Subsidizing Enjoyable Education

Robert Dur\textsuperscript{1}
Amihai Glazer\textsuperscript{2}

\textsuperscript{1}Department of Economics, Erasmus Universiteit Rotterdam, CESifo, Munich, and Tinbergen Institute,
\textsuperscript{2}Department of Economics, University of California, Irvine.
Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Amsterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

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Subsidizing Enjoyable Education

Robert Dur∗ Amihai Glazer†

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Abstract

We provide an explanation for why both college tuition and government grants to college students are typically means-tested. The critical idea is that attending college is both an investment good and a consumption good. The consumption benefit from education implies that, when tuition and grants are uniform, the marginal rich student is less smart than some poor people who choose not to attend college, thus reducing the social returns to education and increasing the college’s cost of education. Competition in the market for college education results in means-tested tuition. In addition, to maximize the social returns to education government should means-test grants. We thus provide a rationale for means-tested tuition and grants which relies neither on capital market imperfections nor on redistributive objectives.

Keywords: tuition policy, education subsidies, self-selection.
JEL-codes: H52, I2.

∗Tinbergen Institute, Erasmus University Rotterdam, and CESifo, Munich. Address: Department of Economics H 7-21, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. dur@few.eur.nl
†Department of Economics, University of California, Irvine, CA 92697 USA. aglazer@uci.edu
1 Introduction

College tuition and government grants to students are commonly means-tested.\(^1\) The literature offers three main arguments for this: capital market imperfections, redistribution, and price discrimination by monopolistic colleges. None of these arguments is fully satisfactory. First, rather than means-tested tuition fees or grants, providing students with loans (with repayment conditional on future income) is the efficient (and possibly also more equitable) way of dealing with missing capital or insurance markets (Jacobs and Van Wijnbergen (2003)). Second, though optimal redistribution may require means-tested grants (Dur, Teulings, and Van Rens (2004)), the redistribution argument cannot explain why private colleges in a competitive education market charge different tuition to students with different incomes. Third, it is unlikely that exploitation of monopolistic power by colleges can fully explain why tuition varies with income. As Epple, Romano, and Sieg (2004, p. 5) note, “The stylized fact that colleges can extract so much revenue from higher income households is clearly an empirical puzzle given many colleges competing for students. ... More future research is needed to find other compelling explanations for this puzzle.”

We provide a new rationale for why both college tuition and government grants to students are means-tested. The critical idea is that attending college is both an investment which increases future earnings, and a consumption good. One implication of the model is that a rich person of low ability may be willing to pay more for college than would a poor person of high ability. Consequently, when smarter students are less costly or otherwise advantageous to colleges, in a competitive equilibrium colleges may charge poor students a lower price than rich students. Moreover, when the social return to education exceeds the private return, allocative efficiency may require government grants to students to be means-tested.

The idea that education is not merely an investment but also provides consumption benefits is widely acknowledged. As Heckman (2000, p.15) notes: “There is, undoubtedly, a consumption component to education.” Some empirical evidence shows a consumption value of higher education. Lazear (1977), using data on young males in the United States, finds that in-

\(^1\)See the review of tuition policy and student support in 13 countries by the Irish Department of Education and Science (2003). See National Center for Education Statistics (2003) for detailed information about tuition and financial aid to students in the United States.
individuals with much education (M.A.’s and Ph.D’s), push education beyond the level that maximizes the present value of future income, suggesting that education has consumption value. The reverse holds for lower levels of education. Using Dutch data, Oosterbeek and van Ophem (2000) find evidence that schooling is a good that both generates utility and raises future income. Alstadsæter (2004) provides evidence for Norway.

2 Literature

Since Schultz (1960) and Becker (1964) developed the theory of human capital, economists have neglected the consumption benefits from education. Two exceptions are Alstadsæter (2003) and Malchow-Møller and Skaksen (2004), who study optimal taxation and financing of education when education yields both a pecuniary and a non-pecuniary return. Both papers employ a representative agent framework and so abstract from heterogeneity in ability and in wealth among agents, which are crucial features of our model. Bovenberg and Jacobs (2003) allow for consumption benefits from education in a model where agents have heterogeneous ability, but no wealth.

