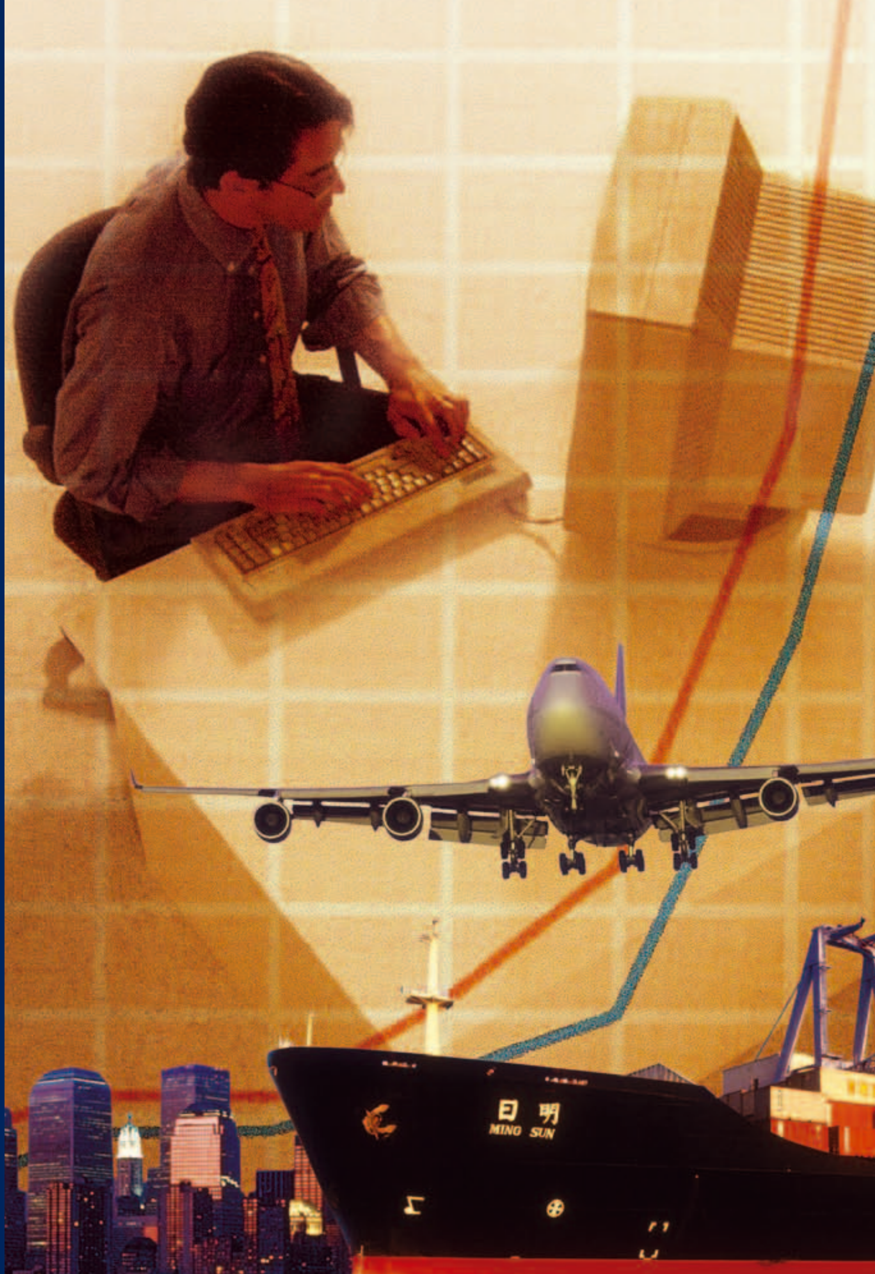
KEVIN PAK

Revenue Management: New Features and Models



Revenue Management:
New Features and Models

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Nieuwe eigenschappen en modellen

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Preface

It has taken me over four years to complete the thesis you're holding in your hands right now. Obviously, such a large project does not come into existence without the support, cooperation and motivation of others. Here I want to thank the people that have been important for my continuing work on this thesis these last couple of years.

First of all, I want to thank my supervisors Rommert Dekker and Nanda Piersma. Nanda, you're responsible for starting this all. First by introducing me to the academic world and second by providing me with the research topic for this thesis. It's a shame that I could only work with you for the first two years of my research. Rommert, I want to thank you for the way you took over Nanda's job for the last two years of my research. Despite your lack of background on Revenue Management you provided me with great insights and research suggestions. At times I stood amazed at your ability to recognize new research opportunities. Next to that, you supported me in any way possible and always made time for me in your already busy schedule.

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Kevin Pak

Rotterdam, March 2005

Contents

Chapter 1

Introduction

1.1 Background and Motivation

Revenue management is currently one of the most successful applications of operations research (OR). Contrary to most other OR applications that are generally concerned with efficiency and cost reduction, revenue management aims directly at the revenue side of the firm. This has gained many managers' attention over the years and will continue to do so in the foreseeable future. Revenue management began as a tool that a handful of major airlines started using after the deregulation of the prices in the U.S. airline industry in 1978. Pricing became a major business tool and airlines started differentiating passengers into classes for which different prices were charged. Because these passengers all make use of the same seat capacity, it became evident that the availability of the classes has to be controlled. This way, the airline company can prevent the plane from filling up with low fare passengers while still offering the competitive low fare. Controlling the availability of the capacity over the various price classes became known as the revenue management (or yield management[1]) problem.

[1] Revenue management and yield management are two terms that are generally used to describe the same problem. In recent years, however, revenue management has become the more common one, whereas yield management has slowly died away.

The level of success of revenue management in the airline industry is enormous and renowned among many professionals. Smith et al. (1992) report a quantifiable benefit of the revenue management systems at American Airlines of over $1.4 billion for a three year period which is more than the net profit for the same three year period. Moreover, Cross (1997) tells the story of how American Airlines' revenue management systems caused the downfall of competitor and once flourishing low-cost airline company PeopleExpress. Insights such as these have led to the general understanding that good revenue management systems are crucial for the survival of any airline company in today's market.

With the success story of the airline industry to back it up, revenue management has found applications in many other industries. Hotels were quick to pick up on the concepts provided by the airline companies, whereas the car rental industry provided another big success story when National Car Rental was saved from liquidation by the implementation of up-to-date revenue management systems (see: Geraghty & Johnson (1997)). Other applications can be found in railways, TV broadcasting, cargo transportation, theatres, gas storage and transportation, cruise lines, telecom providers and more. When giving his view of the future, Peter Bell (1998) even mentioned that he expects revenue management concepts to be applied to almost everything that is sold in the near future. This is underpinned by the fact that major department stores are currently examining the opportunities of revenue management concepts for their specific needs.

The increasing interest in revenue management applications has also led to an increasing number of revenue management professionals and has even given rise to a number of both small and large companies specialized specifically in the topic. At the same time, academic research on the subject of revenue management has increased almost exponentially over the last fifteen years. Several universities now offer revenue management courses and INFORMS, the leading scientific society in OR, has started a Pricing and Revenue Management subsection. In 2002 the Journal of Revenue and Pricing Management was brought to life and recently, Talluri and van Ryzin (2004a) wrote the first real textbook on revenue management. This book provides a unified theory for revenue management problems and will hopefully help to established revenue management as a major OR application in theory as it already is in practice.

Revenue management originates from a real-life practical problem and grew into a popular research topic for academics from that. Throughout the years, academic research has provided theoretical foundations for concepts used in practice and improved and extended the underlying mathematical models. Up until now, the link between practitioners and academics has remained very strong. A good example of this is the annual INFORMS Pricing and Revenue Management conference that attracts practitioners and academics alike, who can share thoughts on difficulties and advances in both fields. The research topics we present in this thesis also stem from practical problems. All topics were first and foremost issues that practitioners wanted to see resolved. For this, we tested and improved existing models (Chapter 3), extended models to deal with new developments in the field (Chapter 4) or constructed new techniques entirely (Chapter 5). With this we hope to contribute to the further development of revenue management techniques for real-life applications.

In the following section we define the term revenue management more closely, and in the final section of this chapter we provide a chapter layout for the remainder of this thesis.

1.2 What is Revenue Management?

Anyone looking for a formal definition of revenue management is likely to find a large number of formulations. These formulations generally fall into two categories: those that define revenue management as the well-defined problem everybody knows from the airline industry and other likewise applications, and those that define revenue management as a much broader field that encompasses just about any activity involved with the sales side of a firm. Although the latter is certainly desirable, it’s the first definition that revenue management is more known for. We go into both definitions in the following sections.

1.2.1 Revenue Management as an Inventory Control Problem

Companies selling perishable goods or services often face the problem of selling a fixed inventory of a product over a finite horizon. If the market is characterized by customers willing to pay different prices for the product, it is often possible to target different customer segments by the use of product differentiation. This creates the opportunity to sell the product to different customer segments for different prices, e.g. charging different prices at different points in time or offering a higher service level for a higher price. In order to prevent the whole inventory to be taken up by the low-price customers, decisions have to be made about what part of the inventory is available for each customer segment. Obviously, you do not want to sell too many items to the low-price customers, but at the same time you do not want items to remain unsold either. This inventory control problem is the most popular and well-known description of the revenue management problem.

Three conditions are generally considered to be necessary for revenue management to be beneficial for a company (see: Weatherford and Bodily (1992)):

- perishable product
- fixed capacity
- possibility for price-differentiation

The fact that the product is perishable and capacity is fixed means that a specific amount of items has to be sold before a certain deadline. Each item that is not sold before the deadline becomes worthless. This means that the seller cannot wait indefinitely for a high paying customer and might be forced to accept low-price customers instead in order to sell all items before the deadline. Further, the possibility for price-differentiation has to be present in order to charge different prices for the same product. The classical example is that of the airline industry, where a fixed number of seats has to be sold before the plane takes off. Airlines generally differentiate passengers into many different price classes based on their time of booking, place of booking, return date etc.

The term revenue management stems from the airline industry and originally also encompassed overbooking and pricing decisions. However, since overbooking and pricing were already well-known topics, the attention was quickly drawn to the new feature in revenue management for which at that time names were used as: seat inventory control, seat allocation, passenger mix, discount allocation and seat management. Not before long, however, revenue management for most people became a much more popular synonym for the inventory control problem. Nowadays, revenue management is almost exclusively used to denote this problem and we will do the same throughout this thesis. In the following section, however, we shortly discuss the broader definition of the term.

1.2.2 A Broader Definition of Revenue Management

Imagine owning a number of tickets for you and your friends for a big stadium event such as a pop concert, a football match or a dance event. Since some of your friends won't be attending the event, you decide to sell their tickets. If you want to do this as profitable as possible, some decisions will have to be made. You can sell the tickets in advance on the internet or at the last minute at the entrance of the stadium. If you sell the tickets on the internet, you have to decide whether it will be for a fixed price or in an auction. What will the fixed price be? Or the starting and reserve prices for the auction? Moreover, will you offer all tickets as one bundle or as separate items? When you sell the tickets at the entrance of the stadium you also have to decide what price to ask. However, you also have to consider what possible counteroffer to accept and when to lower or raise the price as time proceeds. Sales decisions such as when, where, to whom, for how much and in what form to sell your product, are important decisions that every seller has to make. When using a broader definition of the term, revenue management is the art of making all these kinds of sales decisions as good as possible.

Talluri and van Ryzin (2004a) distinguish three categories of sales decisions that make up the revenue management spectrum: structural, price and quantity decisions. Structural decisions are strategic decisions concerning issues such as what to sell and how to do so. Examples of structural decisions are: how to bundle products, how to differentiate a product to target different customer segments, which sales method to use and what general price structure to use. These decisions are generally kept fixed over a longer period of time. After the structural decisions are made, price and quantity decisions are used to optimize revenues on a day-to-day basis. They involve decisions such as: what price to charge at this moment, when to give a discount, what part of the capacity to reserve for each customer segment and whether to accept or reject a specific sales offer.

The structural sales decisions obviously have a huge impact on the day-to-day price and quantity decisions. It is therefore strongly recommended to integrate the various types of sales decisions. In general, though, the structural decisions are considered to be marketing decisions that are set by a totally different department. Since revenue management is known as a strongly OR oriented approach to the day-to-day sales decisions, the structural decisions are not often referred to as revenue management decisions. In fact, the more popular definition of revenue management that we gave in the previous section corresponds directly with what Talluri and van Ryzin call the quantity-based sales decisions. The price-based sales decisions are strongly related to the quantity-based sales decisions, but pricing is a well-known term of its own. This has led to pricing and revenue management coexisting as the two terms used for respectively the day-to-day price and quantity sales decisions. This is evident from the names of the before mentioned Journal of Revenue and Pricing Management and INFORMS' Pricing and Revenue Management subsection.

As said before, the day-to-day price and quantity decisions are strongly related to each other. In fact, as Gallego and van Ryzin (1997) point out, price-based and quantity-based controlling of the sales can conceptually be seen as two different approaches to the same problem. Consider the situation where different prices are charged for different customer segments that make use of the same capacity. When pricing schemes are used to control the sales, sales in a given customer segment can be stimulated by reducing the price. On the other hand, sales can be stopped in a given customer segment by setting a very high price, which leaves the capacity to be used by the other customer segments. Quantity-based control of the sales makes use of a given price structure. The decision is then what part of the capacity to make available for each price that can be charged for the product. Making capacity available for a very low price can be seen as giving a discount and a customer segment can be blocked for further sales by denying it any capacity. For example, offering a 20% discount for a week is conceptually not different from opening up a 20% discount price class for the first 15 customers if this is the number of customers during that week. Whether price or quantity is used as control variable generally depends on the company and the industry it is in.

In this thesis, we use the term revenue management for which it is best known: quantity-based control of the sales. In other words, we see the problem as an inventory control problem. We assume that all structural sales decisions that precede the day-to-day decision making are fixed beforehand by higher management as is usually the case. As said before, quantity- and price-based sales control techniques are generally interchangeable. For the applications that we discuss in this thesis, however, quantity is the common variable to control sales. For these applications, price-based techniques would both stray away too far from current practice and result in much more difficult and impractical models.

1.3 Chapter Layout

In this section we provide the chapter layout for the remainder of this thesis. In Chapter 2 we discuss the well-known airline passenger revenue management problem and give an overview of the OR techniques proposed for this problem. The airline industry is by far the most popular application of revenue management, both in practice as among academics. It is the model after which most other revenue management applications are moulded. In fact, many popular revenue management applications make use of models and techniques first developed for the airline industry. We think that it is important for everyone working with quantity-based revenue management applications to be familiar with the airline problem. An earlier version of Chapter 2 can be found in Pak and Piersma (2002). More recently, Talluri and van Ryzin (2004a) provide a similar overview in their textbook.

In Chapter 3 we present and evaluate models and techniques for revenue management in the hotel industry. Hotel revenue management is not a new topic and has been studied by: Baker and Collier (1999), Bitran and Gilbert (1996), Bitran and Mondschein (1995) and Weatherford (1995) among others. We provide new insights in two ways. First, we introduce and evaluate the use of stochastic programming models next to the existing deterministic models. Further, we do not restrict ourselves to one fixed decision period in which bookings can occur for a given set of booking dates. Instead, we define multiple optimization points at which we simultaneously optimize the complete set of booking dates that are open for booking. This results in a rolling horizon of overlapping decision periods, which conveniently captures the effects of overlapping stays. The research presented in Chapter 3 is based on concepts first put forward in the Master's Thesis by Paul Goldman, which was supervised by Richard Freling. An earlier version of Chapter 3 is provided in Goldman et al. (2002), but in Chapter 3 we provide some major extensions and new computational results.

In Chapter 4 we return to the airline revenue management problem. In this problem the capacities of the business and economy class sections of the plane are traditionally considered to be fixed and distinct capacities. In Chapter 4, we give up this notion and instead consider the use of convertible seats. A row of these seats can be converted from business class seats to economy class seats and vice versa. This offers an airline company the possibility to adjust the capacity configuration of the plane to the demand pattern at hand. Doing so, changes the actual capacity of the plane by which we let go of the fixed capacity property often thought necessary for revenue management. We show how to incorporate the shifting capacity opportunity into a dynamic, network-based revenue management model. We also extend the model to include cancellations and overbooking. The research presented in Chapter 4 is joint work with Gerard Kindervater of KLM Royal Dutch Airlines. An earlier version of the research in Chapter 4 can be found in Pak et al. (2003).

Cargo transportation is a widely recognized application of revenue management. However, the research done for this particular application is very limited. In Chapter 5 we present a new approach to deal with the cargo revenue management problem. This problem differs from the well-known passenger revenue management problem by the fact that its capacity is 2-dimensional, i.e. weight and volume and that the weight, volume and profit of each booking request are random and continuous variables. This means that each cargo shipment is uniquely defined by its weight, volume and profit. Passengers, on the other hand, always take up one seat and belong to a pre-specified price class. Previous papers on the subject of cargo revenue management, e.g. Kasilingam (1996) and Karaesmen (2001), suggest dividing the cargo shipments into groups in order to apply standard passenger revenue management techniques. In Chapter 5, we treat the cargo shipments as the unique items that they are. We formulate the problem as a multi-dimensional on-line knapsack problem and show that a bid-price acceptance policy is asymptotically optimal if demand and capacity increase proportionally and the bid prices are set correctly. Further, we provide a polynomial-time algorithm to obtain the optimal bid prices for a given set of shipments. This research has been conducted in correspondence with KLM Royal Dutch Airlines and is loosely based on the Master's Thesis by Bart Buijtendijk. An earlier version of Chapter 5 can be found in Pak and Dekker (2004).

Finally, Chapter 6 is the last chapter of this thesis in which we summarize our findings and give our concluding remarks.

Chapter 2

The Airline Revenue Management Problem and its OR Solution Techniques

2.1 Introduction

Revenue management originates from the airline industry. In this industry, revenue management has been a major success and has received a lot of attention throughout the years. In fact, the airline revenue management problem has become the prototype for which a revenue management problem is known. Most other well-known applications of revenue management are directly derived from the airline problem and generally use the same kind of mathematical models that were originally constructed for the airline industry. Examples of such are the hotel (see: Baker and Collier (1999), Bitran and Gilbert (1996), Bitran and Monschein (1995), Goldman et al. (2002) and Weatherford (1995)), railroad (see: Ciancimino et al. (1999) and Kraft et al. (2000)) and car rental (see: Geraghty and Johnson (1997)) industries. We think it is important for everyone working with revenue management to be familiar with the airline problem and its solution techniques. We note that we use the term airline revenue management for the problem that airlines face when accepting passengers on the various flights they offer. Although many airlines also carry cargo shipments, passenger transportation is generally considered to be their core business and is the application that revenue management is known for. Opposed to the passenger problem, cargo revenue management is a somewhat underdeveloped area to which we pay attention in Chapter 5.

In this chapter we provide an overview of OR techniques available for the airline revenue management problem. In Section 2.2 we discuss the airline revenue management problem in more detail. We introduce the common booking control policies in a general way in Section 2.3. In the sections 2.4 and 2.5 respectively, we present the underlying mathematical models for airline revenue management in the case of a single flight and a network of flights. Finally, in Section 2.6 we provide a summary and some concluding remarks.

2.2 Airline Revenue Management

After the deregulation of the prices in the U.S. airline industry in 1978, pricing became a major business tool. Airlines were forced to match the prices of the competition, but at the same time did not want to let go of the passengers who were still willing to pay a higher price. For that reason, airlines started differentiating the passengers into different classes for which different prices were charged. Passengers are differentiated into these fare classes based on various features, such as the time of booking, the cancellation options or the inclusion of a Saturday night stay. With all the different fare classes competing for the same seat inventory of the plane, the availability of the classes has to be controlled in order to prevent the plane from filling up with low-fare passengers. This seat inventory control problem is the airline revenue management problem. In Section 2.2.1 we discuss the difficulties in airline revenue management and in Section 2.2.2 some problems closely related to the revenue management problem. Finally, in Section 2.2.3 we discuss some assumptions that are generally made in revenue management research about the behaviour of demand.

2.2.1 Difficulties in Airline Revenue Management

The airline revenue management problem concerns the allocation of the finite seat inventory to the demand that occurs over time before the flight departs. The objective is to find the right combination of passengers on the flights such that revenues are maximized. The desired allocation of the seat inventory has to be translated into a booking control policy, which determines whether or not to accept a booking request when it arrives. It is possible that at a certain point in time it is more profitable to reject a booking request in order to be able to accept a booking request of another passenger at a later point in time. To complicate matters, a passenger can make use of multiple connecting flights to reach his/her destination. These passengers are competing for the same seats as single flight passengers are, but on multiple flights at the same time. Therefore, an airline has to be able to compare requests that take up seats on different flights. The description of the problem provided above, gives us the three major difficulties in airline revenue management:

Uncertain demand

The stochastic nature of demand is obviously one of the major problems in revenue management. The first step for dealing with demand uncertainty is to have a good grasp on the historical data. This means that all demand data has to be registered and stored carefully in order to be used for future decision making. Statistical tools have to be used to model future demand, taking into account such things as the booking patterns of the various types of passengers and seasonal trends. Human interaction is sometimes needed in the case of special events that cannot be prediction based on historical data. The second step is dealing with demand uncertainty in the actual revenue management model. Even when a full statistical distribution is available for the demand, it is not trivial how to include this into a mathematical model that provides a booking control policy to use on-line. Most revenue management models currently used in practice are, in fact, deterministic models for which demand is replaced by a point estimate of the expectation of the demand. Some models have been proposed that do take into account more than just the expectation of the demand, either by stochastic programming models or by simulation techniques.

On-line decision making

Booking requests come in gradually over the booking period. Whenever such a booking request occurs, the decision, whether to accept or reject it, has to be made right away. In order to do so, the airline has to make use of an on-line booking control policy. Important for such a booking control policy is that it always has the current booking information at its disposal. Whenever a seat is sold, this must be recognized in the reservation system immediately. The booking control policy applies for all sales. Tickets, on the other hand, are sold in many different locations. This means that it is very important that all ticket sales are registered in one Central Reservation System (CRS), which contains all sales data necessary for the booking control policy to stay up-to-date. A good booking control policy is one that continually re-adjusts itself according to the current inventory, demand forecasts and time until departure. In practice, however, such full dynamic models are generally not feasible because of the computation time involved. Most models currently used, are static models that generate a desirable allocation of the seats at a certain point in time, typically the beginning of the booking period, based on a demand forecast at that point in time. These static models are usually adjusted a number of times during the booking period according to the situation at hand.

Flight network

Many airline companies offer a large network of flights. Because a passenger can make use of multiple connecting flights to reach his/her destination, the network of flights has to be considered as a whole when finding the right passenger mix. In order to see this, consider a passenger travelling from A to C using flights from A to B and from B to C. If each flight is considered separately, this passenger can be rejected on one of the flights because another passenger is willing to pay a higher fare on this flight. But by rejecting this demand, the airline also loses an opportunity to create revenue for the other flight. If a seat remains unsold on the other flight, it could have been more profitable to accept the passenger to create revenue for both flights. Because yet another passenger can make use of flights from B to C and C to D, the overlap of the flights does not necessarily stop with two flights. Only when the network of connecting flights is considered as a whole, can the various types of passengers be truly compared to each other. Optimizing the whole flight network simultaneously results in a large-scale mathematical programming model that can quickly become computationally demanding.

We note that the network of flights as an airline company defines it, consists of a number of flights that all take off in one pre-specified time frame. An airline generally specifies such a time frame in order to offer passengers the possibility to transfer from one flight to another without too much waiting time. For a return flight, the away and return flights are not considered in the same network. Instead, they are considered as different requests and are treated individually.

2.2.2 Topics Related to Airline Revenue Management

As discussed in Chapter 1, revenue management can be considered to encompass more than just the inventory control problem that we discuss in this chapter. In fact, the term was originally used in the airline industry to include overbooking and pricing decisions. Next to these two problems, also demand forecasting is very important for eventual revenue management decisions. In this section we shortly discuss these three problems closely related to the revenue management problem. For more extensive overviews of the roles of demand forecasting, overbooking and pricing in relation to the revenue management problem, we refer to McGill and van Ryzin (1999) and Talluri and van Ryzin (2004a).

Demand forecasting

Determining what level of inventory to appoint to each fare class depends greatly on demand forecasts for the various fare classes. If the number of high-fare passengers is overestimated, this will result in empty seats, whereas too many low-fare passengers will be accepted on the plane if the number of high-fare passengers is underestimated. An airline needs various kinds of demand forecasts. Next to the number of passengers, also the number of cancellations and no-shows are important. Further, also the booking patterns of the various types of customers should be taken into account. If, for example, only 10% of the expected demand has been realized for a certain fare class with just a few days to go until departure, this can lead to adjustments to the booking control policy if it is not known that this specific fare class tends to fill up at the last moment. The trouble is that in practice particularly the high-fare booking requests tend to come in late. These are generally business travellers that need to be somewhere on relatively short notice. Leisure travellers on the other hand, are much more flexible in choosing the time and destination of their trip and often do so many months in advance.

One of the mistakes often made about forecasting, is that it is sufficient to provide a point estimate of the expectation. Such a point estimate is rarely accurate and provides little information otherwise. Information about the distribution of the demand is much more insightful. This way, deviations from the expected value can be ascribed a certain probability of occurring. However, practical problems for even obtaining an accurate point estimate should not be underestimated. A major issue in airline demand forecasting, for example, is that observed demand is censured in the way that only the number of accepted bookings is observed. This means that the upward potential for the demand generally remains unknown. Also the effects of the booking control policy on demand behaviour are generally hard to evaluate. If a fare class is closed for booking, this can divert a potential passenger to another flight, another fare class or a competitor. Such effects should be taken into account, but are difficult to observe in practice. Further, airline demand differs for each day of the week and shows strong seasonal trends. It has even shown to be correlated to the state of the economy.

