Anti-dumping, Intra-industry Trade and Quality Reversals

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Anti-dumping, Intra-industry Trade and Quality Reversals*

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Abstract

We examine an export game where two firms (home and foreign), located in two different countries, produce vertically differentiated products. The foreign firm is the most efficient in terms of R&D costs of quality development and the foreign country is relatively larger and endowed with a relatively higher income. The unique (risk-dominant) Nash equilibrium involves intra-industry trade where the foreign producer manufactures a good of higher quality than the domestic firm. This equilibrium is characterized by unilateral dumping by the foreign firm into the domestic economy. Two instruments of anti-dumping (AD) policy are examined, namely, a price undertaking (PU) and an anti-dumping duty. We show that, when firms' cost asymmetries are low and countries differ substantially in size, a PU leads to a quality reversal in the international market, which gives a rationale for the domestic government to enact AD law. We also establish an equivalence result between the effects of an AD duty and a PU.

JEL Classification: F12, F13

Keywords: anti-dumping duty, intra-industry trade, price undertaking, product quality, quality reversals

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1 Introduction

Times have changed in the world of anti-dumping activities. While only a few developed countries, mainly the EC and US, were users of anti-dumping action less than a decade ago, anti-dumping is now used by both developed and developing countries. Recent statistics reveal that 97 members of the World Trade Organization (counting the EC as a single member) have incorporated anti-dumping provisions in their national laws (WTO, 2002; Blonigen and Prusa, 2003). Assessing the current situation, the European Communities conclude: “Anti-dumping is now a ‘global’ instrument and every country is now both a potential user and a potential target of anti-dumping action” (EC, 2002a, p.1). International organizations are particularly worried about the increasing human and financial resources that successful anti-dumping investigations require.

The proliferation of countries adopting anti-dumping provisions seem to parallel the accelerating globalization of the international economy, involving more trade with transition and developing countries. This is highlighted in the numerous anti-dumping investigations which show considerable differences in the types of products made worldwide. Because of lower quality standards in some countries, their local firms produce and export goods whose quality is inferior to that of Western firms. It is commonly believed that since low quality goods command a lower price, they are prime candidates for AD petition against their producers.

The various market outcomes of anti-dumping (AD) have been extensively reviewed in Blonigen and Prusa (2003). Bearing in mind that AD actions by a government sanction findings of dumping by exporting firms into a particular country, a number of stylized facts have inspired our framework of analysis:

- Of the 1105 measures in force reported in June 2001, 21.8% are maintained by the US and 19.8% by the EC (WTO, 2002, Table IV.6). In terms of cases per dollar of imports, India’s intensity of AD use is seven times the US figure, and Argentina’s is twenty times (Finger, Ng and Wangchuk, 2000). The heterogeneity of countries involved suggests important asymmetries between firms participating in bilateral trade.

- There is convincing evidence that a significant proportion of trade is characterized by different levels of quality. For example, Greenaway et al. (1994, 1995) show that over two thirds of all intra-industry trade in the UK involves trade of vertically differentiated goods.

- Petitions filed by a US industry against imports concern products which are usually classified
under 10 digit subheadings of the Harmonized Tariff Schedule of the United States. At this level of disaggregation, sources of supply of this product in a domestic market are a few firms. Even in large trading blocs like the US or EC, it is common that the case concerns two players, a local and a foreign producer. See, for example, USITC (2001, 2002b), or EC (2002b, p. 25 and p. 48).

• Hearings and public reports reveal that, besides prices, perceived quality differences are important in most AD cases (USITC, 2001, 2002a, 2002b, 2003).

• Anti-dumping protection, via either duties or price undertakings, gives rise to a variety of strategic interactions between market participants, in particular regarding quality leadership in the international market.

The importance of these features varies from country to country and no single theory can describe the complexity of the various cases. However, the main elements that we shall include in our model are: (i) endogenous quality choice, (ii) asymmetries between firms and between countries, (iii) firms’ strategic responses to AD policies, (iv) intra-industry trade, and (v) quality reversals.

We analyze an international trade game between two firms located in different countries that produce quality-differentiated goods. Domestic and foreign consumers have heterogenous preferences for the sole product attribute, quality. This quality-differentiated good is supplied at home and abroad by a local firm and by imports from the foreign producer. Quality development is costly and firms are asymmetric in regard to R&D costs needed for quality development. Both markets are not totally served in equilibrium, implying endogenous market sizes and are asymmetric in that they differ in size and in the distribution of consumer tastes. We study a three-stage game. In the first stage, governments opt for free trade or enact anti-dumping law. This AD regulation includes two popular instruments, namely, AD duties and price undertakings. In the second stage, firms select the qualities to be produced, and incur the fixed costs; the third stage is an export game where firms compete in prices.

We show the existence of a unique (risk-dominant) free trade equilibrium that is characterized by intra-industry trade. The foreign firm, which is the most efficient, produces a good of higher quality than the domestic firm. Since consumers across countries differ in their concern for quality, unilateral dumping by the foreign firm into the domestic market arises in equilibrium. In this context, there may be a rationale for an AD policy by the domestic government. We find that the effects of a price undertaking hinge upon the extent of asymmetries in countries’ size and wealth as
well as in firms’ R&D costs. When countries are of similar size, a price undertaking (PU) cannot be justified, neither on the basis of home firm’s profits nor on the basis of domestic consumer welfare. When country sizes differ substantially and cost asymmetries are low, a PU leads to a quality reversal in the international market, which increases not only home firm’s profits but also domestic welfare. This provides a rationale for the domestic government to enact anti-dumping law based on social welfare considerations. If cost asymmetries are large instead, the foreign firm continue to produce high quality but exits the domestic economy as a result of a PU. This increases the profits of the domestic firm and thus gives a rationale for an AD policy in response to firm lobbying. We also examine the effects of AD duties and establish an equivalence result with a PU.

Our paper is a contribution to the study of anti-dumping in oligopolistic industries. The literature on anti-dumping is extensive and the reader is referred to the survey of Blonigen and Prusa (2003) and the book of Feenstra (2003) for a detailed discussion of this work. Various papers (see e.g. Ethier and Fisher, 1987; Fisher, 1992; Leidy and Hoekman, 1990; and Reitzes, 1993) have examined how the presence of AD protection gives strategic firms incentives to alter their price or output decisions vis-à-vis free trade to influence the AD outcome. This may lead to higher or lower welfare depending on the existing market structure. The distinctive feature of our paper is that we examine the effects of anti-dumping legislation in an international market where firms produce vertically differentiated products. In this connection, the paper most closely related to our work is by Vandenbussche and Wauthy (2001). In a game of one-way trade between a domestic and a foreign firm, they show that, relative to free trade, a PU gives the foreign firm incentives to be more aggressive and become the quality leader in the international market. AD law leads in this case to lower social welfare for the home country. In their paper the competing local price is used as a proxy for the normal value of a good under a PU, and this blocks the exports of the foreign low-quality firm. Our paper differs from theirs, among other things, in that we examine a model of two-way trade, which allows us to use a more standard definition of dumping. Anderson et al. (1995) examine a variant of the reciprocal dumping model of Brander and Krugman (1983) where two governments can enact AD law or not. They find that government impose no law in equilibrium if they maximize welfare. Moreover, though an individual firm has an incentive to lobby for AD law, consumer welfare increases and firm profits fall if laws are bilaterally enacted.