Our paper has some similarity with Wickelgren (2001). He argues that given past discrimination, non-discriminatory employers or universities may voluntarily practice affirmative action in their hiring or admission decisions. The reason is that a person who overcame discrimination is likely more able than someone in the same position who faced no such obstacle. Likewise, in our paper, colleges charge lower tuition to poor students, who have higher expected ability than the rich students they displace. This stems, however, from the consumption benefit from education rather than from past discrimination.

De Fraja (2003) argues that high-potential individuals from groups with relatively few high-potential individuals (‘disadvantaged’ groups) should receive higher government grants, since grant provision to these groups entails lower budgetary cost (lower inframarginal subsidies). As we shall see, such a result also appears in our model, even when we assume that ability and student’s wealth are uncorrelated. We identify two other reasons (which hold even when the government budget constraint does not bind, and when inframarginal subsidies are inexpensive) for why government should give larger grants to poorer people.
3 Assumptions

College students differ in two ways. First, students differ in ability, denoted by $a$. Second, at the start of their college career, students differ in wealth, $w$. Each person knows his own ability, but the colleges and the government do not; the colleges and government can only observe a student’s wealth. We assume that students’ ability and wealth are distributed according to the joint density function $f(a, w)$. In Sections 4 and 5, we assume that the distribution is uniform, that is, $f(a, w) = f$ for all $(a, w)$. Section 6 discusses the implications of relaxing this assumption.

For simplicity, we consider a two-period model. In period 1 a person can choose whether to attend college. In period 2, a person who did not attend college has income $a$. A person who attended college earns $a + p(a)$, where $p'(a) > 0$: the return to college increases with ability.\(^2\) We also assume $p''(a) = 0$. As we shall see, this assumption ensures that we can safely ignore opportunities to work in period 1 for persons who do not attend college.

For simplicity, suppose consumption of goods occurs only in period 2; since we assume perfect capital markets, that simplification does not affect our results. Consumption in period 2 by a person with initial wealth $w$ who did not attend college is $a + w$. Let college tuition to a person with wealth $w$ be $t(w)$. It follows that consumption by a person with initial wealth $w$ who attended college is $a + p(a) + w - t(w)$.

The utility from consuming goods is given by the function $v(\cdot)$, with the usual properties $v'(\cdot) > 0$, $v''(\cdot) < 0$, and $v'''(\cdot) \geq 0$. The consumption benefit from attending college is $b$. This benefit can reflect the opportunities to date members of the opposite sex, to enjoy the sports facilities, to take part in the excitement of school football games, to live away from home, and so on. For convenience, we assume that utility is separable in consumption goods and the consumption benefit of education. Necessary for our results is that education is not an inferior good.

The topic we address becomes interesting if a college prefers to enroll smart students. This can arise for many reasons. 1) Peer group effects within colleges can make increased attendance by smart students benefit all other students (see Epple and Romano (1998)). 2) Faculty may find it more pleasant or interesting to teach smart students, and so a college may attract

\(^2\)Most empirical studies find complementarity between ability and education, see e.g. Harmon, Oosterbeek and Walker (2003), Dur and Teulings (2004), and the studies mentioned therein.
better faculty, or attract a given quality of faculty at lower cost, the better are its students.\footnote{At Yale University, the “faculty was astonished and delighted by the leap in academic ability” of freshmen after it changed undergraduate admission policies in the 1960’s. See “The birth of a new institution: How two Yale presidents and their admissions directors tore up the ‘old blueprint’ to create a modern Yale” by Geoffrey Kabaservice, \textit{Yale Alumni Magazine}, December 1999, (http://www.yalealumnimagazine.com/issues/99_12/admissions.html)} 3) Studious students may be less likely to engage in behavior (such as drunkenness) which may impose costly legal liability on the college. 4) Attracting smarter students may enhance a college’s prestige.

We find that a simple but fruitful approach is to suppose that a college’s costs decline with the quality of its students. Let a college’s cost of educating a student with ability $a$ be $c(a)$, with $c’(a) < 0$. Throughout, we assume perfect competition in the market for college education.

