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Generational Accounting, Solidarity and Pension Losses*

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Abstract

The creeping stock market collapse eroded the wealth of funded pension systems. This led to political tensions between generations due to the fuzzy definition of property rights on the pension funds wealth. We argue that this problem can best be resolved by the introduction of *generational accounts*. Using modern portfolio and consumption planning theory we show that the younger generations should have the higher equity exposure due to their human capital. Capital losses should be distributed smoothly over lifetime consumption. When stock markets are depressed equity should be bought, savings and consumption should be scaled down equiproportionally, and retirement should be postponed. Portfolio investment restrictions are quite costly.

1 Introduction

Many OECD country's pension systems are currently in a state of crisis. The creeping stock market collapse has eroded the wealth of funded pension systems. The apparent opulence of the funded pensions due to the stock market increases in the nineties lead to demands from politicians and CEO's to cream skim the pension funds. As a result, many funds gave away pension holidays to deliver for some of the company's stakeholders. As a consequence, many funds

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are still trying to cope with the subsequent stock market decline. Some funds currently no longer guarantee the defined benefits. Many funds have temporarily stopped to index the pensions and premia have shot up. Who should pay for the losses due to the stock market decline: Current retirees, workers or unborn generations?

We analyze the situation using modern portfolio theory as presented in Bodie, Merton and Samuelson (1992). This literature shows that the younger generations should have a higher equity exposure than the older generations due to the fact that the younger have a higher stock of human capital. Our focus is different, however, and looks at the implications for pension systems. One finds that capital market losses should be distributed smoothly over lifetime consumption and saving, with similar implications for longevity shocks. When stock markets are depressed equity should be bought, savings and consumption should be scaled down equiproportionally, and retirement should be postponed.

To enable pension funds and society to achieve these optimal lifetime consumption-saving patterns, we argue that two elements are crucial. One is the more obvious requirement that the pension system should operate on a funded basis. The other less obvious requirement, witness the prevalence of pension funds with diffuse stakeholder influence, is to clearly define the property rights of each generation on the pension fund's wealth, so that political decision making on the distribution of losses can be avoided. This paper makes a plea for the application of a *system of generational accounts* for pension funds, similar to Auerbach and Kotlikof's (1987) idea for the public sector. All premiums paid by one generation should be administered in a separate account. Also, the fund's investments must be separately administered, so that each generation's property rights on the investment returns are clearly defined.

Keeping separate track of each generation's investment is crucial, as the risk profile of optimal investment policies differs between generations. This ensures optimal intertemporal smoothing behavior, and introduces an important automatic macro economic stabilizer. That is, for a capital loss, the percentage point increase in the pension premium should be equal to the percentage decrease in expected pensions after the date of retirement, or equivalently, all planned future consumption is reduced by the same percentage. Another important adjustment and smoothing device is to delay the retirement age. Other systems like the defined benefit system or pay-as you-go system inefficiently put the whole burden of adjustment on the current working generation, thereby stimulating the boom bust cycle. We calculate that the optimal decrease in consumption due to the fall in stock prices in 2001-2002 the period is about 11%. By definition, generational accounts exploit intra-generational solidarity, just like life insurance policies do in the market place and do not bank on infeasible inter-generational solidarity. Nevertheless it is of interest to discuss a mild form of intergenerational solidarity where society invests in stocks on behalf of future generations. It is shown to be indeed optimal that young generations are exposed to stock market uncertainty even before their entry on the labor market. We discuss the adverse selection problems that come with this type of policy. We also discuss a simple response to resolve capital losses without need of generational solidarity, which is to allow for a flexible retirement age.

Pension funds require public supervision regarding their operations. However, supervision should not impose restrictions on the funds' investment policies, such as that all equity investment should be with EU companies, as was suggested by several EU member states, or that pension funds should not be allowed to invest in stocks at all, popular advice in depressed equity markets. We calculate the cost of the latter restriction, that a fund would only be allowed to hold an all bond portfolio. We calculate that real labor income should go up permanently by at least 12% to offset the negative effect on expected utility.

The following section lays out the assumptions of the model that we apply throughout this paper. In the subsequent sections, we work through all topics that are raised in this introduction.

2 The model of portfolio selection

To derive portfolio rules for funded systems we build on the literature which started with Merton's (1969) and Samuelson's (1969) analyses of lifetime portfolio selection for an individual. More recently this literature has been extended in labor economics by investigating the expected benefits from human capital and by modeling the uncertainty regarding future wage income, see Bodie, Merton and Samuelson (1992) and more recently Menoncin (2003). We take into account the anticipated returns on human capital on the optimal composition of portfolio. For our purposes it suffices to take labor income for certain and we assume labor supply to be fixed. Our focus and contribution is with regard to the implications one can derive from this framework for funded pension plans.