Our paper is also related to the work explaining how product quality matters in international trade. The monopoly problem is discussed in Musa and Rosen (1978) and Krishna (1987) and oligopoly versions of this model have recently received substantial attention in the international
trade literature. The papers most closely related to our work are Motta et al. (1997), Herguera et al. (2002) and Zhou et al. (2002). Motta et al. (1997) analyze the introduction of trade between two countries that produce different quality levels. They show that the quality leader maintains its position after the opening up to international trade. Our paper contributes to this work on quality leadership by showing that there is a unique equilibrium in the export game and that firms’ cost asymmetries are crucial to sustain quality leadership in the market. In addition, our paper focuses on positive and normative issues of AD policy. Herguera et al. (2002) study optimal trade policy in a model with one-way trade. Zhou et al. (2002) study the implications of an export policy in a third-market model where cost differences between the exporting countries are very large. In contrast, our model allows for small costs asymmetries as well. This is an important difference because it gives the government an incentive to enact AD policy that can lead to a reversal in quality leadership.

The paper is organized as follows. Next section presents the details of our model. Section 3 solves for the free trade equilibrium and establishes the conditions for dumping. Section 4 examines AD legislation in the form of price undertakings while Section 5 establishes the equivalence result between AD duties and price undertakings. Section 6 concludes. All proofs are relegated to the Appendix to ease the reading of the paper.

2 The Model

We examine an international trade game between two firms producing goods that are vertically differentiated. These two firms, located in two different countries, produce goods for their own market and, eventually, for exports. The firm located in the foreign (home) country is referred to as the foreign (home) firm and all foreign variables are denoted by an asterisk 

We index both countries by \( i = 1, 2 \) where subscript 1 refers to the home country and subscript 2 to the foreign country. Let \( q_i^* \) denote the quality of the product manufactured by the foreign firm to be sold in country \( i, i = 1, 2 \). Likewise, let \( q_i \) be the quality of the product manufactured by the home firm to be sold in country \( i, i = 1, 2 \). As firms incur fixed R&D costs of quality development we assume flexible production (Eaton ad Schmitt, 1994), that is, once firms invest in the necessary technology and organize their facilities to develop and produce one basic product, they can produce various downgrades of this basic product at no cost. The idea is modelled via the following specification of R&D costs: domestic firm’s costs of producing variants \( q_1 \) and \( q_2 \) are \( C(q_1, q_2) = c \max\{q_1, q_2\}^2 / 2 \); likewise, foreign firm costs of producing variants \( q_1^* \) and \( q_2^* \) are given
by $C(q_1^*, q_2^*) = c^* \max\{q_1^*, q_2^*\}^2 / 2$, where $c$ and $c^*$ are development cost parameters which measure R&D efficiency. We assume $c^* = 1$ without loss of generality and $c > 1$, that is, the home firm is less efficient than the foreign firm in developing any level of quality.\(^1\) Once the quality of the goods to be offered is determined, we assume that production takes place at a common marginal cost which is normalized to zero.\(^2\)

Assume a population of measure 1 at home and a measure $m^* \geq 1$ in the foreign country. Consumers buy at most one unit and have preferences given by the following quasi-linear (indirect) utility function: $U = \theta q - p$, if she buys a unit of a good of quality $q$ at price $p$, and 0 otherwise. Parameter $\theta$ is consumer specific and measures the utility a consumer derives from consuming a unit of quality. Assume that $\theta$ is uniformly distributed over $[0, \bar{\theta}]$ at home, and over $[0, \lambda^* \bar{\theta}]$ abroad, with $\lambda^* \geq 1$, $\bar{\theta} > 0$. Tirole (1988) shows that $\theta$ is the inverse of the marginal utility of income so our assumption $\lambda^* \geq 1$ implies that foreign consumers have higher incomes on average and more sophisticated tastes. Our specification of demand thus captures both size and income differences between countries via $m^*$ and $\lambda^*$.\(^3\) Finally, we assume there are transaction costs associated to parallel trade which render goods arbitrage unprofitable for consumers.

We study a three-stage game. In the first stage of the game, governments opt for free trade or enact anti-dumping law. In the second stage, firms choose the quality of the goods to be produced, and incur the fixed costs. Finally, firms engage in an export game where they compete in prices. The appropriate solution concept is subgame perfect equilibrium. The model is solved by backward induction.

## 3 Trade Equilibrium

The assumptions of our model depict a situation in which a domestic firm, located in a smaller and poorer country, considers, besides supplying its own market, to export to a larger and richer country. It faces competition from a foreign firm, which is more efficient. Given this, the following

\(^1\) Cost asymmetries across firms in different countries may capture differences in the available production technologies as well as in the costs of labor and capital. They are important here because they allow us to pin down a unique equilibrium in qualities. The specification of the cost function could be more general without affecting results qualitatively. For example, Moraga-Gonzále and Viaene (2005) use cost functions with a degree of homogeneity $k \geq 2$ in qualities. While larger $k$ values affect results quantitatively, they do not alter them qualitatively.

\(^2\) This cost specification captures the distinctive features of pure vertical differentiation models, where the costs of quality improvements mainly fall on fixed costs and involve only a small or no increase in unit variable costs (see Shaked and Sutton, 1982, 1983). The normalization adopted here is without loss of generality provided that the main bulk of costs falls on fixed costs rather than on variable costs. Adding small marginal costs of production makes computations cumbersome and obscures the presentation of the results.

\(^3\) As we will see later, country asymmetries in regard to demand may play an equilibrium selection role for some export strategy profiles; moreover they are important for the effects of anti-dumping policy.
three questions that arise are: Once both countries open up to international trade, what is the pattern of trade that emerges in equilibrium? What are the product qualities that are produced by each firm in equilibrium? For which of the two firms, if any, is it optimal to dump its good in the international market? We address these three issues in what follows.

To solve the export game outlined above, it is useful to obtain preliminary results that describe firms’ pricing and quality decisions for different export strategies. These export strategy profiles lead to the different patterns of international trade in Table 1.

<table>
<thead>
<tr>
<th>Foreign Firm</th>
<th>Export (E)</th>
<th>Not export (NE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home firm</td>
<td>Intra-industry trade</td>
<td>Duo. abroad, mono. at home</td>
</tr>
<tr>
<td>Not export (NE)</td>
<td>Mono. abroad, duo. at home</td>
<td>Autarky</td>
</tr>
</tbody>
</table>

Table 1

This matrix contains four cells, each corresponding to a different export strategy profile. The demands firms face are different across international trade patterns, and so are their optimal quality and pricing decisions. The games we analyze have typically two Nash equilibria, one where the foreign firm is quality leader and one where the domestic firm is quality leader. To select amongst equilibria, we use the risk-dominance criterion of Harsany and Selten (1988).

3.1 Export Game

Autarky (NE,NE)

The case of autarky is one where each firm produces only for its own market. In this case, simple derivations lead to the demands at home and abroad:

\[
D_1(p_1, q_1) = 1 - \frac{p_1}{\theta q_1} \quad (1)
\]

\[
D_2(p^*_2, q^*_2) = m^* \left( 1 - \frac{p^*_2}{\lambda^* \theta q^*_2} \right) \quad (2)
\]

Maximization of profits leads to the following result:

**Lemma 1** In autarky the domestic firm produces a good of quality \( q_1 = \bar{q}/4c \) and charges \( p_1 = \bar{q}_1/2 \), while the foreign firm produces a good of quality \( q^*_2 = m^* \lambda^* \bar{q}/4 \) and charges \( p^*_2 = \bar{q}^* \lambda^* q^*_2/2 \).

\^4This criterion is also used in Motta et al. (1997) and Moraga-González and Viaene (2005)
Though both firms are monopolists in their own country, the foreign firm always produces the highest quality because it is more efficient and produces in a larger and richer country where the average consumer is willing to pay more for each unit of quality.

**International duopoly abroad and monopoly at home (E,NE)**

This is a situation where the home firm produces a good of quality $q_1$ for its domestic market and exports a good of quality $q_2$ to the foreign country; the foreign firm produces just for its local market.