Overbooking

Airlines often have to cope with cancellations and no-shows. Therefore, in order to prevent a flight from taking off with vacant seats, airlines tend to overbook a flight. This means that the airline books more passengers on a flight than the capacity of the plane allows. Smith et al. (1992) report that in the airline industry approximately 50% of all bookings eventually turn into cancellations or no-shows and about 15% of all seats on sold-out flights would be unused without overbooking. Research on overbooking in the airline industry goes back as far as 1958 (see: Beckmann (1958)). The major issue in overbooking is comparing the negative effects of an empty seat with that of a denied boarding. The actual value of a denied boarding is, however, not transparent since it can also involve a certain loss of goodwill. Overbooking is closely related to the revenue management problem in the sense that both are concerned with the level of accepted bookings. Nevertheless, the two practices are generally approached separately.

Pricing

The importance of pricing for the revenue management process is evident. The existence of the different fare classes is the starting point for revenue management concepts to be applied. The general price structure underlying the fare classes is very important in determining the eventual allocation of the passengers. The price structure is generally fixed for a longer period of time by strategic and marketing decision makers. Especially competition plays an important role in setting this price structure. Important to realize is that the prices as communicated to the public, do not always reflect the profit margin that the airline company makes on a ticket. Selling tickets by internet, for example, generally generates a much higher margin for the airline than when a sales agent is used. Also, large companies often receive profound discounts on the actual fares for using specific airlines. Since it is often difficult to use the actual margin generated by a booking request in the on-line decision process, the fare level associated with a fare class is often adjusted to reflect the average margin generated by the fare class over the various sales channels.

As discussed in Chapter 1, it is also possible to control the day-to-day sales by continuously changing the prices, which is often called dynamic pricing. In fact, as Gallego and van Ryzin (1997) point out, there is a natural duality between quantity- and price-based controlling of the sales. In this case, closing a low fare class can be interpreted as raising the price, whereas opening a low fare class can be interpreted as giving a discount. Dynamic pricing techniques seem to have entered the airline industry with low cost airlines such as easyJet and Ryan Air, who do not make use of a pre-specified price structure but instead change their prices over time. However, such pricing schemes can also be controlled by quantity decisions, e.g. accept 20 requests for a low price, accept 25 requests for a medium price and finally accept 10 requests for a high price. We restrict ourselves to such quantity-based techniques since price-based techniques quickly result in far more difficult models for the problem that we consider. For models for a dynamic pricing approach to airline revenue management, we refer to: Chatwin (2000), Feng and Gallego (1995, 2000), Feng and Xiao (2000a, b), Gallego and van Ryzin (1994, 1997), Kleywegt (2001), Maglaras and Meissner (2004), Brumelle and Walczak (2003), You (1999) and Zhao and Zheng (2000) among others. See also Bitran and Caldentey (2003) who provide an overview of pricing models for revenue management problems.

2.2.3 Assumptions about Demand Behaviour

For each study, assumptions have to be made. In our discussion of the airline revenue management problem for example, we already assumed that the seat inventory, the general price structure and the booking period are all fixed and known. These are all relatively straightforward assumptions (see Chapter 4 for a relaxation of the fixed capacity assumption). Some of the assumptions generally made in airline revenue management, however, are not that straightforward. These are the assumptions made about the behaviour of the demand. The following three assumptions are usually made when concentrating on the airline revenue management problem:

- no cancellations, no-shows and overbooking
- no group bookings
- demand is independent of the booking control policy used

The first two assumptions are easy to comprehend. The first simply states that no attention goes out to the overbooking problem. The overbooking and revenue management problems are usually considered separately and keeping overbooking out of the picture, allows us to concentrate better on the revenue management problem. The second assumption says that we don't consider group bookings. The difference between group bookings and normal bookings is that accepting a group booking leads to a large inventory jump. Not all mathematical models that we discuss in the following sections can cope with this. In practice, small groups can usually be treated as individual bookings whereas decisions concerning large groups can be made outside the model.

The third assumption about the behaviour of demand is one that has caused much debate. It states that demand is independent of the booking control policy that is used. This means that a passenger has a strict preference for one specific fare class, no matter what classes are available. It also means that a rejected booking request is lost forever even if other fare classes are still available. This is not very realistic. In practice, a passenger presumably bases his/her choice on the available fare classes. And whenever a booking request is rejected it is likely to try another fare class or flight to reach his/her destination. When a passenger books in a higher fare class because a lower class is not available, this is called buy-up behaviour. When a passenger makes use of another flight entirely, it is called diversion. Customer choice behaviour like this has been the subject of a number of studies in revenue management, such as: Algers and Besser (2001), Andersson (1998), Belobaba and Farkas (1999), Belobaba and Weatherford (1996), Bodily and Weatherford (1995), Brumelle et al. (1990), Pfeifer (1989), Talluri and van Ryzin (2004b), Weatherford et al. (1993), You (2001) and Zhao and Zheng (2001). The models discussed in these studies are all single flight models often even with no more than two fare classes. Network models are generally too sizeable already for such extensions.

Standard revenue management models as we discuss in the following sections do not consider customer choice behaviour. However, a great deal of flexibility is generally brought into these models by the use of practical experience. A great deal of the customer choice behaviour can, for example, be captured artificially in the demand forecasts. How important it is to be aware of the problem, however, becomes evident from Cooper et al. (2004). They describe what is called the spiral-down effect which leads to an ever decreasing number of high-fare passengers when revenue management practitioners do not take buy-up behaviour into account.

2.3 Booking Control Policies

Before we discuss the mathematical models used for airline revenue management, we first introduce the types of booking control used to translate the models into practical on-line policies. The goal of a booking control policy is to obtain the right passenger-mix such that revenues are maximized. There are two approaches to this problem. The first is to pre-specify a desirable allocation of the seats over the various types of passengers, and the second is to approximate the value for which each seat can be sold. These two approaches have resulted in two popular and widely used booking control policies: booking limits and bid prices.

2.3.1 Booking Limits

A booking-limit control policy, limits the number of passengers to accept in each fare class. Thus, if the booking limit for class Q is set to 35, than the number of passengers accepted in class Q is no more than 35. This way the total seat inventory can be partitioned over the various fare classes in order to obtain the desired passenger-mix. Booking requests are accepted as long as the booking limit has not been reached. Also used in practice, is a dual definition of a booking-limit policy, known as a protection levels policy. For this policy, a pre-specified number of seats are protected for each fare class. Booking limits and protection levels both partition the seat inventory over the different types of passengers and are basically equivalent.

Booking limits (and protection levels) are considered to provide a robust solution. That is, the partitioning they provide can be held fixed for as long as no changes in the demand distribution are foreseen. This is supported by Cooper (2002) who proves that a fixed booking-limit policy is asymptotically optimal as demand and capacity increase proportionally and the right booking limits are used. More surprising, Cooper also shows that adjusting the booking limits during the booking process can lead to a lower expected revenue for the policy. Nevertheless, in practice, booking limits tend to be adjusted on a constant basis in order to stay up-to-date with unforeseen fluctuations in demand that would otherwise reduce the performance of a fixed policy.

The problem with pre-specifying a partitioning of the seat inventory is that demand can deviate from what is taken into account. Considering the fact that more high-fare bookings can arrive than accounted for, it is counterintuitive to set a booking limit for them. In fact, whenever a high-fare booking request arrives and seats are still available, it should always be accepted. Therefore, booking limits (and protection levels) are often nested. Nested booking limits, limit the availability of the seats in a hierarchical manner. This means that any type of booking for which the booking limit has been reached, can tap into the seat inventory reserved for any lower valued type of booking. This way, the nested booking limit for the highest fare class is set equal to the full capacity of the plane. The nested booking limit for the second highest fare class is set to the full capacity of the plane minus the initial booking limit for the highest fare class and so on.

For a single flight the different types of passengers can simply be nested by fare class. For a network of flight, however, determining a nesting order is not trivial. The booking request with the highest fare is not necessarily the one with the highest contribution to the network of flights as a whole. The question is how to rank two requests that make use of different units of capacity. Williamson (1992) studies different nesting techniques for network models. She finds that nesting by straightforward variables such as fare class and fare level are outperformed by nesting based on an approximation of the opportunity costs of the seats used. Such approximations of the opportunity costs can be obtained from the mathematical programming models that we discuss in Section 2.5. The contribution of each booking request to the network of flights is then approximated by its fare minus the opportunity costs of the seats it uses. A drawback of this nesting technique is that the opportunity costs change over time as the occupation of the flights change. Therefore, the nesting order will have to be adjusted throughout the booking process.

Many of the mathematical models proposed to construct booking limits provide as good a non-nested partitioning of the seats as possible given a certain demand distribution. After that, a nesting technique is applied to provide nested booking limits. We note, however, that this approach can easily lead to overprotection of the more profitable booking requests. Because these booking requests can make use of the seats appointed to lower booking requests, the need to value the upward potential of their demand has decreased. An exception to this is when all lower requests arrive before the higher requests, as is the case when a discount is given for advance booking. In this case, there is no nested capacity for the higher requests to make use of.

2.3.2 Bid Prices

A bid price policy is a type of control that is directly linked to the opportunity costs (or replacement costs) of a seat. For this policy, a bid price is set for each flight which specifies a threshold value for which a seat can be sold at this point of time. A booking request is accepted only if its fare exceeds the sum of the bid prices of the flights it uses. The value of a bid price typically depends on the remaining capacity, the remaining time and expectations about demand. In a way, bid prices are much simpler to implement than booking limits. For a bid-price policy, only one threshold value has to be stored for each flight. Booking limits, on the other hand, are fare class based. This means that a booking limit has be stored for every possible type of booking, which are generally many per flight.

A drawback of bid prices is that they do not provide a robust solution. A bid-price policy only performs well if it is continuously updated. If one bid price would apply over a longer period of time, this would imply that not one request is accepted for any of the fare classes that fall under the bid price. Likewise, it does not restrict the number of bookings accepted for the fare classes that do exceed the bid price, even if they do so only marginally. However, if the policy is updated frequently enough, it can open or close fare classes for booking by lowering or raising the bid price.

Talluri and van Ryzin (1998) provide a theoretical framework for the use of bid prices. They show that a bid-price policy is only optimal in the case when the opportunity costs of a combination of flights are equal to the sum of the opportunity costs of the individual flights. This kind of linearity does not hold in general. They show, however, that a bid-price policy is asymptotically optimal as demand and capacity increase proportionally and the right bid prices are used.

2.4 Single Flight Models

In this section we give an overview of the mathematical models presented throughout the literature for single flight revenue management. We distinguish two categories of models: static and dynamic. Static models provide a fixed booking control policy based on the total future demand for each fare class as predicted at the time of optimization. These are generally booking-limit or protection-level policies. Brumelle and McGill (1993) show that under the assumption that booking requests come in sequentially in order of increasing fare level, i.e. low-fare bookings arrive before high-fare bookings, static models actually provide an optimal policy as long as no change in the probability distributions of demand is foreseen. When demand comes in sequentially in order of increasing fare level, the booking period can be divided into intervals for which all booking requests belong to the same fare class. The number of booking requests to accept for a fare class can then be fixed for the time interval its requests come in, since no additional information on any of the other fare classes will arrive to change the situation at hand. The sequential arrival assumption is not entirely implausible in the single flight case where discount fares are often offered for booking in advance. In general, however, this assumption will not hold.

Dynamic models approximate the opportunity costs of a seat at any given point in time based on the remaining capacity, the remaining time and expectations about demand. Dynamic models can be seen as a bid-price policy for which the bid price is recalculated every time a new booking request occurs. In Section 2.4.1 we discuss the static models and in Section 2.4.2 the dynamic models.

2.4.1 Static Models

Littlewood (1972) was the first to propose a solution method for the airline revenue management problem. He considered a single flight with two fare classes. The idea of his scheme is to equate the marginal revenues in each of the two fare classes. He suggests closing down the low fare class when the certain revenue from selling another low-fare seat is exceeded by the expected revenue of selling the same seat at the higher fare. That is, low-fare booking requests should be accepted as long as

$$r_2 \geq r_1 \Pr(D_1 > x_1), \tag{2.1}$$

where r_1 and r_2 are the high and low fare levels respectively, D_1 denotes the demand for the high fare class, x_1 is the number of seats to protect for the high fare class and $\Pr(D_1 > x_1)$ is the probability of selling all protected seats to high-fare passengers. The smallest value of x_1 that satisfies the above condition is the number of seats to protect for the high fare class, and is set as the protection level. This can also be seen as setting nested booking limits for the high and low fare classes equal to $y_1 = C$ and $y_2 = C - x_1$, where C is the seat capacity of the plane.

Belobaba (1987) suggests using Littlewood's rule for multiple fare classes. Assume that there are n fare classes indexed from high to low, that is $r_1 > r_2 > \ldots > r_n$. Then, to construct a nested booking limit for fare class j, Belobaba suggests using Littlewood's rule to systematically compare class j with each higher fare class k. This way, protection levels are obtained for the number of seats to protect from class j for class $k < j$, denoted by x_k^j, as follows:

$$\Pr(D_k > x_k^j) = \frac{r_j}{r_k}. \tag{2.2}$$

A nested booking limit for class j is then given by: $y_j = C - \sum_{k=1}^{j-1} x_k^j$, which is the total capacity minus the seats protected from class j for the more profitable fare classes. Belobaba introduces the term expected marginal seat revenue (EMSR) for the approach. The method is known as the EMSR-a method. An alternative method often used in practice is the EMSR-b method. This method aggregates the demand of the higher fare classes rather than the individual protection levels. For this method, the combined protection level for the classes 1 to j-1 is given by:

$$\Pr(\overline{D}_{j-1} > x_{j-1}) = \frac{r_j}{\overline{r}_{j-1}}, \tag{2.3}$$

where $\overline{D}_{j-1}$ is the aggregated demand for the first j-1 classes and $\overline{r}_{j-1}$ is the weighted average of the fares of the first j-1 classes. A nested booking limit for class j is obtained by: $y_j = C - x_{j-1}$. Although the EMSR-a and EMSR-b methods are popular and easy to use in practice, they do not provide the theoretical optimal solution for the multiple fare class problem. In order to obtain the number of seats to protect from class j, the EMSR-a method aggregates the individual protection levels obtained for the fare classes higher than j. This is not optimal because of the statistical averaging effect that occurs when aggregating random variables. The EMSR-b method aggregates the demand before constructing a joint protection level for all classes higher than j, but approximates the revenue obtained by the higher fare classes somewhat roughly by its weighted average.

Under the assumption that booking requests occur in order of increasing fare, static models that provide the theoretical optimal solution method do exist. Such optimal policies have been presented independently by Brumelle and McGill (1993), Curry (1990) and Wollmer (1992). Curry uses continuous demand distributions and Wollmer uses discrete demand distributions. The approach Brumelle and McGill propose, is based on sub-differential optimization and admits either discrete or continuous demand distributions. They show that an optimal set of nested protection levels must satisfy the conditions:

$$\delta_{+} ER_j(x_j) \leq r_{j+1} \leq \delta_{-} ER_j(x_j), \qquad \forall\, j = 1, 2, ..., n\text{-}1, \tag{2.4}$$

where $ER_j(x_j)$ is the expected revenue from the j highest fare classes when x_j seats are protected for those classes and δ_+ and δ_- are the right and left derivatives with respect to x_j respectively. These conditions express that a change in x_j away from the optimal level in either direction will produce a smaller increase in the expected revenue than an immediate increase of r_{j+1}. The same conditions apply for discrete and continuous demand distributions. Notice, that it is only necessary to set n-1 protection levels when there are n fare classes on the flight, because no seats will have to be protected for the lowest fare class. Brumelle and McGill show that under certain continuity conditions the conditions for the optimal nested protection levels reduce to the following set of probability statements:

$$\begin{aligned} r_2 &= r_1 \Pr(D_1 > x_1) \\ r_3 &= r_1 \Pr(D_1 > x_1 \cap D_1 + D_2 > x_2) \\ &\vdots \\ r_n &= r_1 \Pr(D_1 > x_1 \cap D_1 + D_2 > x_2 \cap ... \cap D_1 + D_2 + ... + D_{n-1} > x_{n-1}). \end{aligned} \tag{2.5}$$

These statements have a simple and intuitive interpretation, much like Littlewood's rule, and are based on the idea of equating the marginal revenues in the various fare classes. Robinson (1995) finds the optimality conditions when the assumption of a sequential arrival order with monotonically increasing fares is relaxed into a sequential arrival order with an arbitrary fare order. Furthermore, Curry (1990) provides an approach to apply his method to a network setting. He suggests nesting the various fare classes on each route of connecting flights and applying the single flight model to each individual route.

Van Ryzin and McGill (2000) introduce a simple adaptive approach for finding protection levels for multiple nested fare classes, which has the distinctive advantage that it does not need any demand forecasting. Instead, the method uses historical observations to guide adjustments to the protection levels. They suggest adjusting the protection level x_j upwards after each flight if all the fare classes up to j reached their protection levels, and downwards if this has not occurred. They prove that under reasonable regularity conditions, the algorithm converges to the optimal nested protection levels. This scheme of continuously adjusting the protection levels has the advantage that it does not need any demand forecasting and therefore is a way to get around all the difficulties involving this practice. However the updating scheme does need a sufficiently large sequence of flights to converge to a good set of protection levels. In practice, such a start-up period will generally not be granted when there are profits to be made.

The solution methods in this paragraph are all static. This class of solution methods can be optimal under the sequential arrival assumption as long as no change in the probability distributions of the demand is foreseen. However, information on the actual demand process can reduce the uncertainty associated with the estimates of demand. Hence, repetitive use of a static method over the booking period based on the most recent demand and capacity information is the general way to proceed.

2.4.2 Dynamic Models

Contrary to static models, dynamic models do not specify a fixed policy for use over a longer period of time. Instead, they monitor the state of the booking process over time and decide on acceptance of a particular booking request when it arrives, based on the state of the booking process at that point in time.

Lee and Hersh (1993) study the standard version of the problem. They consider a discrete-time dynamic programming model, where demand for each fare class is modelled by a non-homogeneous Poisson process. Using a non-homogeneous Poisson process gives rise to the use of a Markov decision model in such a way that, at any given time t, the booking requests before time t do not affect the decision to be made at time t except for the capacity that has already been used. The states of the Markov decision model are only dependent on the time until the departure of the flight and on the remaining capacity. The booking period is divided into a number of decision periods. These decision periods are sufficiently small such that not more than one booking request arrives within such a period. The state of the process changes every time a decision period elapses or the available capacity changes. Let $V_t(c)$ be the optimal expected revenue that can be generated given a remaining capacity of c seats and with t time units left to go. Further, let R_t be a random variable, with $R_t = r_j$ if a booking request for class j occurs with t time units to go, and $R_t = 0$ otherwise. Then $V_t(c)$ must satisfy the Bellman equation:

$$\begin{aligned} V_t(c) &= E\left[\max_{x\in\{0,1\}}\left\{R_t x + V_{t-1}(c-x)\right\}\right] \\ &= V_{t-1}(c) + E\left[\max_{x\in\{0,1\}}\left\{\left(R_t - \Delta V_{t-1}(c)\right)x\right\}\right], \end{aligned} \tag{2.6}$$

where x is the decision variable that determines whether to accept ($x = 1$) or deny ($x = 0$) a booking request, and $\Delta V_{t-1}(c)$ is the expected marginal value of a seat, given by: $\Delta V_{t-1}(c) = V_{t-1}(c) - V_{t-1}(c-1)$. Boundary conditions for the problem are:

$$V_0(c) = 0, \qquad \forall\ c = 0, 1, \ldots, C,$$
$$V_t(0) = 0, \qquad \forall\ t = 0, 1, \ldots, T,$$

since there is no revenue to be gained when there is no capacity or time left.

From (2.6) Lee and Hersh derive the optimal control policy, which is to accept a class j request, with t time units to go and capacity c available, if and only if:

$$r_j \geq \Delta V_{t-1}(c). \tag{2.7}$$

This decision rule says that a booking request is only accepted if its fare exceeds the opportunity costs of the seat, defined here by the expected marginal value of the seat. Lee and Hersh show that solving the model under the decision rule given by (2.7) results into a booking policy that can be expressed as a set of critical values for either the remaining capacity or the time until departure. For each fare class the critical values provide either an optimal capacity level for which booking requests are no longer accepted in a given decision period, or an optimal decision period after which booking requests are no longer accepted for a given capacity level. The critical values are monotone over the fare classes. Lee and Hersh also provide an extension to their model to incorporate group arrivals.

Kleywegt and Papastavrou (1998) demonstrate that the problem can also be formulated as a dynamic and stochastic knapsack problem (DSKP). Their work is aimed at a broader class of problems than only the single flight revenue management problem considered here, and includes the possibility of stopping the process before time 0 with a given terminal value for the remaining capacity, waiting costs for capacity unused and a penalty for rejecting an item. Their model is a continuous-time model, but they only consider homogeneous arrival processes for the booking requests. In a recent paper Kleywegt and Papastavrou (2001) extend their model to allow for group arrivals.

Subramanian et al. (1999) extend the model proposed by Lee and Hersh to incorporate cancellations, no-shows and overbooking. They also consider a continuous-time arrival process as a limit to the discrete-time model by increasing the number of decision periods. Liang (1999) reformulates and solves the Lee and Hersh model in continuous-time. Van Slyke and Young (2000) also obtain continuous-time versions of Lee and Hersh' results. They do this by simplifying the DSKP model to the more standard single flight revenue management problem and extending it for non-homogeneous arrival processes. They also allow for group arrivals. Brumelle and Walczak (2003) extend the problem with non-homogeneous semi-Markov arrivals which allows for correlated demand between the fare classes. They also include group arrivals and overbooking. Lautenbacher and Stidham (1999) link the dynamic and static approaches. They demonstrate that a common Markov decision process underlies both approaches and formulate an omnibus model which encompasses the static and dynamic models as special cases.

2.5 Network Models

Airlines typically offer many different flights that can be combined by a passenger in order to reach his/her final destination. This means that an airline offers different route/fare-class combinations, also known as origin-destination/fare-class (ODF) combinations. As discussed in Section 2.2.1, this creates the problem of comparing requests for routes that make use of different flights. One way to do this is to distribute the revenue of a route over its individual flights, which is called prorating, and apply single flight models to the individual flights. Williamson (1992) investigates different prorating strategies, such as prorating based on mileage and on the ratio of the local fare levels. She concludes, however, that mathematical programming models that take into account the network of flights as a whole can produce a considerable revenue gain over simple prorating techniques.

In Section 2.5.1 we first discuss the structure of the theoretically optimal control policy before we present the most common mathematical programming models for network revenue management in Section 2.5.2. In sections 2.5.3 and 2.5.4 we discuss how nested booking-limit and bid-price policies can be constructed from the mathematical programming models. We finish this section in Section 2.5.5 with some dynamic approximation schemes for the network revenue management problem.

2.5.1 Optimal Control

Just as for the single flight problem, a dynamic formulation with its optimal control policy can be formulated for the network revenue management problem. In order to do so, consider a flight network consisting of m flights and n route/fare-class combinations. Let $\mathbf{r}$

$= (r_1, r_2, \ldots, r_n)^{\mathrm{T}}$, $\mathbf{D} = (D_1, D_2, \ldots, D_n)^{\mathrm{T}}$ and $\mathbf{c} = (c_1, c_2, \ldots, c_m)^{\mathrm{T}}$ be the fares, demand and capacities of the various route/fare-class combinations and flights. Further, define the matrix $\mathbf{A} = [a_{ij}]$, such that $a_{ij} = 1$ whenever route/fare-class combination j uses flight i and $a_{ij} = 0$ otherwise. We denote the j^{th} column of $\mathbf{A}$ by $\mathbf{A}_j$, which gives the capacities used by route/fare-class combination j. Since there are generally multiple fare classes for each route and these are all considered to be different 'products', multiple route/fare-class combinations will have the same incidence vector $\mathbf{A}_j$. In order to obtain the Bellman equation for the optimal expected revenue function $V_t(\mathbf{c})$, define $\mathbf{R}_t = (R_{t,1}, R_{t,2}, \ldots, R_{t,n})^{\mathrm{T}}$ as a random vector for which $R_{t,j} = r_j$ if a request for class j occurs at time t, and $R_{t,j} = 0$ otherwise. Finally, let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ be the control variable denoting whether to accept a class j request ($x_j = 1$) or not ($x_j = 0$). Then, $\mathbf{x}$ is restricted to the set $X_t(\mathbf{c}) = \left\{\mathbf{x} \in \{0,1\}^n : \mathbf{Ax} \le \mathbf{c}\right\}$ and the Bellman equation is given by:

$$V_t(\mathbf{c}) = E\left[\max_{\mathbf{x} \in X_t(\mathbf{c})} \left\{\mathbf{R}_t^{\mathrm{T}}\mathbf{x} + V_{t-1}(\mathbf{c} - \mathbf{Ax})\right\}\right], \tag{2.8}$$

with the boundary condition:

$$V_0(\mathbf{c}) = 0, \qquad \forall\ \mathbf{c}.$$

The optimal control policy can easily be obtained from (2.8) and says to accept a class j booking request, with t time units to go and capacity $\mathbf{c}$ available, if and only if sufficient capacity is available and:

$$r_j \ge V_{t-1}(\mathbf{c}) - V_{t-1}(\mathbf{c} - \mathbf{A}_j)\,. \tag{2.9}$$

The optimal control policy provided here is a natural extension of the one presented in Section 2.4.2 for the single flight case. Computationally, however, there is a major difference between the two. Whereas the single flight optimal control policy can be calculated by backward induction for reasonable problem instances, this is computationally far from feasible for the network case. Because of the size and the combinatorial aspects of the network revenue management problem, obtaining a value for one specific $V_t(\mathbf{c})$ is already hard. In the next section we provide mathematical programming models that can be used to approximate the optimal expected revenue for a given situation.