\section{Market equilibrium}

A student with ability $a$ and wealth $w$ attends college if:

$$v[a + p(a) - t(w) + w] + b \geq v(a + w). \quad (1)$$

Let $a^*(w)$ denote the ability of a student with wealth $w$, who, in equilibrium, is indifferent about attending college. In a competitive equilibrium, it holds for all $w$ that:

$$v \{a^*(w) + p[a^*(w)] - t(w) + w\} + b = v[a^*(w) + w], \quad (2)$$

$$t(w) = \int_{a^*(w)}^{\hat{a}(w)} f(a,w)c(a)da. \quad (3)$$

The first equation describes, for each level of wealth the students who attend college. Since smarter students have a higher return to education, $p’(a) > 0$, a person with wealth $w$ and with $a \geq a^*(w)$ attends college.\footnote{Allowing persons who do not attend college to work in period 1 changes equation (2) to

$$v \{a^*(w) + p[a^*(w)] - t(w) + w\} + b = v[2a^*(w) + w].$$

Under our assumption that $p$ is a linear function of $a$, none of our results are affected. When, however, $p(a)$ is concave, the highest ability students may prefer working in period 1 over attending college, because their opportunity cost of education may exceed their return to education. We abstract from this.}
second equation describes the equilibrium level of tuition. Perfect competition implies that tuition for a student with wealth $w$ equals the expected cost of educating a student with wealth $w$.

Consider first education with no consumption value, $b = 0$. Then, equation (2) reduces to

$$p[a^*(w)] = t(w).$$

Therefore, a person will attend college only if the return to education is higher than or equal to tuition. Using (3), we can verify that when $b = 0$, equilibrium tuition is independent of wealth, $t'(w) = 0$. For suppose wealthier students would pay higher tuition, $t'(w) > 0$. Then equation (4) would imply that $a^*$ increases with wealth $w$. Given that the distribution of ability and wealth is uniform, this means that richer students would on average be smarter. As $c'(a) < 0$, the right-hand side of equation (3) then implies that a college’s expected average cost is lower when admitting richer students. Hence, if $t'(w) > 0$, the expected cost per student declines with the wealth of the student body, whereas tuition increases with student’s wealth. Clearly, when $t'(w) > 0$, the zero-profit condition (3) is violated for some $w$. A similar argument applies for $t'(w) < 0$, and for any other nonuniform tuition policy. Only when tuition is uniform, $t'(w) = 0$, can the zero-profit condition hold for all levels of wealth. With uniform tuition, the average ability of students and, hence, the expected cost per-student, will be independent of the wealth of students. So if attending college has no consumption value, tuition will be uniform.

When $b > 0$, condition (2) implies that some students with $p(a) < t(w)$ attend college. Though this reduces their lifetime consumption of goods, they enjoy the consumption benefit, $b$, of college.

**Proposition 1:** If college education has consumption value ($b > 0$), colleges charge higher tuition to richer students ($t'(w) > 0$).

**Proof:** See Appendix.

The intuition behind Proposition 1 is straightforward. By the concavity of $v(\cdot)$, the marginal utility of consuming goods declines with wealth, so that a rich person is more willing than is a poor person to reduce consumption of goods in return for the consumption benefits from education. With uniform tuition, the least able poor student in college will therefore be smarter than the least able rich student. This also implies that poor students will on average be smarter than rich students. As a college’s cost of education declines with student’s ability, in the competitive equilibrium colleges charge
lower tuition to poorer students. In equilibrium, the rich will nevertheless be over-represented in college. For equal representation would imply equal expected ability and, hence, uniform tuition in a competitive equilibrium.

In equilibrium, some persons who avoid college are smarter than some who attend college. Since the smarter persons get a higher return from education, aggregate output is higher when they attend college. When, however, externalities are absent, the equilibrium is Pareto-efficient. Forcing a poorer but smarter person to replace a richer but dumber student at the same tuition rate would reduce the utility of both students. Though the smarter person would gain a higher rate of return and would enjoy the same consumption benefit from education, payment of the tuition fee reduces the utility from consumption goods of the poor student more than it reduces the utility of the rich student.