Since there is just one good sold in the home country, this country’s demand is given by (1) as before. By contrast, since two variants are sold in the foreign country, we need to calculate the demand faced by each firm. Note that the explicit derivation of each firm’s demand abroad depends on whether the foreign firm’s good is of higher or of lower quality than the domestic variant. Consider first the case where the foreign firm sells a good of quality $q^*_2$ in its own market and faces competition from low-quality exports, i.e., $q^*_2 > q_2$. Denote by $\tilde{\theta}$ the buyer who is indifferent between buying high quality or low quality. From the buyers’ utility function, it follows that $\tilde{\theta} = (p^*_2 - p_2) / (q^*_2 - q_2)$. Denote by $\hat{\theta}$ the consumer indifferent between acquiring the low-quality good or nothing, that is, $\hat{\theta} = p_2 / q_2$. Hence, the high-quality good is demanded by those consumers such that $\tilde{\theta} \leq \theta \leq \hat{\theta}$. Likewise the low-quality variant is demanded by those buyers such that $\hat{\theta} \leq \theta < \tilde{\theta}$. As $\theta$ is uniformly distributed on $[0, \lambda^*\tilde{\theta}]$, foreign demands for high- and low-quality goods are readily obtained:

$$D_2(\cdot) = m^* \left( \frac{p^*_2 - p_2}{\lambda^*\tilde{\theta}(q^*_2 - q_2)} - \frac{p_2}{\lambda^*\tilde{\theta}q_2} \right), \quad D^*_2(\cdot) = m^* \left( 1 - \frac{p^*_2 - p_2}{\lambda^*\tilde{\theta}(q^*_2 - q_2)} \right).$$  \hspace{1cm} (3)

Consider now the opposite case where home firm’s exports are of higher quality, i.e., $q_2 > q^*_2$. Demands can be calculated as in (3) and have the following form:

$$D_2(\cdot) = m^* \left( 1 - \frac{p^*_2 - p_2}{\lambda^*\hat{\theta}(q^*_2 - q_2)} \right), \quad D^*_2(\cdot) = m^* \left( \frac{p^*_2 - p_2}{\lambda^*\hat{\theta}(q_2 - q^*_2)} - \frac{p^*_2}{\lambda^*\hat{\theta}q^*_2} \right).$$  \hspace{1cm} (4)

Using these demands in (3) and in (4) we can solve the game for the different quality equilibria using the Harsanyi-Selten criterion. As equilibrium profits depend on $c$ and on the product $\lambda^*m^*$, we find that for each product $\lambda^*m^*$ there exists a threshold value of the home firm’s cost parameter denoted $\overline{\tau}(\lambda^*m^*)$ such that below $\overline{\tau}(\cdot)$ the domestic firm is quality leader in the international market whereas above $\overline{\tau}(\cdot)$ the foreign firm produces the high-quality good instead.
Lemma 2. If an equilibrium exists where the home firm is the only exporter, then \( q_1 = q_2 \); moreover for every \( \lambda^* m^* \) there exists \( \tau(\lambda^* m^*) \) such that: (i) \( q_1 = q_2 > q_2^* \) for all \( c < \tau(\lambda^* m^*) \); (ii) \( q_1 = q_2 < q_2^* \) for all \( c > \tau(\lambda^* m^*) \).

This result has two important implications. First, the exporting firm produces a single product for the international market. Any other quality profile is ruled out by the cost function that represents flexible production and the possibility that firms leapfrog the quality produced by the competitor. The second implication of Lemma 2 is that when the home firm is the sole exporter of goods, though it is less efficient, it becomes quality leader in the international market for low cost asymmetries. The reason is that the firm can use larger world revenues to compensate for its larger development costs. The leadership of the domestic firm, however, is difficult to sustain when its relative inefficiency increases.

International duopoly at home and monopoly abroad (NE,E)

When the foreign firm is the sole exporter and the home firm produces for its domestic market only, one good is sold in the foreign country and thus the demand for this good is given in (2). In contrast, two variants are sold in the home country; as before the demand faced by each firm in the home country depends on which firm is the quality leader. Derivations as above can be repeated for the case where the home firm sells a good of quality \( q_1 \) in its own market and faces competition from exports of a good of lower quality, i.e., \( q_1 > q_1^* \).

\[
D_1(\cdot) = 1 - \frac{p_1 - p_1^*}{\theta(q_1 - q_1^*)}, \quad D_1^*(\cdot) = \frac{p_1 - p_1^*}{\theta(q_1 - q_1^*)} - \frac{p_1^*}{\theta q_1^*},
\]

(5)

In the opposite case where foreign exports are of higher quality, i.e., \( q_1^* > q_1 \), we obtain the following demands:

\[
D_1(\cdot) = \frac{p_1^* - p_1}{\theta(q_1^* - q_1)} - \frac{p_1}{\theta q_1}, \quad D_1^*(\cdot) = 1 - \frac{p_1^* - p_1}{\theta(q_1^* - q_1)}
\]

(6)

Given these demands the following result offers a necessary condition for an equilibrium to exist.

Lemma 3. If an equilibrium exists where the home firm produces for its local market only and the foreign firm exports goods then \( q_2^* = q_1^* > q_1 \).

As in the preceding trade pattern, there are only two qualities in the international market. Moreover, when the foreign firm is the sole exporter it is always the quality leader; this is because this firm is more efficient and can spread the development costs over the world market.

Intra-industry trade (E,E)
When both firms export, the demands faced by each firm depend on which of the two firms is quality leader. Demands for the different cases are given in the expressions (3) to (6). The following result is a necessary condition for an intra-industry trade equilibrium.

**Lemma 4** If an equilibrium exists with intra-industry trade, then \( q_1 = q_2 < q^*_1 = q^*_2 \).

Hence, only two variants can be sold in an equilibrium with intra-industry trade. Moreover, the quality leader in the international market is the most efficient foreign producer. Summarizing, the restrictions the concept of equilibrium imposes on the range of qualities produced are given in Table 2:

<table>
<thead>
<tr>
<th>Foreign Firm</th>
<th>Export (E)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Home firm</td>
<td>( q_1 = q_2 &lt; q^<em>_1 = q^</em>_2 )</td>
<td>( q_1 = q_2 &lt; (&gt;)q^*_2 \text{ if } c &gt; (&gt;)\bar{c}(\lambda^<em>m^</em>) )</td>
</tr>
<tr>
<td>Not export (NE)</td>
<td>( q_1 &lt; q^<em>_1 = q^</em>_2 )</td>
<td>( q_1 &lt; q^*_2 )</td>
</tr>
</tbody>
</table>

Table 2

Two important observations follow from Table 2. One, an individual firm will produce a single variant for its own market and for the export market. Two, whether a firm is quality leader in the international market depends on the parameters of the model, in particular on country size and income asymmetries and on firms’ costs asymmetries.

### 3.2 Equilibrium under Free Trade

Having computed firms’ equilibrium payoffs under different export strategy profiles, we are now ready to examine the equilibria of the export game. Denote by \( \pi_{i,j} \) the profits of the domestic (foreign) firm under export strategy profile \((i, j)\), \( i, j = E, NE \). We find that \( \pi^*_{E,E} > \pi^*_{E,NE} \) and that \( \pi^*_{NE,E} > \pi^*_{NE,NE} \), which implies that exporting is a dominant strategy for the foreign firm. Likewise exporting is a dominant strategy for the home firm since \( \pi_{E,E} > \pi_{NE,E} \) and \( \pi_{E,NE} > \pi_{NE,NE} \). As a result the game is dominance solvable and the unique equilibrium involves two-way trade.