2.5.2 Mathematical Programming Models

The mathematical programming models that we provide in this section are aimed at finding the seat allocation that maximizes the total expected revenue of the network and satisfies the capacity constraints on the various flights. The models can therefore be used to approximate the value $V_t(\mathbf{c})$ for a given t and $\mathbf{c}$. This approximation can be used directly for the accept/deny decision. However, the models generally also provide the means to construct booking-limit and bid-price policies, which provide much more practical booking control policies. The main difference between the models that we present in this section is in the way they account for the stochastic nature of demand.

The first full network formulation of the network revenue management problem has been proposed by Glover et al. (1982). They formulate the problem as a minimum cost network flow problem with one set of arcs corresponding to the flights and another set corresponding to the route/fare-class combinations. The method is aimed at finding the flow on each arc in the network that maximizes revenue, without violating the capacity constraints on the flights and upper-bounds posed by the demand forecasts for the route/fare-class combinations. A drawback of the network flow formulation is that it cannot always discriminate between the routes chosen from an origin to a destination. Therefore, this formulation only holds when passengers are path-indifferent. The advantage of the formulation is that it is easy to solve and can be re-optimized very fast.

A formulation of the problem that is able to distinguish between the different routes from an origin to a destination, is given by the integer programming model underlying the network flow formulation. It is common practice to solve the LP relaxation of this model rather than the integer programming problem, since the integer programming problem is usually hard to solve when the number of decision variables and constraints is large and the LP relaxation provides a good approximation for it. The LP relaxation of the model is known as the deterministic linear programming (DLP) model and approximates the optimal expected revenue as follows:

$$V_t^{DLP}(\mathbf{c}) = \quad \max \quad \mathbf{r}^{\mathrm{T}}\mathbf{x} \qquad (2.10)$$
$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \le \mathbf{c}$$
$$0 \le \mathbf{x} \le E[\mathbf{D}],$$

where $E[\mathbf{D}]$ denotes the expected demand for the various route/fare-class combinations and $\mathbf{x}$ gives the partitioning of the seat inventory. In this model, the demand is treated as if it takes on a known value, e.g. as if it is deterministic, and no information on the demand distributions is taken into account. Accordingly, the model produces the optimal seat allocation if the expected demand corresponds perfectly with the actual demand. The allocation of the seats obtained by the DLP model can be used as booking limits for an on-line policy. Chen et al. (1998) show that $V_t^{DLP}(\mathbf{c})$ provides an upper-bound for the optimal expected revenue $V_t(\mathbf{c})$. In sections 2.5.3 and 2.5.4 we discuss how to obtain nested booking limits and bid prices based on the mathematical programming models presented in this section.

The DLP model is a deterministic model and will never reserve more seats for a higher fare class than the airline expects to sell on average. In fact, it does not recognize the fact that $\Pr(D_j \ge k) \ge \Pr(D_j \ge k+1)$ at all. In order to determine whether reserving more seats for more profitable route/fare-class combinations can be rewarding, it is necessary to incorporate the stochastic nature of demand in the model. Wollmer (1986) develops a model which incorporates probabilistic demand into a network setting. It makes use of the same expected marginal revenue principles as Littlewood and Belobaba do for the single flight case. Therefore the model is called the EMR model. It is formulated as follows:

$$V_t^{EMR}(\mathbf{c}) = \quad \max \quad \sum_{j=1}^{n} r_j \sum_{k=1}^{M_j} z_j^k \Pr(D_j \geq k) \tag{2.11}$$

$$\text{s.t.} \quad x_j = \sum_{k=1}^{M_j} z_j^k \qquad \forall\, j = 1, 2, \ldots, n$$

$$\mathbf{Ax} \leq \mathbf{c}$$

$$0 \leq z_j^k \leq 1 \qquad \forall\, j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, M_j,$$

where M_j is an upper-bound to the number of seats allocated to route/fare-class combination j. A possible value for M_j is for example the smallest capacity remaining on any of the flights j uses. The decision variable z_j^k takes on the value 1 when k seats or more are allocated to the route/fare-class combination j and 0 otherwise. The coefficient of z_j^k in the objective function represents the expected marginal revenue of allocating an additional k^{th} seat to the route/fare-class combination. A drawback of the EMR model is clearly the large amount of decision variables, which makes the model impractical to use. Also, the EMR model makes use of the full distribution function of the demand for each route/fare-class combination. Obtaining such a full distribution is often difficult and the accuracy with which this can be done is often dubious. Chen et al. (1998) show that $V_t^{EMR}(\mathbf{c})$ provides a lower-bound for the optimal expected revenue $V_t(\mathbf{c})$.

De Boer et al. (2002) introduce a variant to the EMR model for which they use a coarser demand discretization. They link the model to stochastic programming techniques and it is therefore known as the stochastic linear programming (SLP) model. It incorporates the stochastic nature of demand by discretizing it to a limited number of values: $d_j^1 < d_j^2 < \ldots < d_j^{N_j}$, where N_j is the number of discretization points for route/fare-class combination j. The SLP model is given by:

$$V_t^{SLP}(\mathbf{c}) = \quad \max \quad \sum_{j=1}^{n} r_j \sum_{k=1}^{N_j} z_j^k \Pr(D_j \geq d_j^k) \tag{2.12}$$

$$\text{s.t.} \quad x_j = \sum_{k=1}^{N_j} z_j^k \qquad \forall\, j = 1, 2, \ldots, n$$

$$\mathbf{Ax} \leq \mathbf{c}$$

$$0 \leq z_j^1 \leq d_j^1 \qquad \forall\, j = 1, 2, \ldots, n$$

$$0 \leq z_j^k \leq d_j^k - d_j^{k-1} \qquad \forall\, j = 1, 2, \ldots, n, k = 2, 3, \ldots, N_j.$$

The decision variables z_j^k each accommodate for the part of the demand D_j that falls in the interval $(d_j^{k-1}, d_j^k]$. Summing the decision variables z_j^k over all k, gives the total number of seats allocated to route/fare-class combination j. The EMR model is a special case of the SLP model that can be obtained by letting $d_j^1 = 1$ and $d_j^k - d_j^{k-1} = 1$ for all $k = 2, 3, ..., M_j$. The SLP formulation is, however, more flexible because it allows a reduction of the number of decision variables by choosing a limited amount of demand scenarios. If only the expected demand is considered as a possible scenario, the SLP model reduces to the DLP model. In fact, the DLP and EMR models can be seen as the two extremes that can be obtained from the SLP model. The first is obtained by considering only one demand scenario, the latter by considering all possible scenarios. The fact that $V_t^{DLP}(\mathbf{c})$ provides an upper-bound and $V_t^{EMR}(\mathbf{c})$ provides a lower-bound for the optimal expected revenue $V_t(\mathbf{c})$, supports the use of the SLP model which encompasses both models. De Boer et al. (2002) show that the SLP model produces better results than the DLP model and that including more than 3 or 4 demand scenarios in the SLP model does not increase the performance significantly.

The mathematical programming models discussed in this section are capable of capturing the combinatorial aspects of the network revenue management problem. We discuss how to derive nested booking limits and bid prices from the models in the next sections.

2.5.3 Constructing Nested Booking Limits

The mathematical programming models presented in the previous section, provide an allocation of the seat inventory over the various types of demand. It seems natural to interpret this allocation as a set of non-nested booking limits. How to obtain nested booking limits, however, is not trivial in a network environment. In order to obtain nested booking limits, a nesting order has to be determined first. Williamson (1992) suggests nesting the route/fare-class combinations by the incremental revenue that is generated if an additional seat is made available for the route/fare-class combination while everything else remains unchanged. For the DLP model, she approximates this by the dual price of the corresponding demand constraint. De Boer et al. (2002) stick to Williamson's idea of using the net contribution to network revenue of the route/fare-class combinations to determine a nesting. However, since their SLP model does not have the demand restrictions present in the DLP model, they approximate the opportunity costs of a route/fare-class combination by the sum of the dual prices of the capacity constraints it uses. An approximation of the net contribution to network revenue, $\bar{r}_j$, is then obtained by subtracting this from the fare level:

$$\bar{r}_j = r_j - \boldsymbol{\mu}^{\mathrm{T}} \mathbf{A}_j, \tag{2.13}$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_m)^{\mathrm{T}}$ denotes the dual prices of the capacity constraints. For the DLP model this nesting method is equivalent to Williamson's approach. The advantage of this method over Williamson's, is that it can be applied for both deterministic and stochastic programming models.

After determining a nesting order on each flight, nested booking limits can be constructed. Let H_j^i be the set of route/fare-class combinations that have a higher rank than j on flight i, i.e. those route/fare-class combinations on flight i that are deemed more profitable for the network than j. Then the nested booking limit for route/fare-class combination j on flight i is given by:

$$y_j^i = c_i - \sum_{k \in H_j^i} x_k \ . \tag{2.14}$$

This illustrates that nested booking limits are obtained from non-nested booking limits by allowing route/fare-class combinations to make use of all seats on the flight except for the seats reserved for higher ranked combinations. Since the capacity, and therefore also the nested booking limit, is defined per flight, the number of seats available for a certain route/fare-class combination can vary over the different flights it uses. Because of this, a booking request is only accepted if its nested booking limits on all flights it uses, allow for it.

Nested booking limits clearly have their advantages as opposed to non-nested booking limits. However, although widely used in practice, the methods discussed above for obtaining nested booking limits do have their shortcomings. First of all, an optimal allocation of the seats as provided by one of the mathematical programming models is adjusted to allow nesting. This disregards the fact that the solution of the model would have looked differently should it have provided the optimal nested allocation of the seats. Secondly, the nested booking limits are defined per flight, whereas the contribution of the nesting scheme to the whole network should be taken into account.

In a recent study, Bertsimas and de Boer (2001) introduce a simulation-based method for obtaining nested booking limits using stochastic gradients. They define the expected revenue as a function of the booking limits and their aim is to find those booking limits that optimize the function. The DLP model is used to generate an initial solution which takes the combinatorial aspects of the network into account and by which a nesting order can be determined. After that, the solution is gradually improved to make up for factors such as the stochastic nature of demand and nesting. The search direction is determined by the gradient of the expected revenue function. Because the expected revenue function is not known, it is approximated by means of simulation. The expected revenue generated by a set of booking limits is approximated by its average over a sequence of simulated demand realizations. The gradient of the function is approximated by the change in expected revenue caused by a small deviation in the booking limits.

Using simulation to find the right booking limits has the advantage that just about any demand pattern can be accounted for. The difficulty with this method is, however, to estimate the expected revenue function adequately. Bertsimas and de Boer also provide a way to construct bid prices in this manner.

Whereas Bertsimas and de Boer (2001) use a discrete model of capacity and demand, van Ryzin and Vulcano (2004) use a continuous model. This creates the possibility to go around all the problems concerning the discrete model. For the continuous model, van Ryzin and Vulcano are able to compute the gradient of the expected revenue function exactly instead of using first-difference estimates. In addition, they are able to prove that stochastic gradient methods are at least locally convergent.

2.5.4 Constructing Bid Prices

For a bid-price control policy, a bid price is set for each flight in the network reflecting the opportunity costs of a seat. A booking request is accepted only if its fare exceeds the sum of the bid prices of the flights it uses. Therefore, a booking request for route/fare-class combination j, is accepted if and only if there is sufficient capacity and:

$$r_j \geq \boldsymbol{\mu}^{\mathrm{T}} \mathbf{A}_j, \tag{2.15}$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_m)^{\mathrm{T}}$ are the bid prices set for the m flights. Looking at the optimal control policy provided by (2.9), it is easy to see that a bid-price policy provides the optimal control structure if: $V_{t-1}(\mathbf{c}) - V_{t-1}(\mathbf{c} - \mathbf{A}_j) = \boldsymbol{\mu}^{\mathrm{T}} \mathbf{A}_j$. This is the case when the opportunity costs of the capacity taken up by the booking request are equal to the sum of the opportunity costs of the individual flights.

The general way to approximate the opportunity costs of a seat is by the dual price of the capacity constraint of the flight in one of the mathematical programming models presented in Section 2.5.2. Notice that this measure is equivalent to the approximation de

Boer et al. (2002) use for the opportunity costs of a route/fare-class combination. Williamson (1992) uses the dual prices of the DLP model as bid prices, whereas de Boer et al. (2002) use the bid prices of their SLP model.

Talluri and van Ryzin (1999) introduce a simulation-based method to construct bid prices. The idea is to incorporate more stochastic information into the DLP model by replacing the expected demand by the random vector itself. They simulate a sequence of demand realizations and for each realization determine the optimal seat allocation. This can be done by applying the DLP model with the realization of the demand taking the place of the expected demand as the upper-bound for the number of bookings to accept. This way, each simulated demand sequence provides an optimal allocation of the seats with the corresponding dual prices. The bid price for a flight is then simply defined as the average value of the dual prices of the flight. This is known as the randomized linear programming (RLP) method. This method has the appealing advantage that it accounts for the stochastic nature of demand by ways of simulation. Simulation offers the flexibility to model various properties of the demand that would otherwise be very hard or even impossible to model.

A disadvantage of a bid-price control policy is that the bid prices have to be adjusted continuously. If a bid price is not adjusted regularly, this means that there is no limit to the number of bookings that can be accepted for a route/fare-class combination once it is open for bookings, i.e. once its fare exceeds the opportunity costs. This can lead to flights filling up with passengers that only marginally contribute to the network revenue. Talluri and van Ryzin (1998) provide a theoretical framework for the use of bid prices. They show that although bid prices are not optimal in general, they are asymptotically optimal when capacity and demand increase proportionally.

2.5.5 Dynamic Approximate Schemes

The booking control policies discussed thus far for the network revenue management problem are all static methods. They construct a booking-limit or bid-price policy for the entire booking horizon. Although the values are generally re-adjusted a number of times during the booking process, a model that would actually consider the fact that the policy is adjusted over time would probably provide different values in the first place. A dynamic programming approach to the network revenue management problem is computationally far from feasible. However, some dynamic approximation schemes have been suggested.

Chen et al. (1998) are the first to provide a dynamic approximation scheme for the network problem. They deduct the optimal control policy given by (2.9) and suggest to approximate the values $V_{t-1}(\mathbf{c})$ and $V_{t-1}(\mathbf{c} - \mathbf{A}_j)$ directly whenever a booking request for route/fare-class combination j is made. To approximate the values, the objective value of a mathematical programming model can be used for both situations. Subtracting the objective values gives the opportunity costs to which the fare can be compared. Chen et al. (1998) argue that the opportunity costs are overestimated by the DLP model and underestimated by the EMR model. Based on this idea, they formulate the following acceptance scheme for a booking request for route/fare-class combination j:

reject if: $r_j \leq \mu_{EMR}$, otherwise

accept if: $r_j \geq \mu_{DLP}$, otherwise

accept if: $r_j > x$, with x random from the interval $[\mu_{EMR}, \mu_{DLP}]$,

where μ_{EMR} and μ_{DLP} are the opportunity costs of the request as approximated by the EMR and DLP models. Evaluating the two models in two different states every time a booking request comes in obviously requires too much computation time for an on-line policy. Therefore, Chen et al. propose a method to estimate the value function of a model for each possible state beforehand. They evaluate the model on a carefully selected limited number of points in the state space and use these observations to estimate the value function of the model over the entire state space. The selection of the points is based on an Orthogonal Array method, and Multivariate Adaptive Regression Splines are used to estimate the value function of the model. With an approximation of the value function of each model available at any time, decisions can be made on-line. The difficulty of this scheme is to obtain good estimations of the value functions of the mathematical programming models.

For their simulation-based method, discussed in Section 2.5.3, Bertsimas and de Boer (2001) also make use of an expected future revenue function. They divide the booking period into smaller time intervals. This way, the computationally cumbersome simulation-based method for constructing booking limits can be applied to the current time interval only, taking into account the expected future revenue as a function of the remaining capacity. In order to estimate the expected future revenue function, the values for this function are evaluated over a carefully selected limited number of points in the state space and interpolated to the entire state space.

Just as Chen et al. (1998), Bertsimas and Popescu (2003) propose to approximate the values $V_{t-1}(\mathbf{c})$ and $V_{t-1}(\mathbf{c} - \mathbf{A}_j)$ directly in order to obtain an estimation of the opportunity costs of a request. They use the network flow formulation of the problem, proposed by Glover et al. (1982), to do this. This model is easy to solve and can be re-optimized very efficiently. A drawback of the network flow formulation is that it only holds when passengers are path-indifferent. Bertsimas and Popescu extend their method with a roll-out policy using simulation techniques. For this, multiple future demand realizations are simulated for which the revenues obtained by the current policy can be easily computed. Averaging the revenues over the simulated demand realizations provides an approximation of the expected future revenue that is used to approximate the current opportunity costs. This method evades any of the problems encountered for booking control policies discussed in the previous sections. However, although very interesting and insightful in its own right, the computation time involved in the method is such that it generally prevents it from being useful in practice.

2.6 Summary and Conclusion

In this chapter, we present the airline revenue management problem as it is generally considered. We discuss the difficulties concerning uncertain demand, on-line decision making and dealing with a network of flights. We also discuss topics closely related to the revenue management problem such as demand forecasting, overbooking and pricing. Further, we provide an overview of OR techniques for airline revenue management under the most common assumptions about demand behaviour. We distinguish two popular and widely used booking control policies: booking limits and bid prices. Both policies have their advantages and their disadvantages. Most notably, booking limits provide a robust solution for a longer period of time, but encounter problems with nesting. Bid prices on the other hand are closely related to the theoretically optimal control policy but need to be adjusted frequently. Neither of the policies is optimal, but they can be proven to be asymptotically optimal and are easy to use in practice.

The mathematical models used for single flight and network revenue management problems, are somewhat different. For the single flight case, simple static models based on the expected marginal value of a seat can be easily constructed for use in practice. Dynamic models can also be constructed and their optimal policy can be proven to possess nice structural properties as long as group arrivals and overbooking are not allowed. For practical use, however, the dynamic models are generally less feasible.

Because of the combinatorial aspects involved with the network revenue management problem, mathematical programming models are generally used for this problem. We present three of such models that vary in the manner that the uncertainty of the demand is accounted for. Based on the mathematical programming models, nested booking limits and bid prices can be constructed. We discuss various ways to do so that all have their stronger and weaker aspects. Dynamic models are computationally not feasible for the network problem. We do discuss some dynamic approximation schemes. However, the question remains whether the problems they encounter in order to get around the computational difficulties do not outweigh the positive effect of capturing the dynamics of the booking process.

Despite the level of attention for the airline revenue management problem throughout the years, some issues are still open for improvement. Recent studies have concentrated on finding better ways to construct booking limits and bid prices in a network environment (see: Bertsimas and de Boer (2001) and van Ryzin and Vulcano

(2004)) or estimating the expected future revenue function for dynamic approximation schemes for the network problem (see: Bertsimas and de Boer (2001), Bertsimas and Popescu (2003) and Chen et al. (1998)). These problems certainly provide issues for future research. The majority of the current research in airline revenue management, however, is aimed at relaxing one or more of the assumptions about demand behaviour discussed in Section 2.2.3 (see the references given in Section 2.2.3). Although conceptually very interesting, these studies are generally restricted to the single flight and often even two fare class case of the problem, which is far from practice.

As mentioned in the introduction, the airline revenue management problem as discussed in this chapter forms the basis for most other well-known revenue management applications. The models presented in Sections 2.4 and 2.5 are widely used for applications in the hotel, railroad and car rental industries with only minor adjustments.

Chapter 3

Models and Techniques for Hotel Revenue Management using a Rolling Horizon

3.1 Introduction

After the airline industry, the hotel industry is the second most popular application area of revenue management. As the success story of revenue management started to reach beyond the airline industry, hotels were quick to recognize the similarities between selling the seats on a plane to different types of passengers for different prices, and the situation hotels face when renting rooms. In this chapter, we provide models and techniques for hotel revenue management. We present both deterministic and stochastic mathematical programming models of which the latter has not been applied before to the hotel revenue management problem. We construct booking-limit and bid-price control policies based on the deterministic and stochastic models. We also construct randomized bid prices by simulation techniques. The performance of the booking control polices is evaluated in a simulated environment. Unlike previous studies, we extend the booking control policies with a rolling horizon of overlapping decision periods which conveniently captures the effects of overlapping stays.

In Section 3.2 we discuss the hotel revenue management problem and the literature presented on it. We present the mathematical models in Section 3.3 and the booking control policies based on them in Section 3.4. In Section 3.5 we describe the test case we construct in order to evaluate the different booking control policies. Finally, we present the results of our simulation study in Section 3.6 and conclude this chapter in Section 3.7.

3.2 Hotel Revenue Management

In the hotel industry different prices are charged for a room depending on features such as: the time of booking, company affiliation, multiple-day stays and the intermediary sales agent. This way, hotels offer the same rooms to different types of guests for different prices. While hotel managers would like to fill their hotels with highly profitable guests as much as possible, it is generally also necessary to allow for less profitable guests in order to prevent rooms from remaining vacant. The number of low-price guests in the hotel should be managed carefully, however, in order to be able to allocate as many high-price guests as possible. The booking requests come in gradually over time and the decision whether to accept a certain booking request or not has to be made at the moment it comes in. This is known as the hotel revenue management problem and resembles the airline problem discussed in the previous chapter.

In Section 3.2.1 we shortly discuss some of the difficulties in hotel revenue management and in Section 3.2.2 we provide an overview of the related literature on the problem.

3.2.1 Difficulties in Hotel Revenue Management

Many of the problems concerning revenue management in the airline industry, i.e. uncertain demand, on-line decision making and network effects, also apply to the hotel industry. The uncertainty of demand and the on-line decision making process are difficulties that directly translate to the hotel revenue management problem. They bring stochastic and dynamic aspects into the problem. Also the network effects caused in the airline industry by the network of flights that an airline typically offers, has its correspondence in the hotel industry. These network effects are caused by the fact that guests in a hotel can stay multiple days such that their stays can overlap. Consider for example, one guest staying from Monday to Friday and another from Thursday to Sunday. Then, the guests compete for the same room capacity on Thursday and Friday. Although a hotel manager would like to fill his hotel with guests who pay a high price, it can be more profitable to accept a lower price request when this request would also fill up capacity for other days where demand is low. Conceptually, a guest that stays in a hotel for multiple days can be seen as a passenger that uses multiple flights to reach his/her destination. However, whereas the flights offered by an airline span a finite network that is observed at a given point in time, the different stays in a hotel can continue to overlap over an infinite horizon. This means that no fixed decision period can be determined without missing some of the effects of the overlapping stays. Therefore, we apply the booking control policies over a rolling horizon of overlapping decision periods where all booking types that are open for booking are taken into account.

Another problem in hotel revenue management is that the room rates do not always provide the total profit associated to a guest. Often, profits generated by the bar, restaurant, casino or other facilities that a hotel offers, can be quite significant. Therefore, it is very interesting for a hotel to recognize the true profitability of a booking request. Tools such as a client ID card can provide a means for this. In practice, however, it is very difficult to estimate the full profitability of a booking request beforehand. For the remainder of this chapter we consider the profitability of a booking request to be given by its room rate.

Next to the fact that we assume the profitability of a booking request to be given by the price it pays for the room, we make use of the common assumptions in quantity-based revenue management that the room capacity and the general price structure are given and fixed. We also assume that there is a limit to the number of days that a guest can stay in the hotel and to the number of days that a booking can be made in advance. The first does not have to be imposed by the hotel explicitly but can rather be a natural bound observed in practice caused by the fact that guests will eventually leave the hotel. The second is generally determined by the reservation system that has a limited planning horizon. Further, we consider the general assumptions about demand behaviour. This means that we do not consider group bookings, cancellations and no-shows and assume that demand is independent of the booking control policy used. This last assumption has been the subject of debate but is difficult to relax. We refer to Section 2.2.3 for some references to studies that relax the assumption for the airline problem.

3.2.2 Literature

Hotel revenue management has received attention in a number of papers. Bitran and Mondschein (1995) and Bitran and Gilbert (1996) concentrate on the room allocation problem at the targeted booking day. This means that they concentrate on the stays that start on the current day itself. The hotel manager has to decide whether or not to accept a guest who shows up without a reservation (walk-in), taking into account the number of reservations already made and the potential number of remaining walk-ins. They formulate this problem as a stochastic and dynamic programming model. Bitran and Gilbert also provide three simple heuristics to construct booking control policies that can be used during the booking period.

Weatherford (1995) concentrates on a booking control policy for the booking period. He constructs booking limits based on a deterministic mathematical programming model similar to the one discussed in Section 2.5 for the airline problem. The booking limits are nested based on the dual prices of the model such that a booking request can always make use of the capacity reserved for any booking request that is considered to be less profitable. A drawback of this policy is that it considers demand to be deterministic. Weatherford allows for multiple-day stays and optimizes the model for a decision period consisting of a fixed set of target booking days. He does not account for overlapping stays that fall outside the set of target booking days. Nevertheless, he shows that taking into account multiple-day stays explicitly produces better results than when they are considered as a set of single-day stays.

Baker and Collier (1999) compare the performance of five booking control policies: two simple threshold approaches, the deterministic nested booking-limit policy proposed by Weatherford, the deterministic nested booking-limit policy that includes overbooking and finally a deterministic bid-price policy obtained from Weatherford's deterministic model. Baker and Collier compare the performance of the booking control policies under 36 hotel operating environments by means of simulation and advise on the best heuristic for each operating environment.

In this chapter we concentrate on the booking control problem. This makes our work comparable to the work of Weatherford (1995) and Baker and Collier (1999). Unlike these previous studies, we use the booking control policies over a rolling horizon of decision periods, such that all overlap between the different types of stay can be accounted for. Also, next to the booking control policies based on the well-known deterministic model, we obtain bid prices that account for demand uncertainty by simulation and we obtain nested booking limits and bid prices based on a mathematical programming model that accounts for the stochastic nature of demand. As Baker and Collier (1999), we compare the performance of the different methods in a simulated environment.