Our explanation for means-tested college tuition contrasts with the idea that colleges aim to attract poor and minority students for reasons of diversity or of equity. Differentiation of tuition by colleges for these reasons could make the average poor student in college less able than the average rich student, and some current evidence shows that difference (Rothstein (2004)). But other evidence shows, as our model predicts, that poor students enrolled in college are of higher average ability than rich students, and that means-tested financial aid can increase the quality of the student body. At a National Press Club event centered around the book America’s Untapped Resource: Low-Income Students in Higher Education, a former College Board official claimed that “The fact is, the dumbest rich kids have as good a chance of going to college as the smartest poor kids.”5 Consistent with this view, a study by the Maryland Higher Education Commission found that community college students who receive need-based financial aid perform as well as or better than their wealthier peers. For example, 74.3 percent of the low-income students who received financial aid returned for a second year of study at their community college, transferred, or earned a credential, compared to 62.5 percent of non-recipients. Similarly, 40.5 percent of new full-time freshman who received need-based financial aid transferred to a public four-year institution and/or earned a community college degree within five years of matriculation, as opposed to about one-third of non-recipients.6

6See Janis Battaglini, “A comparison of the retention, transfer and graduation rates of need-based financial aid recipients at Maryland public col-

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The history of means-tested financial aid at Yale University offers another instructive example. In the Class of 1957, before Yale offered means-tested financial aid and before it practiced needs-blind admissions, graduates of private schools (who were overwhelmingly wealthy) constituted more than 60 percent of the Class of 1957. But they constituted less than half the membership of Phi Beta Kappa (the most prestigious national honor society) and one-sixth of the membership of Tau Beta Pi, the national engineering honor society. The largest feeder schools (Andover, Exeter, Lawrenceville, Hotchkiss, and St. Paul’s, all of which are private), sent about 20 percent of the class; but each accounted for only one of the 64 members of Phi Beta Kappa. Other traditional feeder schools such as Groton, Hill, Kent, St. Mark’s, St. George’s, and Taft contributed no members to Phi Beta Kappa.

In 1963 Yale greatly increased its financial aid, and by 1966 adopted a fully needs-blind admissions policy: the University no longer rejected qualified applicants who could not afford Yale’s costs, eliminated any quota on the number of scholarship students, and placed no limit on the amount of money available for grants and loans. The class entering in 1966 was composed of 58 percent public school students, a higher percentage than ever before, and a jump from 52 percent the previous year; financial aid jumped to nearly $1 million, 30 percent above what it had been the year before; gift aid from the University increased by almost 50 percent. This class entered with higher SAT scores than ever before; a student who scored its mean SAT verbal mark of 697 would have been at the 75th percentile of the class that entered four years before.

5 Government means-tested grants

So far, we ignored externalities of education. As a result, the market equilibrium is Pareto-efficient and government intervention is unnecessary. Suppose now that, in addition to the private return \( p(a) \), there is a public return to education \( \lambda p(a) \). Of course, only the private benefits, not the social returns,
affect a person’s decision to attend college, or a college’s tuition policy. The resulting underinvestment in human capital can be removed by subsidies. We shall see in this section that the consumption benefit from education implies that optimal subsidies will be means-tested rather than uniform.

One reason the social return to education may exceed the private return is taxation. If each student ignores the effect of his education on government’s tax revenues, and if the cost of education is not fully deductible at the same rate as the returns to education are taxed, taxation results in underinvestment in education (see, among others, Boadway, Marceau, and Marchand (1996), Anderson and Konrad (2003), and Bovenberg and Jacobs (2003)). Education can also generate externalities in production. For instance, if innovation increases with the knowledge workers gained in college, and innovations are afforded imperfect patent protection, then the private return to education is less than the social return. Such externalities are an important feature of models of endogenous economic growth (Lucas (1988), Romer (1986), 1990)). Recent empirical evidence is provided by among others Moretti (2004) and Teulings and Van Rens (2003), and surveyed in Sianesi and Van Reenen (2003).

