Optimal Storage Rack Design for a 3-dimensional Compact AS/RS

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ABSTRACT AND KEYWORDS	
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Optimal Storage Rack Design for a 3-dimensional Compact AS/RS

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Abstract

In this paper, we consider a newly-designed automated storage and retrieval system (AS/RS). The system consists of an automated crane taking care of movements in the horizontal and vertical direction. A gravity conveying mechanism takes care of the depth movement. The aim of the research was to facilitate the problem of optimal design and performance evaluation of the system. We estimate the crane's expected travel time for single command cycles. From the expected travel time, we calculate the optimal ratio between three dimensions that minimize the travel time for a random storage strategy. Finally, we illustrate the findings of the study by a practical example.

1. Introduction

Although their application is still limited, compact storage systems become increasingly popular for storing products (Van den Berg and Gademann 2000 and Hu et al. 2005), with relatively low unit-load demand, on standard product carriers. Their advantage is the full automation, making it possible to retrieve and store unit loads around the clock, on a relatively small floor area. In principle, every load can be accessed individually, although some shuffling may be required. They are also used to automatically presort unit loads within the system, so that these loads can rapidly be retrieved when they are needed.

Several compact storage system technologies exist with different handling systems taking care of the horizontal, vertical and depth movements. In this paper, we calculate the travel time and investigate the optimal dimensions for minimizing the travel time under a random storage strategy, for a given storage capacity, of the compact storage system as sketched in Figure 1. This system has been designed for several application areas.

The compact storage system consists of a storage/retrieval (S/R) machine taking care of movements in the horizontal and vertical direction (the S/R machine can drive and lift simultaneously). A gravity conveying mechanism takes care of the depth movement. Conveyors work in pairs: unit loads on one conveyor flow to the rear end of the rack, in the neighboring conveyor unit loads flow to the S/R machine. At the backside of the rack, an inexpensive simple elevating mechanism lifts unit loads from the down conveyor to the upper conveyor, one at a time.

[Insert Figure 1 here]

The innovation of the system is in its cheap construction: no motor-driven parts are used for the conveyors and the construction of the lifting mechanisms is simple as well. The unit loads move by (controlled) gravity. Potential application areas are also innovative. We have studied applications in dense container stacking at a container yard and the Distrivaart project in the Netherlands, where pallets are transported by barge shipping between several suppliers and several supermarket warehouses. This project has actually been implemented and has resulted in a fully automated storage system on a barge (see Figure 2).

[Insert Figure 2 about here]

The throughput capacity of the system depends on not only the physical design, the speeds of handling systems used, but also on the dimensions of the system and the storage and retrieval strategy used. We assume that only single cycles are carried out (in fact, we investigate only retrievals, since storage and retrieval are likely to be decoupled in these systems) and that the storage strategy is random. This is more or less a worst-case scenario, since in reality pre-sorting is often possible. Although finding the S/R machine travel time is not too difficult for the general case, finding closed-form expressions for the three dimensions that minimize the total travel time is more complicated. Analytically, we have been able to find these dimensions for the case that the rack is SIT (i.e. length and height of the rack are equal in horizontal and vertical travel time of the S/R machine respectively). For the none-square-in-time (NSIT) case, we have to rely on different methods. For a given total storage space, we use the nonlinear solver of What'sBest®7.0 (LINDO optimization software for Excel users) to find the optimal dimensions and the corresponding travel time. After considering a wide range of total storage space values, we propose regression formulas for estimating the expected travel time (for single-command cycles) and the optimal dimensions.

This paper is organized as follows. In the next section, we review literature concerning travel time models for AS/RS and mention assumptions and notations used in the paper. In Section 3, we present the travel time models for estimating the expected single-command travel time. In Sections 4 and 5, we find the optimal rack's dimensions that minimize the travel time. We illustrate the results found in Section 5 by an example in Section 6. The effect of fixing one dimension on the optimal travel time is mentioned in Section 7. Finally, we conclude and propose some potential directions for future research in Section 8.

2. Literature review, assumptions and notations

A considerable number of papers exist that analyze AS/RS performance (e.g. estimating expected travel time, rack's dimensions, system throughput, etc.). Figure 3 lists major problem characteristics and solution methods used in AS/RS performance models in the literature. The following common assumptions are commonly used (see also Bozer and White 1984, 1990, 1996 Ashayeri et al. 2002, Foley et al. 2004):

- The S/R machine is capable of simultaneously moving in vertical and horizontal direction at constant speeds. Thus, the travel time required to reach any location in the rack is approximated by the Tchebyshev metric. In contrast, in manual-pick order-picking systems, which use humans to retrieve items from storage area, the travel distance (or, equivalently, travel time) is measured by the Euclidean metric.

- The rack is considered to have a continuous rectangular pick face, where the depot (also: *I/O point*) is located at the lower left-hand corner.

In this section, we will review recent publications (i.e. published after 1995) concerning AS/RS performance analysis. We will discuss the publications mainly based on the system characteristics embedded in the model and solution methods applied. For a general review on the design and control of automated material handling systems, we refer to Johnson and Brandeau (1996). For an overview of travel time models for AS/RS published before 1995, it is advisable to see Sarker and Babu (1995).

