30	48	0	-	+	35	-	$\hat{\underline{S}}_3, \hat{\underline{S}}_4, \underline{S}_4$
2	29	50	-	--	++	20	++	$\hat{\underline{S}}_2, \underline{S}_2$
3	28	44	0	+	+	40	--	$\hat{\underline{S}}_3, \hat{\underline{S}}_4, \underline{S}_4$
4	27	40	++	++	--	15	-	$\hat{\underline{S}}_3$(two times), $\underline{S}_3$
5	26	41	-	0	+	50	+	$\hat{\underline{S}}_3$, finally optimal
6	25	46	0	0	++	5	++	$\hat{\underline{S}}_5, S_5$
7	24	40	+	++	0	5	++	$\hat{\underline{S}}_3, \hat{\underline{S}}_4, \underline{S}_4$
8	23	43	--	+	++	5	0	$\hat{\underline{S}}_2, \underline{S}_2$
9	22	38	-	+	+	5	--	$\hat{\underline{S}}_3$
10	21	37	-	--	-	10	+	$\hat{\underline{S}}_2, \underline{S}_2$
11	20	35	0	+	0	45	++	$\hat{\underline{S}}_3, \underline{S}_3$
12	19	41	+	+	++	15	+	$\hat{\underline{S}}_3, \underline{S}_3$
13	18	36	-	+	+	20	0	$\hat{\underline{S}}_3, \underline{S}_3$
14	17	32	0	0	--	30	++	$\hat{\underline{S}}_3, \underline{S}_3$
15	16	28	0	0	+	40	+	$\hat{\underline{S}}_3, \underline{S}_3$
16	15	25	0	0	0	25	0	$\hat{\underline{S}}_3, \underline{S}_3$
17	14	26	0	++	++	10	+	$\hat{\underline{S}}_3, \underline{S}_3$
18	13	24	--	-	-	25	-	$\hat{\underline{S}}_2, \underline{S}_2$
19	12	23	+	++	+	5	+	$\hat{\underline{S}}_3, \underline{S}_3$
20	11	20	0	+	0	10	-	$\hat{\underline{S}}_3, \underline{S}_3$

As can be read from $\hat{P}_2$ and table 5.2 we have, by rejecting profile number 2, rejected the factory with the highest construction costs. Therefore we define

$$(5.7) \quad \underline{S}_2 = [11,48, -, -,--, 5,--] \quad \text{and } P_2 = \hat{P}_2$$

Management wants next a capacity of at least 20 million units per year to meet the export orders ($w_1 \leq 20$). For the same reason, it would like to have the best possible transportation facilities ($w_5 \geq ++$). We thus get

$$(5.8) \quad \hat{\underline{S}}_3 = [20,48, -, -,++, 5,--]$$

As can be seen in table 5.2, there is only one profile (number 6) left in this solution, by which the potence matrix reduces to

$$(5.9) \quad \hat{P}_3 = \begin{bmatrix} 25 & 46 & - & - & ++ & 5 & -- \\ 25 & 46 & - & - & ++ & 5 & -- \end{bmatrix}$$

Management judges this loss in potence too heavy and wonders what the effect is of a less perfect transportation system. The model then proposes the following solution:

$$(5.10) \quad \hat{\underline{S}}_3 = [20,48, -, -, 0, 5,--]$$

There are more profiles left in this solution, viz. the numbers 1,3,5,6 and 7. The potence matrix can be written as:

$$(5.11) \quad \hat{P}_3 = \begin{bmatrix} 30 & 40 & + & ++ & ++ & 40 & ++ \\ 24 & 48 & - & - & 0 & 5 & -- \end{bmatrix}$$

Management thinks the proposal is good enough to justify the shifts in the potence matrix, by which we can formulate:

$$(5.12) \quad \underline{S}_3 = [24,48, -, -, 0, 5,--] \quad \text{and } P_3 = \hat{P}_3$$

In the following step transportation is required to be slightly better and the possibility to enter the local market must be 'not too bad' ($w_7 \geq 0$). This solution is judged to 'outweigh' the shifts in the potence matrix. Then the fourth solution is determined as

$$(5.13) \quad \underline{S}_4 = [25,46, -, 0, +, 5, +]$$

for which the fifth and sixth profiles are still feasible. Finally management wants too face a local market which exceeds the 5 million units sales in $\underline{S}_4$. Consequently, the final solution becomes

$$(5.14) \quad \underline{S}_5 = [26,41, -, 0, +,50, +]$$

which is the fifth profile.

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