Figures 1 and 2 illustrate the various firm payoffs. The difference between these two Figures is the extent of cost asymmetries between the firms. In Figure 1, the home firm is not very inefficient compared to the foreign firm. This implies that the home firm is leader in quality for the export strategy profile \( \{E, NE\} \) (c.f. Lemma 2). Figure 2 is constructed assuming that firm cost asymmetries are large; in this case the foreign firm is quality leader for any export strategy profile.
Let us define $\mu = q^*/q$, with $\mu > 1$ since $q^* > q$. Variable $\mu$ represents the quality gap between the two firms’ variants and measures the degree of product differentiation. Then:

**Proposition 1** There is a unique free trade equilibrium of the export game. This equilibrium involves intra-industry trade and is characterized as follows: (i) The home firm produces a good of quality

$$q = \frac{\bar{\theta}(1 + \lambda^* m^*) \mu^2 (4\mu - 7)}{(4\mu - 1)^3}$$

that is sold locally and exported to the foreign country; (ii) the foreign firm produces a good of higher quality

$$q^* = 4\bar{\theta}(1 + \lambda^* m^*)\frac{\mu(4\mu^2 - 3\mu + 2)}{(4\mu - 1)^3}$$
for its own market and for exports to the home country; (iii) \( \mu \) is the quality gap between the variants and is the solution to

\[
\frac{4(4\mu^2 - 3\mu + 2)}{\mu^2(4\mu - 7)} = \frac{1}{c}.
\] (9)

(iv) These goods sell at prices

\[
p_1 = \frac{\bar{q}(\mu - 1)}{4\mu - 1}; \quad p_2 = \lambda p_1
\]

\[
p_2^* = \frac{2\lambda\bar{q}^*(\mu - 1)}{4\mu - 1}; \quad p_1^* = p_2^*/\lambda,
\]

in the two countries. (v) Market demands for each firm are

\[
D_1 = \frac{\mu}{4\mu - 1}; \quad D_2 = m^* - \frac{\mu}{4\mu - 1}
\]

\[
D_1^* = \frac{2\mu}{4\mu - 1}; \quad D_2^* = m^* - \frac{2\mu}{4\mu - 1}
\]

(vi) The Grubel-Lloyd measure of two-way trade (in value) is

\[
GL = 100 \left\{ 1 - \left| \frac{p_2D_2 - p_1^*D_1^*}{p_2D_2 + p_1^*D_1^*} \right| \right\} = 100 \left\{ 1 - \frac{|\lambda^*m^* - 4\mu|}{\lambda^*m^* + 4\mu} \right\}
\] (14)

(vii) Domestic firm’s profits are \( \pi = q^*q/8 \) and foreign firm’s profits are \( \pi^* = 2cq^*q \).

In the market equilibrium of Proposition 1, variable \( \mu \) is the unique solution to the third degree polynomial in (9). Besides being a measure of product differentiation, \( \mu \) relates to the extent of price competition between the firms. To show this, take the ratio (11) to (10) to compute the relative prices in each market: \( p_1^*/p_1 = 2\mu \) and \( p_2^*/p_2 = 2\mu \). An increase in \( \mu \) widens the quality gap, thereby mitigating competition and increasing relative prices in both countries. For any \( \mu \), demands are positive so the market equilibrium involves intra-industry trade in vertically differentiated products, like in the contributions of Falvey (1981) and Shaked and Sutton (1984). The Grubel-Lloyd index depends on the primitive parameters of the model: the relative country size \( m^* \), the relative taste difference \( \lambda^* \), and the relative cost difference \( c \) (via the solution to (9)).

### 3.3 Conditions for Dumping

A product is considered as being dumped if its export price to a particular country is less than a “normal value,” standard used by the WTO, or less than a “fair value,” standard used by the U.S. government. There are different ways of calculating a product’s “normal” or “fair” value. The more standard definition of dumping, the one that is advanced by the WTO (see the WTO website), is when a company exports a product at a price lower than the price it normally charges on its own.
home market. In our framework, this definition of dumping leads to a bilateral comparison of \( p_1 \) with \( p_1^* \), and \( p_2 \) with \( p_2^* \) in (10) and (11), which yields the following result:\(^5\)

**Proposition 2** Under free trade only unilateral dumping arises; in particular, the foreign firm dumps its high-quality goods into the domestic market.

This Proposition leads to a number of observations. First, dumping arises because cross-country differences in the distribution of tastes provide the foreign firm with incentives to cut its export price relative to the price it charges in its own market. Second, traditional treatments of dumping have shown the possibility of reciprocal dumping based on transportation costs (Anderson *et al.* 1995; Brander and Krugman, 1983; Weinstein, 1992). Here dumping is unilateral and the introduction of transportation costs or import tariffs does not undermine this result, unless they are large enough to offset the influence of the cross-country differences in the distribution of tastes. Third, a popular belief is that low-quality goods are those that are dumped in the export markets because they usually command a lower price. In our setting, the reverse occurs: it is the high-quality good that is dumped into the smaller and poorer country.

To substantiate a case for anti-dumping law the local firm must also suffer injury from dumped imports. In our framework, the share of imports in the domestic economy is \( D_1^* / (D_1 + D_1^*) = 2/3 \) (see Proposition 1). It is clear that this market share is sufficiently large to justify a demand for AD legislation. Whether the government positively responds to such a demand depends on its objectives. We use the weighted sum of consumer surplus and firms’ profits as a measure of social welfare:

\[
W = \beta_1 CS + \beta_2 \pi, \; \beta_j \in \{0, 1\}, j = 1, 2
\]  

(15)

When revenues accrue from the imposition of an AD duty, we add them to (15). The weights \( \beta_1 \) and \( \beta_2 \) characterize policymaker’s preferences. If \( \beta_1 = \beta_2 = 1 \) we have the usual definition of social welfare; if \( \beta_1 = 1 \) and \( \beta_2 = 0 \), the government cares only about consumer surplus; finally when \( \beta_1 = 0 \) and \( \beta_2 = 1 \), only firm’s profits matter.\(^6\) Domestic consumer surplus is given by:

\[
CS_1 = \int_{p_1/q}^{p_1^*} (xq - p_1^*)dF(\theta) + \int_{p_1/q}^{p_1^* - p_1} (xq - p_1)dF(\theta)
\]

\(^5\) Another possibility is to consider the competing local price as a proxy for the normal value. Though this is characterized as the “lay” definition of dumping (Weinstein, 1992) and the method has been used in theory (Vandenbussche and Wauthy, 2001) and applied in a number of cases, among others in Mexico (Niels, 2004), this is a definition of last resort, useful only when other methods of calculations are not possible.

\(^6\) We exclude the case \( \beta_1 = 0 \) and \( \beta_2 = 0 \).
where $F(\theta)$ is the cumulative distribution function over the interval $[0, \bar{\theta}]$. Using equilibrium prices in (10), consumers surplus can be written more conveniently as:

$$\text{CS}_1 = \frac{\bar{\theta} \mu^2 (4\mu + 5) q}{2(4\mu - 1)^2}. \quad (16)$$

4 Anti-dumping Policy

Two AD policy instruments are commonly used by governments, namely, price undertakings and anti-dumping duties. The former is more commonly used in the EU while the latter is observed more frequently in the US. A price undertaking is a binding commitment to raise export prices so that either the dumping or the injury suffered from dumped imports by the domestic country is eliminated (GATT, 1991, p. 74). An anti-dumping duty equalizes the price that consumers in different countries pay for the same good by means of an import tariff. The analysis that follows compares price undertakings and anti-dumping duties and offers an equivalence result between these two AD instruments. Both are desirable on welfare grounds for certain parameter configurations. In particular, they can lead to a quality reversal in the international market whereby the low-quality firm becomes the producer of high quality. We begin with the analysis of a price undertaking because of its simpler derivations.