Consider a government that aims to maximize national output net of the costs of college education. The government’s objective is thus

\[
\max \int \int f(a, w)\,da\,dw + \int \int f(a, w)[a + (1 + \lambda)p(a) - c(a)]\,da\,dw. \tag{5}
\]

Let government affect behavior by providing grants, \(g(w)\), to students, which can be conditioned on their wealth. In equilibrium, students’ demand for college education becomes:

\[
v \{a^*(w) + p[a^*(w)] + g(w) - t(w) + w\} + b = v[a^*(w) + w]. \tag{6}
\]

Tuition is still given by (3). For simplicity, we assume that the government has a given budget, denoted by \(G\), for student grants:

\[
\int \int f(a, w)g(w)\,da\,dw \leq G. \tag{7}
\]

We will consider both a binding and a non-binding budget constraint.
Consider first education which has no consumption benefit \((b = 0)\). Then, as we saw in the previous section, in equilibrium all persons whose return to education exceeds the tuition attend college. Moreover, tuition is independent of a student’s wealth and equals the expected cost of education. Hence, if externalities are absent \((\lambda = 0)\), optimal student grants are zero \((g(w) = 0\) for all \(w)\). For providing grants would induce people whose return to education is lower than the expected costs of college education to attend college. When the social returns to education exceed the private returns \((\lambda > 0)\), optimal student grants are positive, so that students internalize the externality of their education on national output. Optimal student grants are independent of student’s wealth. If grants varied with student’s wealth, students receiving high grants would on average be less smart than students receiving low grants. This would result in lower output (since \(p'(a) > 0\)) and higher cost of college education (since \(c'(a) < 0\)) than when the government spends the same budget on uniform grants.

Consider next education which has consumption value \((b > 0)\).

**Proposition 2**: If college education has consumption value \((b > 0)\), optimal student grants are means-tested \((g'(w) < 0)\).

**Proof**: See Appendix.

The intuition behind Proposition 2 follows. The consumption benefit from education implies that with uniform grants (or without grants) some poor people not attending college are smarter than the least able rich student. Since the return to education increases with student’s ability, a grant to a poor student has higher social benefits than a grant to a rich student, as it induces smarter students to attend college. When the government’s budget constraint is not binding, the grant policy must induce the social return to education to equal the marginal cost of education for the marginal student at any given level of wealth:

\[
(1 + \lambda) p[a^*(w)] - c[a^*(w)] = 0,
\]

implying that college education becomes independent of student’s wealth. Tuition will therefore be uniform. We can see from equation (6) that (8) requires that poorer students get higher grants. In the more plausible case where the government budget constraint binds (the shadow cost of public funds is positive), achieving full equality of education by means-tested government grants is not optimal, and so tuition increases in student’s wealth. For three reasons optimal government grants decrease with student’s wealth.
First, as with a non-binding budget constraint, in the absence of a grant some poor people who do not attend college are smarter than some rich students, so that the social return to increasing education of the poor is larger. Second, because the marginal utility of income declines with income, the poor respond more than the rich to an increase in government grants. Hence, a given increase in college participation is attained at lower cost. Third, an increase in the grant to rich students involves a higher budgetary cost than a similar increase in the grant to poor students, as the rich are more numerous in college than the poor. In other words, concentrating grant provision on poor students reduces the government’s cost, as fewer grants are provided to students who would choose to attend college in the absence of the grant.

Optimal government grants need not be positive for all students as the consumption benefit from education induces some students whose private return to education is lower than tuition to attend college. If, however, the externalities from education are sufficiently large, social returns exceed tuition, and optimal grants to all students will be positive.

6 Non-uniform distribution of wealth and ability

We assumed that students’ ability and wealth are uniformly distributed. Clearly, relaxing this assumption may affect our results. Consider first our result on a college’s tuition policy. We saw that, when education is enjoyable, tuition increases with student’s wealth as rich college students are on average less smart than poor college students. Clearly, with a non-uniform distribution, this need not hold. Though the marginal rich student will have lower ability than poor students, the rich may on average be smarter than poor students, for instance when students’ ability and wealth are strongly positively correlated. Then, in the competitive equilibrium tuition may decline with wealth. Only when the consumption benefit from education is sufficiently large, average ability will decline with student’s

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8 There is some evidence for greater price elasticity of demand for higher education among poor people, see e.g. McPherson and Schapiro (1991) and Kane (1994). However, Cameron and Heckman (1999), Dynarski (2000), and Stanley (2003) find no or the reverse effect.