- *Storage rack.* Storage shape may influence the performance of AS/RS. It is proved that under the random storage assignment and with a constant AS/RS speed, the SIT rack is the optimal configuration (Bozer and White 1984). However, this is not necessarily true for other storage assignments. Pan and Wang (1996) propose a framework for the dual-command cycle continuous travel time model under the class-based assignment. The model is developed for SIT racks using a first-come-first-serve (FCFS) retrieval sequence rule. Floy and Frazelle (1991) derive the distribution of dual-command travel time for SIT rack with uniform distributed turnover. Recently, Park et al. (2005) propose the distribution of the expected dual-command travel time and throughput of SIT racks with two storage zones: high and low turnover. Ashayeri et al. (1997, 2002) compute the expected cycle time for an S/R machine where racks can be either SIT or NSIT. Park et al. (2003a) compute the mean and variance of single and dual-command travel times for NSIT racks with turnover-based storage assignment. They also show how to adjust the model if the class-based storage assignment is used. In general, AS/RSs have racks of equally-sized cells. However, in some cases, a higher utilization of warehouse storage can be archived by using unequal sized cells. Lee et al. (1999, 2005) develop travel time models for a rack with unequal cells under a random storage assignment, and both single and dual-command cycles. They also compare the proposed continuous-rack model with a discrete-rack model (through simulation) and conclude that the differences in expected travel times are small.

- *Storage assignment.* Using class-based and dedicated storage assignments may lead to a substantial saving on the travel time of the S/R machine (see Section 1.3.2). For a two-class-based storage assignment rack, Kouvelis and Papanicolaan (1995) develop expected command cycle time formulas for both single and dual-command cycles. They also present explicit formulas for the optimal boundary of the two storage areas in the case of single-command cycles. As exact expressions of the throughput are often lengthy and cumbersome, Foley et al. (2004) derive formulas bounding and approximating the throughput of a mini-load system with exponential distributed pick time and either uniform or turnover-based storage assignment. They report that for typical configurations, the worst-case relative error for the bounds is less than 4%.

- *S/R machine operational issues*. With one shuttle, the S/R machine can at most execute two commands (storage and retrieval) in one travel cycle. Single and dual-command cycles are studied in most of studies in the literature (for example, single-command cycles in Kim and Seidmann 1990, Park et al. 2003a; dual-command cycles in Foley and Frazelle 1991, Pang and Wang 1996). By using multiple shuttles, the S/R can perform more than two commands in one travel cycle, and thus the system performance can be enhanced. Meller and Mungwattana (1997) present analytical models for estimating the throughput in multi-shuttle AS/RS systems. Potrč et al. (2004) present heuristics travel time models for AS/RS with equal-sized cells in height and randomized storage under single- and multi-shuttle systems. Almost all studies concerning AS/SR assume that the S/R speed is constant. Certainly it is not true in practice (Hwang and Lee 1990), although the impact of accelerating and decelerating is limited (especially for large racks). Chang et al. (1995) propose a travel time model of S/R machines by considering the speed profiles that exist in real-word applications. They consider the system under random storage assignment, single and dual-command cycles. Chang and Wen (1997) extent this travel time model to investigate the impact on the rack configuration. The results demonstrate that the optimal rack configuration of the single-command cycles is still SIT whereas the dual-command cycles may not be. Wen et al. (2001) also adjust the travel time model in Chang et al. (1995), but for the class-based and turnover-based storage assignment.

- *Solution approach*. Most of the travel time models were developed based on statistical analysis and simulation (for example, Hausman et al. 1976, Graves et al. 1977, Bozer and White 1984, Foley et al. 2002, 2004). Lee (1997) uses a single-server model with two queues to estimate the throughput of a mini-load system, where the cycle times are assumed to be independent, identical, and exponentially distributed (iid) random variables, while requests arrive according to a Poisson process. Simulation results in this study show that the method performs well and can be easily adapted for other AS/RS. However, Hur et al. (2004) claim that the exponential distribution of travel times does not reflect the dynamic aspect of the system. They propose to use an M/G/1 queuing model (also with a single server and two queues). According to their computational results, the proposed approach gives satisfactory results with very high accuracy. Park et al. (1999) study an end-of-aisle order-picking system with inbound and outbound buffer positions (a mini-load system with a horse-shoe front-end configuration). They model the system as a two-stage cyclic queueing system consisting of one general and one exponential server queue with limited capacity. They assume that the S/R machine always executes dual-command cycles and that the dual-command cycle times are independent of each other. With known results for a two-stage cyclic queueing system, they obtain closed form expressions for the stationary probability and the throughput of the system. To compute the mini-load system throughput, the distribution of order arrivals is needed (usually the pick time distribution is assumed to be exponential or uniform, see for example Bozer and White 1990, 1996, Foley and Frazelle 1991). However, this information is not completely available at the designing phase (only partial information is known). Foley et al. (2002) determine upper and lower throughput bounds for mini-load systems under several different types of the partial information: no information, mean only, and NBUE (i.e. New Better than Used in Expectation, roughly it means that the mean pick time of a partially processed bin is smaller than the mean pick time from a new bin).