4.1 Price Undertakings

Under a price undertaking, the foreign firm must set an export price that is equal to its local price, i.e., $p^*_1 = p^*_2 = p^*$. Given this constraint, the demands faced by the different firms are:

$$D_1(\cdot) = \frac{p^* - p_1}{\bar{\theta}(q^* - q)} - \frac{p_1}{\bar{\theta}q}; \quad D_2(\cdot) = m^* \left( \frac{p^* - p_2}{\lambda^* \bar{\theta}(q^* - q)} - \frac{p_2}{\lambda^* \bar{\theta}q} \right)$$

$$D^*_1(\cdot) = 1 - \frac{p^* - p_1}{\bar{\theta}(q^* - q)}; \quad D^*_2(\cdot) = m^* \left( 1 - \frac{p^* - p_2}{\lambda^* \bar{\theta}(q^* - q)} \right);$$

The profits of the foreign and the home firm are then given by:

$$\pi^* = p^* (D^*_1 + D^*_2) - \frac{q^* s^2}{2}$$

$$\pi = p_1 D_1 + p_2 D_2 - \frac{c q^2}{2}.$$
as well. Given this and using the first order condition $d\pi^*/dp^* = 0$ we can solve for equilibrium prices:

$$
p_1 = p_2 = \frac{p^* q}{2q^*}
$$

$$
p^* = \frac{\lambda^*(1 + m^*)}{\lambda^* + m^*} \frac{2q^*(q^* - q)}{4q^* - q}
$$

Anticipating these equilibrium prices, firms select qualities to maximize reduced-form profits:

$$
\pi^* = 4\tilde{\theta}\lambda^*(1 + m^*)^2 \frac{q^* (q^* - q)}{(4q^* - q)^2} - \frac{q^*}{2}
$$

$$
\pi = \frac{\tilde{\theta}\lambda^* (1 + m^*)^2 q^* (q^* - q)}{(4q^* - q)^2} - \frac{cq^2}{2}
$$

The first order conditions with respect to qualities are

$$
\frac{d\pi^*}{dq^*} = \frac{4\tilde{\theta}\lambda^*(1 + m^*)^2 q^* (4q^* - 3q^* q + 2q^2)}{(4q^* - q)^3} - q^* = 0 \quad (17)
$$

$$
\frac{d\pi}{dq} = \frac{\tilde{\theta}\lambda^* (1 + m^*)^2 q^* (4q^* - 7q^*)}{(4q^* - q)^3} - cq = 0 \quad (18)
$$

To simplify matters, let us again define $\mu = q^*/q$, with $\mu > 1$ since $q^* > q$. Using $\mu$, the ratio of first order conditions (17) and (18) leads to an expression exactly identical to (9). Hence, an important conclusion is that a price undertaking affects neither the equilibrium degree of product differentiation $\mu$ nor the extent of international price competition.

Plugging $\mu$ in (17) and (18) allows us to isolate equilibrium qualities under a price undertaking:

$$
q = \frac{\tilde{\theta}\lambda^*(1 + m^*)^2}{c(\lambda^* + m^*)} \frac{\mu^2(4\mu - 7)}{(4\mu - 1)^3}
$$

$$
q^* = \frac{4\tilde{\theta}\lambda^* (1 + m^*)^2}{(\lambda^* + m^*)} \frac{\mu(4\mu^2 - 3\mu + 2)}{(4\mu - 1)^3}
$$

Since $\mu$ is unaltered, the qualities in (19) and (20) can easily be compared to those under free trade in (7) and (8). It is readily seen that a price undertaking leads to a decrease in the quality of both variants.

Using $\mu$ again equilibrium (hedonic) prices are as follows:

$$
\frac{p_1}{q} = \frac{p_2}{q} = \frac{p^*}{2q^*}
$$

$$
\frac{p^*}{q^*} = \frac{\lambda^*(1 + m^*)}{\lambda^* + m^*} \frac{2\tilde{\theta}(\mu - 1)}{4\mu - 1}
$$

A comparison of these prices with those in Proposition 1 reveals that, compared to free trade, a price undertaking leads to an increase in all (hedonic) prices in the domestic market and in a decrease in all (hedonic) prices in the foreign market.
Equilibrium demands are given by

\begin{align}
D_1 &= \frac{\lambda^* (1 + m^*)}{\lambda^* + m^*} \frac{\mu}{4\mu - 1}; \quad D_2 = \frac{m^* (1 + m^*)}{\lambda^* + m^*} \frac{\mu}{4\mu - 1}; \\
D_1^* &= \frac{2\mu}{4\mu - 1} - \frac{m^* (\lambda^* - 1)}{\lambda^* + m^*} \frac{2\mu - 1}{4\mu - 1}; \quad D_2^* = \frac{2m^* \mu}{4\mu - 1} + \frac{m^* (\lambda^* - 1) 2\mu - 1}{\lambda^* + m^*} \frac{4\mu - 1}{4\mu - 1}
\end{align}

(23)

It is clear from (23) and (24) that the world demand for low-quality products faced by the domestic firm, \((D_1 + D_2)\), is not affected by a price undertaking; however, the distribution of quantities sold in the world changes in such a way that the domestic firm’s local sales increase and its exports decrease. Likewise, \((D_1^* + D_2^*)\) is similar under both free trade and a price undertaking but, in the latter case, the exports of the foreign firm decrease while its local sales increase by the same amount. More importantly, the unique feature of a price undertaking resides in the expression for \(D_1^*\) in (24): \(D_1^*\) can become zero for certain parameter configurations, which forces the foreign firm out of the export market.

Let us suppose for the moment that a price undertaking does not alter the international trade pattern, i.e., \(D_1^*\) remains strictly positive. This is the case if the following (sufficient) condition holds.

**Condition 1.** \(\lambda^* < 2m^*/(m^* - 1)\)

Condition 1 is always satisfied whenever countries are of similar size \((m^* = 1)\). If this condition holds, we are fully equipped to examine the effects of a price undertaking on firms’ profits and consumer surplus. Using \(\mu\), the first order conditions (17) and (18), and the expressions for qualities in (19) and (20), the reduced-form profits of the firms as follows:

\begin{align}
\pi^* &= 2cq^*q \\
\pi &= \frac{q^*q}{8}
\end{align}

(25)\hspace{1cm}(26)

Since we know that qualities are lower under a price undertaking, it is clear that both firms profits are lower than under free trade. Hence, when Condition 1 holds, there is no incentive for the domestic firm to lobby for an implementation of AD legislation in the form of a PU \((\beta_1 = 0, \beta_2 = 1)\) in (15)). Moreover, since a price undertaking does not change \(\mu\) and decreases \(q\), we conclude that domestic consumers lose from it. The following Proposition summarizes the main results.

**Proposition 3** Assume that Condition 1 holds. Then, compared to free trade, a price undertaking imposed by the domestic government results in: (i) a decrease in the quality of both variants; (ii) an
increase (decrease) in the prices of both variants in the domestic (foreign) country; (iii) a decrease in the profits of both firms, and (iv) in a decrease in consumer surplus. Anti-dumping policy in the form of a price undertaking is not justified in this case.

Suppose now that a price undertaking leads to the exit of the high-quality foreign firm from the domestic market. Solving the equation $D_1^* \leq 0$ in $\lambda^*$ and assuming $m^* \geq 2\mu/(2\mu - 1)$ yields $\lambda^* \geq m^*(4\mu - 1)/(m^*(2\mu - 1) - 2\mu)$. Since the RHS of this expression is monotonically decreasing in $\mu$, and since the solution to (9) is monotonically increasing in $c$, but bounded below by 5.25123 there is a sufficient condition for which the foreign firm refrains from exporting its high-quality good. This condition is the following:

**Condition 2.** $\lambda^* > 1.91m^*/(0.9m^* - 1)$, and $m^* > 1/0.9$.

What kind of trade equilibrium does prevail in the presence of a price undertaking when condition 2 holds? To answer this question we refer to Lemma 2, in particular to cell $\{E, NE\}$ in Tables 1 and 2. In the proof of this Lemma we show that when the foreign firm sells only locally while the domestic firm sells its good both locally and internationally, if cost asymmetries are sufficiently large then the foreign firm will remain quality leader in the international market. The profits of the local firm increase while domestic consumer surplus decreases in this case simply because of the monopoly position of the home firm in the domestic market. Implementation of anti-dumping legislation in this case can only be a response to home firm lobbying for AD law ($\beta_1 = 0, \beta_2 = 1$ in (15)).