9 This third reason has also been identified by De Fraja (2003) as an efficiency rationale for reverse discrimination in education.
wealth, and so tuition will increase with student’s wealth.

Next consider our result on means-tested government grants. When the government budget constraint does not bind, allowing for more general distribution functions does not affect our result. This is seen by inspecting (6) and (8). Equation (8) implies optimality requires $a^*$ to be independent of wealth, independent of how ability and wealth are distributed over the population. Equation (6) then implies that grants should decrease with wealth.

Our result on government grants may differ when the government budget constraint binds. First, the effect of an increase in grants to persons with a given wealth depends on the density of students at the margin for that wealth, $f[a^*(w), w]$. (See the first term in first-order condition (10) in the Appendix.) Allowing for more general distribution functions implies that this term need no longer be independent of student’s wealth $w$. Second, the rich need not necessarily outnumber the poor in college, and so the cost of inframarginal subsidies may be higher for poorer groups. (See the second term in first-order condition (10) in the Appendix.) Thus, the trade-off between increasing the social benefits from education and the budgetary cost of grant provision may be affected. Since, at the margin, poorer students are still smarter than richer students, our main argument, that grants to poorer students have higher social benefits than grants to richer students, still holds. This is the more so, the higher is the consumption benefit from education. Hence, when the consumption benefit from education is sufficiently large, grants will still decrease with student’s wealth.

7 Conclusion

We showed that the consumption benefit from college may explain why both college tuition and government grants to college students are typically means-tested. The general effect we identified can apply in areas outside of college. We can think of a professional conference. If the intent is to attract people interested in the contents of the conference, then the organizers may want to hold it in a location which is unattractive for a vacation. Similarly, consider the effects of the move of the German capital from Bonn to Berlin. Bonn was an unattractive location, while Berlin is a highly attractive city in which to work and live. Therefore, governmental offices in Bonn may have attracted officials dedicated to public policy; offices in Berlin would also attract people who want a government job not because they like the job, but for the
opportunity to work in Berlin.

8 Appendix

Proof of Proposition 1:
Let (1) hold with equality and replace $a$ with $a^*$. Totally differentiating with respect to $a^*$ and $w$ results in:

$$
\frac{da^*}{dw} = -\frac{[1 - t'(w)]v'[a^* + p(a^*) - t(w) + w] - v'(a^* + w)}{[1 + p'(a^*)]v'[a^* + p(a^*) - t(w) + w] - v'(a^* + w)}.
$$

(9)

Note that (2) implies that if $b > 0$, then for all $w$:

$$a^* + p(a^*) - t(w) + w < a^* + w.
$$

Hence, since $p'(a^*) > 0$ and $v''(\cdot) < 0$, the denominator of (9) is always positive. The sign of the numerator depends on the value of $t'(w)$.

Suppose that $t'(w) \leq 0$. Then (9) implies that $da^*/dw < 0$, and so the right-hand side of (3) increases with $w$. Since $t'(w) \leq 0$ implies that the left-hand side of (3) weakly decreases with $w$, the zero-profit condition (3) will be violated for some $w$.

If $t'(w) > 0$, $da^*/dw$ may be positive, namely when $t'(w)$ is very large, see (9). This implies that the right-hand side of equation (3) decreases with $w$. Since $t'(w) > 0$ implies that the left-hand side of (3) increases with $w$, this cannot hold in a competitive equilibrium. Only if $t'(w) > 0$ for all $w$, but not too large so that $da^*/dw < 0$ for all $w$, will both the left-hand side and the right-hand side of (3) increase with $w$. Note that $t'(w) < 1$, because $t'(w) \geq 1$ would imply $da^*/dw > 0$. Note also that $0 < t'(w) < 1$ and $p'(a^*) > 0$ imply that $-1 < da^*/dw < 0$.