3.3 Mathematical Programming Models

The booking control policies that we consider for the hotel revenue management problem are based on an optimal allocation of the demand for a fixed set of target booking days. In this section, we present two mathematical programming models that can provide such an allocation. The models differ in the way that they take demand into account. In Section 3.3.1 we discuss a deterministic model that only considers a point estimate of the demand. In Section 3.3.2 we present a stochastic model that considers a discretization of the demand distribution.

3.3.1 Deterministic Model

The deterministic model that we consider is the same as Weatherford (1995) proposes for his nested booking-limit policy. This model uses the expected demand as a point estimate of the demand and obtains the optimal allocation of the rooms for it. This means that it treats demand as if it were deterministic and equal to its expectation. To formulate the model, we note that each type of booking request is defined by three aspects: its price class, its starting day and its length of stay. We consider n of such booking types. Further, we let the decision period of the model consist of m target booking days. Let $\mathbf{r} = (r_1, r_2, \ldots, r_n)^{\mathrm{T}}$, $\mathbf{D} = (D_1, D_2, \ldots, D_n)^{\mathrm{T}}$ and $\mathbf{c} = (c_1, c_2, \ldots, c_m)^{\mathrm{T}}$ denote the prices, demand and capacities of the various booking types and days. Further, we define the matrix $\mathbf{A} = [a_{ij}]$, such that $a_{ij} = 1$ if booking type j stays on day i and $a_{ij} = 0$ otherwise. We denote the j^{th} column of $\mathbf{A}$ by $\mathbf{A}_j$, which gives the capacities used by booking type j and consists of exactly as many 1 entries as the number of days that the guest wishes to stay in the hotel. The deterministic mathematical programming model can now be formulated as follows:

$$\begin{aligned} \max \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} && (3.1) \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \le \mathbf{c} \\ & 0 \le \mathbf{x} \le E[\mathbf{D}], \end{aligned}$$

where $E[\mathbf{D}]$ denotes the expected demand for the various booking types and $\mathbf{x}$ gives the partitioning of the room capacity. The objective of the model is to maximize revenues under the restriction that the total number of reservations for a day does not exceed the room capacity for that day. The number of rooms allocated to each booking type is restricted by the level of the demand, which in this model is replaced by its expectation. If demand actually corresponds with its expectation, the deterministic model provides the optimal room allocation.

Since it is usually difficult to optimize an integer programming model, the decision variables are generally not restricted to be integer. Although the constraint matrix is not totally unimodular, previous experiences of Williamson (1992) and de Boer et al. (2002) with the LP model in (3.1) show that when demand and capacity is integer, the optimal solution often is as well. When the model produces a fractional solution, however, it will generally not take much effort to produce an integer solution by applying branch-and-bound techniques.

3.3.2 Stochastic Model

The deterministic model approximates the distribution of the demand by a point estimate of its expectation. This means that it assumes $\Pr(D_j \geq d) = 1 \quad \forall \quad d \leq E[D_j]$ and $\Pr(D_j \geq d) = 0 \quad \forall \quad d > E[D_j]$, $\forall$ j = 1, 2, …, n. In reality, however, the distribution function is a smoothly decreasing function. Using such a rough estimate as the deterministic model does, means that the probability of another type j arrival is overestimated (by the value 1) as the number of arrivals has not reached its expectation yet and underestimated (by the value 0) as the expected number of arrivals has been reached. In this section we present a mathematical programming model that approximates the distribution function of demand more smoothly. This stochastic model was first introduced by De Boer et al. (2002) for the airline industry. The model incorporates the stochastic nature of demand by discretizing it to a limited number of values: $d_j^1 < d_j^2 < \ldots < d_j^{N_j}$, where N_j is the number of discretization points for booking type j. The stochastic model is then given by:

$$\max \quad \sum_{j=1}^{n} r_j \sum_{k=1}^{N_j} z_j^k \Pr(D_j \geq d_j^k) \tag{3.2}$$

$$\begin{aligned}
\text{s.t.} \quad & x_j = \sum_{k=1}^{N_j} z_j^k && \forall j = 1, 2, \dots, n \\
& \mathbf{Ax} \leq \mathbf{c} && \\
& 0 \leq z_j^1 \leq d_j^1 && \forall j = 1, 2, \dots, n \\
& 0 \leq z_j^k \leq d_j^k - d_j^{k-1} && \forall j = 1, 2, \dots, n, k = 2, 3, \dots, N_j.
\end{aligned}$$

The decision variables z_j^k each accommodate for the part of the demand D_j that falls in the interval $(d_j^{k-1}, d_j^k]$. Note that z_j^{k+1} will only be nonzero after z_j^k has reached its upper-bound of $d_j^k - d_j^{k-1}$, since $\Pr(D_j \geq d_j^k) \geq \Pr(D_j \geq d_j^{k+1})$. Summing the decision variables z_j^k over all k, gives the total number of rooms allocated to booking type j. As for the deterministic model, the decision variables of the stochastic model are not restricted to be integer.

The deterministic model can be obtained from the stochastic model by limiting the number of discretization points to one and setting it equal to the expected demand. Likewise, the EMR model introduced by Wollmer (1986) for the airline industry, can be obtained by fully discretizing the demand distribution for all possible values. De Boer et al (2002) show, however, that for the airline industry 3 or 4 discretization points suffice to capture most of the extra revenue generated by considering the stochastic nature of the demand. A discretization point can be interpreted as a possible demand scenario. We say that such a demand scenario occurs whenever the demand exceeds the level of the demand of the scenario. For each scenario k defined for booking type j, the level of the demand, i.e. d_j^k, and the probability that the scenario occurs, i.e. $\Pr(D_j \geq d_j^k)$, have to be determined. Note that the scenario probabilities are not mutual exclusive and therefore do not sum to 1.

3.4 Booking Control Policies

The mathematical programming models presented in the previous section form the basis for the booking control policies that we consider. In Section 3.4.1 we discuss how we construct nested booking limits from the models and in Section 3.4.2 how we obtain bid prices. In Section 3.4.3 we discuss how to use these booking control policies over a rolling horizon of decision periods.

3.4.1 Nested Booking Limits

The number of rooms allocated to each booking type by the models from the previous section, can easily be interpreted as booking limits. These limits can be used as the maximum number of booking requests to accept for each booking type during the booking period. It is never optimal, however, to reject a booking request when rooms are still available for other less profitable booking types, even if its own booking limit has been reached. Therefore, each booking type should be allowed to tap into the rooms allocated to any booking type that is less profitable. When this is allowed, the booking limits are called nested. In order to form nested booking limits, the different booking types need to be ranked by their contribution to the overall revenue of the hotel. When such a ranking is determined, a nested booking limit for a booking type can be determined by the sum of the rooms allocated to it and every other, lower ranked booking type.

It is not trivial what measurement to use when determining a nesting order for the different booking types. Using the price class does not take into account the length of the stay. Such a measurement will rank guests who are willing to pay more for one day above guests who are willing to pay a little less for multiple days, whereas the overall revenue generated by the multiple days will most likely be higher. Nesting by the complete revenue generated by the stay does take into account the length of the stay. But this measurement does not account for the load factors of the different days. Certain days can be very busy and always fully booked, whereas other days can be mainly vacant. A stay that occupies many busy days should be valued differently from a stay that uses mainly days with a lot of vacant rooms. One way to take into account all of these aspects, is to use the dual prices obtained from the underlying allocation model. The dual price corresponding to the capacity restriction for a day reflects the expected gain that can be obtained if one additional room were available on that day. It can be interpreted as the opportunity costs of a room. Adding the dual prices of all the days used by a stay, gives an indication of the opportunity costs of the stay. A measurement for nesting is then obtained by subtracting these opportunity costs from the revenue generated by the stay. Thus, a nesting order is based on:

$$\bar{r}_j = r_j - \boldsymbol{\mu}^{\mathrm{T}} \mathbf{A}_j, \tag{3.3}$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_m)^{\mathrm{T}}$ denotes the dual prices of the capacity constraints. Nested booking limits can now easily be constructed for the deterministic and stochastic models, which we will call the Deterministic Nested Booking Limits (DNBL) and the Stochastic Nested Booking Limits (SNBL) methods.

3.4.2 Bid Prices

The second type of booking control policy we study in this paper is the bid-price policy. This policy directly links the opportunity costs of a stay to the acceptance/rejection decision. A bid price is constructed for every day to reflect the opportunity costs of renting a room on that day. As before, we estimate the opportunity costs of renting a room on a particular day by the dual price of the capacity constraint for that day in the underlying mathematical programming model. A booking request is only accepted if the revenue it generates is greater than the sum of the bid prices of the days it uses. This means that a booking request for type j, is accepted if and only if there is sufficient capacity and:

$$r_j \geq \boldsymbol{\mu}^{\mathrm{T}} \mathbf{A}_j, \tag{3.4}$$

This way, bid prices can be constructed from the deterministic and stochastic models, which we will call the Deterministic Bid Prices (DBP) and the Stochastic Bid Prices (SBP) methods.

Next to the DBP and the SBP policies, we formulate one more bid-price policy. This policy was first suggested by Talluri and van Ryzin (1999) for the airline industry and makes use of simulation techniques to account for the stochastic nature of demand. Note that the deterministic model as presented in (3.1), can provide the optimal room allocation for any given demand realization by substituting it for the expected demand. Talluri and van Ryzin suggest to randomly simulate a sequence of demand realizations. Then, for each simulation the dual prices associated with the optimal allocation can be obtained. Averaging the dual prices over the different demand realizations provides a simple method to obtain bid prices that take different realizations of the demand into account. We call this the Randomized Bid Prices (RBP) method.

3.4.3 Rolling Horizon

The mathematical programming models that we presented in Section 3.4.1 provide an allocation of the rooms for a fixed decision period. However, we use them over a rolling horizon of decision periods. We assume that booking requests cannot be made more than F days in advance, and that the longest possible stay in the hotel consists of M days. The booking requests that come in at day t can then start their stay in the hotel at day t at the earliest and at day $t + F$ at the latest. The last possible day that a booking request can end, is at day $t + F + M$. Therefore, if a booking control policy is determined at day t, the decision period we consider is given by the time interval $[t, t + F + M]$. Within this decision period all overlap between the different booking types are taken into account, except for the overlap at the end of the interval corresponding to the stays that fall partly outside of the decision period. However, these are the booking types for which booking has only just opened. First of all, hardly any booking requests will come in this early in the booking process except for some extremely early bookings. Second, since booking has just started for those target booking days, the hotel will be nearly empty for those days and critical decisions will not have to be made yet. By the time critical decisions have to be made for these days, the decision period will have rolled forward and have captured all overlap between the neighbouring days.

The booking control policy is constructed at different points in time. Every time a new policy is constructed, the decision period rolls forward. The booking limits for the booking types that were already available for booking are adjusted, while new booking limits are constructed for the booking types that have just opened for booking. In the same manner, the bid prices for the days already in the decision period are adjusted, while new bid prices are constructed for the newly added days.

3.5 Description of the Test Case

The performance of the different booking control policies is evaluated by ways of simulation. In this section, we discuss the simulation environment. This is chosen such that it reflects the situation described to us by a hotel in the Netherlands. We consider a hotel with a total capacity of 150 identical rooms. These rooms can be rented out in 10 different price classes, described in Table 3.1. We consider that the maximal length of a stay is 7 days and that a booking request can be made at most 13 weeks in advance. We do not allow overbooking and do not consider cancellations, no-shows or group bookings. The booking control policies that we consider are updated on a weekly basis. But we shortly discuss the effects of doing this less or more often.

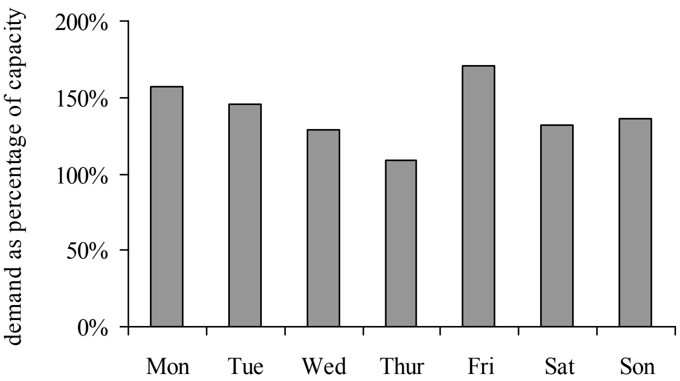
Table 3.1: Price Classes.

	Class	Price
1	Tourist Rate Tours & Groups	$ 50
2	Tourist Rate Low Budget	$ 75
3	Tourist Rate Packages	$ 110
4	Tourist Rate Medium Budget	$ 120
5	Rack Rate	$ 250
6	Corporate Rate Liaison Corporation	$ 75
7	Corporate Rate Management	$ 125
8	Corporate Rate Salesperson	$ 100
9	Corporate Rate MCI	$ 175
10	Corporate Rate other	$ 150

We simulate the arrivals of booking requests by a non-homogeneous Poisson process with intensities dependent on the price class, the starting day of the stay (e.g. Monday, Tuesday, etc.) and the time until the target booking day. We allow for different booking patterns for the different price classes to account for low tourist classes to book early in the booking process and high corporate classes to book at the end of the booking process among others. Further, we let some days, e.g. Friday, be more busy than other days, e.g. Thursday. In order to let the arrival intensities fluctuate over time, we divide the booking period into 10 smaller periods each with a constant arrival intensity. Just as Baker and Collier (1999) and Bitran and Mondschein (1995), we do not consider the length of the stay to be of influence on the arrival intensity. Instead, we model the length of the stay of each arrival by a logistic distribution with a parameter dependent on the price class and the starting day of the stay. A full presentation of the arrival intensities and parameter values for the logistic distribution is provided in Appendix 3.A.

The arrival intensities and the parameters for the logistic distribution are chosen to reflect a busy period in the hotel in which on average the total demand exceeds the capacity of the hotel. This is the situation in which revenue management produces the highest gains in revenue. The average level of the demand per day of the week expressed as a percentage of the capacity is presented in Figure 3.1. Note that one booking request can generate demand for multiple days. Figure 3.1 shows that in our test case Friday is the busiest day of the week with an average demand which exceeds capacity by more than 70%. On Thursday the average demand is only just above the capacity of the hotel, which makes it the least busy day of the week.

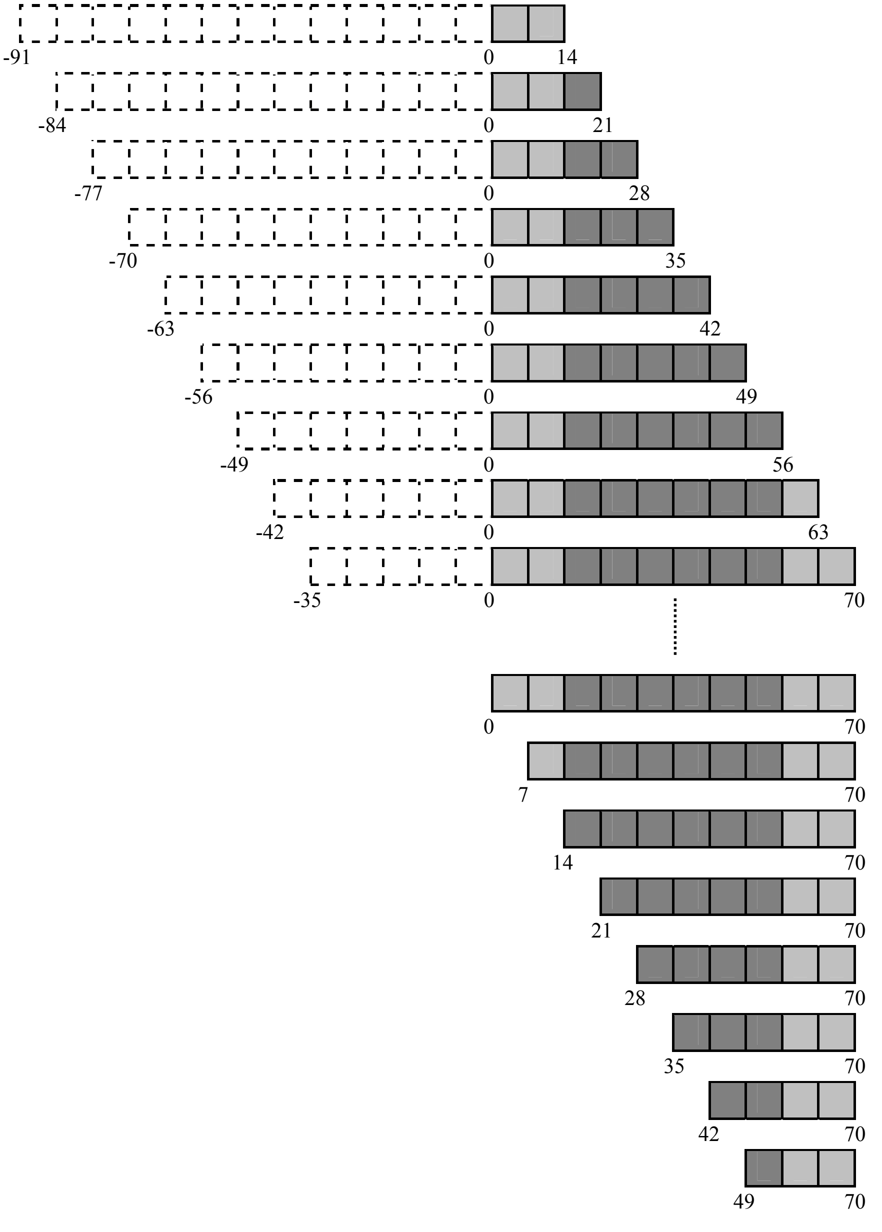
Figure 3.1: Average demand per day of the week as percentage of the capacity.



We compare the performance of the different booking control policies over a 6 week period. Since the hotel is empty at the beginning of the simulation, the booking control policies that are constructed at that time are not representative for the general situation. To be representative, they would have to consider the overlap of the guests already in the hotel based on prior booking decisions. In order to do so, we make use of a start-up period in which the simulation can reach a steady state before the start of the 6 evaluation weeks. Since the booking control policies remain fixed for a week and the maximum length of stay is also a week, the guests that are accepted by the initial non-representative booking control policies can overlap until the end of the second week. In order to refrain from all such guests in the evaluation period, we choose the start-up period to consist of 2 weeks. Likewise, we also use a cool-down period of 2 weeks to capture the overlap at the end of the evaluation period.

We consider the beginning of the first day of the start-up period to coincide with time $t = 0$, and the beginning of the second day with $t = 1$. This means that the first day of the start-up period takes place on the time interval $[0, 1)$. Then, because a booking request can be made 13 weeks in advance, the booking process starts at $t = -91$. At that moment, booking control policies are derived for all booking requests that can come in that week. Note that, since booking requests can book at most 91 days in advance and can stay at most 7 days in the hotel, booking control policies that are set in the week $[t, t + 7)$ have to consider the decision period $[t, t + 105)$. However, since we do not consider any arrivals before time $t = 0$, the decision period at time $t = -91$ only consists of $[0, 14)$. Likewise, the decision period considered at $t = -84$ is $[0, 21)$ and so on. Because our test case only spans 6 evaluation weeks plus two times 2 weeks to start-up and cool-down, the maximum length of the decision period in our test case will be 70 days. A graphical illustration of the rolling decision periods is given in Figure 3.2. In this figure, the start-up and cool-down periods are coloured light and the actual evaluation period is coloured dark.

Figure 3.2: Illustration of the rolling decision periods in the test case.



In conclusion, we summarize the aspects that define the simulation environment in Table 3.2.

Table 3.2: Specification of the test case.

10 price classes
150 identical rooms
max 7 day stay
max 90 days in advance booking
6 week evaluation period
2 week start-up and cool-down periods
demand exceeds capacity
different booking patterns for the different price classes
demand is independent of the booking control policies
no overbooking
no cancellations and no-shows
no group bookings

3.6 Computational Results

In Section 3.4, we specified the following five booking control policies:

- Deterministic Nested Booking Limits (DNBL)
- Deterministic Bid Prices (DBP)
- Stochastic Nested Booking Limits (SNBL)
- Stochastic Bid Prices (SBP)
- Randomized Bid Prices (RBP)

We evaluate the performance of these five booking control policies when they are applied to the simulated environment discussed in the previous section. In Section 3.6.1 we report the results for the DNBL, DBP and RBP methods which are all based on the deterministic mathematical programming model. In Section 3.6.2 we report results for the SNBL and SBP methods for various demand discretizations for the stochastic mathematical programming model. The computations discussed in the following sections are performed on a Pentium III 550 MHz personal computer (256 MB RAM), using CPlex 7.1 to optimize the mathematical programming models. All computation times are reported in seconds.

3.6.1 Deterministic and Randomized Booking Control Policies

The deterministic and randomized booking control policies make use of the deterministic model provided in (3.1) to construct a booking control policy. The DNBL and DBP policies do so by optimizing the model once for the expected demand. The RBP policy optimizes the model a number of times for a set of randomly generated demand realizations and uses the average of the dual prices provided by the model. We use 100 demand realizations for this randomization technique. We measure the performance of the booking control policies over another 100 simulated arrival processes using equal random numbers for all policies. We compare the results with those obtained by a simple First Come First Serve (FCFS) policy and with the optimal acceptance policy which can be determined with hindsight. The performance of the booking control policies is presented in Table 3.3. The averages and standard deviations presented in Table 3.3 are taken over the 100 simulations.

Table 3.3: Performance of the FCFS, DNBL, DBP, RBP and ex-post optimal booking control policies.

	FCFS	DNBL	DBP	RBP	Ex-post Optimal
Revenue					
Average	601750	663209	652702	689212	754628
Std. Dev.	5369	7045	6481	7932	7923
Minimum	586975	645465	634050	671970	734740
Maximum	613880	682720	670845	707960	769160
% of Ex-post Optimal					
Average	79.75	87.89	86.50	91.33	100
Std. Dev.	0.883	0.503	0.542	0.462	0
Minimum	77.71	86.79	85.18	90.00	100
Maximum	81.46	89.33	87.98	92.24	100
Load Factor	0.987	0.936	0.941	0.926	0.995
Average Comp. Time (sec.)	0	10.72	8.67	1121.19	-

Table 3.3 shows that there is a considerable gap between the average revenue obtained by a simple FCFS policy and the optimal revenue that can be obtained. Averaged over the 100 simulation runs, the FCFS policy does not yield more than 79.75% of the optimal revenue. The DNBL and DBP policies close this gap up to 87.89% and 86.50% respectively. As can be expected, the RBP policy which accounts for the stochastic nature of demand performs even better. On average the RBP policy obtains 91.33% of the optimal revenue. This means that the differences with the DNBL and DBP policies are respectively 3.44% and 4.83% of the optimal revenue, which in this test case corresponds to more than \$26.000 and \$36.500 respectively. In fact, the minimum revenue obtained by the RBP policy for any of the 100 simulations, is greater than any of the revenues obtained by the other policies which means that the RBP policy outperforms the other policies for

all 100 simulations. When we use *t*-tests to test the hypotheses that its average performance does not differ from those of the DNBL or DBP policies, this is rejected with a *p*-value of 0.000. When we compare the DNBL and DBP policies, it turns out that also the DNBL policy outperforms the DBP policy for all cases. Also for these two policies the *t*-test rejects the hypothesis that the average performance of the policies is equal with a *p*-value of 0.000.

To provide more insights in the performance of the booking control policies, we show the average number of bookings accepted by the five booking control policies for each price class in Figure 3.3.

Figure 3.3: Number of accepted booking requests per price class for the FCFS, DNBL, DBP, RBP and ex-post optimal booking control policies.

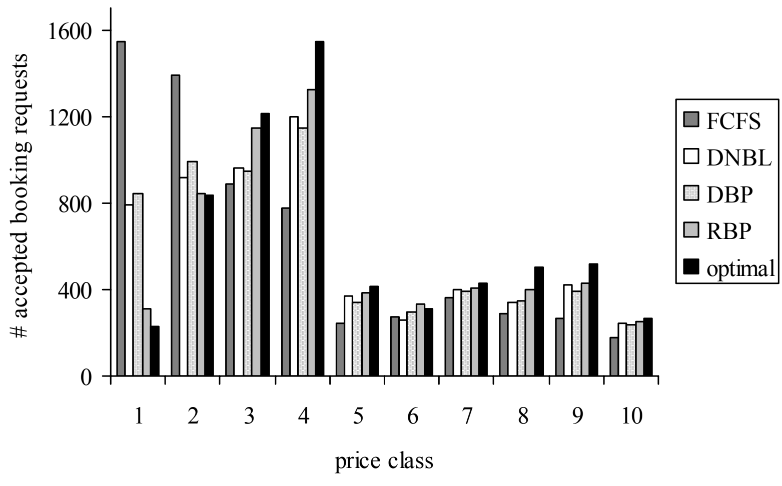


Figure 3.3 shows that the FCFS policy accepts far too much of the early, low-price tourist booking requests of classes 1 and 2. Because of this, it does not have enough capacity left for the high-price bookings. The DNBL and DBP policies seem to make the same mistake,

but to a smaller extent. The RBP policy follows the optimal acceptance policy reasonably well.

Of the booking control policies that we consider, the RBP policy obtains the best results. This is not surprising since it is the only policy thus far that accounts for the stochastic nature of demand. A drawback of the RBP policy, however, is the computation time involved with the method. We chose to use 100 demand realizations to obtain the randomized bid prices. This shows directly in the average computation time of the RBP policy which is more than 100 times larger than that of the DNBL and DBP policies. The computation time involved with the RBP policy grows proportionally to the number of demand realizations that is used to obtain the bid-price approximation. Therefore, one way to reduce the computation time of the RBP policy is to reduce the number of demand realizations used for the randomization. However, doing so will also reduce the accuracy and reliability of the manner by which demand is accounted for in the policy.