When cost asymmetries are sufficiently low, by contrast, a quality reversal takes place: the domestic firm becomes the high-quality producer in the unique (risk-dominant) equilibrium of the export game (see Lemma 2). As high-quality production is highly profitable both domestic firm’s profits and consumer surplus increase compared to free trade. In this case, a price undertaking is desirable whatever the preferences of the government are. Summarizing:

**Proposition 4** Assume that Condition 2 holds. Then, for every pair $(\lambda^*, m^*)$ there exists a $\tau(\lambda^*m^*)$ such that: (i) If $c < \tau(\lambda^*m^*)$, relative to free trade, a price undertaking results in a quality reversal so that the domestic firm becomes the quality leader in the international market. In this case, anti-dumping policy increases the domestic firm’s profits and social welfare. As a result, anti-dumping policy is justified for any preference of the government. (ii) If $c \geq \tau(\lambda^*m^*)$ a price undertaking by the domestic government results in an equilibrium where the foreign firm exits the
domestic market but holds its quality leadership. In this case, compared to free trade, the domestic firm’s profits increase while consumer surplus decreases. As a result, anti-dumping policy in the form of a price undertaking can only be rationalized on the basis of lobbying by the domestic firm.

Proposition 4 is illustrated in Figure 3. Panel (a) depicts home firm’s profits; the thicker curve represents free trade while the broken line a price undertaking. For high cost asymmetries, in particular for this example $c > 1.15$, the foreign firm produces high quality for its local market only, while the home firms sells a low-quality good both locally and abroad. For low cost asymmetries ($c < 1.15$) a quality reversal takes place and the home firm produces a good of higher quality than that of the foreign firm. When Condition 2 holds, the profits of the local firm are always greater under a PU than under free trade. An implication is that these higher profits could potentially finance the future adoption of new technologies and of cost-reducing investments. If this is the case, the quality leadership of the domestic firm could then be sustained in the long-run.

Panel (b) represents the usual measure of social welfare ($\beta_1 = \beta_2 = 1$) where the thicker curve represents welfare under free trade while the broken line under AD intervention. For low cost asymmetries, welfare is higher under a PU than under free trade. This is the result of the quality reversal we explained above. High cost asymmetries sustain an equilibrium where the home firm is a domestic monopolist and this decreases social welfare relative to free trade.

![Figure 3: Effects of a price undertaking (λ* = 4, m* = 3, θ = 100)](image-url)
4.2 Anti-dumping Duties

The other popular instrument of AD legislation involves the imposition of a duty that equalizes the price that consumers in different countries pay for the same good. Given that the price the foreign firm charges locally is \( p^*_1 \) and the export price is \( p^*_2 \), an anti-dumping policy in the form of a duty involves a commitment by the domestic government to levy an *ad valorem* duty \( t \) that equalizes the price that is paid by consumers in different countries:

\[
p^*_1(1 + t) = p^*_2 \tag{27}
\]

With an anti-dumping duty \( t \), demands faced by the foreign and domestic firms are, respectively:

\[
D_1(.) = p^*_1(1 + t) - p^*_1 - \frac{p^*_1}{\bar{q}} ; \quad D_2(.) = m^* \left( \frac{p^*_2 - p^*_1}{\lambda^* \bar{q}} - \frac{p^*_2}{\lambda^* \bar{q}} \right) \tag{28}
\]

\[
D_1^\star(.) = 1 - \frac{p^*_1(1 + t) - p^*_1}{\bar{q}} ; \quad D_2^\star(.) = m^* \left( 1 - \frac{p^*_2 - p^*_1}{\lambda^* \bar{q}} \right) \tag{29}
\]

Firms maximize profits given by

\[
\pi^* = p^*_1 D_1^\star + p^*_1(1 + t) D_2^\star - \frac{q^*}{2}
\]

\[
\pi = p_1 D_1 + p_2 D_2 - \frac{c q^2}{2}
\]

anticipating the anti-dumping duty \( t \) given by:

\[
t = \frac{p^*_2 - p^*_1}{p^*_1} \tag{30}
\]

Plugging (30) in (29) and taking the first order condition \( d\pi^*/dp_1^* \) yields:

\[
\frac{d\pi^*}{dp_1^*} = 1 - \frac{p^*_2 - p^*_1}{\bar{q}} = D_1^\star(.)
\]

The RHS of this first order condition is simply the domestic demand for high quality. As long as \( D_1^\star(.) \) is greater than zero, the profits of the foreign firm are increasing in its export price \( p^*_1 \). The optimal pricing behavior of the foreign firm is then to set \( p^*_1 = p^*_2 \), which implies that tariff revenues are zero, as has been shown earlier, for example in Feenstra (2003). In summary we obtain the following equivalence result.

**Proposition 5** *An anti-dumping duty imposed by the domestic government results in an equalization of international prices. Hence, anti-dumping duties and price undertakings are equivalent in our model.*
5 Conclusions

We have presented a model of international trade where two firms located in two different countries produce quality-differentiated goods for their local markets and, eventually, for exports. An important feature of our model is the existence of size and income differences across, and in firms’ R&D cost structures. We have shown that, under free trade, the unique (risk-dominant) Nash equilibrium involves intra-industry trade; in addition, the most efficient firm, the foreign, is the quality leader in the international market. Since consumers in different countries differ in their concern for quality, in equilibrium, unilateral dumping by the foreign firm into the domestic country occurs. In this context we have looked for a rationale for AD law.

When countries differ substantially and firms cost asymmetries are low, then a PU leads to a quality reversal in the international market. This results not only in much greater profits for the home firm but also in greater social welfare, which gives the domestic government incentives to enact anti-dumping law. By contrast, if firms’ cost asymmetries are large instead, we have found that a PU results in the exit of the foreign firm from the export market, which confers the local firm a monopoly position in the domestic market. This provides a rationale for the introduction of AD policy only based on home firm lobbying. Finally, we have found no rationale for price undertakings if countries are of similar size, or if income differences are small, since they lead to lower profits for the local firm and lower social welfare. When AD duties are considered we have derived an equivalence result between AD duties and PU.

6 Appendix

Proof of Lemma 1: The proof follows from simple profit-maximization.  ■

Proof of Lemma 2: Let the home firm produce goods of quality $q_1$ and $q_2$ and the foreign firm a good of quality $q_2^*$. There are 6 possible quality configurations. Consider that $q_2^* > q_2 > q_1$. We note that given optimal pricing of the home firm $p_1 = \bar{\theta}q_1/2$, this firm profits are monotonically increasing in $q_1$; as a result, the firm would gain by deviating and increasing $q_1$. The same argument applies if $q_2 > q_1 > q_2^*$ and $q_2 > q_2^* > q_1$. This leaves us with three more cases to consider. Consider now $q_1 > q_2 > q_2^*$. In this case, given qualities, firms choose their prices to maximize profits given

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as follows:

\[
\pi = m^* \left( \frac{1}{\lambda^*} \frac{p_2 - p_2^*}{\theta(q_2 - q_2^*)} \right) p_2 + \left( 1 - \frac{p_1}{\theta q_1} \right) p_1 - \frac{c q_1^2}{2}
\]

\[
\pi^* = m^* \left( \frac{p_2 - p_2^*}{\lambda^*} \frac{1}{\theta(q_2 - q_2^*)} - \frac{p_2^*}{\lambda^*} \theta q_2^* \right) p_2 - \frac{q_2^2}{2}
\]

