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A NOTE ON MULTIPLE OBJECTIVE PROGRAMMING AND REDUNDANCY

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Abstract

A number of correspondences between multiple objective programming and redundancy is noted. It appears that these problems are closely related. Consequently, attempts to solve one of them may benefit from developments relating to the other problem.

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A multiple objective programming problem (see for an overview e.g. Starr and Zeleny [1977] and Nijkamp and Spronk [1979]) encompasses a vector of distinct objective functions

$$(1)\qquad \underline{G}(\underline{x}) = \begin{bmatrix} g_1(\underline{x}) \\ \vdots \\ g_k(\underline{x}) \\ \vdots \\ g_K(\underline{x}) \end{bmatrix} \qquad (\underline{x} : n \times 1)$$

subject to the constraints

$$(2)\qquad A\underline{x} \leq \underline{b} \qquad (A: m\times n \quad b: m\times 1)$$

An individual constraint ℓ from (2) is defined to be redundant (see e.g. Gal [1975] and Telgen [1977]) if and only if $\hat{u}_\ell \geq 0$ where

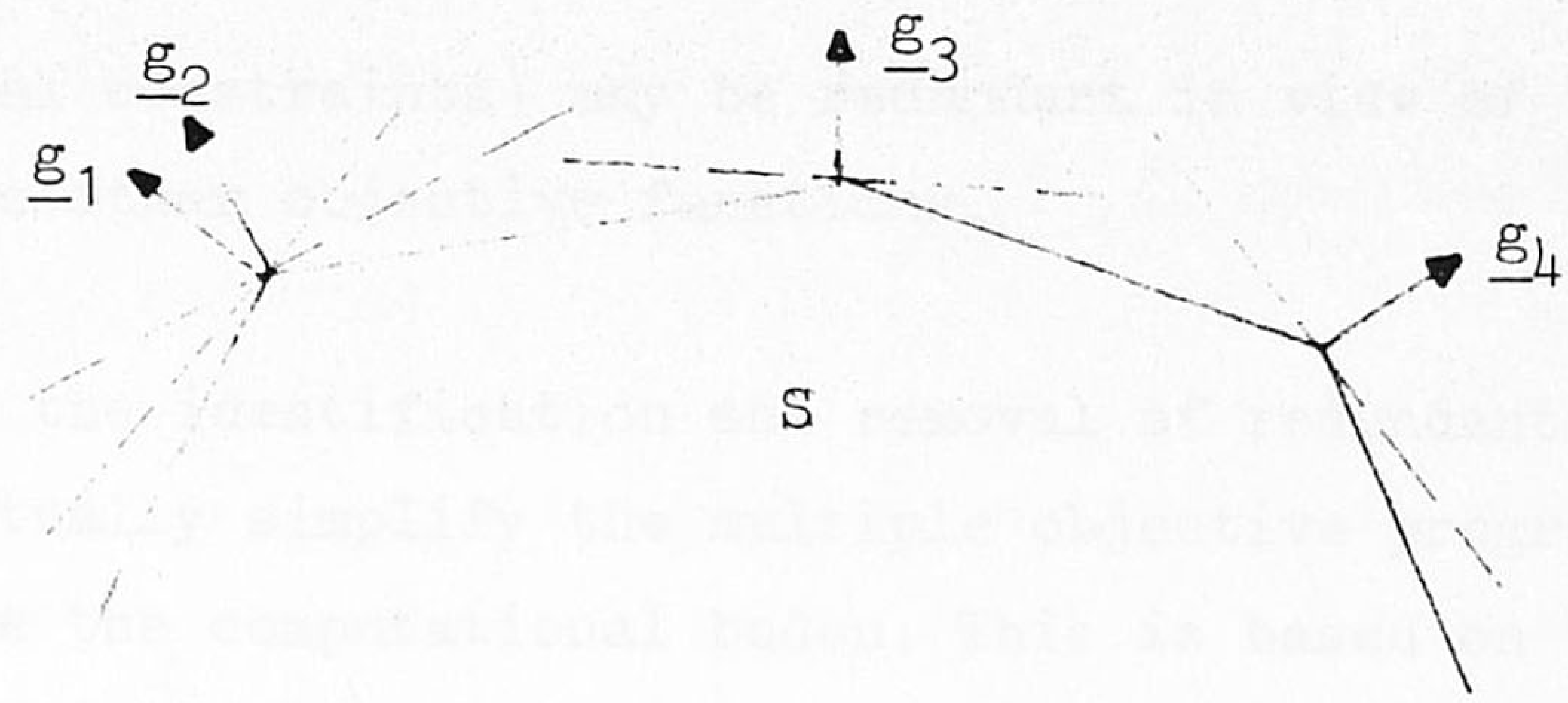
$$(3)\qquad \begin{cases} \hat{u}_\ell = \min b_\ell - \sum_{j=1}^{n} a_{\ell j} x_j \\ \text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall\, i \neq \ell \end{cases}$$

A number of relations and interactions between multiple objective programming and redundancy should be noted.