Tests show that even when only 1 demand simulation is used to approximate the RBP bid prices, the policy performs better than the DNBL and DBP policies and obtains 90.15% of the optimal revenue on average. This increases to 91.08% when 5 demand realizations are used and to 91.23% for 10 demand realizations, which is already very close to the average performance when 100 simulations are used. When the number of demand realizations is further increased to 20, the average performance of the RBP policy is increased to 91.29% of the optimal revenue. This shows that the additional increase when 20 demand realizations are used instead of 10 is only marginal, whereas the computation time will be doubled. The standard deviation of the performance gives an indication of the robustness of the policy. When only 1 demand realization is used, the standard deviation is 0.558 and larger than those of the DNBL and DBP policies. However, for 10 demand realizations the standard deviation of the RBP policy is 0.641 which is about equal to the standard deviation when 100 realizations are used. This indicates that in our test case neither the average performance nor the robustness of the RBP policy suffers much from reducing the number of demand realizations used for the randomization to 10.

When a bid-price policy is set, this means that no booking request is accepted in the hotel whose price does not live up to the bid price. This means that a certain price class is either open or closed for booking. This does not provide a way to accept some of the low-price booking requests unless the bid price is adjusted a number of times during the booking process. It is therefore to be expected that a bid-price policy is sensitive to the frequency by which it is adjusted over the booking period. Booking limits do not have this problem and are considered to be more robust. In this test case we update the booking control policies weekly. In order to test the sensitivity of the booking control policies to the updating frequency, we also evaluate the performance of the booking control policies when they are updated daily and once every two weeks. The results are presented graphically in Figure 3.4. We note that the results of the RBP policy are those obtained when only 10 demand realizations are used for the randomization technique. As discussed above, this hardly reduces the performance of the policy, whereas it greatly reduces the computation time which is especially demanding when the policy is updated every day.

Figure 3.4: Performance of the DNBL, DBP and RBP booking control policies as a function of the number of days between the policy updates.

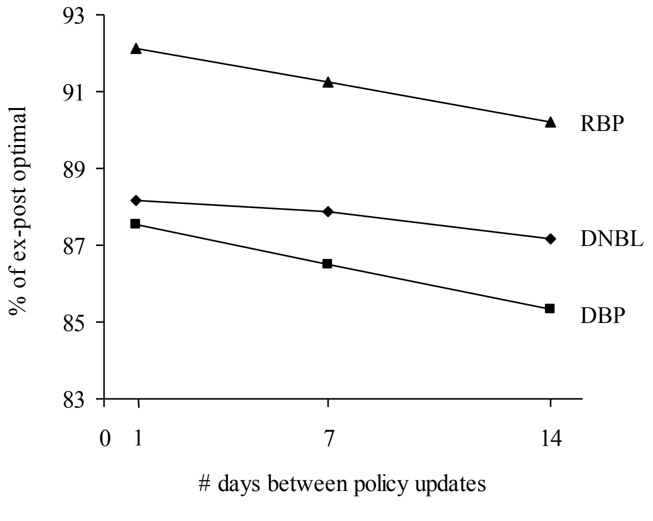


Figure 3.4 shows that the DBP and RBP policies are indeed more sensitive to the frequency by which the policies are updated than the DNBL policy. This results from the slope of the lines presented in Figure 3.4, which are steeper for the two bid-price polices. This means that the difference between the performance of the DNBL and DBP policies is reduced and between the DNBL and RBP policies is enlarged as the update frequency is increased. When the booking control policies are updated once every two weeks, the DNBL policy outperforms the DBP policy by 1.84% of the optimal revenue. When the policies are updated once every week this is 1.39% and for every day this is only 0.61%. The differences between the DNBL and RBP policies are 3.01%, 3.34% and 3.98% when the days between the policy updates are reduced from 14 to 7 to 1 respectively. Note, however, that although it is possible in practice to update a booking control policy on a daily basis, e.g. during the night, it is hardly possible to do so more frequently. Our findings show that the DNBL still outperforms the DBP policy when the policies are updated daily.

3.6.2 Stochastic Booking Control Policies

The results presented in the previous section, show that it can be rewarding to take the stochastic nature of demand into account when constructing a booking control policy. Whereas the RBP policy needs a number of model optimizations, and therefore computation time, to set its bid prices, the stochastic model given in (3.2) provides a way to do so with only one optimization. However, before the model can be applied, the manner by which the demand distribution is approximated has to be determined.

Based on the findings of de Boer et al (2002), who show that for the airline industry 3 or 4 demand scenarios suffice to capture most of the extra revenue generated by considering the stochastic nature of the demand, we consider only 3 demand scenarios for the stochastic model. These demand scenarios can be interpreted as a low, an average and a high demand scenario. We define the level of the average demand scenario as the expected demand and define the levels of the low and high demand scenarios as k times the standard deviation away from the expected demand. For determining the scenario probabilities, we define the variable:

$$z_j = \frac{D_j - \mu_j}{\sigma_j}, \tag{3.5}$$

where D_j is the demand for booking type j and μ_j its mean and σ_j its standard deviation. Then, by the central limit theorem, z_j is approximately distributed as a standard normal distribution. Now, because we can write $D_j \geq \mu_j + k\sigma_j$ as: $z_j \geq k$, we can approximate the probability that the demand exceeds the average demand plus k times the standard deviation by $N(-k)$, where $N(.)$ is the distribution function of the standard normal distribution. This way we can easily obtain the scenario probabilities for each scenario spread k.

In Figure 3.5 we show the performance of the SNBL and SBP policies for values of scenario spreads ranging from k = 0.1 to k = 2. Note that when k = 2, the spread between the low and high demand scenarios is large, since $\Pr(D_j \geq \mu_j - 2\sigma_j) = 0.9773$ and $\Pr(D_j \geq \mu_j + 2\sigma_j) = 0.0227$. The performance of the booking control policies is measured with respect to the ex-post optimal revenue. Each evaluation point reflects the average revenue obtained over the same 100 simulated booking processes used in the previous section. We also include the average revenues obtained by the DNBL, DBP and RBP policies in Figure 3.5. The full performance information for the SNBL and SBP policies is provided in Table 3.4.

Figure 3.5: Performance of the SNBL and SBP booking control policies for different spreads between the demand scenarios.

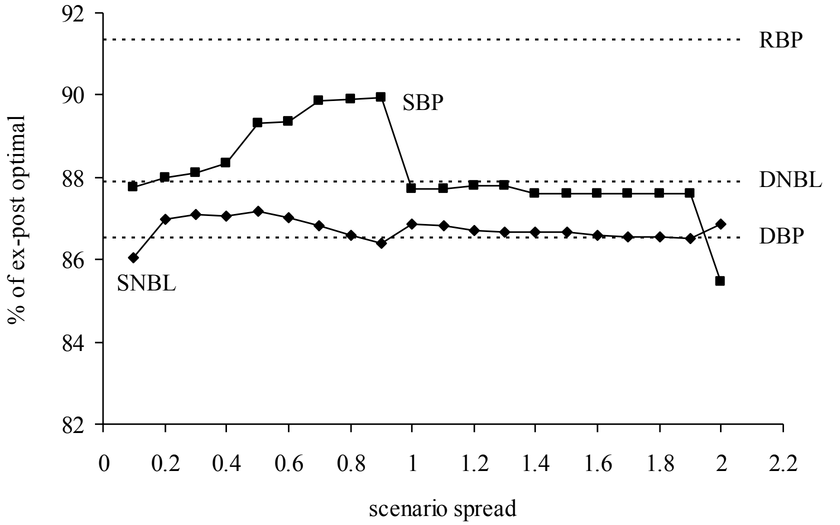


Table 3.4: Performance of the SNBL and SBP booking control policies for different spreads between the demand scenarios.

	SNBL				SBP			
k	% of Optimal	Std. Dev.	Load Factor	Comp. Time	% of Optimal	Std. Dev.	Load Factor	Comp. Time
0.1	86.03	0.635	0.813	27.17	87.77	0.608	0.951	24.91
0.2	87.00	0.552	0.834	27.17	87.99	0.591	0.950	24.93
0.3	87.09	0.567	0.842	27.20	88.12	0.592	0.949	25.03
0.4	87.05	0.552	0.846	27.19	88.36	0.616	0.936	24.97
0.5	87.18	0.542	0.859	27.43	89.31	0.569	0.929	25.13
0.6	87.01	0.544	0.859	27.24	89.34	0.563	0.928	25.10
0.7	86.82	0.565	0.868	27.19	89.86	0.578	0.926	24.89
0.8	86.58	0.566	0.869	27.22	89.89	0.564	0.926	25.04
0.9	86.38	0.589	0.878	27.19	89.94	0.563	0.926	25.08
1.0	86.88	0.560	0.892	27.04	87.71	0.660	0.950	24.92
1.1	86.84	0.553	0.892	27.09	87.74	0.643	0.950	24.97
1.2	86.69	0.569	0.894	27.04	87.79	0.600	0.949	24.84
1.3	86.69	0.582	0.894	27.12	87.81	0.587	0.949	24.90
1.4	86.69	0.576	0.894	27.05	87.60	0.611	0.946	24.83
1.5	86.67	0.581	0.894	27.03	87.59	0.604	0.946	24.83
1.6	86.59	0.565	0.894	27.10	87.60	0.602	0.946	24.92
1.7	86.57	0.582	0.894	27.09	87.60	0.592	0.946	24.86
1.8	86.57	0.550	0.894	27.11	87.62	0.602	0.945	25.05
1.9	86.53	0.566	0.894	27.22	87.61	0.599	0.945	25.04
2.0	86.85	0.612	0.905	26.99	85.47	0.724	0.962	24.88

The results portrayed in Figure 3.5, show that the booking-limit policy does not benefit from the use of the stochastic model at all. In fact, any variant that we consider of the SNBL policy is outperformed by the DNBL policy. The bid-price policy, on the other hand, does benefit from the use of the stochastic model. Its average performance rises steadily when the spread between the three demand scenarios is increased up until 89.94%

of the optimal revenue for k = 0.9. If the demand spread is increased to 1, however, its average performance falls back to 87.71%. When the scenario spread is increased to 2 the performance of the SBP policy is even worse than for the DBP policy. This shows that the SBP policy is very sensitive to the discretization of the demand chosen in the underlying model. The SNBL policy does not seem to have this problem, since its performance is relatively robust over the different scenario spreads that we consider. Except for k = 2, the revenues obtained by the SNBL policy are, however, much less than the revenues obtained by the SBP policy. Both policies are outperformed by the RBP policy. However, Table 3.4 shows that although the SNBL and SBP policies need about 2.5 times as much computation time as the DNBL and SBP policies, they are still much faster than the RBP policy.

The fact the booking-limit policy does not benefit from the use of the stochastic model seems surprising but has been observed before for the airline industry by Williamson (1992) and de Boer et al. (2002). They argue that this can mainly be ascribed to the fact that the booking limits are nested. When the stochastic nature of demand is taken into account explicitly, this often results in booking limits that are more protective for the higher price classes. However, nesting is already a method to deal with additional demand for these price classes, such that combining both methods quickly results in overprotection of the higher price classes. We do observe an increase in the performance of the bid-price policy when it is based on the stochastic model. This can be expected since a bid-price policy does not make use of nesting techniques that can counter the effects of the stochastic model. Williamson (1992) and de Boer et al. (2002) do not find such positive results for the SBP policy. This can be due to the differences between the network structure considered for the airline and hotel test cases. However, since the SBP policy depends highly on the chosen demand discretization, one can also easily obtain different performance observations for different model specifications.

3.7 Summary and Conclusion

In this chapter, we study different booking control policies for the hotel revenue management problem. The main difference that sets the hotel revenue management problem apart from the airline problem, is that the combinatorial effects present in the problem are not due to the multiple flights that are used by a passenger, but rather by the overlap between the guests in the hotel that stay for multiple days. This overlap is not restricted to a fixed time horizon. Therefore, we define a rolling horizon of decision periods which conveniently captures the overlap between the different booking requests for the hotel.

We consider five booking control policies that are applied over the rolling horizon of decision periods. Bid prices and nested booking limits are constructed based on the well-known deterministic mathematical programming model. Further, we also obtain bid prices and nested booking limits based on a stochastic mathematical programming model that has not been used for hotel revenue management before. Finally, randomized bid prices are obtained by ways of simulation. The performance of the different booking control policies is evaluated in a simulated environment. The results show that the randomized bid prices strictly outperform all other policies and reach up to 91.33% of the optimal revenue that can be obtained. However, this policy is also computationally very demanding and therefore not always useful in practice. We show that in our test case the number of demand realizations used for the randomization technique can be chosen as low as 10 without sacrificing much of the performance and reliability of the policy. This is a means to reduce the computation time of the policy.

When the booking limits and bid prices are obtained from the deterministic model, the booking limits provide better results. However, since the bid prices are more sensitive to the frequency by which the policy is updated, the gap between the performance of the booking-limit and bid-price policies can be reduced when the policies are updated more frequently. Nevertheless, the booking limits still provide better results when the policies are updated every day. We show that the booking-limit policy does not benefit from the use of the stochastic model. This is in line with prior findings by Williamson (1992) and de Boer et al. (2002) for the airline industry, and can be explained by the nested nature of the policy which already accounts for the stochastic nature of demand in its own way. The bid-price policy, on the other hand, can benefit greatly from the use of the stochastic model. We show that the stochastic bid prices can outperform all other policies except for the randomized bid prices. However, the stochastic bid prices are very sensitive to the demand distribution considered in the underlying mathematical programming model. In fact, when the demand discretization is not chosen beneficially, the stochastic bid prices can perform very badly. The deterministic booking-limit policy does not have this problem and even provides relatively robust results for different updating frequencies. Therefore, the deterministic booking-limit policy still is one of the most attractive policies to use in practice.

Following the results presented in this chapter, the stochastic mathematical programming model seems an interesting object for further research. It would be interesting to see whether insights can be obtained concerning the relation between the demand discretization chosen in the model and the performance of the bid prices. Such insights might take away some of the uncertainties involved with adapting the stochastic bid-price policy for practical use. Further, research opportunities lie in extending the booking control policies presented here to include aspects left out of this study, such as overbooking and group bookings. From the airline revenue management literature we know, however, that some aspects of the revenue management problem, such as competition and buy-up behavior, can be particularly hard to incorporate into a model.

3.A Simulation Parameters for the Hotel Test Case

In this section we provide the simulation parameters that we use to construct the hotel test case. As described in Section 3.5, we simulate the demand for the different price classes by a non-homogeneous Poisson process with arrival intensities dependent on the price class, the starting day of the stay and the time until the target booking day. In order to do this, we divide the booking period into 10 smaller periods each with a constant arrival intensity. For the simulation itself, we divide each day during the booking period into 10,000 smaller time intervals in which no more than one arrival can occur. We assume that the length of the stay is independent of the arrival intensity and model it by a logistic distribution with a parameter dependent on the price class and the starting day of the stay.

In the Table 3.5 we present the average number of booking requests in the test case per day of the week for each price class in each time period. In Table 3.6 we provide the parameters of the logistic distribution used to model the length of the stays.

Table 3.5: Average demand per day of the week for each price class and time period.

Panel A: Monday

Price	Time Period										
Class	1	2	3	4	5	6	7	8	9	10	Total
1	1,5	1	0,75	0,5	0,5	0,5	0,2	0,02	0,025	0,005	5
2	3	3	2,25	2,25	2,25	0,75	0,75	0,45	0,15	0,15	15
3	0,25	0,25	0,25	0,25	0,5	1,5	1	0,25	0,25	0,5	5
4	0,75	0,6	0,9	1,05	0,45	0,3	3	4,5	0,45	3	15
5	5	0	0	0	0	0	0	0	3	2	10
6	0,02	0,04	0,04	0,06	0,04	0,2	0,4	0,8	4,4	14	20
7	2	2	4	0	2	2	2	2	2	2	20
8	0,3	0,3	0,3	0,3	0,3	6	6	3	3	10,5	30
9	0	0	0	0	3	3	3	0	6	15	30
10	0,6	0,15	0,15	0,15	0,15	0,15	0,9	0,15	0,15	0,45	3
Total	13,42	7,34	8,64	4,56	9,19	14,4	17,25	11,17	19,425	47,605	153

Panel B: Tuesday

Price Class	Time Period 1	2	3	4	5	6	7	8	9	10	Total
1	0,9	0,6	0,45	0,3	0,3	0,3	0,12	0,012	0,015	0,003	3
2	1	1	0,75	0,75	0,75	0,25	0,25	0,15	0,05	0,05	5
3	0,15	0,15	0,15	0,15	0,3	0,9	0,6	0,15	0,15	0,3	3
4	0,3	0,24	0,36	0,42	0,18	0,12	1,2	1,8	0,18	1,2	6
5	1	0	0	0	0	0	0	0	0,6	0,4	2
6	0,05	0,1	0,1	0,15	0,1	0,5	1	2	11	35	50
7	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5	2,5	25
8	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,2	9	10
9	0	0	0	0	0	0	10,5	0	7	17,5	35
10	0,8	0,2	0,2	0,2	0,2	0,2	1,2	0,2	0,2	0,6	4
Total	6,8	4,89	4,61	4,57	4,43	4,87	17,47	6,912	21,895	66,553	143

Panel C: Wednesday

Price Class	Time Period 1	2	3	4	5	6	7	8	9	10	Total
1	6	4	3	2	2	2	0,8	0,08	0,1	0,02	20
2	6	6	4,5	4,5	4,5	1,5	1,5	0,9	0,3	0,3	30
3	1	1	1	1	2	6	4	1	1	2	20
4	1,5	1,2	1,8	2,1	0,9	0,6	6	9	0,9	6	30
5	2,5	0	0	0	0	0	0	0	1,5	1	5
6	0,004	0,008	0,008	0,012	0,008	0,04	0,08	0,16	0,88	2,8	4
7	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	3
8	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,3	13,5	15
9	0	0	0	0	0	0	0,6	0	0,4	1	2
10	0,6	0,15	0,15	0,15	0,15	0,15	0,9	0,15	0,15	0,45	3
Total	18,054	12,808	10,908	10,212	10,008	10,74	14,33	11,74	5,83	27,37	132

Panel D: Thursday

Price Class	Time Period 1	2	3	4	5	6	7	8	9	10	Total
1	0,9	0,6	0,45	0,3	0,3	0,3	0,12	0,012	0,015	0,003	3
2	5	5	3,75	3,75	3,75	1,25	1,25	0,75	0,25	0,25	25
3	0,15	0,15	0,15	0,15	0,3	0,9	0,6	0,15	0,15	0,3	3
4	1	0,8	1,2	1,4	0,6	0,4	4	6	0,6	4	20
5	4	0	0	0	0	0	0	0	2,4	1,6	8
6	0,01	0,02	0,02	0,03	0,02	0,1	0,2	0,4	2,2	7	10

7	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	5
8	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,15	0,3	13,5	15
9	0	0	0	0	0	0	2,4	0	1,6	4	8
10	0,6	0,15	0,15	0,15	0,15	0,15	0,9	0,15	0,15	0,45	3
Total	12,31	7,37	6,37	6,43	5,77	3,75	10,12	8,112	8,165	31,603	100

Panel E: Friday

Price	Time Period										
Class	1	2	3	4	5	6	7	8	9	10	Total
1	6	4	3	2	2	2	0,8	0,08	0,1	0,02	20
2	12	12	9	9	9	3	3	1,8	0,6	0,6	60
3	1,2	1,2	1,2	1,2	2,4	7,2	4,8	1,2	1,2	2,4	24
4	4	3,2	4,8	5,6	2,4	1,6	16	24	2,4	16	80
5	0	1,5	1,5	1,5	0,75	0,75	0,75	0,75	4,5	3	15
6	0,001	0,002	0,002	0,003	0,002	0,01	0,02	0,04	0,22	0,7	1
7	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	1
8	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,02	0,9	1
9	0	0	0	0	0	0	0	0	0,2	0,8	1
10	0,2	0,05	0,05	0,05	0,05	0,05	0,3	0,05	0,05	0,15	1
Total	23,511	22,062	19,662	19,463	16,712	14,72	25,78	28,03	9,39	24,67	204

Panel F: Saturday

Price	Time Period										
Class	1	2	3	4	5	6	7	8	9	10	Total
1	10,5	7	5,25	3,5	3,5	3,5	1,4	0,14	0,175	0,035	35
2	4,8	4,8	3,6	3,6	3,6	1,2	1,2	0,72	0,24	0,24	24
3	1,2	1,2	1,2	1,2	2,4	7,2	4,8	1,2	1,2	2,4	24
4	0,6	0,48	0,72	0,84	0,36	0,24	2,4	3,6	0,36	2,4	12
5	3	0	0	0	0	0	0	0	1,8	1,2	6
6	0,001	0,002	0,002	0,003	0,002	0,01	0,02	0,04	0,22	0,7	1
7	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	2
8	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,02	0,04	1,8	2
9	0	0	0	0	0	0	0,3	0	0,2	0,5	1
10	0,8	0,2	0,2	0,2	0,2	0,2	1,2	0,2	0,2	0,6	4
Total	21,121	13,902	11,192	9,563	10,282	12,57	11,54	6,12	4,635	10,075	111

Panel G: Sunday

Price Class	Time Period 1	2	3	4	5	6	7	8	9	10	Total
1	12	8	6	4	4	4	1,6	0,16	0,2	0,04	40
2	5	5	3,75	3,75	3,75	1,25	1,25	0,75	0,25	0,25	25
3	1,1	1,1	1,1	1,1	2,2	6,6	4,4	1,1	1,1	2,2	22
4	0,75	0,6	0,9	1,05	0,45	0,3	3	4,5	0,45	3	15
5	2	0	0	0	0	0	0	0	1,2	0,8	4
6	0,003	0,006	0,006	0,009	0,006	0,03	0,06	0,12	0,66	2,1	3
7	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	2
8	0,04	0,04	0,04	0,04	0,04	0,04	0,04	0,04	0,08	3,6	4
9	0	0	0	0	0	0	2,1	0	1,4	3,5	7
10	0,8	0,2	0,2	0,2	0,2	0,2	1,2	0,2	0,2	0,6	4
Total	21,893	15,146	12,196	10,349	10,846	12,62	13,85	7,07	5,74	16,29	126

Table 3.6: Parameter values for the logistic distribution.

Price Class	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	0,17	0,17	0,17	0,18	0,18	0,20	0,20
2	0,70	0,70	0,70	0,50	0,50	0,60	0,60
3	0,21	0,19	0,17	0,17	0,18	0,19	0,20
4	0,35	0,35	0,50	0,50	0,50	0,50	0,40
5	0,50	0,55	0,50	0,50	0,55	0,60	0,60
6	0,90	0,50	0,90	0,90	0,80	0,90	0,99
7	0,50	0,90	0,90	0,50	0,70	0,90	0,99
8	0,50	0,50	0,90	0,90	0,99	0,99	0,99
9	0,90	0,98	0,90	0,90	0,90	0,85	0,90
10	0,24	0,25	0,25	0,19	0,24	0,20	0,24

Chapter 4

Airline Revenue Management with Shifting Capacity

4.1 Introduction

Airline revenue management is the practice of controlling the booking requests that an airline company receives such that the planes are filled with the most profitable passengers. One of the essential assumptions in the traditional airline revenue management problem is that the capacities of the flights are fixed and given. In this chapter, we relax this assumption and extend the problem to include flexible capacities. Traditionally, airlines determine the capacities of the flights well before the booking process starts. However, making such capacity decisions dynamically during the booking process has gained the interest of both academics as practitioners in recent years. For example, Berge and Hopperstad (1993) and Listes and Dekker (2002) present methods to apply the fleet assignment problem in a dynamic manner, which is known as demand-driven-dispatch (D^3). A method to assign capacity where it is needed in a less drastic manner than swapping planes, is provided by the use of convertible seats. A row of these seats can be converted from business class seats to economy class seats and vice versa. This offers an airline company the possibility to adjust the capacity configuration within a plane, i.e. between the business and economy class sections, to the demand pattern at hand. In this study, we show how to incorporate the shifting capacity opportunity offered by the convertible seats into a dynamic, network-based revenue management model. We also extend the model to include cancellations and overbooking. The revenue potential of the shifting capacity opportunity is evaluated in a simulated environment.

In Section 4.2 we introduce the convertible seats and the shifting capacity opportunity they provide. A mathematical formulation of the problem is given in Section 4.3. We first provide the formulation of the standard airline revenue management problem and extend this for the shifting capacity decision and cancellations and overbooking. In Section 4.4 we describe the test case that we construct to evaluate the revenue potential of the various ways to deal with the shifting capacity opportunity. We present the computational results of the test case in Section 4.4 and conclude this chapter in Section 4.5.

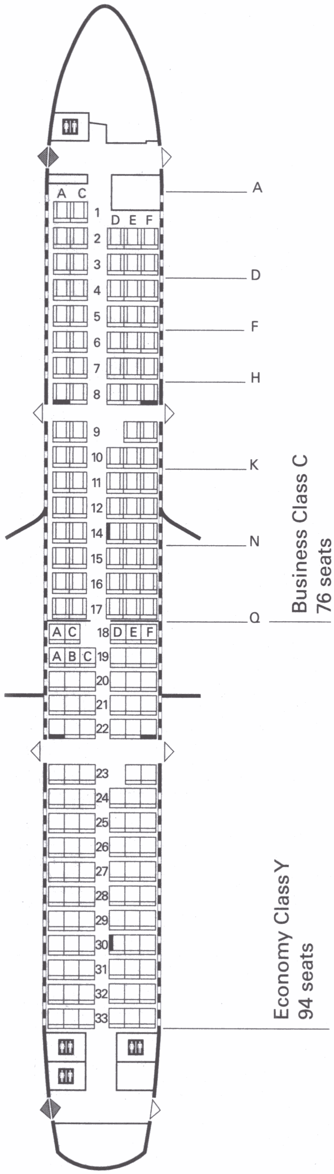
4.2 Shifting Capacity

In the traditional airline revenue management problem, the capacities of a plane and its different sections, i.e. business and economy class, are considered to be fixed and given. Despite of this, airline companies are not unfamiliar with the practice of shifting capacity from the business to the economy class. This is done by ‘upgrading’ individual passengers from economy to business or by ‘moving the curtain’ between the two sections. A drawback of these procedures is that passengers that pay for the economy class get the luxury of the business class (or at least a business class seat) for free. An airline company should prevent this from happening on a large scale because of the danger that people will anticipate on this and start booking economy class with the probability to be given a business class seat instead of booking business class in the first place. Therefore, upgrading and moving the curtain are not desirable tactics to apply on a large scale.