In the above-mentioned publications, there are only two travel directions are considered (vertical and horizontal). However, situations exist where the S/R machine can travel in three orthogonal directions simultaneously, i.e. vertical, horizontal and cross-aisle direction. Park and Webster (1989b) propose a conceptual model that can help a warehouse planner in the design of 3-dimensional, pallet storage systems. Park and Webster (1989a) deal with the problem of finding a rule for assigning rack locations to product turnover classes to minimize the expected travel time. In these publications, however, the rack dimensions are given or, in other words, the problem of determining the optimal rack dimensions is neglected. For the AS/RS described in Section 1, the S/R machine can only travel vertically and horizontally. However, there is another travel time/direction associated with each travel cycle of the S/R machine; it is time needed to convey the load to the pick position or to reveal an empty location to store the load. For that reason, we also use the terminology 3-dimensional compact storage for our system. We have not found any literature on travel time estimation and/or optimal system dimensioning for this or similar AS/RS types. In the following section, we will step by step estimate the single-command travel time of the S/R machine for the system that we introduced in Section 1.

3. Travel time estimation

Besides the common assumptions mentioned in the previous section, we use the following explicit assumptions for our travel time model:

- The S/R machine operates on a single-command basis (multiple stops in the aisle are not allowed).

- The total storage space, the speed of the conveyor (s_c), as well as the S/R machine's speed in the horizontal (s_h) and vertical direction (s_v), are known.

- The S/R machine travels simultaneously in the horizontal and vertical direction. In calculating the travel time, constant velocities are used for the horizontal and vertical travel: no accelerating or decelerating effects. These effects should be taken into account for the cases of short travel distances. However, in our model they are reflected (or included) in the pick-up/ deposit time.

- We use random storage. That is, any point within the pick face is equal likely to be selected for storage or retrieval.

- The pick-up and deposit (P/D) time for a given load is known and constant. The P/D time is identical for all loads.

The length (*L*), the height (*H*) of the rack and the perimeter (or length 2*S*) of the conveyor form three orthogonal dimensions of the system. Without loss of generality, we suppose that the travel time to the end of the rack is always no less than the travel time to the highest location in the rack: $\frac{H}{s_v} \leq \frac{L}{s_h}$. To standardize the system, we define the following quantities.

$t_c = \frac{2*S}{s_c}$: length (in time) of the conveyor.

$t_h = \frac{L}{s_h}$: length (in time) of the rack.

$t_v = \frac{H}{s_v}$: height (in time) of the rack.

$$T = \max\{t_h, t_v, t_c\}$$

$b = \min\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$. Note that $0 \le b \le 1$ and $b = 1$ iff $t_h = t_v = t_c$.

a is the remaining element (besides b and 1) of the set $\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$, thus $0 < b \le a \le 1$.

For determining the optimal dimensions of the rack, we suppose that the total storage space (or capacity of the warehouse) V is given. Therefore $2*H*L*S$ is a constant. As a result $t_h t_v t_c = V$ is also a constant.

Assume that the retrieval location is represented by (x, y, z), where X, Y and Z refer to the movement directions of the S/R machine and conveyor. We can see that the S/R machine's travel time for single-command cycles (*ESC*) consists of the following components.

- Time needed to go from the depot to the pick position and to wait for the pick to be available at the pick position (if the conveyor circulation time is larger than the travel time of the S/R machine), W. In other words, W is the maximum of the following quantities:
  - time needed to travel horizontally from the depot to the pick position,
  - time needed to travel vertically from the depot to the pick position,
  - time needed for the conveyor to circulate the load from the current position to the pick-up position, R.
- Time needed for the S/R machine to return to the depot, U.
- Time needed for picking up and dropping off the load, c (assumed to be constant).

Hence, the expected travel time can be expressed as follows:

$$ESC = E(W) + E(U) + c \qquad \textbf{(1)}$$

As c is a constant, it does not have any influence on the rack layout so from now on we will not consider this component.

As proven by Bozer and White (1984), in the case of a 2-dimensional rack, the travel time from a random pick location to the depot can be calculated as:

$$E(U) = \left(\frac{\beta^2}{6} + \frac{1}{2}\right) t_h, \qquad \textbf{(2)}$$

where $\beta = \frac{t_v}{t_h} (\beta \leq 1)$ is the shape factor of the rack (recall that we assume $t_h \geq t_v$).

Let $F(w)$ denote the mass probability function that W is less than or equal to w. We assume that the x, y, z coordinates are independently generated, where: $0 < x \leq a$, $0 < y \leq b$ and $0 < z \leq 1$ (that is, we consider the 'normalized' rack). Similar to the case of 2-dimensional racks (see Bozer and White 1984), we have:

$$F(w) = \mathrm{P}(W \leq w) = P(X \leq w).P(Y \leq w).P(Z \leq w)$$

Furthermore, as we use randomized storage; the location coordinations are uniformly distributed. Therefore,

$P(Z \leq w) = w$, with $0 \leq w \leq 1$

$$P(X \leq w) = \begin{cases} w/a & \text{if } 0 \leq w \leq a \\ 1 & \text{if } a < w \leq 1 \end{cases}$$

and

$$P(Y \leq w) = \begin{cases} w/b & \text{if } 0 \leq w \leq b \\ 1 & \text{if } b < w \leq 1 \end{cases},$$

Hence,

$$F(w) = \begin{cases} w^3/ab & \text{if } 0 \leq w \leq b \\ w^2/a & \text{if } b < w \leq a \\ \mathrm{w} & \text{if } a < w \leq 1 \end{cases}$$

$$\Rightarrow f_W(w) = \begin{cases} 3w^2/ab & \text{if } 0 \leq w \leq b \\ 2w/a & \text{if } b < w \leq a \\ 1 & \text{if } a < w \leq 1 \end{cases}$$

Therefore,

$$E(W) = T \int_{w=0}^{1} g(w) w dw = T\left(\int_{w=0}^{b} \frac{3w^3}{ab} dw + \int_{w=b}^{a} \frac{2w^2}{a} dw + \int_{w=a}^{1} w dw \right)$$

$$\Rightarrow E(W) = \left(\frac{b^3}{12a} + \frac{a^2}{6} + \frac{1}{2} \right) \quad \textbf{(3)}$$

From (1), (2) and (3), it is possible now to find the single-command travel time if we know the relative magnitude of each dimension compared to others (i.e. which one is the longest, shortest). And therefore the ratio between three dimensions which minimizes the expected travel time can be determined. To facilitate the analysis, we distinguish two situations: SIT racks (Section 4) and NSIT racks (Section 5).