Let t denote the age of the individual. Individuals work from $t = 0$ till a fixed retirement age $t = T$ and live on a pension from $t = T$ till their determinate death at $t = D$. For simplicity, we ignore any other uncertainty than the return on investment. Furthermore, we assume that labor income while working is constant, and we normalize it to unity. Individuals have a Constant Relative Risk Aversion (CRRA) utility function. Since the labor supply and retirement decision are exogenous in our analysis, we neglect any disutility of working. Hence, consumption is the only argument of the utility function:

$$\begin{aligned} \overline{U}(t) &\equiv \int_t^D e^{-\beta(s-t)} u[C(s)]\, ds \qquad (1) \\ u(C) &\equiv \frac{1}{1-\theta} C^{1-\theta}, \end{aligned}$$

where $\overline{U}(t)$ is lifetime utility at age t conditional on the path of consumption $C(s)$ during the rest of the individual's life; $u(C)$ is the individual's instantaneous utility, where θ is the coefficient of relative risk aversion; β is the subjective discount rate.

In order to finance her consumption while retired, an individual saves labor income. She has two options for investment: A risk free bond with a return ρ, or risky equity with price $P(t)$. Suppose the $\log P(t)$ evolves according to a random walk with drift μ and normally distributed innovations with variance $\sigma^2 dt$. Thus the price of the risky asset follows a geometric Brownian motion:

$$dP(t) = \bar{\mu}P(t)dt + \sigma P(t)dW(t),$$

where $\bar{\mu} \equiv \mu + \frac{1}{2}\sigma^2$ is the expected return on the equity (the factor $\sigma^2/2$ is a consequence of Ito's lemma or alternatively follows from the mean of a lognormal distribution), and $W(t)$ is a Wiener process. We assume that there is a trade off between return and risk: $\bar{\mu} > \rho$.

Let $S(t)$ be the wealth of the individual and let $f(t)$ be the fraction of her savings that the individual is willing to invest on the stock market. Then, the law of motion of savings for a working individual reads:

$$\begin{aligned} dS(t) &= [f(t)\bar{\mu} + \{1 - f(t)\}\rho] S(t) dt + [Y(t) - C(t)]dt \\ &\quad + \sigma f(t) S(t) dW(t), \end{aligned} \tag{2}$$

where $Y(t)$ denotes labor income; $Y(t) = 1$ while working until the time of retirement T, while $Y(t) = 0$ from retirement T until death at time D. The first term on right hand side is the expected return on investment, the second term is the change in wealth due to saving or dissaving, and the final term reflects the volatility of stock prices. The budget constraint of the individual can be written as an initial and a terminal condition on $S(t)$:

$$S(0) = 0, \ S(D) \geq 0.$$

Individuals start working without any wealth and are not allowed to die in debt.

From the literature we calibrate the parameters of the model. We choose an active working life T of forty years and fifteen years of retirement, $D - T = 15$. The coefficient of relative risk aversion θ plays a crucial role in our analysis. Experimental economics, see e.g. evidence in Newbery and Stiglitz (1981, ch.7), and real life experiments such as reported in Beetsma and Schotman (2001) suggest values between one (logarithmic) and ten. Indirect evidence on the attitudes towards risk from the equity premium literature such as presented in Campbell's review (1999) yield less precise estimates.[1] The real business cycle literature typically experiments with the values in the range between one and ten, see e.g. Cooley and Prescott (1995), Rouwenhorst (1995) and Backus et al. (1995). We benchmark our calculations using $\theta = 10$. The value of the rate of time preference β is set at two percent per year, which is a fair compromise between Cooley and Prescott (1995), Rouwenhorst (1995) and Backus et al. (1995). From Campbell's review and on basis of the revealing study by Dimson, Marsh, and Staunton (2002) providing the long run statistical properties of different countries' asset market performances, we set the return on riskless bonds ρ equal to 2%, the drift in equity μ is calibrated to 6% and the standard deviation of equity returns σ is taken equal to 20% per year, so that equity has an expected return of 8%.

[1] For example, Table 5 from Campbell for eleven countries implies estimates that range from minus 295 to plus 7215 (with mean value 1010, standard error of the mean 662, and a standard deviation of 2198).

Table 1: Parameter Calibration

Population Parameters		
T	40	retirement age
D	55	time of death
Preference Parameters		
θ	10	relative risk aversion
β	0.02	time preference
Asset Return Parameters		
ρ	0.02	bond return
μ	0.06	drift
σ	0.2	standard deviation
$\bar{\mu}$	0.08	mean equity return

3 A generation's optimal policy

The derivation of a generation's savings and portfolio choice is along the lines of Bodie et al. (1992) using the continuous time dynamic programming technique. In Appendix I a detailed derivation is provided. Here we outline the results. A generation chooses $C(t)$ and $f(t)$ to maximize the expected utility of its members $E\left[\overline{U}(t)\right]$ subject to the dynamic budget constraint (2). Let $U(t, S)$ denote the optimal expected utility as a function of the state variables of the system, t and S. The first order conditions for a maximum entail:

$$\begin{aligned} C(t) &= U_S^{-\frac{1}{\theta}} \\ f(t) &= -\frac{\bar{\mu}-\rho}{\sigma^2}\frac{U_S}{SU_{SS}}, \end{aligned} \tag{3}$$

where we suppress the arguments of U for brevity. In the Appendix, we show that these first order conditions imply:

$$f(t) = \frac{\bar{\mu}-\rho}{\theta\sigma^2}\frac{S(t)+\frac{1}{\rho}\left(1-e^{-\rho\left(T-\hat{t}\right)}\right)}{S(t)}, \tag{4}$$