We note that the problem of the home firm is separable in \( p_1 \) and \( p_2 \). Taking the first order conditions \( \partial \pi / \partial p_1 = 0 \), \( \partial \pi / \partial p_2 = 0 \) and \( \partial \pi^* / \partial p_2^* = 0 \) and solving the reaction functions in prices yields:

\[
p_1 = \frac{\bar{\theta} q_1}{2}; \quad p_2 = \frac{2 \lambda^* \bar{\theta} q_2^* (q_2 - q_2^*)}{4q_2 - q_2^*}; \quad p_2^* = \frac{\lambda^* \bar{\theta} q_2^* (q_2 - q_2^*)}{4q_2 - q_2^*}
\]

Anticipating optimal pricing, it is easy to see now that the profits of the home firm are monotonically increasing in \( q_2 \). Indeed profits are given by

\[
\pi = \frac{4 m^* \lambda^* \bar{\theta} q_2^* (q_2 - q_2^*)}{(4q_2 - q_2^*)^2} + \frac{\bar{\theta} q_1}{4} - \frac{c q_1^2}{2}
\]

and

\[
\frac{\partial \pi}{\partial q_2} = \frac{4 m^* \lambda^* q_2^* \bar{\theta} (4q_2^2 - 3q_2 q_2^* + 2q_2^2)}{(4q_2 - q_2^*)^3} > 0 \text{ since } q_2 > q_2^*.
\]

As a result, the home firm would gain by deviating and increasing \( q_2 \).

We now turn to consider \( q_2^* > q_1 > q_2 \). In this case we note that the foreign firm would obtain zero profits and would thus gain by deviating and choosing \( q_2^* < q_2 \). To see this note that firms profits are as follows:

\[
\pi = m^* \left( \frac{p_2^* - p_2}{\lambda^*} \frac{1}{\theta(q_2^* - q_2)} \right) p_2 + \left( 1 - \frac{p_1}{\theta q_1} \right) p_1 - \frac{c q_1^2}{2}
\]

\[
\pi^* = m^* \left( 1 - \frac{p_2^* - p_2}{\lambda^*} \frac{1}{\theta(q_2^* - q_2)} \right) p_2 - \frac{q_2^2}{2}
\]

Taking the first order conditions \( \partial \pi / \partial p_1 = 0 \), \( \partial \pi / \partial p_2 = 0 \) and \( \partial \pi^* / \partial p_2^* = 0 \) and solving the reaction functions in prices yields:

\[
p_1 = \frac{\bar{\theta} q_1}{2}; \quad p_2 = \frac{\lambda^* \bar{\theta} q_2^* (q_2^* - q_2)}{4q_2^* - q_2}; \quad p_2^* = \frac{2 \lambda^* \bar{\theta} q_2^* (q_2^* - q_2)}{4q_2^* - q_2}
\]

Substituting these prices into the profits functions yields:

\[
\pi = \frac{m^* \lambda^* \bar{\theta} q_2^* (q_2^* - q_2)}{(4q_2^* - q_2)^2} + \frac{\bar{\theta} q_1}{4} - \frac{c q_1^2}{2}
\]

\[
\pi^* = \frac{m^* \lambda^* \bar{\theta} q_2^{*2} (q_2^* - q_2)}{(4q_2^* - q_2)^2} - \frac{q_2^*}{2}
\]
The first order conditions with respect to quality $\partial \pi / \partial q_1 = 0$, $\partial \pi / \partial q_2 = 0$ and $\partial \pi^* / \partial q_2^* = 0$ can be written as:

$$
\frac{\partial \pi}{\partial q_1} = \frac{\theta}{4} - cq_1 = 0
$$

$$
\frac{\partial \pi}{\partial q_2} = \frac{\lambda^* \theta q_2^* (4q_2^* - 7q_2)}{(4q_2^* - q_2)^3} = 0
$$

$$
\frac{\partial \pi^*}{\partial q_2^*} = \frac{4\lambda^* \theta q_2^* (4q_2^* - 3q_2^2 + 2q_2^2)}{(4q_2^* - q_2)^3} - q_2^* = 0.
$$

From $\partial \pi / \partial q_2 = 0$, it follows that $q_2 = 4q_2^*/7$. Substituting this into the equation $\partial \pi^* / \partial q_2^* = 0$ and solving for $q_2^*$ yields $q_2^* = 7\lambda^* \theta/4$ and thus $q_2 = \lambda^* \theta/6$. Substituting $q_2$ and $q_2^*$ into the expression for profits yields $\pi^* = 0$.

We now prove that the foreign firm would gain by deviating and producing a good of quality $q_2^* < q_2$ (downward leapfrogging). From the arguments above, equilibrium pricing would be as follows:

$$
p_1 = \frac{\bar{q}_1}{2}; p_2 = \frac{2\lambda^* \theta q_2(q_2 - q_2^*)}{4q_2 - q_2^*}; p_2^* = \frac{\lambda^* \theta q_2^*(q_2 - q_2^*)}{4q_2 - q_2^*}
$$

Given optimal pricing and the quality choices of the home firm, the foreign firm profits can be written as follows:

$$
\pi^d = \frac{\lambda^* \theta \lambda^* \theta^2 q_2^* (\lambda^* \theta^2 - q_2^2)}{(4\lambda^* \theta - 3q_2^2)^2} - q_2^* \lambda^* \theta^2.
$$

The first order condition is

$$
\frac{\lambda^* \theta}{4(\lambda^* \theta - 3q_2^2)^2} - q_2^* = 0
$$

From this equation we can isolate $q_2^*$ and plug it into the expression for $\pi^d$. This yields:

$$
\pi^d = \frac{\lambda^* \theta^2 q_2^* (36q_2^2 - \lambda^* \theta(9q_2^2 - 2\lambda^* \theta))}{8(2\lambda^* \theta - 3q_2^2)^3} > 0 \text{ since } q_2^* < \frac{\lambda^* \theta}{6}.
$$

We are left with the case $q_1 > q_2^* > q_2$. However, it is easy to see that this case is similar to the previous one. Note that firms profits would be given by the expression (31) and thus the arguments above also hold here. This completes the proof that the quality that the home firm offers abroad must be equal to the quality it sells domestically, i.e., $q_1 = q_2 = q$.

Building in this remark, we note that two quality configurations can be part of an equilibrium:

(i) $q > q_2^*$ and (ii) $q_2^* > q$. Consider first $q > q_2^*$. The firms would choose prices to maximize:

$$
\pi = m^* \left(1 - \frac{p_2 - p_2^*}{\lambda^* \theta(q - q_2^*)}\right) p_2 + \left(1 - \frac{p_1}{\lambda^* \theta}\right) p_1 - \frac{q^2}{2}
$$

$$
\pi^* = m^* \left(\frac{p_2 - p_2^*}{\lambda^* \theta(q - q_2^*)} - \frac{p_2^*}{\lambda^* \theta q_2^*}\right) p_2^* - \frac{q_2^2}{2}
$$
The solution to this problem is

\[ p_1 = \frac{\bar{q}_q}{2}; \quad p_2 = \frac{2\lambda^* \bar{q}_q (q - q_2^*)}{4q - q_2^*}; \quad p_2^* = \frac{\lambda^* \bar{q}_q^2 (q - q_2^*)}{4q - q_2^*} \]  

(33)

Anticipating equilibrium prices, qualities should be chosen to maximize:

\[ \pi = \frac{4m^* \lambda^* \bar{q}_q^2 (q - q_2^*)}{(4q - q_2^*)^2} + \frac{\bar{q}_q}{4} - \frac{cq^2}{2} \]

\[ \pi^* = \frac{4m^* \lambda^* \bar{q}_q^* (q - q_2^*)}{(4q - q_2^*)^2} - \frac{q_2^*}{2} \]

First order conditions are

\[ \frac{\partial \pi}{\partial q} = \frac{4m^* \lambda^* \bar{q}_q (4q^2 - 3qq_2^* + 2q_2^2)}{(4q - q_2^*)^3} + \frac{\bar{q}_q}{4} - cq = 0 \]  

(34)

\[ \frac{\partial \pi^*}{\partial q_2^*} = \frac{m^* \lambda^* \bar{q}_q^2 (4q - 7q_2^*)}{(4q - q_2^*)^3} - q_2^* = 0. \]  

(35)

Unfortunately, these equations cannot be solve analytically for \((q, q_2^*)\) and we are therefore led to use numerical methods in what follows. Numerical simulations show that the solution to the system of equations (34)-(35) with accompanying prices given in (33) is indeed an equilibrium for all parameters: for this we have checked that firms profits are positive and that a single firm does not have incentives to leapfrog the rival’s choice of quality.