4. Optimal dimensions for the square-in-time (SIT) rack

As shown in Bozer and White (1984): "*For 2-dimensional racks, the expected travel time will be minimized if the rack is SIT*". Suppose that this type of rack is used we further consider two situations:

- when the conveyor's length is the largest dimension (Section 4.1),
- when the conveyor's length is the shortest dimension (Section 4.2).

4.1 Conveyor's length is the largest dimension (SIT_CL)

In this case, we have $T = t_c$, $a = b$ (thus $\beta = 1$), $t_h = at_c$, $t_v = at_c$ and $a^2 t_c^{\ 3} = V$. From (1) and (2):

$$\begin{cases} E(U) = \frac{2}{3} a t_c \\ E(W) = \left(\frac{a^2}{4} + \frac{1}{2} \right) t_c \end{cases}$$

$$\Rightarrow ESC_{SIT\_CL} = \left(\frac{a^2}{4} + \frac{2}{3}a + \frac{1}{2} \right) t_c \quad \textbf{(4)}$$

At this point, our problem turns out to be the following constrained-optimization problem:

$$\begin{aligned} &\textit{Minimize} && f_{SIT\_CL}(a, t_c) = \left(\frac{a^2}{4} + \frac{2}{3}a + \frac{1}{2} \right) t_c \\ &\textit{subject to} && D = \left\{ (a, t_c) \middle| a^2 t_c^3 = V,\ 0 < a \le 1, t_c \ge 0 \right\} \end{aligned}$$

We use the Lagrangian multiplier method to include the constraint $a^2t_c^{\,3}=V$ in the objective function and obtain: $L(a,t_c,\lambda)=\left(\frac{a^2}{4}+\frac{2}{3}a+\frac{1}{2}\right)t_c+\lambda\left(a^2t_c^{\,3}-V\right)$, where λ is the Lagrangian multiplier. The critical points of $L(a,t_c,\lambda)$ must be the solutions of the following system:

$$\begin{cases}\dfrac{\partial L(a,t_c,\lambda)}{\partial a}=0\\ \dfrac{\partial L(a,t_c,\lambda)}{\partial t_c}=0\\ \dfrac{\partial L(a,t_c,\lambda)}{\partial \lambda}=0\end{cases}\Leftrightarrow\begin{cases}\left(\dfrac{a}{2}+\dfrac{2}{3}\right)t_c+2a\lambda t_c^{\,3}=0\\ \dfrac{a^2}{4}+\dfrac{2}{3}a+\dfrac{1}{2}+3\lambda a^2t_c^{\,2}=0\\ a^2t_c^{\,3}-V=0\end{cases}\Leftrightarrow\begin{cases}\lambda=-0.46\big/\sqrt[3]{V^2}\\ a=0.72\\ t_a=t_b=0.89\sqrt[3]{V}\\ t_c=1.24\sqrt[3]{V}\end{cases}$$

It is easy to see that the sufficient condition for the critical point to be the minimum point is satisfied (meaning that Hessian matrix *H* is positive semi-definite at the critical point). Thus, this critical point is the minimum point and the optimal value is $ESC^{*}_{SIT\_CL}=1.38\sqrt[3]{V}$.

We conclude:

> *"Given an SIT rack with a total storage capacity V and provided that the conveyor's length t_c is the longest dimension, the estimated travel time of the S/R machine will be minimized if $t_v:t_h:t_c\equiv 0.72:0.72:1$ and the optimal travel time is $1.38\sqrt[3]{V}$ ".*

4.2 Conveyor's length is the shortest dimension (SIT_CS)

In this case $a=b\ (\text{so } \beta=1), T=t_h=t_v,\ t_c=bt_h$ and $bt_h^{\,3}=V$. From (2) and (3) we have:

$$\begin{cases}E(v)=\dfrac{2}{3}t_h\\ E(w)=\left(\dfrac{b^2}{4}+\dfrac{1}{2}\right)t_h\end{cases}$$

$$\Rightarrow ESC_{SIT\_CS}=\left(\frac{b^2}{4}+\frac{7}{6}\right)t_h \qquad \textbf{(5)}$$

At this point, our problem turns out to be the following constrained-optimization problem:

$$\begin{aligned} &\textit{Minimize} \quad f_{SIT\_CS}(b,t_h)=\left(\frac{b^2}{4}+\frac{7}{6}\right)t_h \\ &\textit{subject to} \quad D=\left\{(b,t_h)\middle| bt_h^3=V,\ 0<b\le 1, t_h\ge 0\right\} \end{aligned}$$