where $\hat{t} \equiv \min(T, t)$. The second term in the numerator $\rho^{-1}\left(1-e^{-\rho\left(T-\hat{t}\right)}\right)$, is a generation's net discounted value of human capital. Hence, the denominator is the sum of its financial and human capital, or equivalently, the net discounted value of all its expected future consumption. The final factor in equation (4) is therefore the inverse of the ratio of financial to total capital of a generation. When retired, $T \geq t \geq D$, the human capital is fully depleted and all future consumption of that generation has to be financed from its is financial capital. Thus for all $T \geq t \geq D$, the proportion of investment in the risky asset follows the **static** mean variance portfolio choice dictum:

$$f(T) = \frac{\bar{\mu}-\rho}{\theta\sigma^2}.$$

It also is the celebrated rule from Merton (1969) and Samuelson (1969) for dynamic intertemporal asset allocation. The share is increasing in the risk premium $\bar{\mu} - \rho$ and decreasing in the degree of riskiness of the risky asset σ^2 and the coefficient of relative risk aversion θ. During the working life $t < T$, however, when the net discounted value of labor income is still positive, the share of investment in the risky asset is higher than during retirement. It is proportional to the inverse of the financial wealth relative to the total wealth of a generation. This implies that $f(t)$ can even be larger than unity. Hence, $f(t)$ is not really a share. The higher investment in the risky asset at young ages fits the layman's intuition. However, the reason for this investment rule is not the standard argument that young people face a longer time horizon to make up for losses in the value of their investment, compare Merton (1969) and Samuelson (1969) for the refutation of that argument. The real reason is that when a generation enters the labor market, its financial capital is only a small part of its total wealth. The largest part is human capital. Thus going short in bonds while young is not as risky as it appears, since this is counterbalanced by the stock of human capital. The stock of human capital works like a buffer. Since human capital is largest when young, a young generation can take on more risk than an older generation. Hence, the young invest a greater share of their financial capital in equity. The optimal investment policy is to distribute risk exposure "evenly" over the lifetime of a generation. For example, consider two investment policies: investing 50 Euro in stocks and 50 Euro in bonds during two years or investing 100 Euro in stocks in the first year and 100 Euro in bonds during the second year. Though the expected return of both policies is equal (approximately, we ignore accumulated interests for simplicity), the riskiness of the second policy is larger. An "even" distribution of risk exposure over the lifetime requires the amount of money invested on the stock market to be a fixed fraction of discounted wealth, or equivalently, the discounted value of expected future consumption. Hence, since the initial stock of savings is low, the "share" of savings invested on the stock market should be high. Using the parameter values calibrated above, one finds that for the pensioners $f(T) = 0.15$. The sensitivity with respect to the parameter choices is easily investigated, e.g. at $\theta = 2$ one gets $f(T) = 0.75$.

4 High leverage for young generations

Multiplication of equation (4) by $S(t)$ yields an expression for the amount invested in equity, $I(t)$:

$$I(t) = \frac{\bar{\mu} - \rho}{\theta\sigma^2}\left[S(t) + \frac{1}{\rho}\left(1 - e^{-\rho\left(T-\hat{t}\right)}\right)\right]. \tag{5}$$

The value of investment at the beginning of the working career, $I(0)$, can be calculated easily, using $S(0) = 0$. Taking the benchmark parameter values gives $I(0) = 4.13$, which means that at beginning of one's career a generation should borrow four times one's yearly wage and invest this in the equity markets.

Similar numbers can be found in Bodie, Merton, and Samuelson (1992). The investment in equity has to be financed by simultaneous borrowing of the risk free asset. This may appear quite high since it is a risky strategy not unlike a highly leveraged hedge fund. In practice, young persons are credit constrained, so this strategy may be infeasible[2]. Nevertheless, one has only to incorporate loans for college and the mortgages to realize that the leverage (in terms of financial wealth) of young and middle aged is often quite considerable. In particular, the investment in housing paid for by mortgages, contributes to efficient risk taking at younger ages, since house and stock prices are highly correlated. In fact the young effectively purchase a call option on the macro economic performance of the economy with their human capital as collateral.

5 Intergenerational solidarity

A frequently raised objection against generational accounts is that these rule out intergenerational solidarity. Judging by the current political and generational stalemate in countries like Italy and Germany one can wonder whether this solidarity is feasible at all. The problem was formally analyzed within an overlapping generations economy by Hendricks, Judd and Kovenock (1980), who showed that the core in such models is empty, implying that there is no political support for intergenerational transfers and taxes. Nevertheless, it is useful to analyze the physical constraint on intergenerational solidarity.

Intergenerational solidarity is equivalent to spreading the stock market risk over a longer time period than just a generation's lifetime. The longer the time period over which the risk can be spread, the better it can be diversified, and hence, the lower is standard deviation. A fundamental insight here is that the opportunities for this diversification are asymmetric. Including uncertainty on future stock prices in a generation's portfolio is theoretically impossible. The information on the realization of P_t becomes available only after the death of the generation. Hence, it can not adjust its consumption to these realizations.