Another way for shifting business and economy class capacities is provided by so-called convertible seats. By a simple procedure, a row of these seats can be converted from economy class to business class seats and vice versa. When a row is converted from business to economy class, the number of seats in the row is increased and the width of each seat is decreased. The distances between the rows, however, remain the same. An example of this is given in Figure 4.1. In this figure, taken from the Swiss Air Lines 2002 Timetable, an Airbus 321 plane is shown equipped with convertible seats. Each row of seats can be used as either six economy class seats or five business class seats. The table included in Figure 4.1 gives a number of possible configurations of the plane varying from no business class seats in configuration A to 76 business class seats in configuration Q.

Because a passenger that has booked for the economy class indeed gets an economy class seat, the drawback previously mentioned for upgrading and moving the curtain does not apply when the convertible seats are used. Moreover, extra seats become available whenever a business class row is converted into an economy class row. The convertible seats can be used without any serious consequences which makes the plane very flexible in coping with different demand patterns. These different demand patterns can occur among flights that are flown on different days of the week or on different times of the day. Also seasonal differences can be encountered, as can differences caused by unforeseen events such as a decline in demand during an economic regression.

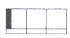
Figure 4.1: Airbus 321 with convertible seats.



Convertible seats rows 1–17
Position of movable class divider

Pos.	C-Class	Y-Class	Total
A	0	186	186
D	12	170	182
F	22	158	180
H	32	146	178
K	46	130	176
N	61	112	173
Q	76	94	170

Each seat is equipped with a satellite telephone.



Seats with removable armrests for passengers with reduced mobility.

Non-smoking seats only!

Lavatory.

In this chapter, we provide a model to incorporate the shifting capacity opportunity offered by the convertible seats into the traditional revenue management problem. The booking control policy that we use to extend for the shifting capacity decision is a dynamic approximation scheme, which is re-optimized for every new booking request. The underlying model is the standard deterministic mathematical programming model for network revenue management. However, as Talluri and van Ryzin (1999), we extend the model to account for the stochastic nature of demand by ways of simulation. A nice additional property of the dynamic approximation scheme that we construct, is that it can easily handle group bookings since it can just as easily compute the opportunity costs of one as of multiple seats. Most other revenue management policies do not have this property. On the other hand, we recognize that when a flight network is large, the computation time involved with the dynamic approximation scheme often prevents it from being useful for practical use. However, it still is an interesting and scarcely studied solution method that can be useful for smaller problem instances. Additionally, it provides a way to study the revenue potential of the shifting capacity opportunity with as little interference as possible of the inefficiencies that the more common booking control policies, such as booking limits and bid prices, generally cope with. We note that the underlying mathematical programming model that we use also provides the means to construct booking limits and bid prices in much the same way as the standard revenue management models.

In our booking control policy, we allow for cancellations and overbooking in much the same way as Bertsimas and Popescu (2003) do. Overbooking is not always incorporated in revenue management research, but it is important to do so in combination with the shifting capacity decision. When determining an overbooking policy, one should take into account the fact that one booking can block an entire row of seats from becoming available for the other section of the plane. Also, it is interesting to see if in some cases it is profitable to deny one or two accepted bookings to board on the flight such that the row becomes available for the other section, even though there are costs involved by doing so. For illustration, we describe a test case in which one plane is used for a series of flights and compare the results obtained in a simulated environment when the shifting capacity decision is made (i) beforehand and is kept fixed over all flights, (ii) before each flight, and (iii) dynamically during the booking process of each flight.

Finally, we note that the shifting capacity opportunity that we discuss in this chapter, is a way to allocate capacity where it is needed. In this respect this study is the first step towards the integration of revenue management and dynamic capacity management. In the airline industry dynamic capacity management is generally associated with the fleet assignment problem, which is aimed at assigning the different types of planes to the different flights such that revenues are maximized. When this is done dynamically, i.e. when the fleet assignment is changed to match the actual demand when departure time closes, this is also known as demand-driven dispatch (D^3) (see: Berge and Hopperstad (1993) and Listes and Dekker (2002)). By using planes equipped with convertible seats, airline companies will be able to fine-tune the capacity allocation started with the fleet assignment and will be able to match capacity and demand even better.

4.3 Problem Formulation

The essential decision to be made in airline revenue management is whether or not to accept a booking request when it arrives. In order to make this decision, the direct revenue gained by accepting the request has to be compared to the revenue that can be expected to be gained from the seats it uses if the request is not accepted, i.e. the opportunity costs of the seats. For determining the opportunity costs of the seats, it is important to have a good estimation of the future demand for the various routes and price classes. Further, it should be taken into account that a route requested by a customer can consist of multiple flights. This means that different routes can make use of the same flights. Therefore, in order to get a good approximation of the opportunity costs, the combinatorial effects of the whole network of flights have to be considered.

We formulate the problem under the standard assumptions in airline revenue management that the general price structure and the booking period are fixed and known. Further, we also keep to the common assumption that demand is independent of the booking control policy used. However, unlike many other revenue management studies, we do allow for group bookings and cancellations and overbooking. Most importantly, we relax the assumption that the capacities of the business and economy sections of the plane are fixed. Instead, we consider the capacity to be shifted between the two sections.

In Section 4.3.1 we give the traditional formulation for the network revenue management problem. In sections 4.3.2 and 4.3.3 we extend the model for the shifting capacity decision and cancellations and overbooking respectively.

4.3.1 Traditional Problem Formulation

In traditional airline revenue management, the business and economy sections of the plane are considered to be distinct and fixed. This means that the different sections of the plane do not have to be modelled explicitly, but can conceptually be seen as two different flights in the network with distinct demand. We note that a booking request for a route/fare-class combination explicitly belongs to a business or an economy price class. We model demand as a sequence of booking requests over time and we measure time in discrete intervals counting backwards, i.e. at time 0 the process ends. The problem formulation that we use in this chapter is slightly different from the one used in Chapter 2. Considering the fact that we refrain from a fixed capacity in the following sections, we no longer define the state of the process by the remaining capacity, but by the number of accepted booking requests instead.

Consider a flight network consisting of n route/fare-class combinations and $2m$ flights, where m is the actual number of flights in the network which is multiplied by 2 when the business and economy sections are considered to be different flights. Let $\mathbf{r} = (r_1, r_2, \ldots, r_n)^T$, $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$ and $\mathbf{c} = (c_1, c_2, \ldots, c_{2m})^T$ be the fares, accepted booking requests and capacities of the various route/fare-class combinations and flights. Further, define the matrix $\mathbf{A} = [a_{ij}]$, such that $a_{ij} = 1$ whenever route/fare-class combination j uses flight i and $a_{ij} = 0$ otherwise. Define $V_t(\mathbf{u})$ as the optimal expected revenue that can be obtained with t time units to go and $\mathbf{u}$ booking requests already accepted and $\mathbf{R}_t = (R_{t,1},$

$R_{t,2}$, …, $R_{t,n})^{\mathrm{T}}$ as a random vector for which $R_{t,j} = r_j$ if a request for class j occurs at time t, and $R_{t,j} = 0$ otherwise. Finally, let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$ be the control variable denoting whether to accept a class j request ($x_j = 1$) or not ($x_j = 0$). Then, $\mathbf{x}$ is restricted to the set $X_t(\mathbf{u}) = \{\mathbf{x} \in \{0,1\}^n : \mathbf{A}(\mathbf{u}+\mathbf{x}) \le \mathbf{c}\}$ and the Bellman equation for the optimal expected revenue function $V_t(\mathbf{u})$, is given by:

$$V_t(\mathbf{u}) = E\left[\max_{\mathbf{x}\in X_t(\mathbf{u})} \left\{\mathbf{R}_t^{\mathrm{T}}\mathbf{x} + V_{t-1}(\mathbf{u}+\mathbf{x})\right\}\right], \tag{4.1}$$

with the boundary condition:

$$V_0(\mathbf{u}) = 0, \qquad \forall\ \mathbf{u}.$$

The optimal control policy can easily be obtained from (4.1) and says to accept a booking request for route/price-class combination j, with t time units to go and $\mathbf{u}$ booking requests already accepted, if and only if sufficient capacity is available and:

$$r_j \ge V_{t-1}(\mathbf{u}) - V_{t-1}(\mathbf{u}+\mathbf{e}_j), \tag{4.2}$$

where $\mathbf{e}_j$ is the j^{th} column of a $n\times n$ identity matrix. The left hand side of (4.2) denotes the direct revenue associated with the booking request, whereas the right hand side denotes the estimated opportunity costs of the seats taken up by the booking request.

The optimal control policy given by (4.2) has been derived in some form or another for both discrete as well as continuous time by a number of people, including Bertsimas and Popescu (2003), Chen et al. (1998), Lautenbacher and Stidham (1999), Lee and Hersh (1993), Liang (1999), Subramanian et al. (1999), Talluri and Van Ryzin (1998, 1999) and Van Slyke and Young (2000). The difficulty, however, is to approximate $V_t(\mathbf{u})$. Let $\mathbf{D} = (D_1, D_2, \ldots, D_n)^{\mathrm{T}}$ be the demand of the various route/price-class combinations for the remaining time periods. Then if **D** is known, $V_t(\mathbf{u})$ can be obtained from the following mathematical programming model:

$$
\begin{aligned}
V_t(\mathbf{u}) = \max \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\
\text{s.t.} \quad & \mathbf{A}(\mathbf{u}+\mathbf{x}) \le \mathbf{c} \\
& 0 \le \mathbf{x} \le \mathbf{D} \qquad \text{integer},
\end{aligned}
\tag{4.3}
$$

where $\mathbf{x} = (x_1, x_2, ..., x_n)^{\mathrm{T}}$ determines the number of booking requests that are accepted for each route/price-class combination. The mathematical programming model in (4.3) provides a linear system of equations that can be solved by standard IP optimization techniques such as branch and bound.

In practice, **D** will not be known. One way to obtain an approximation for $V_t(\mathbf{u})$ is to replace **D** with its expected value. However, this does not take into account the stochastic nature of demand. A stochastic model has been proposed by Wollmer (1986). But this model is computationally intractable because of the large number of decision variables. A reduced version of Wollmer’s model, which considers only a limited number of discretization points for the demand distribution, is proposed by De Boer et al. (2002). The simplest method to incorporate the stochastic nature of demand is probably proposed by Talluri and Van Ryzin (1999). They simulate a sequence of demand realizations and compute the optimal future revenue for each of these realizations by the model given in (4.3). They approximate $V_t(\mathbf{u})$ by averaging the outcomes.

It is common practice to construct booking-limit or bid-price policies based on a mathematical programming model as provided above. Such booking control policies can be used for the on-line accept/deny decisions for a longer period of time until the policies are updated. In this study, however, we do not make use of such a policy to bridge the time between model optimizations. Instead we use a dynamic approximation scheme that directly obtains an approximation of $V_t(\mathbf{u})$ by optimizing the mathematical programming model for every booking request that comes in. Such a dynamic approximation scheme will not always be useful in practice, unless the problem instance is small. However, it is an interesting and scarcely studied method which provides us the means to study the revenue potential of the shifting capacity opportunity without interference caused by the booking control policy that is used.

An additional advantage of the dynamic approximation scheme is that it can deal with group bookings in a natural way. Note that when a booking request is made for k seats on route/price-class combination j, with t time units to go and $\mathbf{u}$ booking requests already accepted, it should be accepted if and only if sufficient capacity is available and:

$$kr_j \geq V_{t-1}(\mathbf{u}) - V_{t-1}(\mathbf{u} + k\mathbf{e}_j) \,. \tag{4.4}$$

This is a natural extension of the optimal decision rule presented in (4.2). Most booking control policies have difficulties with decisions concerning multiple units of capacity at once, since they are based on small deviations of the value function. These small deviations are often estimated by the dual prices of the underlying mathematical programming model. The dynamic approximation scheme, on the other hand, can just as easily approximate $V_{t-1}(\mathbf{u} + k\mathbf{e}_j)$ as $V_{t-1}(\mathbf{u} + \mathbf{e}_j)$.

4.3.2 Problem Formulation with Shifting Capacity

In this section we extend the standard airline revenue management problem with the shifting capacity decision. We use the same notation as in the previous section with the difference that the capacity $\mathbf{c}$ is no longer a constant, but is now dependent on the shifting capacity decision. We note that the optimal acceptance policy does not change, but the underlying mathematical programming model does. Assume that each plane has got a limited number of possible capacity configurations collected in the state space Y. Let m be the number of flights and $\mathbf{y} = (y_1, y_2, ..., y_m)^{\mathrm{T}} \in Y$ be the shifting capacity vector which denotes the chosen capacity configuration for each plane. Further, let the capacities be defined as a function of **y**, **c**(**y**). Then, for a given demand vector **D**, $V_t(\mathbf{u})$ can be obtained by:

$$
\begin{aligned}
V_t(\mathbf{u}) = \max \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} && (4.5)\\
\text{s.t.} \quad & \mathbf{A}(\mathbf{u}+\mathbf{x}) \le \mathbf{c}(\mathbf{y}) \\
& 0 \le \mathbf{x} \le \mathbf{D} \qquad \text{integer}, \\
& \mathbf{y} \in Y,
\end{aligned}
$$

where **x** determines the number of requests accepted for each route/price-class combination and **y** determines the capacity configurations of the planes.

Unless **c**(**y**) and Y are of a very specific form, model (4.5) will not be a system of linear equations. Therefore, it can be very hard to optimize the model. However, we show that the specifications of **c**(**y**) and Y that can be encountered in practice, are such that model (4.5) generally reduces to a system of linear equations. In order to see this, we describe the situation of a plane that has got two sections; a business and an economy class section. Then a plane which is equipped with convertible seats usually has a number of seats which are fixed for both sections along with a number of rows which can be used as either business or economy class rows. Assume that the fixed seat capacity for the business class is given by c_b and for the economy class by c_e. Further, let there be R rows of convertible seats which can each be used for either b_b business class seats or b_e economy class seats. Then we can define:

$$
\mathbf{c}(y) = \begin{pmatrix} c_b + b_b y \\ c_e + b_e(R-y) \end{pmatrix}, \qquad \text{with } Y = \{y \in \mathcal{N} : 0 \le y \le R\}.
$$

In this case, y denotes the number of convertible rows appointed to the business class section. Note that when we consider multiple flights, the capacity configurations of the flights will have to be denoted by a vector **y** instead of only a number y.

In order to present the model (4.5) with this specific formulation of $\mathbf{c}(\mathbf{y})$, we let $\mathbf{c}_b$, $\mathbf{c}_e$, $\mathbf{b}_b$, $\mathbf{b}_e$ and $\mathbf{R}$ be vectors of dimension $m\times 1$, such that they contain the shifting capacity information for all flights in the network. Further, we also partition n, $\mathbf{r}$, $\mathbf{x}$, $\mathbf{A}$, $\mathbf{u}$ and $\mathbf{D}$ into a part that contains the information concerning the business class and a part that contains the information concerning the economy class. Then we can define:

$$
\begin{aligned}
V_t(\mathbf{u}) = \max \quad & \mathbf{r}_b{}^{\mathrm{T}}\mathbf{x}_b + \mathbf{r}_e{}^{\mathrm{T}}\mathbf{x}_e \\
\text{s.t.} \quad & \mathbf{A}_b(\mathbf{u}_b + \mathbf{x}_b) - \mathbf{b}_b^{\mathrm{T}}\mathbf{y} \le \mathbf{c}_b \\
& \mathbf{A}_e(\mathbf{u}_e + \mathbf{x}_e) + \mathbf{b}_e^{\mathrm{T}}(\mathbf{R} - \mathbf{y}) \le \mathbf{c}_e \\
& 0 \le \mathbf{x}_b \le \mathbf{D}_b \qquad \text{integer}, \\
& 0 \le \mathbf{x}_e \le \mathbf{D}_e \qquad \text{integer}, \\
& 0 \le \mathbf{y} \le \mathbf{R} \qquad \text{integer},
\end{aligned}
\tag{4.6}
$$

where $\mathbf{x}_b$ and $\mathbf{x}_e$ determine the number of requests accepted for each route/price-class combination in the business and economy class and $\mathbf{y}$ determines the configurations of the planes. Model (4.6) consists of a linear system of equations and can therefore be optimized by the same procedures as model (4.3). Furthermore, the model has the same number of capacity constraints as model (4.3) and only has m more decision variables, where m is the number of flights in the flight network. Note that, although model (4.6) provides a configuration of the planes for every time it is optimized, only at the end of the booking period, at time 0, the planes will be physically converted into the desirable configuration.

4.3.3 Problem Formulation with Cancellations and Overbooking

In the airline industry, a large amount of bookings typically get cancelled before departure. Therefore, in order to prevent the planes from taking off with empty seats, airline companies overbook the flights. Whenever overbooking is applied, there is a probability that not all bookings can get on the plane. This can happen intentionally when a low fare booking is denied boarding in favour of a high fare booking, or accidentally when the number of cancellations is overestimated. However, there will be a penalty cost involved with denying an accepted booking to board. These can consist of all kinds of costs such as accommodation costs or loss of goodwill. The penalty costs normally prevent airline companies from taking too much risk with overbooking. It is interesting to see, however, if it is worthwhile to take more risk of bearing the costs of a denied boarding if this means that the entire row is shifted from the economy to the business class section of the plane or vice versa.

A deviation from the formulation before is that we define the variables $\overline{\mathbf{x}}$, $\overline{\mathbf{u}}$ and $\overline{\mathbf{D}}$ as the net values of **x**, **u** and **D**, where we define the net value as the number of booking requests corrected for the number of cancellations. This means that, if 30 booking requests have been accepted for route/price-class combination j, of which 6 will be cancelled in the future, then $\mathbf{u}$ = 30 but $\overline{\mathbf{u}}$ = 24. Obviously it is not known in advance which bookings will be cancelled. However, we can substitute $\overline{\mathbf{u}}$ and $\overline{\mathbf{D}}$ by expected or simulated values. Finally, let $\mathbf{q}_b = (q_{b,1}, q_{b,2}, \ldots, q_{b,n_b})^{\mathrm{T}}$ and $\mathbf{q}_e = (q_{e,1}, q_{e,2}, \ldots, q_{e,n_e})^{\mathrm{T}}$ denote the penalty costs of each route/price-class combination in the business and economy class sections. Then, we define the following mathematical programming model to obtain $V_t(\mathbf{u})$ when cancellations and overbooking are taken into account:

$$
\begin{aligned}
V_t(\mathbf{u}) = \max \quad & \mathbf{r}_b^{\mathrm{T}}\overline{\mathbf{x}}_b + \mathbf{r}_e^{\mathrm{T}}\overline{\mathbf{x}}_e - \mathbf{q}_b^{\mathrm{T}}\mathbf{z}_b - \mathbf{q}_e^{\mathrm{T}}\mathbf{z}_e && (4.7)\\
\text{s.t.} \quad & \mathbf{A}_b(\overline{\mathbf{u}}_b + \overline{\mathbf{x}}_b - \mathbf{z}_b) - \mathbf{b}_b^{\mathrm{T}}\mathbf{y} \le \mathbf{c}_b \\
& \mathbf{A}_e(\overline{\mathbf{u}}_e + \overline{\mathbf{x}}_e - \mathbf{z}_e) + \mathbf{b}_e^{\mathrm{T}}(\mathbf{R} - \mathbf{y}) \le \mathbf{c}_e \\
& 0 \le \overline{\mathbf{x}}_b \le \overline{\mathbf{D}}_b \quad \text{integer,} \\
& 0 \le \overline{\mathbf{x}}_e \le \overline{\mathbf{D}}_e \quad \text{integer,} \\
& 0 \le \mathbf{y} \le \mathbf{R} \quad \text{integer,} \\
& 0 \le \mathbf{z}_b \quad \text{integer,} \\
& 0 \le \mathbf{z}_e \quad \text{integer,}
\end{aligned}
$$

where $\mathbf{z}_b$ and $\mathbf{z}_e$ determine which bookings in the business and economy class sections are denied boarding when the plane takes off.

4.4 Description of the Test Case

In order to evaluate what the added value can be of using a revenue management policy that exploits the shifting capacity opportunity offered by the convertible seats, we apply the models presented in the previous section to a simulated test case. In this section we discuss the simulation environment. The simulation environment that we construct is chosen to reflect insights obtained from professionals in the airline industry. This means that the results that we obtain in this specific setting give an indication of what one can expect to find in practice.

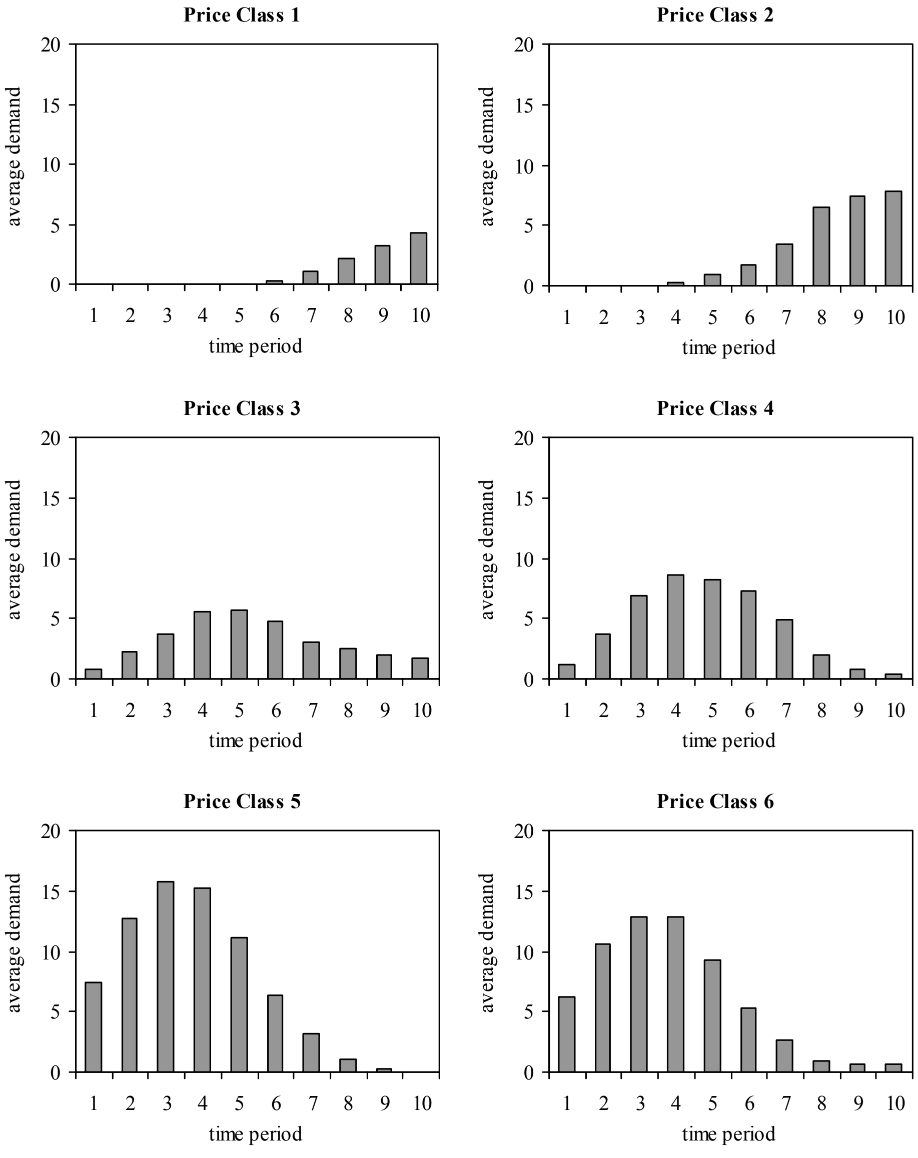
The test case consists of a single flight that is flown three times by the same plane. Each flight is characterized by its own typical demand pattern. These three flights can be interpreted to be the same flight in different seasons, on different days of the week, or on different times of the day. Specifically, we model one base flight together with one flight that has more business class and less economy class passengers, and one flight that has less business and more economy class passengers. The plane that is used for this test case has a total of 35 rows of passenger seats that can all be used for either five business class seats or six economy class seats. This resembles the Airbus 321 depicted in Figure 4.1 very much. We consider two price classes in the business class and four in the economy class. The prices are given in Table 4.1. Also given in Table 4.1 is the average demand of each price class for all three flights. Flight 2 is the base flight. For the first flight, the average business class demand is defined as 30% above the business class demand on the base flight, and the average economy class demand is defined as 30% below the average economy class demand on the base flight. For the third flight this is the other way around.

Table 4.1: Price classes.

			Average Demand		
Class	Class Type	Price	Flight 1	Flight 2	Flight 3
1	Business	$ 400	14.3	11	7.7
2	Business	$ 350	36.4	28	19.6
3	Economy	$ 250	22.4	32	41.6
4	Economy	$ 200	30.8	44	57.2
5	Economy	$ 150	51.1	73	94.9
6	Economy	$ 100	43.4	62	80.6

Next to the fact that the six price classes differ in price and demand level, they also have a specific booking pattern. The arriving booking requests are modelled by a non-homogeneous Poisson process. This is done by partitioning the booking period into ten smaller time periods each with a constant arrival rate. Demand for the two business classes is assumed to realize at the end of the booking period, whereas demand for the two cheapest price classes is modelled to occur at the beginning of the booking period. Graphical presentations of the arrival patterns of the price classes for the base flight are included in Figure 4.2. For the other two flights, the booking patterns are the same only with different demand levels. For sake of simplicity, we do not model booking requests for multiple seats, but only consider single seat bookings.