In a fashion similar to SIT racks, we obtain:

$$\begin{cases} b=0.97 \\ t_c=0.98\sqrt[3]{V} \\ t_v=t_h=1.01\sqrt[3]{V} \end{cases}$$

The optimal value is $ESC^*_{SIT\_CS}=1.42\sqrt[3]{V}$. We can conclude:

> *"Given an SIT rack with a total storage capacity V and provided that the conveyor's length t_c is the shortest dimension, the estimated travel time of the S/R machine will be minimized if $t_v:t_h:t_c\equiv 1:1:0.97$ and the optimal travel time is $1.42\sqrt[3]{V}$ ".*

Comparing two situations, we can see the rack where the conveyor's length is the longest dimension provides a shorter expected (single-command) travel time. Therefore, we can draw the following general conclusion for the SIT rack:

> **Proposition 1** *"Given an SIT rack with a total capacity V, the expected travel time of the S/R machine will be minimized if $t_v:t_h:t_c\equiv 0.72:0.72:1$ and the optimal travel time is $ESC^*_{SIT}=1.38\sqrt[3]{V}$ ".*

5. Optimal dimensions for none-square-in-time (NSIT) rack

For this case, we make a distinction between the following situations:

- the conveyor's length is the longest dimension (NSIT_CL),
- the conveyor's length is the medium dimension (NSIT_CM),
- the conveyor's length is the shortest dimension (NSIT_CS).

If the conveyor's length is the longest dimension then we have: $T=t_c$, $t_h=at_c$, $t_v=bt_c\left(\text{thus } \beta=\frac{b}{a}\right)$ and $abt_c^3=V$. From (2) and (3) we have:

$$ESC_{NSIT\_CL} = \left(\frac{b^2}{6a^2}+\frac{1}{2}\right)at_c + \left(\frac{b^3}{12a}+\frac{a^2}{6}+\frac{1}{2}\right)t_c = \left(\frac{b^3+2b^2}{12a}+\frac{a^2}{6}+\frac{a}{2}+\frac{1}{2}\right)t_c$$

Similarly, if the conveyor's length is the medium dimension: $T = t_h, t_v = bt_h$, $(\text{thus } \beta = b)$, $t_c = at_h$ and $abt_h^3 = V$:

$$ESC_{NSIT\_CM} = \left(\frac{b^3}{12a}+\frac{a^2}{6}+\frac{b^2}{6}+1\right)t_h$$

And if the conveyor is the shortest dimension: $T = t_h, t_v = at_h$, $(\text{thus } \beta = a)$, $t_c = bt_h$ and $abt_h^3 = V$:

$$ESC_{NSIT\_CS} = \left(\frac{b^3}{12a}+\frac{a^2}{3}+1\right)t_h$$

It is easy to see that $ESC_{NSIT\_CL} \le ESC_{NSIT\_CM} \le ESC_{NSIT\_CS}\ \forall \left(0 < b \le a \le 1, V > 0\right)$. It means that the systems where the conveyor is the shortest or medium dimension cannot provide a better solution compared to the system where the conveyor is the longest dimension. For this reason, from now on, we can ignore ESC_{NSIT_CS} and ESC_{NSIT_CM}.

The problem of finding the optimal ESC_{NSIT_CL} turns out to be the following constrained-optimization problem:

$$\begin{array}{ll} \textit{Minimize} & f_3(a,b,t_c) = \left(\frac{b^3+2b^2}{12a}+\frac{1}{6}a^2+\frac{a}{2}+\frac{1}{2}\right)t_c \\ \textit{subject to} & D = \left\{(a,b,t_c) \middle| abt_c^{\ 3} = V,\ 0 < b < a \le 1, t_c \ge 0, V > 0\right\} \end{array}$$

It is hard to solve this problem analytically. For this reason, we opt for the numerical optimization. For a given total storage capacity, *V*, we used the nonlinear optimization module built in What'sBest to find the optimal dimensions as well as the optimal estimated single cycle time of the S/R machine. We carried out an extensive number of experiments (on a very wide range of V: from 10 to 2000 cubic time units). From the experimental results found:

- The optimal ratio between three dimensions does not depend on the system capacity *V*: ESC_{NSIT_CL} reaches the optimum if $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$.
- In order to estimate the relation between the system capacity *V* and the optimal estimated travel time $ESC^*_{NSIT\_CL}$, we carried out a regression analysis (on SPSS). In the analysis, the total storage capacity varied from 10 to 2000 (cubic time units). We used different curve fitting models and found that the optimal estimated travel time is best estimated by the following relation: $ESC^*_{NSIT\_CL} \approx 1.38\sqrt[3]{V}$. The standard errors of the estimate is less than 10^{-5}.
- When the system is cubic-in-time (all dimensions are equal in time), it is easy to find that $ESC^*_{cubic\_in\_time} = 1.42\sqrt[3]{V}$. Interestingly, $ESC^*_{cubic\_in\_time} = ESC^*_{SIT\_CS}$.
- As shown in Figure 4, there is a difference between the overall optimal value and the other optimums with some restrictions on the dimensions. However, the gap is very small; the difference between the cubic-in-time configuration and the optimal one is:

$$\left[\left(1.42\sqrt[3]{V} - 1.38\sqrt[3]{V}\right) \Big/ 1.38\sqrt[3]{V}\right] * 100\% \approx 2.90\% .$$

- The reason that the cubic-in-time rack is not optimal is that the travel time consists of two components (see Section 3). The travel time from the depot to the pick location depends on the movement times on all three directions, but the time needed to go back to the depot depends only on the vertical and horizontal travel time.