Contrary to future shocks, including past shocks to stock prices is feasible, but only under strict conditions. For this type of solidarity to work, a generation is supposed to have invested in the risky asset before the start of its career. Instead of a deterministic starting value of its financial capital, $S(0) = 0$, a generation borrows to buy stocks at some date $t < 0$ (at birth, say). Hence, $S(0)$ is a random variable with a positive expectation due to risk premium on the investment in equity, $\bar{\mu} - \rho > 0$. Though the expectation is positive, the starting value of $S(0)$ will be negative for some realizations. The optimal investment policy maximizes the expected value of the starting utility, $\mathrm{E}[U(0, S(0))]$, where we take expectations over $S(0)$. This system exposes pension funds to

[2]Credit constraints may reflect the risk of a fall in stock prices leaving the young with negative financial wealth. The collateral value of their human capital may then become distressed since the knowledge that one has to service a high debt for the rest of one's life, creates severe moral hazard problems regarding work effort, so that no bank would be prepared to finance this gearing with no other collateral.

even stronger adverse selection problems than the high leverage for young generations considered in the previous section. If stock prices have risen during the investment period prior to labor market entry, people will love to participate in the pension fund. However, if stock prices have come down, nobody will voluntary enter the fund upon entering the labor market. The problem that the young can walk away, is analogous to the hold up problem in Industrial Organization. Only government regulation to impose mandatory participation can solve this problem, if it finds support for such a rule.

Suppose participation can be enforced. Mathematically, the problem of entry upon birth into the pension scheme is not very different from the standard problem treated in the previous section. One only has to include an additional term for the time span between the date of birth, $t = -B$, and labor market entry, $t = 0$. During this period, a generation has no labor income: $Y(t) = 0$. For a clean comparison with the case discussed till sofar, we ignore consumption before labor market entry. The initial condition $S(0) = 0$ is now replaced by the condition $S(-B) = 0$. For the rest, the problem remains exactly the same. By a similar derivation as before, a generation's optimal investment in the risky asset satisfies for $-B \leq t \leq 0$:

$$I(t) = \frac{\bar{\mu} - \rho}{\theta\sigma^2}\left[S(t) + \frac{1}{\rho}\left(1 - e^{-\rho T}\right)e^{\rho t}\right],$$

while equation (5) remains valid for $I(t)$ after the entry on the labor market at $t = 0$. For $S(t) = 0$, the investment in equity is lower at $t = -B$ than at $t = 0$, since the net discounted value of labor income is smaller for $t < 0$. However, there is still a strong incentive to invest in stocks, showing the value of diversifying risk to the period before labor market entry.

It may seem an unrealistic idea to let a generation invest in stocks even before its entry on the labor market. It is therefore important to realize that in the current systems without generational accounts many pension funds do actually apply this type of intergenerational solidarity, by levying equal premiums for all age groups. The fall in stock prices has forced many pension funds to raise these premia, to compensate for losses in their investment portfolio. Hence, a new generation entering the labor market right now has to carry part of the burden of stock market misfortune from past generations.

To break the deadlock between generations, there is another alternative available which is sort of a go between investments ante a career and spreading the risk to afterlife. This is the flexible retirement age. With some fortune, a generation can retire early. But in case of bad luck, an extension of the date of retirement allows a generation to internalize the hedge against the downside risk. Consider the effects of extending one's career. It is immediate that the share of equity investment for the retired is unaffected. For the active generations an extension of the retirement age increases the risk taking since from (4)

$$\partial f(t) / \partial T > 0, \text{ for } t < T.$$

This effect is similar to the case of labor flexibility treated more formally in

Bodie et al. (1992), by endogenizing the retirement decision. Moreover, from (16) in the Appendix one sees that consumption increases as well

$$\partial C(t) / \partial T > 0, \text{ for } t < T.$$

Thus as a remedy against equity losses an extension of T not only may lessen the severity of capital losses, it also provides for spreading risk over a longer period and has macro stabilizing effects, since it boosts current spending. Currently, many European governments are trying to push this idea as one of the methods to alleviate the pension problems. Interestingly, the absence of a retirement age in the US provides this cushion as an automatic stabilizer.

6 The welfare loss of ruling out investment in stocks

Funded pension plans in The Netherlands and the UK do not operate under direct portfolio restrictions. Historically this was quite different. For example, the fund for teachers in Holland is still suffering from the obligation to swallow all loans placed by the government. France and Spain have recently tried to limit equity investments of any EU based pension fund to EU stocks, thereby foregoing the benefits from investment in high growth Asian countries that do not suffer from the inverted population pyramid. We investigate the social cost of such type of regulation. Suppose that some legal authority rules out any investment of pension funds in equity. This rule would restrict the set of policies available to a generation, and therefore reduce the maximum achievable expected utility of its members. By how much do we have to increase its labor income to offset this negative effect? In the Appendix, we derive the following approximate expression for the required relative wage increase:

$$\frac{1}{4}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 \frac{D}{\theta} + O\left(D^2\right). \tag{6}$$

For the benchmark parameter values the permanent increase in labor income needed to offset the loss due to the obligation to invest all wealth in the risk free asset is 12% (respectively 10% if one uses the exact expression, see equation (17)). This boils down to almost five years extra work. If the coefficient of relative risk aversion θ is lowered to two, the welfare loss becomes a staggering 61% (both for the approximation and the exact formula).