Figure 4.2: Arrival patterns of the demand for the various price classes.



When we consider cancellations and overbooking, we model each booking request to have a probability that it will be cancelled. This cancellation probability is dependent on the price class of the booking request and is the overall probability that the request is cancelled somewhere between the time of booking and the end of the booking period. We assume the cancellation probability to be homogeneous over time, such that a booking request that is made t time periods before the end of the booking period and that has cancellation probability p, has a cancellation probability per time unit of p/t. This way, we are able to model the cancellations by a homogeneous Poisson process. We model the two business classes to have a cancellation probability of 10%, the first two economy classes of 12.5% and the two cheapest price classes of 15%. The simulated demand is increased proportionally to these percentages in order to keep the net demand on the same level as in the previous section.

As discussed in Section 4.3.3, the penalty costs of denying an accepted booking to board have to be taken into account when overbooking is allowed. We set the penalty costs at \$500 for all price classes. This is more than the maximum revenue that can be obtained from any price class, which means that it is never profitable to accept an extra high price booking if this means that another booking has to be denied boarding. With the shifting capacity opportunity, however, it can be profitable to deny one or two bookings to board if this makes the entire row available for the other section of the plane. In conclusion, we summarize the aspects that define the simulation environment in Table 4.2.

Table 4.2: Specification of the test case.

3 consecutive flights with different demand levels
1 plane
35 convertible rows
5 business / 6 economy class seats per row
2 business / 4 economy price classes
different booking patterns for the different price classes
different cancellation probabilities for the different price classes
Overbooking is allowed
$500 penalty costs for a denied boarding
demand is independent of the booking control policies
no group bookings

4.5 Computational Results

In this section we present computational results in order to show how the models described in the Section 4.3 can be used and what the added value can be of using a revenue management policy that exploits the shifting capacity opportunity offered by the convertible seats. We do this by evaluating the performance of three different booking control policies to the simulated test case discussed in Section 4.4. The first booking control policy does not make use of the fact that the capacity can be shifted between the business and economy class sections, the second does, but only before the start of the booking period, and the third fully integrates the revenue management and shifting capacity decisions. The computational results when cancellations and overbooking are not allowed are presented in Section 4.5.1. In Section 4.5.2 we present the results when cancellations and overbooking are allowed.

4.5.1 Results without Cancellations and Overbooking

We compare the performance of three different revenue management policies that differ in the manner in which they deal with the shifting capacity opportunity. All three policies are dynamic approximation schemes in the sense that the opportunity costs are estimated anew for every booking request that comes in. The first policy does not incorporate the shifting capacity decision into the accept/deny decision. For this policy, the capacity remains fixed over all three flights and the traditional model presented in (4.3) is used to estimate the opportunity costs. The capacity configuration of the plane is fixed at the configuration obtained when model (4.6) is used to optimize all three flights at once based on their expected demand. This is done before the booking period, and thus the revenue management process, starts. We call this policy the Fixed Capacity (FC) policy. The second policy does make use of the shifting capacity opportunity. For each flight, which has its own specific demand pattern, a new capacity configuration is determined for the plane. However, this configuration is not changed during the booking period. Before the booking period starts, model (4.6) is used to determine the capacity configuration based on the expected demand and during the booking period, model (4.3) is used to estimate the opportunity costs. The third policy makes use of the shifting capacity opportunity dynamically. It fully integrates the shifting capacity and revenue management problems and continually uses model (4.6) to estimate the opportunity costs. This means that the actual configuration of the plane will be known only at the end of the booking period. We refer to the second policy as the Shifting Capacity (SC) policy and the third policy as the Dynamic Shifting Capacity (DSC) policy.

All three policies are dynamic approximation schemes. We apply them in two different ways that differ in the manner that the stochastic nature of demand is accounted for. We distinguish the Deterministic Dynamic Approximation Scheme (DDAS) and the Randomized Dynamic Approximation Scheme (RDAS). For the DDAS the opportunity costs are approximated based on the expected future demand. That is, in the underlying mathematical programming model, either model (4.3) or (4.6), the demand vector is replaced by its expectation. For the RDAS a number of future demand realizations are simulated. For each demand realization the opportunity costs are determined and the estimation of the opportunity costs that is used by the policy is defined as the average opportunity costs over all simulations. The RDAS accounts for the stochastic nature of demand, whereas the DDAS does not. However, the computation time involved with the RDAS increases proportionally with the number of demand realizations that are considered. Therefore, we restrict ourselves to 10 demand realizations for each opportunity costs approximation. This is only a small number of realizations and we suggest using more in practice in order to add reliability to the method. However, in the test case provided in Chapter 3 we have seen that taking more than 10 demand realizations for the randomization hardly increased the average performance and robustness of the randomized policy. Although this is a different test case, we continue to use 10 demand realizations for the randomization technique in order to reduce the computation time.

Combining the DDAS and RDAS with the FC, SC and DSC policies, gives us a total of six booking control policies. In order to test the policies, we simulate 100 complete booking processes for all three flights. In Table 4.3 we report the overall performance of the six booking control policies when they are applied to the 100 simulated booking processes. We also give the optimal revenues that can be determined with hindsight for each simulation. The results are generated on a Pentium III 550 MHz personal computer (256 MB RAM), using CPlex 7.1 to optimize the mathematical programming models. All computation times are measured in seconds.

Table 4.3: Performance of the booking control policies without cancellations and overbooking.

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Revenue							
Average	121470	124957	125114	121688	125197	126032	128258
Std. Dev.	3422	2873	2976	3479	3037	3048	3444
Minimum	113050	118550	118250	113150	118500	118500	120050
Maximum	129950	130200	130800	130100	131100	131700	136250
% of Ex-post Optimal							
Average	94.78	97.49	97.60	94.95	97.67	98.30	100
Std. Dev.	1.393	1.107	0.783	1.339	1.120	0.751	0
Minimum	89.28	93.94	95.42	89.59	94.13	96.25	100
Maximum	98.22	99.58	99.57	98.34	99.77	99.74	100
Average Comp. Time (sec.)	3.85	3.75	6.58	37.24	34.82	62.50	-

Table 4.3 shows that the FC policies that do not take into account the shifting capacity opportunity are clearly outperformed by the SC and DSC policies that do exploit the shifting capacity opportunity. The FC policy obtains 94.78% and 94.95% of the optimal revenue that can be generated when it is applied as DDAS and RDAS respectively. This is a gap of 2.71% and 2.72% of the optimal revenue with the SC policy when it is applied as DDAS and RDAS. The DSC, which is the most sophisticated policy, performs best and reaches up to 97.6% and 98.3% of the optimal revenue. In our small test case this means that the FC policies are outperformed by some \$1162-\$1448 per flight by the policies that account for the shifting capacity opportunity. As these flights can be flown one or even multiple times a day, and seeing that the extra revenues can be even more for bigger planes, this can lead to a substantial gain in revenues for the airline company. Whereas the FC policies are outperformed by the SC policies by 2.71% and

2.72% of the optimal revenue, the DSC only performs 0.11% and 0.63% better than the SC policies. In fact, all four policies that account for the shifting capacity opportunity perform within 0.81% of each other. This shows that most of the revenue potential offered by the convertible seats can be captured by adjusting the capacity configuration once for each flight. Adjusting it dynamically, i.e. continuously during the booking process, only creates a marginal improvement in the performance of the policy. However, doing so does increase the computation time with some 75%-80%.

When focusing on the differences between the DDAS and RDAS results, Table 4.3 shows that the FC and SC policies do not seem to benefit much from the randomization technique. The improvement in the performance of the FC and SC policies is 0.17% and 0.18% of the optimal revenue respectively. These improvements seem hardly worthwhile if we consider the fact that the computation time of the RDAS is 10 times the computation time of the DDAS. And this difference in computation time grows proportionally with the number of demand realizations that are used for the randomization technique. The DSC policy benefits more from the randomization. This can be explained from the fact that the additional decisions it makes concerning the capacity configuration of the plane benefit from the additional information provided by the randomization technique about the demand distribution. However, also for the DSC policy, the average performance is only increased by 0.70% of the optimal revenue. The standard deviations of the RDAS results are not larger than those of the DDAS results, which indicates that the RDAS is not less reliable than the DDAS when only 10 demand randomizations are used.

We use t-tests to test the statistical significance of the differences in the performance of the booking control policies. The hypothesis that the difference in the average performance of two booking control policies is equal to 0, is rejected for all policy combinations with a p-value of 0.000 except when the DDAS/DSC and the RDAS/SC methods are compared. This case results in a p-value of 0.428. We also apply a sign test which counts the number of times a policy outperforms another policy and tests whether this corresponds to a Binomial distribution with parameter 0.5. Also this test rejects the hypothesis with a p-value of 0.000 for almost all cases. The only exceptions occur when the DDAS/SC, DDAS/DSC and RDAS/SC methods are compared.

In order to see where the differences in the results come from, we include the average capacity configurations and load factors of the flights for the different booking policies in Table 4.4. For both the business and the economy class sections, this table reports the average number of rows appointed to it, the average number of passengers and the average load factor defined as the number of passengers as a percentage of the total capacity of the plane. We note that for the DSC policy the average number of business and economy rows does not necessarily sum to the total number of rows on the plane because for this policy a row which remains empty is not appointed to any of the two sections. For the FC and SC policies the number of rows appointed to the business and economy class sections is known before the booking process starts and is kept fixed no matter whether they are filled or not.

Table 4.4: Capacity configurations, number of passengers and load factors for the booking control policies without cancellations and overbooking.

Panel A: Flight 1

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	10	11.06	10	10	11.17	11.15
# Passengers	46.59	46.59	47.86	46.62	46.61	48.57	49.42
Load Factor	0.266	0.266	0.273	0.266	0.266	0.278	0.282
Economy Class							
# Rows	25	25	23.94	25	25	23.83	23.85
# Passengers	143.11	143.11	142.06	143.05	143.03	141.46	141.96
Load Factor	0.681	0.681	0.676	0.681	0.681	0.674	0.676
Overall Load Factor	0.948	0.948	0.950	0.948	0.947	0.951	0.958

Panel B: Flight 2

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	8	7.79	10	8	8.01	8.13
# Passengers	39.70	37.65	37.18	39.69	37.69	38.23	39.55
Load Factor	0.227	0.215	0.212	0.227	0.215	0.218	0.226
Economy Class							
# Rows	25	27	27.21	25	27	26.99	26.87
# Passengers	149.10	161.09	162.00	148.73	160.52	160.92	161.22
Load Factor	0.710	0.767	0.771	0.708	0.764	0.766	0.768
Overall Load Factor	0.937	0.982	0.984	0.935	0.980	0.985	0.994

Panel C: Flight 3

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	5	5.24	10	5	5.56	5.76
# Passengers	28.71	24.41	25.17	28.71	24.43	26.79	28.15
Load Factor	0.164	0.139	0.144	0.164	0.140	0.153	0.161
Economy Class							
# Rows	25	30	29.76	25	30	29.44	29.24
# Passengers	149.59	179.43	177.42	149.41	179.30	175.62	175.44
Load Factor	0.712	0.854	0.845	0.711	0.854	0.836	0.835
Overall Load Factor	0.876	0.994	0.989	0.876	0.993	0.989	0.996

Panel D: Total

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	7.67	8.03	10	7.67	8.25	8.35
# Passengers	38.33	36.22	36.73	38.34	36.24	37.86	39.04
Load Factor	0.219	0.207	0.210	0.219	0.207	0.216	0.223
Economy Class							
# Rows	25	27.33	26.97	25	27.33	26.75	26.65
# Passengers	147.27	161.21	160.49	147.06	160.95	159.33	159.54
Load Factor	0.701	0.768	0.764	0.700	0.766	0.759	0.760
Overall Load Factor	0.920	0.975	0.974	0.919	0.974	0.975	0.983

Table 4.4 shows that when the shifting capacity opportunity is exploited, the average number of business class rows ranges from 5 on the third flight with a low business class demand to 10 on the first flight with a high business class demand. The FC policy fixes the number of business class rows at 10 and economy class rows at 25, which is a good configuration when the business class demand is high, but results in empty business class seats when the business class demand is low. This is reflected in the average load factors of the flights. For the first flight, the load factors of the FC policies are the same as those for the SC policies and only a little less than those of the DSC policies. For the third flight, however, the total load factors of the FC policies are more than 11% under those of the SC and DSC policies. Combined over the three flights the load factors of the FC policies are at least 5% less than for the SC and DSC policies.

4.5.2 Results with Cancellations and Overbooking

In this section we present computational results when cancellations and overbooking are included in the test case. We model the cancellation probability of a booking request by a homogeneous Poisson process. The two business classes have a cancellation probability of 10%, the first two economy classes of 12.5% and the two cheapest price classes of 15%. The simulated demand is increased proportionally to these percentages in order to keep the net demand on the same level as in the previous section. The penalty costs of denying an accepted booking to board is set at \$500 for all price classes. This is more than the maximum revenue that can be obtained from any price class, which means that it is never profitable to accept an extra high price booking if this means that another booking has to be denied boarding. However, with the shifting capacity opportunity it can be profitable to deny one or two bookings to board if this makes the entire row available for the other section of the plane.

We simulate 100 booking processes to which we apply the same six booking control policies as in the previous section. The underlying mathematical programming models are now adjusted to include cancellations and overbooking as described in Section 4.3.3. The overall performance of the six policies and the optimal results that can be determined with hindsight are presented in Table 4.5.

Table 4.5: Performance of the booking control policies with cancellations and overbooking.

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Revenue							
Average	120656	123663	124260	120829	123848	125154	128172
Std. Dev.	3607	2850	2805	3669	2844	2801	3630
Minimum	110150	116050	117150	110050	115800	117600	119900
Maximum	129600	128700	130600	129750	129300	130000	135850
% of Ex-post Optimal							
Average	94.20	96.56	97.02	94.33	96.70	97.71	100
Std. Dev.	1.548	1.489	1.085	1.549	1.456	1.103	0
Minimum	90.48	90.82	93.86	90.40	91.39	94.67	100
Maximum	98.07	99.46	99.37	98.19	99.38	99.59	100
Average Comp. Time (sec.)	5.65	5.33	9.86	52.05	49.11	84.41	-

The results presented in Table 4.5 show that the booking control policies perform less when we consider cancellations and overbooking. This is not because less revenue is available, which is contradicted by the very small differences between the optimal revenues of both cases, but because the cancellations further complicate the problem. Interesting to observe is that the FC and DSC policies both obtain some 0.6% of the optimal revenue less when cancellations and overbooking are allowed, whereas the SC policy obtains almost 1% less. Apart from this, the differences in the performance of the various policies show very much the same patterns as without cancellations and overbooking. The differences between the FC policies and the SC policies change from 2.71% and 2.72% to 2.36% and 2.37% of the optimal revenue when applied as DDAS and RDAS respectively. The difference between the FC and DSC policies does not change at all for the DDAS and increases from 3.35% to 3.38% of the optimal revenue for the

RDAS. The increase in the performance caused by the randomization techniques of the RDAS are also similar to those observed without cancellations and overbooking, and are 0.13%, 0.14% and 0.69% of the optimal revenue for the FC, SC and DSC policies respectively. We note that all differences in the performance of the booking control policies are statistically significant with a *p*-value of 0.000 when tested by the means of *t*-tests and sign tests as discussed in the previous section.

In Table 4.6, we present the average number of denied boardings per flight for the six booking control policies. Unlike the FC and SC policies, the DSC policy is able to shift the capacity during the booking process. Table 4.6 clearly shows that this flexibility can lead to both a very low as a very high level of denied boardings. When the probability that the demand exceeds its expected value is neglected (DDAS), the DSC policy is able to shift the capacity such that very few accepted bookings are denied boarding. However, when the upward potential of demand is taken into account (RDAS), the DSC policy is willing to take risks and incur some denied boardings in order to generate additional revenue in the other section of the plane. Note that in both cases, the DSC policy performs better than the other policies, even when it endures additional penalty costs.

Table 4.6: Average number of denied boardings per flight.

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Flight 1	0.300	0.300	0.180	0.350	0.330	0.540	0
Flight 2	0.320	0.440	0.210	0.250	0.260	0.520	0
Flight 3	0.220	0.220	0.170	0.150	0.310	0.550	0
Total	0.280	0.320	0.187	0.250	0.300	0.537	0

Finally, in Table 4.7 we present the average capacity configurations and load factors of the flights for the different booking control policies. The capacity configurations and load factors of the flights show no large deviations from those obtained for the case without cancellations and overbooking, except for the fact that the DSC policies tend to appoint some more seats for the business class. This can be seen most clearly for the first flight, where the DSC policies appoint more than a complete row extra to the business class.

Table 4.7: Capacity configurations, number of passengers and load factors for the booking control policies with cancellations and overbooking

Panel A: Flight 1

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	10	11.03	10	10	11.13	11.38
# Passengers	47.48	47.48	48.91	47.61	47.60	49.76	50.89
Load Factor	0.271	0.271	0.279	0.272	0.272	0.284	0.291
Economy Class							
# Rows	25	25	23.97	25	25	23.87	23.62
# Passengers	142.41	142.41	142.08	142.25	142.24	141.77	140.52
Load Factor	0.678	0.678	0.677	0.677	0.677	0.675	0.669
Overall Load Factor	0.949	0.949	0.956	0.949	0.949	0.959	0.960

Panel B: Flight 2

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	8	7.88	10	8	8.04	8.22
# Passengers	39.97	37.40	37.03	40.00	37.44	37.96	39.95
Load Factor	0.228	0.214	0.212	0.229	0.214	0.217	0.228
Economy Class							
# Rows	25	27	27.12	25	27	26.96	26.78
# Passengers	148.82	160.88	161.41	148.25	159.93	160.94	160.68
Load Factor	0.709	0.766	0.769	0.706	0.762	0.766	0.765
Overall Load Factor	0.937	0.980	0.980	0.935	0.976	0.983	0.993

Panel C: Flight 3

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	5	5.3	10	5	5.51	5.82
# Passengers	28.85	23.95	24.94	28.85	24.18	26.60	28.39
Load Factor	0.165	0.137	0.143	0.165	0.138	0.152	0.162
Economy Class							
# Rows	25	30	29.7	25	30	29.49	29.18
# Passengers	148.91	178.62	176.83	148.52	178.24	175.99	175.08
Load Factor	0.709	0.851	0.842	0.707	0.849	0.838	0.834
Overall Load Factor	0.874	0.987	0.985	0.872	0.987	0.990	0.996

Panel D: Total

	DDAS			RDAS			Ex-post Optimal
	FC	SC	DSC	FC	SC	DSC	
Business Class							
# Rows	10	7.67	8.07	10	7.67	8.23	8..47
# Passengers	38.77	36.28	36.96	38.82	36.41	38.11	39.74
Load Factor	0.222	0.207	0.211	0.222	0.208	0.218	0.227
Economy Class							
# Rows	25	27.33	26.93	25	27.33	26.77	26.53
# Passengers	146.72	160.64	160.11	146.34	160.14	159.57	158.76
Load Factor	0.699	0.765	0.762	0.697	0.763	0.760	0.756
Overall Load Factor	0.920	0.972	0.974	0.919	0.971	0.978	0.983

4.6 Summary and Conclusion

In this chapter we introduce convertible seats into the airline revenue management problem. These seats create the opportunity to shift capacity between the business and economy class sections of a plane. We formulate a mathematical programming model to account for the shifting capacity opportunity that we use both in a deterministic and in a randomized dynamic approximation scheme. The mathematical programming model is not much harder than traditional network revenue management models and is also extended to incorporate cancellations and overbooking.

We provide a test case where a single plane is used for multiple flights with different demand patterns. The test case shows that the shifting capacity opportunity gives a rise in revenues of more than 3.3% of the optimal revenue that can be obtained. When the shifting capacity decision is made only once before each flight, the extra revenues are still more than 2.8% of the optimal revenue. The shifting capacity opportunity also increases the load factor of the plane from 92% to more than 97%. When cancellations and overbooking are taken into account these results remain the same. We also observe that taking the shifting capacity opportunity into account can result in a policy that is less careful with respect to overbooking. This is because the opportunity costs of a booking can become very large whenever the booking is blocking an entire row from becoming available for the other section of the plane. Therefore, it can be worthwhile in some cases to take the risk of a denied boarding. Further, we see that randomization of the booking control policy in order to account for the stochastic nature of demand, increases the performance of the policies, but never more than 0.7% of the optimal revenue that can be obtained. The computation time of such a randomized policy is however considerably larger then for a deterministic policy.

This study provides a way to model the shifting capacity decision and gives an indication of the added value of doing so. The booking control policies that we used are computationally very cumbersome and will not always be applicable in practice in this exact way. Therefore, a study on computationally less demanding booking control policies could prove useful. For this, one can think of bid prices that serve as approximations of the opportunity costs for a longer period of time or nested booking limits that determine the number of booking requests to accept for each price class. Both can be based on the models introduced in this study. Further, we acknowledge that our test case is but an initial one and many more can be constructed to obtain further insights. For example, our test case consists of a single flight, which gives us the opportunity to illustrate things more clearly, and does not include multiple seat booking requests. Finally, we would like to mention that most extensions to the standard airline network revenue management problem that are suggested throughout the literature can be applied to the model that we provide in this study as well. This comes forth from the fact that our model still resembles the standard revenue management models very much.

Chapter 5

Cargo Revenue Management: Bid Prices for a 0-1 Multi Knapsack Problem

5.1 Introduction

Revenue management gained popularity in the airline industry where passengers that compete for the same seat in a plane generally pay different prices. The principles of revenue management can also be applied to cargo transportation. Although this is already done in practice, little research can be found on cargo revenue management. The solution techniques used for the cargo revenue management problem are therefore basically the same as the techniques constructed for the well-known passenger revenue management problem. However, cargo revenue management differs from the passenger revenue management problem in a number of ways. Particularly, by the fact that its capacity constraint is 2-dimensional, i.e. weight and volume, and that the weight, volume and profit of each booking request are random and continuous variables. Passenger revenue management solution techniques are generally based on the fact that the booking requests can be categorized in different price classes. Cargo shipments, however, are not defined over such price classes but are uniquely defined by their profit, weight and volume. Without this kind of categorization into distinct classes, the question is whether a revenue management policy can be constructed for the unique cargo booking requests which is still simple and effective for practical use.

In this chapter, we present such a booking control policy. The booking requests are considered to be unique items, which means that we consider a 0-1 decision problem. Since a cargo shipment can use multiple connecting flights to reach its destination and the accept/deny decision has to be made as the booking request occurs, we model the problem as a 0-1 multi-dimensional on-line knapsack problem. We show that a bid-price acceptance policy is asymptotically optimal if demand and capacity increase proportionally and the bid prices are set correctly. Further, we present a heuristic to obtain values for the bid prices based on a greedy algorithm for the 0-1 multi-dimensional knapsack problem proposed by Rinnooy Kan et al. (1993). The performance of the knapsack bid prices is evaluated together with other booking control policies in a simulated environment.

In Section 5.2 we discuss the cargo revenue management problem and its differences with the airline passenger revenue management problem. We provide a mathematical formulation of the problem in Section 5.3. In this section we discuss the optimal control policy, a dynamic approximation scheme for it and the bid-price policy. In Section 5.4 we present a polynomial time algorithm to obtain values for the bid-price policy. In Section 5.5 we discuss the test case that we construct and report the performance of the various booking control policies. Finally, we conclude this chapter in Section 5.6.

5.2 Cargo Revenue Management

The revenue management applications that we discussed in the previous chapters are aimed at accepting those customers that maximize the revenue for the company. Because the capacity is generally fixed and has to be sold before a deadline, there is a trade off between accepting a booking request with a certain revenue and waiting for a more profitable booking request that may or may not come. It is well known that the principles of revenue management can also be applied to cargo transportation (see: Kasilingam (1996)). Although a large part of the cargo capacity is usually consumed by shipments that are determined by long-term contracts, also a certain part of the capacity is generally kept available for on-the-spot sales that tend to be more profitable per kg. Booking requests for these spot sales come in during the booking period and have to be accepted or rejected as they come in. The allocation of the cargo capacity over the long-term contracts and the spot sales is an interesting research topic on itself, but we will not go into this problem in this study. The problem that we consider is that of accepting or rejecting the spot sales as they come in during the booking period.

In Section 5.2.1 we discuss the differences between the cargo and passenger revenue management problems and in Section 5.2.2 we discuss the related literature on cargo revenue management.

5.2.1 Cargo vs. Passenger Revenue Management

Cargo revenue management differs from passenger revenue management in a number of ways. The most important difference is that two passengers who book in the same price class for a flight can be seen as two identical customers, whereas each cargo shipment is unique. The two passengers both take up one seat, have the same ticket options, generate the same revenue, etc. This generally results in revenue management policies that determine how many passengers to accept in each price class, or whether or not a specific price class is open for booking at a certain time. Also for cargo transportation there are usually a number of different product types, e.g. mail, fresh products, live animals, secured products and door-to-door service. However, the profit generated by a shipment does not just depend on the price, but also on the additional costs associated to the shipment, e.g. special packaging, additional trucking and fuel. In addition, cargo booking requests are generally made by a limited number of large customers. These customers have more purchasing power than a potential passenger for a flight. It is no exception that the price for a potential cargo shipment is subject to negotiation before the actual booking request is made. These things combined, mean that next to the weight and volume, also the profit per kg of a cargo shipment is a random variable. The profit, weight and volume all take on continuous values and differ for all shipments. The uncertainty in passenger revenue management lies in the number of passengers that will arrive for each price class, whereas for cargo revenue management each shipment is a unique item with properties that are not known before the time of booking.