[Insert Figure 4 here]

We can make the following conclusion for the NSIT rack:

> ***Proposition 2*** *"Given a NSIT rack with a total storage capacity V, the expected travel time of the S/R machine will be minimized if* $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$ *and the optimal estimated travel time is* $1.38\sqrt[3]{V}$ *".*

Figure 4 shows all eligible possibilities (in both section 3 and 4). We can see that the SIT rack system (i.e. length and height of the rack are equal) results in the overall optimal configuration: it gives the overall shortest estimated single-command cycle time. Now, we are able to state the following proposition:

> **Proposition 3** *Given the 3-dimensional compact AS/RS (as described in Section 1) with a total storage capacity V, the expected single-command travel time of the S/R machine will be minimized if the system dimensions satisfy* $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$ *and the optimal travel time is* $1.38\sqrt[3]{V}$.

6. Effect of fixing one dimension

As shown above, if all three dimensions are 'open', we can find the optimal ratio (with regards to minimizing the estimated travel time) between these dimensions. However, in the Distrivaart project (see Section 1), we could not freely adjust all these dimensions, due to space limitations and equipment standardizations. The previous analysis can also be used to solve the problem with space restrictions. If two dimensions are fixed, then the problem is trivial as all dimensions are defined (given that we know the total system's storage capacity). If only one dimension is fixed, we can still adjust the others to reduce the estimated travel time. Clearly, the resulting optimal travel time can not be shorter than the 'overall' optimum (when we have three 'open' dimensions).

It is straightforward in this case to determine the expected travel time of the S/R machine (e.g. based on formulas (2) and (3)). Figure 5 shows the optimal estimated travel time for different values of the conveyor's length (t_c). From this figure, we can easily see the effect of fixing the conveyor's length. For example, if $t_c = 2\sqrt[3]{V}$ (200% of $\sqrt[3]{V}$), at best we can design a system with an expected travel time of $1.53\sqrt[3]{V}$ (time units), while the 'overall global' optimum, $1.38\sqrt[3]{V}$, is achieved for $t_c = 1.24\sqrt[3]{V}$. Similarly, Figures 6 and 7 show the cases when the rack's length and height (in time) are fixed.

[Insert Figures 5, 6 and 7 here]

7. An example

As an illustrating example, assume that we have to design a 3-dimensional compact system that can store 1000 pallets (other data are given in Table 1). The decision problems are: (1) finding the optimal dimensions of the system and (2) the best position of the S/R machine so that the expected travel time is minimized. The S/R machine either dwells at one end of the rack (A) or between two rack parts (B) (referring to Figure 8).

For situation A: the expected pallet circulation time is S/s_c. Suppose that the length of the conveyors in the left part of the warehouse is $X(0 < X < S)$ (see Figure 8). As pallets are located randomly on the conveyors, in the situation (B) the expected time for a random pallet to be circulated from the current position to the position that the S/R machine can pick it up (the main rack) is:

$$\tau = \left(\frac{X}{S}\right)\left(\frac{X}{s_c}\right) + \left(\frac{S-X}{S}\right)\left(\frac{S-X}{s_c}\right) = \frac{X^2 + (S-X)^2}{Ss_c},$$

where s_c are the conveyor' speed and S is the diameter of the conveyors in situation (A). $\left(\frac{X}{S}\right)$ and $\left(\frac{S-X}{S}\right)$ are the probabilities that the pallet is located in the left-side and the right-side of the warehouse respectively. Applying the Cauchy-Schwarz inequality gives

$$\tau = \frac{X^2 + (S-X)^2}{Ss_c} \geq \frac{\left[X + (S-X)\right]^2}{2Ss_c} = \frac{S}{2s_c}.$$

[Insert Table 1 here]

[Insert Figure 8 here]

This lower bound is tight with equality for $X = S/2$. Therefore, the optimal position, which minimizes the expected single-command travel time, is the middle of the storage rack.

We apply the theorem of Section 5 to calculate the optimal dimensions. We have: $t_c^* = 1.24\sqrt[3]{V} = 10.11$ (*seconds*) and $t_h^* = t_v^* = 0.72 t_c^* = 7.26$ (*seconds*). The rack dimensions must be multiples of the pallet's dimensions. Therefore, we choose the 'practical optimal' dimensions such that they are as closed as possible to the corresponding optimal dimensions found and result in a system with a storage capacity of not less than 1000 pallets (the required capacity). We obtain the practical optimal dimensions: *10* x *8* x *6.5 (seconds)* with an optimal expected travel time of 11.17 (*seconds*).

8. Concluding remarks

In this paper, we discuss a 3-dimensional compact system originating from the Distrivaart project that consists of rotating conveyors and an S/R machine. We extend Bozer and White's method for 2-dimensional rack systems to find the expected single-command travel time of the S/R machine. We found:

- For a given 3-dimensional compact AS/RS (mentioned above) with a total storage capacity V, the optimal rack dimensions are $t_v = t_h = 0.89\sqrt[3]{V}$, $t_c = 1.24\sqrt[3]{V}$, and the optimal travel time is $1.38\sqrt[3]{V}$. Equivalently, the optimal ratio between three dimensions is $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$.
- The cubic-in-time system (i.e. all dimensions are equal in time) is not the optimal configuration (as intuitively we may think). However, it is a good alternative configuration for the optimal one as the resulting expected travel time is only about 3% away from the optimum. This is in line with the findings by Rosenblatt and Eynan (1989) and Chang and Wen (1997) for 2-dimensional SIT racks with single and dual-command cycle respectively. They conclude that "*The expected travel times are fairly insensitive to slight deviations in the optimal rack configuration*".