These numbers account for only part of the welfare loss due to the obligation to invest all wealth in the risk free asset. It captures the welfare loss due to inappropriate restrictions on the supply of risky funds, keeping the risk premium constant. However, the lower supply of risk bearing funds raises the risk premium (macro shortage of risk bearing capital), and thereby the cost of not supplying these funds. Alternatively, the procedure discussed above accounts for the Harberger triangle on the supply side, not for that on the demand side.

7 The consequences of a fall in stock prices

What happens to the distribution of future consumption when stock prices fall, as happened during the period 2001-2002? Consider equation (5). A fall in stock prices by one yearly standard deviation σ reduces the total wealth of a generation, its financial plus its human capital, by:

$$\frac{I(t)\,\sigma}{S(t) + \frac{1}{\rho}\left(1 - e^{-\rho\left(T-\hat{t}\right)}\right)} = \frac{\bar{\mu} - \rho}{\theta\sigma} = 3\%. \tag{7}$$

The numerator is the loss in the value of savings, which is taken relative to the total wealth. A one standard deviation fall in stock prices reduces total wealth by 3%. By the martingale property of stock prices, this fall is expected to be permanent. Since all future consumption is paid for from this wealth, it has to decline by 3% on average, too (see the expression (16) for consumption in the appendix). By the CRRA utility function, this decline should be distributed equally across all periods. Hence, future pensions are reduced by 3%. The rate of saving (or equivalently: pension premium rate) is increased by 3% point. Note that this expression does not depend on the age of a generation t. Due to the optimal investment policy, a fall in stock prices has the same effect on consumption per period for all generations, whether the shock occurs at the start of the career, or just before dieing. Equation (7) implies that the premium rate for each generation follows a random walk. This random walk is the same for all generations, so that the premium rates for each generation move parallel across time. The premium rate upon labor market entry starts at the same level for each generation. Current premium rates differ across generations, though, because each generation has been exposed to a different part of the trajectory of the random walk due to entering the labor market at different points in time.

The holding of equity indeed carries quite some risk. Over a lifetime career equation (7) implies that the standard deviation of the premium rate is as large as

$$\frac{\bar{\mu} - \rho}{\theta\sigma}\sqrt{T} = 19\%.$$

But note that the total expected extra return is also a sizable eleven-, i.e. $\exp(0.0\,6*40) \approx 11$, fold increase of an initial investment. We can also use equation (7) to calculate the consequences for consumption of the fall in stock prices in the period 2001-2002. Stock prices were approximately halved, which is a decline of about 70 log points, or a fall of $0.7/\sigma = 3.5$ yearly standard deviations. Hence, the negative effect on future consumption is a sizeable $3.5 \times 3\% = 10.5\%$.

What is the effect of a fall in stock prices on the portfolio of a generation? Should it buy or sell stocks? Again, we take for granted our assumptions of a constant return on the risk free asset, a constant risk premium and a constant volatility of stock prices, and again, consider equation (5). Since $(\bar{\mu}-\rho)/(\theta\sigma^2) < 1$, a fall in stock prices P_t reduces the right hand side of this expression less than it reduces the value of the investment, simply because the investment in equity is only a fraction of the sum of financial and human capital. Hence, a fall in stock prices reduces the value of the investment in equity below the desired level. This should be offset by buying additional equity. Thus pension funds should be buying in a falling market. In fact this is what happened in the beginning of the recent stock market decline, but supervisors aborted this remixing, or even reversed the process thereby adding to the stock market woes.

The above shows that optimally all generations share when stock markets are depressed, workers and retired alike. Everybody's consumption has to be scaled down by the same percentage, either by higher premiums or by lower pensions. Per contrast, under defined benefit plans solidarity means that retired persons receive a steady income stream unrelated to the performance of the economy. Thus in bad times workers suffer twice, feeding to macro instability.

Lastly, we briefly investigate the 'negative ' effects of longevity. Consider the shock of a sudden increase in longevity. Differentiating (4)

$$\partial f(t)/\partial D = 0, \text{ for } t < T$$

one sees that this does not affect the optimal portfolio mix. One just has to live longer on the same amount of wealth. Thus it stands to reason that consumption

$$\partial C(t)/\partial D < 0, \text{ for } t < T,$$

see (16), falls if life expectancy goes up. If D goes up, the policy response of increasing T discussed in one of the previous sections appears all the more reasonable.

8 Share of equity for mature pension fund

In the previous sections, we have derived the optimal portfolio for a single generation. However, a mature pension fund has participants of all generations, ranging from people who have just started their working career to retirees close to the age of death. Where pension funds do not keep separate track of each generation's account, it might be useful to have some idea whether their share of total investment held in equity nevertheless satisfies the rules for optimal portfolio management per generation. That is: if we add up the optimal portfolios of all generations in the pension fund, what should be the average share of the fund's wealth that is invested in equity? Let $N(t)$ be the size of a generation of age t. Then, the share of investment in stock for the pension funds as a whole, f^*, satisfies:

$$\begin{aligned} f^* &= \frac{\int_0^D f(t)\,S(t)\,N(t)\,dt}{\int_0^D S(t)\,N(t)\,dt} \qquad (8) \\ &= \frac{\bar{\mu}-\rho}{\theta\sigma^2}\frac{\int_0^D \left[S(t)+\frac{1}{\rho}\left(1-e^{-\rho\left(T-\hat{t}\right)}\right)\right]N(t)\,dt}{\int_0^D S(t)\,N(t)\,dt} \end{aligned}$$