We now consider the case \(q < q_2^*\). In this case prices and profits are calculated similarly. Again the first order conditions in quality cannot be solved explicitly and thus we use numerical analysis. Our simulations show that there exists parameter combinations for which this assignment in qualities is also an equilibrium. We see that when \(\lambda^* m^*\) is large, this is always equilibrium; otherwise when \(\lambda^* m^*\) is small we need the cost of the home firm to be relatively large. These observations can be seen in Figure 4, where we have represented the profits firms obtain for these two cases. On the horizontal axis we have cost asymmetries; the first panel captures a situation of where consumer preferences are similar across countries while the second panel shows the profits levels when consumers in the foreign country are willing to pay on average 50% more for one unit of quality.

Since we have sets of parameters for which the two assignments in qualities can be equilibria, we are confronted with the question of which equilibrium is more reasonable. Using the Harsany-Selten risk-dominance notion of refined equilibrium as a selection criterion yields clear-cut results: for every level of \(\lambda^* m^*\), there exists a level of cost \(\bar{c}(\lambda^* m^*)\) such that for all \(c < \bar{c}(\lambda^* m^*)\) the unique refined equilibrium is such that the home firm produces a good of higher quality than
that of the foreign firm; otherwise the home firm is too inefficient and produces low quality. The criterion is represented in Figure X. If we call the equilibrium where the home firm produces high quality “equilibrium 1” and the alternative equilibrium “equilibrium 2”, the left figure represents the quantities $G_{ij}$, which denote the gains to firm $i$ from predicting correctly that firm $-i$ will select equilibrium $j$, $i = j, 1, 2$. The right panel shows the criterion: equilibrium 1 is selected whenever $G_{11}G_{21} > G_{12}G_{22}$ and from the graph it follows that the home firm will produce high quality provided that cost differences are not large.

**Figure 5: Harsany-Selten criterion**

**Proof of Lemma 3:** The proof goes along the lines of that of Lemma 2 and we skip it to save on space. The only difference is that the foreign firm is always the high quality producer (in the
Harsany-Selten refined equilibrium). The reason is that the foreign firm is more efficient than the home firm and in addition it serves two markets in this case.

**Proof of Lemma 4:** We need to rule out any other quality configuration. We note that there are 4! quality configurations but the majority of them can be ruled out easily. First, note that any quality configuration where \( q_1 > q_2 > q_2^* \) cannot be an equilibrium because the home firm would deviate by increasing its quality \( q_2 \). This rules out 4 possible configurations. Likewise, any quality configuration such that \( q_2 > q_1 > q_1^* \) cannot be equilibrium either since the home firm would gain by increasing its quality \( q_1 \). This rules out 4 quality configurations more. The same reasoning can be applied to the foreign firm. Quality configurations such that \( q_2^* > q_1^* > q_1 \) cannot be ruled out since the foreign firm would gain by increasing \( q_1^* \). Likewise, cases where \( q_1^* > q_2^* > q_2 \) cannot be part of an equilibrium because the foreign firm would gain by deviating and increasing \( q_2^* \). These two arguments together rule out 8 quality configurations more. Second, suppose that \( q_2^* > q_2 > q_1^* > q_1 \); again, the foreign firm would gain by increasing its quality \( q_1^* \), which rules out this case. In the same vein, if \( q_1^* > q_1 > q_2^* > q_2 \), the foreign firm would gain by increasing \( q_2^* \). Analogously, if \( q_1 > q_1^* > q_2 > q_2^* \), then the home firm would deviate by increasing \( q_2 \). The home firm would also deviate if \( q_2 > q_2^* > q_1 > q_1^* \), in this case by increasing \( q_1 \). So we are left with only four possible quality configurations which can be part of an equilibrium. We turn to examine these configurations. Consider first the case where \( q_1 > q_2^* > q_1^* > q_2 \). We note that the home firm is a quality leader in the domestic market but sells a low quality good in the foreign market. Using the prices we have derived above in Lemmas 1-3, the profits of the firms can be written as follows:

\[
\pi = \frac{4\theta q_1^2 (q_1 - q_1^*)}{(4q_1 - q_1^*)^2} + \frac{\lambda^* m^* \theta q_2^* q_2 (q_2^* - q_2)}{(4q_2^* - q_2)^2} - \frac{c q_2^2}{2}
\]

\[
\pi^* = \frac{4\lambda^* m^* \theta q_2^*^2 (q_2^* - q_2)}{(4q_2^* - q_2)^2} + \frac{\theta q_1^* (q_1 - q_1^*)}{(4q_1 - q_1^*)^2} - \frac{q_2^*^2}{2}
\]

The home firm chooses \((q_1, q_2)\) to maximize \(\pi\) and the foreign firm selects \((q_1^*, q_2^*)\) to maximize \(\pi^*\).

We note that the firms problem are separable in qualities. Taking the first order conditions and solving the system of four equations yields:

\[
q_1 = \frac{7\theta}{24c}; q_2 = \frac{\lambda^* m^* \theta}{6c}; q_1^* = \frac{\theta}{6c}; q_2^* = \frac{7\lambda^* m^* \theta}{24}
\]

Equilibrium profits would be

\[
\pi = \frac{7\lambda^* m^* \theta^2}{1152}; \pi^* = \frac{7\lambda^* m^* \theta^2}{1152c}.
\]
We now check that the home firm would gain by deviating and producing a higher quality abroad, in particular, we propose the following deviation \( q_1 = q_2 \) chosen to maximize profits. Deviating profits are

\[
\pi^d = \frac{4\theta q_1^2 (1 - q_1^*)}{(4q_1 - q_1^*)^2} + \frac{4\lambda^* m^* \theta q_1^2 (q_1 - q_2^*)}{(4q_1 - q_2^*)^2} - \frac{c q_1^2}{2}
\]

Using the fact that \( q_1^* = \frac{a}{6c} \) and \( q_2^* = \frac{7\lambda^* m^* \theta}{24} \), the first order condition can be written as:

\[
0 = \frac{3456c^3 q_1^2 \theta}{(24cq_1 - \theta)^3} + \frac{720c^2 q_1 \theta^2}{(24cq_1 - \theta)^3} - c \left( 1 + \frac{48\theta^2}{(24cq_1 - \theta)^2} \right) + \frac{192\lambda^* m^* \theta (1152q_1^2 - 252\lambda^* m^* \theta q_1 + 49\lambda^* m^* \theta^2)}{(96q_1 - 7\lambda^* m^* \theta)^3}
\]

Unfortunately, there is no analytical solution for this equation. We have proceeded numerically and checked that the deviating firm always obtains higher profits than equilibrium profits. We note that the remaining assignment in qualities have the same properties, that is, a firm is leader in quality in a market but is a low quality seller in the other market (the other firm vice-versa) and therefore similar arguments rule out these cases.

It remains to check that the Harsany-Selten criterion selects always the foreign firm as a high quality producer. The proof is also based on numerical simulations and is similar to the proof of Lemma 1; we skip it to save on space. ■

References