First, Zionts and Wallenius [1976] have shown that determining whether a given vector is efficient or not, is roughly equivalent to determining whether a given constraint in a linear program is redundant or not.

Second, some interactive methods for the multiple objective programming problem repeatedly add new constraints to the problem in the course of the interactions with a decision maker (see e.g. Nijkamp and Spronk [1978a]). This may give rise to redundant constraints in two ways: the new constraints themselves may be redundant or cause some of the other constraints to become redundant.

Third, the identification of redundant objective functions in linear vector maximum problems (Gal and Leberling [1977]) can be shown to be similar to the identification of redundant constraints. Consider the following figure



Clearly, objective functions $\underline{g}_2$ and $\underline{g}_3$ can be removed without changing the set of efficient solutions. Generally an objective function $g_r(\underline{x}) = \underline{g}_r^T \cdot \underline{x}$ can be removed if there is some $\underline{t} \in \mathbb{R}^{K-1}$ such that

$$(4) \quad \begin{cases} \sum\limits_{\substack{k=1 \\ k \neq r}}^{K} \underline{g}_k \cdot t_k = \underline{g}_r \\ t_k \geq 0 \qquad \forall\, k \neq r \end{cases}$$

Attributing a minimizing (null) objective function and writing the dual problem we obtain:

$$(5) \quad \begin{cases} \max \underline{g}_r^T \cdot \underline{y} = -\min(-\,\underline{g}_r^T \cdot \underline{y}) \\ \text{s.t.} \sum\limits_{=1}^{n} g_{kj} y_j \leq 0 \qquad \forall\, k \neq r \end{cases}$$

Since (5) represents the redundancy definition (3), the objective function $g_r(\underline{x})$ can be removed if (5) has an optimal solution with zero value. Furthermore, note that from (4) it is easily seen, that determining whether a given vector is extreme or not (Zionts and Wallenius [1976]) is equivalent to determining whether a given objective function is redundant or not.

An important advantage is connected with the identification of redundant objective functions. This is, that the decision maker has to reveal his preferences for a smaller number of objective functions than before. However, the other side of the picture is, that the decision maker may not want to remove these mathematically redundant constraints from the preference analysis. A similar argument holds for the following point.

Fourth, goal programming (see e.g. Charnes and Cooper [1977]) is a variant of multiple objective programming, in which some weighted combination of the deviations from a number of prespecified levels of the objective functions has to be minimized. Clearly, each of these prespecified levels (or goal constraints) may be redundant in view of the levels specified for the other objective functions.

Fifth, the identification and removal of redundant constraints may both conceptually simplify the multiple objective programming problem and relieve the computational buden. This is based on theoretical considerations originating from the theory of computational complexity: identifying redundant constraints is equivalent to solving a linear programming problem (Telgen [1977]) and multiple objective programming is generally more difficult because of the non-linearity of the objective function. Therefore, it generally pays to solve an extra linear programming problem to identify redundant constraints.

Furthermore, in the course of some multiple objective programming methods similar subproblems have to be solved a number of times. Simplifying the subproblems by removing redundant constraints reduces the number of computations. This is illustrated by our experiences with a multiple objective programming problem taken from Nijkamp and Spronk [1978] which is based on a case provided by Carlsson [1978].

In the original formulation of this problem, 5 objective functions have to be minimized subject to 29 constraints in 43 non-negative variables. Eight constraints are simply upper and lower bounds. Two other constraints may be stated as generalized upper bounds. Furthermore the model contains 18 equality constraints, which all contain one non-negative variable that does not appear in other constraints. Obviously these variables can be treated as slack variables.

After this preliminary operation the REDUCE option of MPSX can be used, resulting in the identification of 7 redundant non-negativity constraints. A problem of 19 constraints (plus 2 generalized upper bound) in 25 variables (of which 7 are free variables) remains.

Additional reductions can be obtained if a particular objective function is taken into account, which happens several times in the course of solving the problem. For example, with one of these objective functions, the REDUCE option fixes 3 variables causing 3 rows to become superfluous and 3 columns to become vacuous. This reduces the problem to one of 16 constraints (plus 2 generalized upper bounds) in 22 variables (including 7 free variables). The presence of 7 free variables implies that in fact a problem of only 9 constraints in 15 variables has to be dealt with.

It should be noted that the REDUCE option does not necessarily identify <u>all</u> redundant constraints; to achieve this goal more sophisticated methods should be used (see e.g. Gal [1975] and Telgen [1979]).

Clearly, a smaller problem requires less storage capacity and can usually be solved faster. The latter is also an important advantage in interactive multiple objective programming methods, since the interactive process benefits greatly from a prompt reaction.

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