A disadvantage of the method is that we assume that the rack is continuous. This simplification of reality is only justified if the number of storage positions is sufficiently large (see, for example, Graves et al. 1977 and Lee et al. 1999). The quality of the approximation of the real travel time depends on this.

We considered randomized storage only. Clearly, other storage policies (like class-based or dedicated storage) could be considered as well. This is an interesting direction for further research. Another straightforward extension of the research is to analyze the system when the S/R operates in a dual-command basis.

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Tables and Figures

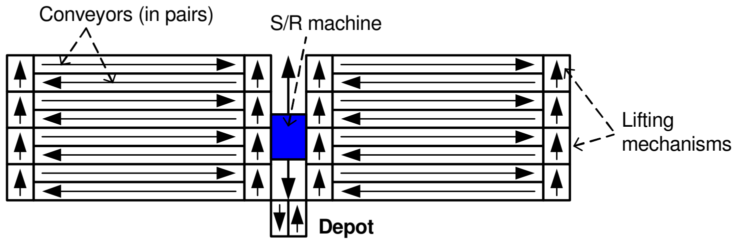

Figure 1 A compact S/RS with gravity conveyors for the depth movements

Figure 2 Distrivaart: a conveyor-supported automated compact storage system on a barge (*source: De Koster and Waals, 2005*).

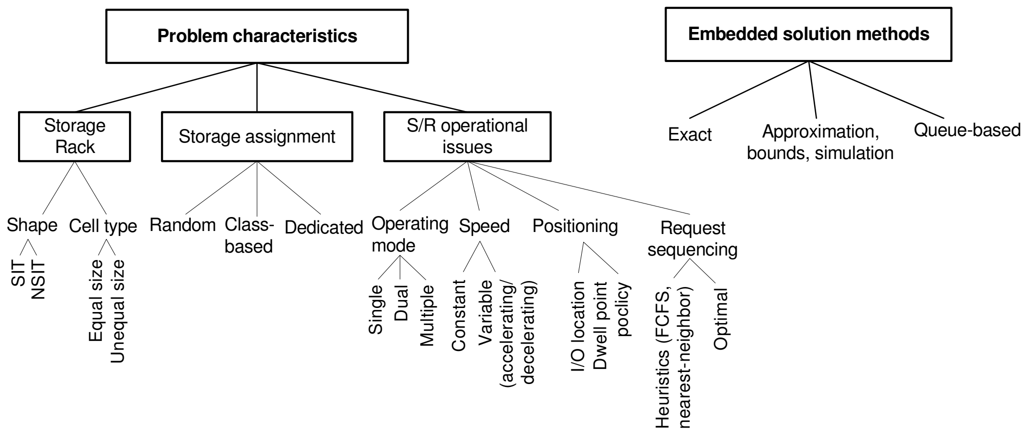

Figure 3 Problem characteristics and solution methods used in AS/RS performance models

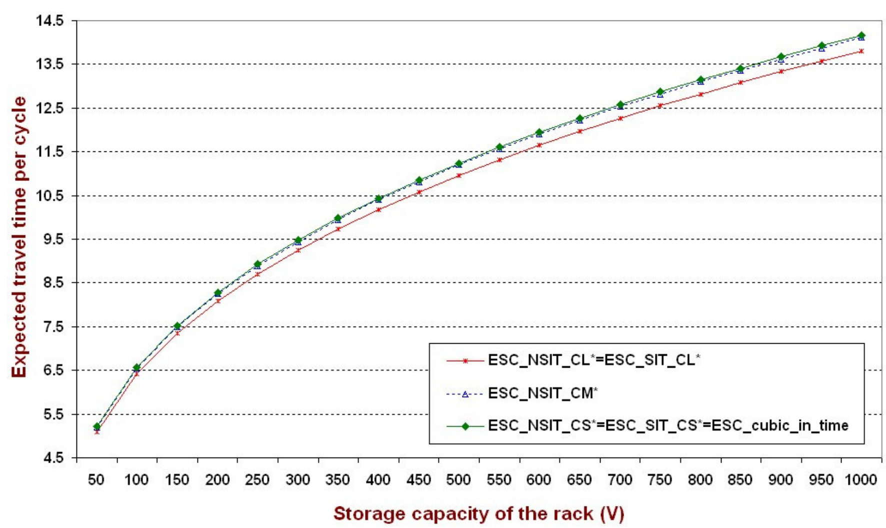

Figure 4 Comparison between optimal expected travel time of SIT and NSIT racks for different values of total storage capacity V