Assume that generations grow at a fixed rate λ, so that the size $N(t)$ of the current generation of age t is $N(0)\,e^{-\lambda t}$. The problem of this expression is that we do not have a deterministic expression for $S(t)$, since it depends on past fortunes on the stock market. Our approach is to assume that the fund has operated under the portfolio constraint that all investments should be made in the riskfree asset in the past. Furthermore, suppose that this restriction is lifted today. Then, how much of its wealth should the pension fund invest in equity? Such was the situation prevailing till a few years ago for the pension fund of the Dutch government employees, one of the largest pension funds in the world. In the Appendix, we show that, as a first order approximation, f^*satisfies:

$$f^* \cong \frac{\bar{\mu} - \rho}{\theta\sigma^2}\frac{D}{D-T} \tag{9}$$

The average ratio of total wealth to savings is approximately equal to total life time D divided by the length of the retirement period $D-T$. Note that the rate of population growth λ is only of second order importance to the value of f^*. Applying this crude approximation to our benchmark parameters suggests that a mature pension fund should invest 55% of its wealth in equity. (using the exact solution (18) from the Appendix, assuming a two percent population growth, $\lambda = 0.02$, yields $f^* = 0.65$, while in case of a static population, $\lambda = 0.00$, the values is $f^* = 0.49$). Hence, the optimal portfolio rules imply that mature pension funds should invest quite a substantial part of their wealth in equity.

9 Discussion

Modern portfolio theory implies that the young should have the higher stock exposure due to their human capital. Capital losses are distributed smoothly over lifetime consumption. When stock markets are depressed equity should be bought, savings and consumption should be scaled down equiproportionally, and retirement should be postponed. These results suggest to introduce funded pensions cum generational accounts. Generational accounts administered by pension funds enables society and generations to adhere to the optimal portfolio choice rules. This is beneficial since such behavior smoothens shocks over time, which inter alia also has a positive effect on the business cycle amplitudes. Generational accounts avoid distribution conflict between generations that might otherwise arise when the stock market goes down. The implications are equitable since all generations share in the fortunes of the economy. Generational accounts do not require the inefficient holding of buffers, and prevent disputes over their ownership. Buffers are analogous to precautionary saving: we save more than is required. This makes sense only for uninsurable hazards. However, for insurable risk, the market provides the cheapest insurance. Since bonds provide insurance against stock market volatility, precautionary saving is an inefficient way to insure these risks.[3]

In the paper we gloss over the uncertainty regarding future labor income. This uncertainty makes human capital a risky investment. At first sight, one

[3] For example, the Dutch supervisory authority PVK bases its policy on the notion of buffers. This required reserve is higher the greater the share a fund has invested in equity.

would expect that the optimal response to a greater riskiness of human capital would reduce the exposure to stock market volatility. However, this is not necessarily true. In the longer run, the evolution of real wages is correlated to the evolution of stock prices. Since in practice desired future disposable income is indexed by future real wages, this correlation reduces the volatility of stock prices relative to desired consumption. Then, a lower relative volatility raises the optimal share of wealth to be invested in equity. This conclusion is turned around when desired future income is indexed by consumption prives. A further issue along these lines is the endogeneity of the retirement decision which we briefly dealt with. The flexibility in the retirement age provides an additional insurance device, as is analyzed extensively in Bodie, Merton, and Samuelson (1992). When a fall stock prices erodes the value of the financial capital, people will adjust by postponing their retirement. The availability of this additional form of insurance induces a higher exposure to stock market volatility.

Finally, the introduction of generation accounts has radical implications for the governance of pension funds. Currently, many European firms are in trouble because the new International Accounting Standards requires them to put large liabilities due to defined benefit pension systems on their balance sheet. Since by definition the pension funds liabilities to a generation must be paid from the available wealth on that generation's account, there can be a clear separation between the responsibility of the employer and that of the pension fund. The employer has neither a responsibility nor a say in the pension policies. Pension liabilities can never pop up on the employer's balance sheet. The counterpart of this argument is that pension fund's wealth is just deferred compensation, owned by the participants in the fund and not by the employer. Hence, employers should be removed from the pension fund's board. This would also facilitate the free movement of labor across Europe which in part is greatly hampered by the current differences in pension systems and sunk contributions.

In summary, we believe that generational accounts allow individuals to benefit from optimal investment policies and remedy a number of the current deficiencies in the definition of property rights.

10 Appendix Derivations

10.1 Derivation of equation (3) and (4)

Optimal utility is an implicit function of time and savings. By Ito's Lemma

$$\begin{aligned} dU(t,S) &= \{U_S(t,S)\left(\left[f\left(t\right)\bar{\mu}+\left(1-f\left(t\right)\right)\rho\right]S\left(t\right)+\left[Y\left(t\right)-C\left(t\right)\right]\right) \\ &\quad +U_t(t,S)+U_{SS}(t,S)\frac{1}{2}\sigma^2 f(t)^2 S(t)^2]\}dt \\ &\quad +\sigma f\left(t\right)S(t)dW(t), \end{aligned} \tag{10}$$

where subscripts denote partial derivatives. The individual chooses $C\left(t\right)$ and $f\left(t\right)$ to maximize her expected utility $E\left[U\right]$ subject to the dynamic budget constraint (2). From (1) by differentiation

$$\frac{d\overline{U}}{dt} = -u\left[C\left(t\right)\right] + \beta U\left(t\right). \tag{11}$$