Proof of Proposition 2:
The government maximizes (5) with respect to $g(w)$ and subject to (3), (6), and (7). In the optimum, for each $w$ it must hold that:

$$
-\frac{da^*(w)}{dg(w)} f[a^*(w), w] \{(1 + \lambda)p[a^*(w)] - c[a^*(w)] - \Lambda g(w)\} - \Lambda \int \int f(a, w) \lambda d\lambda dw = 0,
$$

(10)
where $\Lambda$ is the Lagrange-multiplier for the budget constraint, and

$$
\frac{da^*(w)}{dg(w)} = -\frac{v'(a^*(w) + p[a^*(w)] + g(w) - t(w) + w)}{\{1 + p'[a^*(w)]\} v'(a^*(w) + p[a^*(w)] + g(w) - t(w) + w} < 0,
$$

which follows from (6). For later use, it is convenient to derive how $\frac{da^*(w)}{dg(w)}$ depends on $w$. We can rewrite (11) to:

$$
\frac{da^*(w)}{dg(w)} = -\frac{1}{1 + p'[a^*(w)] + \frac{-v'[a^*(w) + w]}{v'[a^*(w) + p[a^*(w)] + g(w) - t(w) + w}} < 0.
$$

Since $p''(a) = 0$, we only need to know how the last term in the denominator changes when $w$ changes. Straightforward algebra shows that since $v''(\cdot) < 0$ and $v''(\cdot) \geq 0$, the last term in the denominator increases in $w$. Hence, $\frac{da^*(w)}{dg(w)}$ increases in $w$ (is closer to zero, the higher is $w$).

First consider the case where the budget constraint is non-binding, $\Lambda = 0$. Then, first-order condition (10) reduces to:

$$
(1 + \lambda) p[a^*(w)] - c[a^*(w)] = 0.
$$

That is, the optimal grant scheme $g(w)$ is such that for the marginal student from each wealth class, the social return to education equals the marginal cost of education. Clearly, this implies that in the optimum $a^*(w)$ is independent of student’s wealth. Tuition $t(w)$ will therefore also be independent of student’s wealth, see (3). Totally differentiating (6) with respect to $w$ and $g$, keeping $a^*(w)$ and $t(w)$ constant, yields:

$$
\frac{dg}{dw} = -\frac{v'\{a^*(w) + p[a^*(w)] + g(w) - t(w) + w\} - v'[a^*(w) + w]}{v'\{a^*(w) + p[a^*(w)] + g(w) - t(w) + w\}} < 0.
$$

Hence, first-order condition (12) can only be satisfied when grants decrease with student’s wealth, $g'(w) < 0$.

Next consider the case where the budget constraint binds. The first part of first-order condition (10) describes the benefits of increasing grants to students with wealth $w$. Starting from any uniform grant scheme, $g'(w) = 0$, the marginal benefits of grant provision decrease with student’s wealth $w$ since:

1) The term in curly brackets is larger for smaller $w$ since $a^*(w)$ decreases with $w$, and $p'(a) > 0$ and $c'(a) < 0$.  

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2) The term $f[a^*(w), w]$ is the same for all $w$, since students are distributed uniformly.

3) The term $-da^*(w)/dg(w)$ is larger for smaller $w$ by the concavity of $v(\cdot)$, as discussed above.

The second part of (10) describes the budgetary costs of increasing grants to students with wealth $w$. Starting from any uniform grant scheme, $g'(w) = 0$, the marginal cost of grant provision increase with student’s wealth $w$ since $a^*(w)$ decreases with $w$.

Concluding, starting from any uniform grant scheme, $g'(w) = 0$, the marginal benefit of grant provision decrease with student’s wealth, while the marginal cost of grant provision increase with student’s wealth. A uniform grant scheme is therefore suboptimal. Optimal grants decrease with student’s wealth.
9 Notation

a Ability

b Consumption benefit of college education

c(a) College’s cost of educating a student with ability a

f(a, w) Density of the population as a function of ability a and wealth w

g(w) government-provided grant to a student with wealth w

p(a) Return to education of college graduate with ability a

t(w) Tuition at college for a student with wealth w

v(·) Utility from income

w Wealth
References