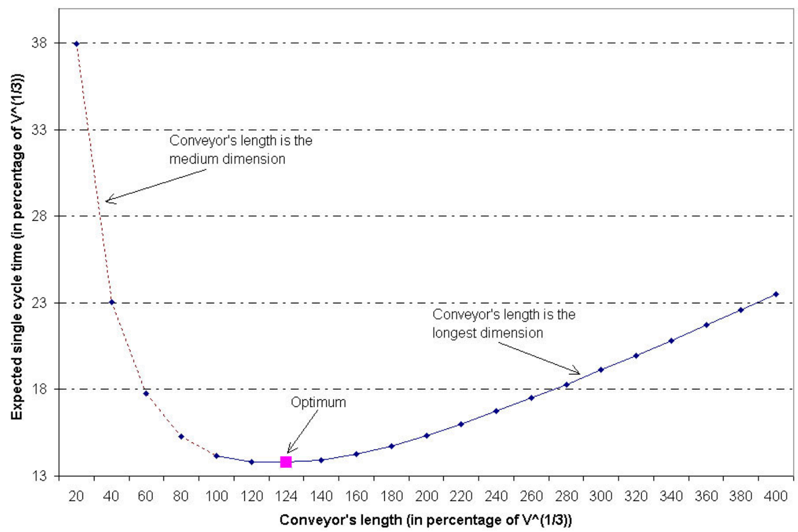

Figure 5 Optimal expected travel time when the conveyor's length is fixed

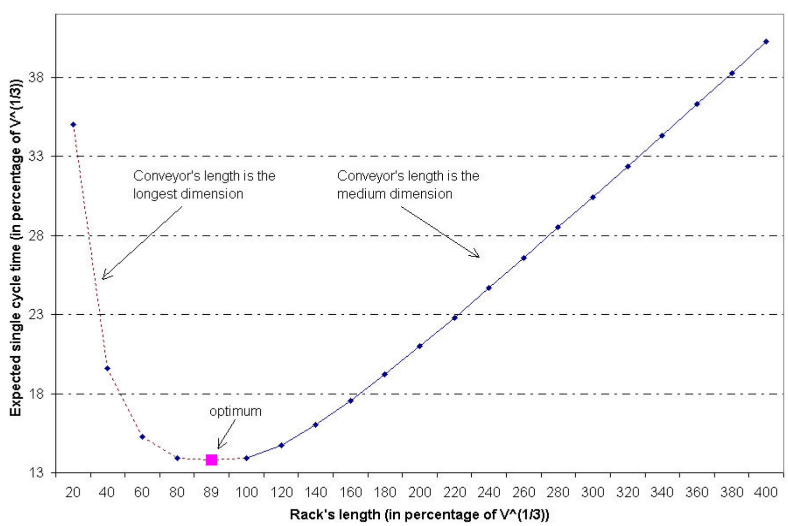

Figure 6 Optimal expected travel time when the rack's length (the longer dimension of the rack) is fixed

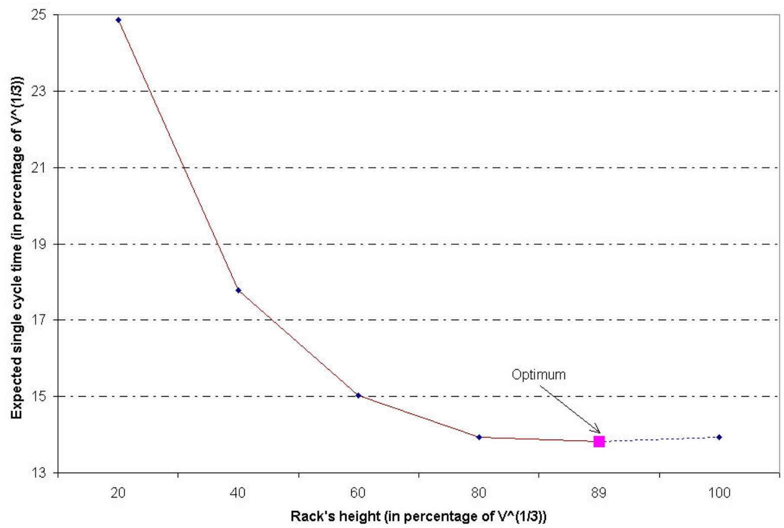

Figure 7 Optimal estimated travel time when the rack's height (the shorter dimension of the rack) is fixed

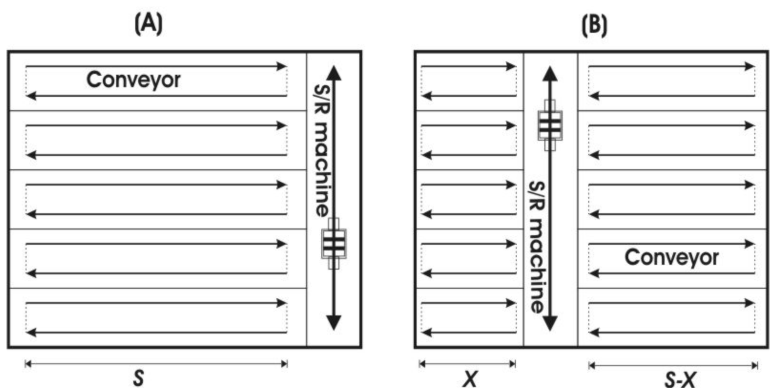

Figure 8 Possible positions of the S/R machine

Table 1 System parameters

Total system capacity (V)		1000 pallets
Storage policy		Random storage
Pallet size in seconds (width x length x height)	Net	*0.4* x *0.4* x *2*
	Gross	*0.5* x *0.5* x *2.17*
S/R machine	Operating policy	Single-command cycle
	Vertical speed (s_v)	0.6 (*meter per second*)
	Horizontal speed (s_h)	2 (*meter per second*)
Conveyors' speed (s_c)		2 (*meter per second*)

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