To maximize expected utility, take expectations in equation (10) and substitute on the left hand side the total differential from (11). This yields:

$$\begin{aligned} & -u\left[C(t)\right] + \beta U\left(t, S\right) \\ = \quad & U_t(t,S) + U_S(t,S)E[dS\left(t\right)] + U_{SS}(t,S)\frac{1}{2}\sigma^2 f(t)^2 S(t)^2. \end{aligned}$$

From (2), we have:

$$E[dS\left(t\right)] = \left(\left[f\left(t\right)\bar{\mu} + \left(1 - f\left(t\right)\right)\rho\right]S\left(t\right) + \left[Y\left(t\right) - C\left(t\right)\right]\right)dt.$$

Rearranging terms, using equation (1) and suppressing reference to the state variables yields:

$$\beta U = \frac{1}{1-\theta}C^{1-\theta} + \left[\left(f\bar{\mu} + \left(1-f\right)\rho\right)S + Y - C\right]U_S + \frac{1}{2}\sigma^2 f^2 S^2 U_{SS} + U_t. \tag{12}$$

Since U is the optimal expected utility, the first order conditions for consumption and the investment policy require the derivatives of the right hand side with respect to C and f to be zero. From these first order conditions we obtain (3). Substitution in equation (12) yields:

$$\beta U = \frac{-\theta}{\theta - 1}U_S^{\frac{\theta-1}{\theta}} + \left(\rho S + Y\right)U_S - \frac{1}{2}\left(\frac{\bar{\mu} - \rho}{\sigma}\right)^2 \frac{U_S^2}{U_{SS}} + U_t. \tag{13}$$

This is a partial differential equation. Its solution takes a particular simple form for a retired person:

$$U\left(t, S\right) = \frac{1}{1-\theta}S^{1-\theta}g\left(t\right), \ g\left(D\right) = 0.$$

For working individuals, when $Y = 1$, the solution is somewhat more complicated:

$$U\left(t, S\right) = \frac{1}{1-\theta}\left[S + \frac{1}{\rho}\left(1 - e^{-\rho(T-t)}\right)\right]^{1-\theta} g\left(t\right). \tag{14}$$

The validity of these solutions is checked easily. Twice differentiating equation (14) and substitution in equation (13) yields a first order differential equation for $g\left(t\right)$:

$$\beta g\left(t\right) = \theta g\left(t\right)^{\frac{\theta-1}{\theta}} - \rho\left(\theta - 1\right)g\left(t\right) - \frac{1}{2}\frac{\theta-1}{\theta}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 g\left(t\right) + g_t\left(t\right), g\left(D\right) = 0. \tag{15}$$

Solving this differential equation and substitution in equation (13) and (14) yields equation (4).

10.2 Derivation of equation (6)

Consider the more general case where $Y(t)$ while working is y. Then utility and consumption satisfy:

$$\begin{aligned} U(t,S) &= \frac{1}{1-\theta}\left[S+\frac{1}{\rho}\left(1-e^{-\rho(T-t)}\right)y\right]^{1-\theta}\left[\frac{\theta}{\alpha}\left(1-e^{-\frac{\alpha}{\theta}(D-t)}\right)\right]^{\theta} \\ C(t) &= \frac{S+\frac{1}{\rho}\left(1-e^{-\rho(T-t)}\right)y}{\frac{\theta}{\alpha}\left(1-e^{-\frac{\alpha}{\theta}(D-t)}\right)} \qquad (16) \\ \alpha &\equiv \beta+\rho(\theta-1)+\frac{1}{2}\frac{\theta-1}{\theta}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 \end{aligned}$$

see equation (14) and (3). The utility at the beginning of the individual's career is $U(0,0)$. In the absence of a risky asset, this utility is equal to case where the risk premium is equal to zero, $\bar{\mu}-\rho=0$, for in that case the optimal policy is not to invest in stocks, $f=0$. Hence, $\alpha=\beta-\rho(1-\theta)$ for that special case. Take $y=1$ and $\alpha=\beta+\rho(\theta-1)+\frac{1}{2}\frac{\theta-1}{\theta}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2$ as the benchmark case. Let y^* be the labor income that yields the same lifetime utility as the benchmark case for $\alpha=\beta-\rho(1-\theta)$; y^* can be solved by setting equal the expressions for $U(0,0)$ in the benchmark and in the special case. Some simplification yields:

$$y^*=\left[\frac{\beta-\rho(1-\theta)}{\alpha}\left(\frac{1-e^{-\frac{\alpha}{\theta}D}}{1-e^{-\frac{\beta-\rho(1-\theta)}{\theta}D}}\right)\right]^{\frac{\theta}{1-\theta}} \qquad (17)$$

The relative wage increase needed to offset the utility loss of an obligation not to invest in equity is $(y^*-1)/1$. A Taylor expansion of y^*-1 in D yields (6).

10.3 Derivation of equation (9)

Total future consumption has to be financed from total wealth, hence:

$$\frac{\theta}{\beta-(1-\theta)\rho}\left(e^{\frac{\rho-\beta}{\theta}t}-e^{\frac{\rho-\beta}{\theta}D-\rho(D-t)}\right)C(0)=\frac{1}{\rho}\left(1-e^{-\rho(T-t)}\right)+S(t)$$

The initial level of consumption follows from setting t equal to zero, using $S(0)=0$:

$$C(0)=\frac{\beta-(1-\theta)\rho}{\theta\rho}\frac{1-e^{-\rho T}}{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}$$

Hence, the stock of savings at age t satisfies:

$$S(t)=\frac{1}{\rho}\left(e^{\frac{\rho-\beta}{\theta}t}-e^{\frac{(1-\theta)\rho-\beta}{\theta}D+\rho t}\right)\frac{1-e^{-\rho T}}{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}-\frac{1}{\rho}\left(1-e^{-\rho\left(T-\hat{t}\right)}\right)$$

Substitution of these relations in equation (8) yields:

$$f^*=\frac{\bar{\mu}-\rho}{\theta\sigma^2}\left[1-\frac{\frac{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}{1-e^{-\rho T}}\int_0^T\left(e^{-\lambda t}-e^{-\rho T+(\rho-\lambda)t}\right)dt}{\int_0^D\left(e^{\frac{\rho-\beta}{\theta}t-\lambda t}-e^{\frac{\rho-\beta}{\theta}D-\rho D+(\rho-\lambda)t}\right)dt}\right]^{-1} \qquad (18)$$

A Taylor expansion of the integrals yields:

$$\begin{aligned}
&\int_0^D \left(e^{\frac{\rho-\beta}{\theta}t-\lambda t} - e^{\frac{\rho-\beta}{\theta}D-\rho D+(\rho-\lambda)t}\right) dt \\
= \; & \frac{1}{2}\frac{\rho\theta-\rho+\beta}{\theta}\left(D^2 - \frac{\rho\theta-2\rho+\theta\lambda+2\beta}{3\theta}D^3\right) + O\left(D^4\right),
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{1-e^{-\rho T}}\int_0^T \left(e^{-\lambda t} - e^{-\rho T+(\rho-\lambda)t}\right) dt \\
= \; & \frac{1}{2}T - \frac{1}{12}\left(2\lambda-\rho\right)T^2 + O\left(T^3\right),
\end{aligned}$$

and

$$\begin{aligned}
&1 - e^{\frac{\rho-\beta}{\theta}D-\rho D} \\
= \; & \left(\rho - \frac{\rho-\beta}{\theta}\right) D - \frac{1}{2}\left(\rho - \frac{\rho-\beta}{\theta}\right)^2 D^2 + O\left(D^3\right).
\end{aligned}$$

Retaining the first two non-zero terms for each expression yields equation (9).

11 References

Auerbach, A., and L. Kotlikof (1987) *Dynamic fiscal policy*, Cambridge University Press, Cambridge.

Backus, D.K., P.J. Kehoe and F.E. Kydland (1995), International business cycles: Theory and evidence, in: T.F. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton, Princeton University Press, 331-356

Beetsma, R.W.J. and P.C. Schotman (2001) Measuring risk attitudes in a natural experiment: data from the television game show Lingo, *Economic Journal*, p 821-848.

Bodie, Z., R.C. Merton and W.F. Samuelson (1992) Labor supply flexibility and portfolio choice in a life cycle model, in: *Journal of Economic Dynamics and Control*, p 427-449.

Caballero, R.J. (1990) Consumption puzzles and precautionary savings, *Journal of Monetary Economics*, p 113-136.

Cambell, J.Y. (1999), Asset prices, consumption, and the business cycle, in: J. Taylor and M. Woodford, eds, *Handbook of Macroeconomics, Vol. 1C, ch. 19,* North Holland, Amsterdam

Cooley, T.F., and E.C. Prescott (1995), Economic growth and business cycles, in: T.F. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton, Princeton University Press, 1-38

De Jong, F. (2003), *Pension Fund Investments: Stocks or Bonds?*, Internal discussion paper, University of Amsterdam.

Dimson, E., P. Marsh, and M. Staunton (2002), *Triumph of the Optimists, 101 Years of Global Investment Returns,* Princeton University Press.

Hendricks, Judd and Kovenock (1980), A note on the core of the overlapping generations model, *Economics Letters 6*, 95-97.

Menoncin, F. (2003), *Optimal real Investment with stochastic income: A quasi-explicit solution for HARA investors*, Internal discussion paper, Universite Catholique de Louvain

Merton, R.C. (1969) Lifetime portfolio selection under uncertainty, the continuous time model, in: *Review of Economics and Statistics*, p 247-257.

Newberry, D.M.G. and J.E. Stiglitz, (1981) *The Theory of Commodity Price Stabilization,* Oxford, Claredon Press

Rouwenhorst, K.G. (1995), Asset pricing implications of equilibrium business cycle models, in: T.F. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton, Princeton University Press, 294-330

Samuelson, P.A. (1969) Lifetime portfolio selection by dynamic stochastic programming, *Review of Economics and Statistics*, p 239-246.

Teulings, C.N., and C.G. de Vries (2003) Pensioenbeleid als automatische destabilisator, *ESB* (in Dutch), p 100-102.