Unlike passenger capacity, the cargo capacity available for the spot sales is generally uncertain. This capacity does not only depend on the capacity taken up by the long-term contracts, but also on the weather conditions, the amount of fuel needed for the trip and more. For a combi-plane, which is used to carry both passengers and cargo, also the number of passengers and the weight and volume of their luggage have to be considered. Further, the actual amount of cargo usually deviates from the initial booking request, especially for the long-term contracts. This means that overbooking is an important aspect of cargo revenue management. In this study, however, we assume that the cargo capacity available for the spot sales is known. This is in line with other revenue management studies, where uncertainties about the capacity and overbooking are usually left out in order to focus on the booking control policy. We notice, however, that common overbooking techniques can be used in combination with the policies that we present in this study.

Passenger revenue management is usually looked at from a network perspective. That is, a passenger that uses multiple connected flights, should be evaluated according to its overall profit to the flight network and not for each flight individually. This is also the case for cargo shipments and the model that we provide takes this into account.

5.2.2 Literature

Although cargo transportation is generally recognized as a natural application for revenue management, it has not received a lot of attention in the literature. In fact, Kasilingam (1996) and Karaesmen (2001) are the only studies that we know that concentrate on cargo revenue management. Kasilingam (1996) concentrates on air cargo and discusses the differences between passenger and cargo revenue management for an airline company. Besides the differences discussed in the previous section, he points out that cargo has the possibility to be shipped among different routes as long as it gets to its destination in time. He also indicates that the number of positions when working with containers is limited. This can be another capacity restriction. Our experience with KLM Royal Dutch Airlines, however, tells us that positioning is hardly a problem for air cargo, since the airline company generally builds pallets itself instead of using containers. The revenue management model that Kasilingam describes is very similar to the standard mathematical programming models known from passenger revenue management. We do not think this kind of model suits the cargo problem since cargo shipments can not be classified into groups (price classes) with identical properties as passengers can.

Karaesmen (2001) formulates the cargo revenue management problem as a continuous linear programming problem and shows that optimal bid prices exist for the problem. In order to obtain bid-price values, she discretizes the state space into weight and volume regions. Such a discretization corresponds with grouping the cargo shipments into a limited number of classes with identical properties. The model proposed by Kasilingham (1996) can then be used to obtain the bid prices. How to group the cargo shipments into distinct classes is not trivial. In this study, we propose a model that does not group the cargo shipments, but instead treats them as the unique items that they are. This leads us to a 0-1 multi-dimensional on-line knapsack problem.

In a series of articles, Kleywegt and Papastavrou investigate what they call the dynamic stochastic knapsack problem (see: Papastavrou et al. (1996), Kleywegt and Papastavrou (1998) and Kleywegt and Papastavrou (2001)). They mention its application to cargo revenue management. However, their models include only one capacity restriction. They choose a dynamic programming approach to the problem which, although theoretically very interesting, is computationally very demanding. Especially when a second capacity restriction is added to this approach, the state space will become intractable for practical use. Our approach is a static but more efficient and practical one.

The bid-price solution technique that we use in this study, has been discussed extensively by Talluri and van Ryzin (1998) for the passenger revenue management problem. They show that bid prices are not optimal in general, but are asymptotically optimal when capacity and demand increase proportionally. Their model differs from ours in the sense that they divide the booking requests into classes in such a way that two requests in the same class always have the same capacity requirement. This is not the case in our problem where every shipment has a random weight and volume. Nevertheless, we show that the asymptotic optimality holds in our case as well.

5.3 Problem Formulation

The cargo revenue management problem as we define it in this study is such that a number of booking requests come in during the booking period. Each booking request is uniquely defined by its weight, volume, profit and the flights it uses. The number of shipments that the airline company can accept on a flight is determined by the weight and volume capacities of the plane. We assume that the capacities are fixed and given and that the actual weight and volume of the shipments do not differ from the booked quantities. We do not consider cancellations and overbooking. Finally, as common in quantity-based revenue management, we assume that demand is independent of the booking control policy used.

In Section 5.3.1 we formulate the theoretical optimal control policy for the problem. A dynamic approximation scheme and the bid-price policy are discussed in Section 5.3.2 and Section 5.3.3 respectively.

5.3.1 Optimal Control

The decision to accept or reject a booking request has to be made at the moment the request comes in. This decision depends on the remaining capacities, the expected future demand and the properties of the booking request. We model the booking requests as a sequence of arrivals over time and measure time in discrete intervals counting backwards, i.e. at time 0 the process ends. We assume each time interval to be small enough such that no more than one request will occur within a time interval. Because a shipment can use more than one flight to reach its destination, we include multiple flights in our model. Assume there are m flights that have weight and volume capacities given by the vectors $\mathbf{c}_w = (c_{w,1}, c_{w,2}, ..., c_{w,m})^T$ and $\mathbf{c}_v = (c_{v,1}, c_{v,2}, ..., c_{v,m})^T$. The capacities cannot be negative and are adjusted every time a booking request is accepted. Define $J_t(\mathbf{c}_w, \mathbf{c}_v)$ as the optimal expected revenue that can be generated with t time units to go and capacities $\mathbf{c}_w$ and $\mathbf{c}_v$ available. We know that $J_t(\mathbf{c}_w, \mathbf{c}_v)$ must satisfy:

$$J_t(\mathbf{c}_w, \mathbf{c}_v) \geq 0 \qquad \forall\ \mathbf{c}_w, \mathbf{c}_v, t$$

$$J_0(\mathbf{c}_w, \mathbf{c}_v) = 0 \qquad \forall\ \mathbf{c}_w, \mathbf{c}_v$$

$$J_t(0, \mathbf{c}_v) = J_t(\mathbf{c}_w, 0) = 0 \qquad \forall\ \mathbf{c}_w, \mathbf{c}_v, t.$$

Further, we know that $J_t(\mathbf{c}_w, \mathbf{c}_v)$ is non-decreasing in t, $\mathbf{c}_w$ and $\mathbf{c}_v$, since it is never a disadvantage to have more time or capacity available. For a more exhaustive analysis of the expected reward function, we refer to Papastavrou et al. (1996), who discuss various special cases of the problem.

In order to formulate the Bellman equation for the problem, define R_t as the random variable that denotes the profit of a booking request that is made with t time periods to go. Note that $R_t = 0$ when no booking request occurs. Also define the random vectors $\mathbf{W}_t = (W_{t,1}, W_{t,2}, \ldots, W_{t,m})^T$ and $\mathbf{V}_t = (V_{t,1}, V_{t,2}, \ldots, V_{t,m})^T$ that give the weight and volume requirements of the booking request on the m flights. This means that if a booking request is made with t time periods to go, $W_{t,i}$ is equal to the weight of the shipment if the shipment uses flight i (i = 1, 2, ..., m) and 0 for all other flights. This is the same for $V_{t,i}$. Further, let x be the control variable denoting whether to accept a booking request (x = 1) or not (x = 0). Then, x is restricted to the set $X_t(\mathbf{c}_w, \mathbf{c}_v) = \{x \in \{0,1\} : x\mathbf{W}_t \leq \mathbf{c}_w, x\mathbf{V}_t \leq c_v\}$ and the Bellman equation is given by:

$$J_t(\mathbf{c}_w, \mathbf{c}_v) = E\left[\max_{\mathbf{x} \in X_t(\mathbf{c}_w, \mathbf{c}_v)} \{R_t x + J_{t-1}(\mathbf{c}_w - x\mathbf{W}_t, c_v - x\mathbf{V}_v)\}\right], \tag{5.1}$$

with the boundary conditions provided above.

Whenever a booking request comes in with t time units to go and capacities $\mathbf{c}_w$ and $\mathbf{c}_v$ available, its profit and its weight and volume requirements are observed. We denote the observed profit by r and the observed weight and volume requirements by $\mathbf{w} = (w_1, w_2, \ldots, w_m)^T$ and $\mathbf{v} = (v_1, v_2, \ldots, v_m)^T$. The optimal control policy provided by (5.1) says to accept the booking request if and only if sufficient capacity is available and:

$$r \geq J_{t-1}(\mathbf{c}_w, \mathbf{c}_v) - J_{t-1}(\mathbf{c}_w - \mathbf{w}, \mathbf{c}_v - \mathbf{v}). \tag{5.2}$$

The left hand side of (5.2) denotes the direct revenue associated with accepting the cargo shipment, whereas the right hand side gives the estimated opportunity costs of the capacities taken up by the shipment.

The optimal control policy is similar to the one known for passenger revenue management presented in Chapter 2. Except that in the passenger case, there is only one capacity dimension and the size of a demand is restricted to a limited number of integer values (and generally even considered to be equal to one). Lautenbacher and Stidham (1999), Lee and Hersh (1993), Liang (1999) and Subramanian et al. (1999), use a dynamic programming approach to the passenger problem for a single flight. Van Slyke and Young (2000) also consider multiple flights, but acknowledge that the complexity introduced by the increased number of dimensions is tremendous. For the cargo problem, which has two capacity dimensions and continuous demand sizes, a dynamic programming approach becomes computationally intractable. In the next section we discuss a dynamic approximation scheme for the problem and in Section 5.3.3 we construct a bid-price policy that is well suited for use in practice.

5.3.2 Dynamic Approximation Scheme

In this section we construct a dynamic approximation scheme that can be used for the on-line accept/deny decision. We propose a method to approximate the optimal expected revenue that can be generated for a given set of capacities and a given time to go, i.e. $J_t(\mathbf{c}_w, \mathbf{c}_v)$. This way, approximations for $J_t(\mathbf{c}_w, \mathbf{c}_v)$ and $J_t(\mathbf{c}_w - \mathbf{w}, \mathbf{c}_v - \mathbf{v})$ can be computed whenever a booking request comes in. These approximations can then be used to apply the control policy provided by (5.2).

If the future demand is known, we can easily compute the optimal future revenue by formulating the problem as a deterministic mathematical programming problem. Assume that we have a given set of n booking requests that define the problem instance Ω. Let the profits of the booking requests be given by the vector $\mathbf{r} = (r_1, r_2, ..., r_n)^{\mathrm{T}}$. Further, define the matrix $\mathbf{A}_w = [a_{w,ij}]$, such that $a_{w,ij}$ is equal to the weight of shipment j if that shipment uses flight i and 0 otherwise. Likewise, define the matrix $\mathbf{A}_v$ for the volume requirements of the shipments. Let $J_t^{\Omega}(\mathbf{c}_w, \mathbf{c}_v)$ be the optimal future revenue that can be generated for problem instance Ω with t time units to go and capacities $\boldsymbol{c}_w$ and $\boldsymbol{c}_v$ available, then $J_t^{\Omega}(\mathbf{c}_w, \mathbf{c}_v)$ can be obtained from the following integer programming problem:

$$
\begin{aligned}
J_t^{\Omega}(\mathbf{c}_w, \mathbf{c}_v) = \quad \max \quad & \mathbf{r}^{\mathrm{T}}\mathbf{x} \\
\text{s.t.} \quad & \mathbf{A}_w\mathbf{x} \le \mathbf{c}_w \\
& \mathbf{A}_v\mathbf{x} \le \mathbf{c}_v \\
& \mathbf{x} \in \{0,1\}^n,
\end{aligned}
\tag{5.3}
$$

where $\mathbf{x} = (x_1, x_2, ..., x_n)^T$ determines whether booking request j is accepted ($x_j = 1$) or not ($x_j = 0$). This model is known to be NP-hard (see Garey and Johnson (1979)). Standard integer programming solution methods, such as branch-and-bound, can be used to solve small problem instances of the model. For bigger problem instances Rinnooy Kan et al. (1993) provide a polynomial time greedy algorithm that provides an asymptotically optimal solution when demand and capacity increase proportionally. We discuss this algorithm in detail in the Section 5.4.

In practice, the future demand is not known. In passenger revenue management it is common practice to formulate a mathematical programming model for the expected demands for the various route/price-class combinations in order to obtain an approximation of the optimal future revenue. However, this is not an option for the cargo revenue management problem since cargo shipments cannot be categorized into distinct classes. Therefore, we suggest to compute the optimal future revenue for a series of simulated demand instances, i.e. $J_t^{\Omega_1}(\mathbf{c}_w, \mathbf{c}_v)$, $J_t^{\Omega_2}(\mathbf{c}_w, \mathbf{c}_v)$, ..., $J_t^{\Omega_k}(\mathbf{c}_w, \mathbf{c}_v)$. An approximation for $J_t(\mathbf{c}_w, \mathbf{c}_v)$ can then be obtained by taking the average value over the simulated instances. Doing this every time a booking request comes in, is not as intractable as dynamic programming. However, it still is computationally very demanding. In the next section, we construct a bid-price policy that does not require any on-line computation time and is therefore easy to use in practice.

5.3.3 Bid Prices

Bid-price acceptance policies are widely used in passenger revenue management. The idea of a bid-price policy is to determine a value for which a unit of capacity can be sold at a certain point in time. This way, the opportunity costs of a booking request can be approximated by the sum of the bid prices of the capacities it uses. The booking request is only accepted if its profit exceeds the opportunity costs. A bid price is determined for every dimension of the capacity, which in our case would mean one weight and one volume bid price for each flight. Optimally, the bid price is a function of the remaining time and capacity. In practice, however, the function is usually approximated by a fixed value that is re-evaluated at fixed points in time.

When the bid prices for the different capacity dimensions are held constant for a longer period of time, the approximation of the opportunity costs of a booking request reduces to a linear combination of the capacity requirements of the request. In order to see this, let $\boldsymbol{\mu}_w = (\mu_{w,1}, \mu_{w,2}, ..., \mu_{w,m})^{\mathrm{T}}$ and $\boldsymbol{\mu}_v = (\mu_{v,1}, \mu_{v,2}, ..., \mu_{v,m})^{\mathrm{T}}$ be the bid prices for the weight and volume capacities of the flights. Then, if a booking request comes in with profit r, and capacity requirements $\mathbf{w}$ and $\mathbf{v}$, it is accepted under the bid-price policy if and only if sufficient capacity is available and:

$$r \geq \boldsymbol{\mu}_w^{\mathrm{T}}\mathbf{w} + \boldsymbol{\mu}_v^{\mathrm{T}}\mathbf{v}, \tag{5.4}$$

which is the sum of the bid prices of the capacities it uses multiplied by the sizes of the capacity requirements. Bid prices are studied extensively for passenger revenue management by Talluri and van Ryzin (1998). They show that when the bid prices are set correctly, a static bid-price policy is asymptotically optimal when the capacities and the demand increase proportionally. We show that this also holds for the cargo problem.

In a probabilistic error analysis, Rinnooy Kan et al. (1993) show that a static bid-price policy is asymptotically optimal for the 0-1 multi-dimensional knapsack problem. They consider a probabilistic version of model (5.3) by letting the profit and the weight and volume requirements of the items in the knapsack be independent identically distributed random variables. Assume that the capacities grow proportionally with the number of items, i.e. $c_{w,i} = n\beta_{w,i}$ and $c_{v,i} = n\beta_{v,i}$, where $\beta_{w,i}$ and $\beta_{v,i}$ are fixed values (i = 1, 2, …, m). Finally, define z_n as the random variable that denotes the optimum solution value of the problem with n items and $z_n(\boldsymbol{\mu}_w, \boldsymbol{\mu}_v)$ as the random variable that denotes the solution value when the items are accepted by the bid-price policy that uses the bid prices $\boldsymbol{\mu}_w$ and $\boldsymbol{\mu}_v$. Then under certain conditions concerning the probability distributions, Rinnooy Kan et al. (1993) show that the sequence $\{z_n(\boldsymbol{\mu}_w, \boldsymbol{\mu}_v)/z_n\}$ converges to 1 with probability one if $\boldsymbol{\mu}_w$ and $\boldsymbol{\mu}_v$ are chosen correctly. For an intuitive clarification, note that it is the combinatorial aspect of the problem that creates a gap between the optimal and the greedy solutions. In fact, if all items were of the same size, the greedy algorithm would provide the optimal solution. Now, by increasing the number of items and the capacity along with it, the size of each individual item becomes less influential and so does the combinatorial aspect of the problem. Eventually, as the number of items goes to infinity, the combinatorial effect dies out.

We argue that the on-line decision problem that we study in this study, can be formulated in the exact same way as discussed above, which means that the asymptotic result holds for our problem as well. As portrait in Section 5.3.1, we can assume that there is a booking request for every decision period without loss of generality. This holds, because we can always consider a booking request to have a profit, weight and volume of value 0. We can now interpret the on-line problem as a 0-1 decision problem for every decision period. When we formulate this as a knapsack problem we let the decision variables in the knapsack correspond to the decision periods in the booking process. This way, all random elements of the problem are modelled in the profit and capacity requirement coefficients, which gives us the model studied by Rinnooy Kan et al. (1993). Thus, also for the cargo revenue management problem, a bid-price policy is asymptotically optimal as demand and capacity increase proportionally and the bid prices are chosen correctly.

5.4 Obtaining Bid Prices

In passenger revenue management, bid prices are often approximated by the dual prices of the LP-relaxation of the underlying mathematical programming model which determines the allocation of the capacity over the demand. The cargo problem, however, is a 0-1 decision problem for which the LP-relaxation is a very crude approximation. In this section we provide polynomial time algorithm to obtain optimal bid prices for a given set of booking requests based on a greedy algorithm for the 0-1 multi-dimensional knapsack problem. In Section 5.4.1 we give the general idea of the greedy algorithm. In Section 5.4.2 we discuss how to obtain bid prices and the computational complexity of doing so.

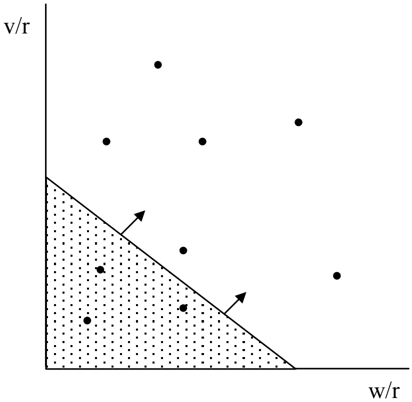
5.4.1 Greedy Algorithm

In order to obtain bid prices, we make use of a greedy algorithm that Rinnooy Kan et al. (1993) present for the 0-1 multi-dimensional knapsack problem. The idea of the greedy algorithm is to weigh each capacity dimension and select those items in the knapsack that have the highest profit per weighted capacity requirement. Assume that $\boldsymbol{\alpha}_w = (\alpha_{w,1}, \alpha_{w,2}, ..., \alpha_{w,m})^{\mathrm{T}}$ and $\boldsymbol{\alpha}_v = (\alpha_{v,1}, \alpha_{v,2}, ..., \alpha_{v,m})^{\mathrm{T}}$ are the non-negative weights for the weight and volume capacities on the flights. As before, let r, $\mathbf{w}$ and $\mathbf{v}$ be the profit and the weight and volume requirements of a booking request. Then the booking requests that are accepted, are those that have the highest ratio:

$$\delta = \frac{r}{\boldsymbol{\alpha}_w^{\mathrm{T}}\mathbf{w} + \boldsymbol{\alpha}_v^{\mathrm{T}}\mathbf{v}} \tag{5.5}$$

In order to understand how the greedy algorithm works, notice that from (5.5) we know that we want to select those booking requests for which the values $\mathbf{w}/r$ and $\mathbf{v}/r$ are small. That is, those requests that do not use a lot of capacity for the profit they provide. For a graphical representation, let each booking request correspond to a point in $\Re_+^{2m}$, given by the coordinates $(w_1/r, w_2/r, ..., w_m/r, v_1/r, v_2/r, ..., v_m/r)$. Then, the greedy algorithm can be regarded as a hyperplane with normal vector $(\alpha_{w,1}, \alpha_{w,2}, ..., \alpha_{w,m}, \alpha_{v,1}, \alpha_{v,2}, ..., \alpha_{v,m})$ that moves upward from the origin and accepts booking request in the order that it encounters them. This is depicted for a single flight in Figure 5.1. For this 2-dimensional case, the hyperplane is reduced to a line with the slope α_w/α_v.

Figure 5.1: Accepting booking requests for a single flight with the greedy algorithm.



For a given set of booking requests, one can simply select those requests that have the highest δ. For the on-line problem, however, the future booking requests are not known. Therefore, a threshold value for δ has to be specified in advance. If $\hat{\delta}$ is the threshold value, then a booking request with profit r and capacity requirements **w** and **v** is accepted if and only if sufficient capacity is available and:

$$\frac{r}{\boldsymbol{\alpha}_w^{\mathrm{T}}\mathbf{w} + \boldsymbol{\alpha}_v^{\mathrm{T}}\mathbf{v}} \geq \hat{\delta}, \qquad (5.6)$$

which is the same as:

$$r \geq \hat{\delta}(\boldsymbol{\alpha}_w^{\mathrm{T}}\mathbf{w} + \boldsymbol{\alpha}_v^{\mathrm{T}}\mathbf{v}). \qquad (5.7)$$

This means that the greedy algorithm can be seen as a bid-price policy, where the bid prices are set equal to $\hat{\delta}\boldsymbol{\alpha}_w$ and $\hat{\delta}\boldsymbol{\alpha}_v$. In terms of Figure 5.1, $\boldsymbol{\alpha}_w$ and $\boldsymbol{\alpha}_v$ define the slope of the line and the threshold value $\hat{\delta}$ determines the height of the line.

5.4.2 Computational Complexity of Finding Bid Prices

The greedy algorithm presented in the previous section gives another interpretation to the well-known bid prices. The bid prices are defined by a set of weights with a corresponding threshold value. For a given set of booking requests, Rinnooy Kan et al. (1993) provide a polynomial time algorithm to determine optimal values for the weights and the threshold value when the number of capacity dimensions is not more than the number of booking requests. They show that the problem becomes NP-hard if the number of capacity dimensions exceeds the number of requests. Since there are only two capacity dimensions in the cargo revenue management problem, this is already ruled out as soon as there are two booking requests per flight. Notice that for a given set of booking requests, each set of weights brings forth an ordering in which the requests are considered for acceptance. However, we need only consider those sets of weights that actually induce a different ordering. That is, we only need to consider those sets of weights that construct a sufficiently different hyperplane such that it encounters the booking requests in a different order as it moves up from the origin. In the case of one flight, and thus 2-dimensions, with n booking requests, there are at most $\binom{n}{2}$ of such changes in the ordering.

Lenstra et al. (1982) provide an algorithm that can be used to determine all possible orderings. In terms of Figure 5.1, the algorithm makes use of the fact that the line that passes through two booking requests, provides exactly that slope for which the requests are swapping places in the ordering. Thus, in order to obtain all orderings, we only have to determine those lines that connect two booking requests. Note that we only have to consider negative slopes, since a positive slope does not provide an ordering of the requests. Let $x_j = (w/r)_j$ and $y_j = (v/r)_j$ denote the weight and volume per profit for booking request j (j = 1, 2, ..., n). Lenstra et al. (1982) suggest to first order the requests by the sequence $\{x_j\}_{j=1}^{n}$ and then to determine the line that passes through requests k and l for all k = 1, 2, ..., n and l = 1, 2, ..., n for which $k < l$ (i.e. $x_k \le x_l$) and $y_k \ge y_l$. For each ordering that can be obtained, one can simply compute the solution of the greedy algorithm by accepting the requests until the capacity is full.

Determining the orderings by the algorithm provided by Lenstra et al. (1982) can be done in $O(n^2 \log n)$ time. Filling the capacity for a specific ordering costs at most $O(n)$ time. Finally, selecting the ordering that produces the highest profit costs $O(\log n)$ time. This means that the total computation time is $O(n^3 \log n)$. When we consider m flights, the total number of exchanges is $O(n^{2m})$. They can be determined in $O(n^{2m} \log n)$ time, such that the total computation time is $O(n^{2m+1} \log n)$. This is polynomial in n for a given number of flights m. For the case of one flight, a schematic overview of the algorithm is given in Figure 5.2. This can easily be extended to the case of m flights.

Figure 5.2: Schematic overview of the algorithm for obtaining optimal bid prices.

Given: A number of n booking requests defined by $\{(x_j, y_j)\}_{j=1}^{n}$, where x_j and y_j are the weight and volume per profit for booking request j.

Step 1: Order the requests by increasing value of x_j.

Step 2: For $k = 1$ to n:

For $l = k+1$ to n and $y_k \geq y_l$:

Let $\gamma = \dfrac{y_l - y_k}{x_l - x_k}$.

For $h = 1$ to n: let $\eta_h = y_h - \gamma x_h$.

Order the requests by increasing value of η_h.

Start accepting requests in this order until no more requests can be accepted.

Let π be the profit obtained and let η be the order value for the last request that is accepted.

Step 3: Find the maximum profit π^* over all orderings.

Let γ^* and η^* be the corresponding slope and order value.

Return: $\mu_w = -\gamma^*/\eta^*$ and $\mu_v = 1/\eta^*$.

The algorithm discussed above, provides a way to obtain the optimal bid prices for a given set of booking requests. In practice, the bid prices have to be set before the demand is known. An easy way to obtain bid prices is to simulate a series of demand sequences, compute the optimal bid prices for each sequence and set the bid prices that are used for the on-line policy to the average values over all sequences.

5.5 Test Case

In this section we present computational results for the solution methods described in the previous sections when they are applied to a simulated test case. The test case that we consider is a realistic one that reflects the situation that we encountered in practice at KLM Royal Dutch Airlines. We obtain results for the bid-price policy when the bid prices are determined by: (i) the algorithm for the 0-1 multi-dimensional knapsack problem and (ii) the dual prices of the LP-relaxation of the problem. We compare the performance of the bid-price policies to that of the dynamic approximation scheme and the ex-post optimal solution that can be obtained with hindsight when all demand is known.

In Section 5.5.1 we first describe the test case that we construct. In Section 5.5.2 we present the computational results which we extend in Section 5.5.3 for different capacity/demand ratios.

5.5.1 Description of the Test Case

The test case that we construct consists of a single flight with a fixed weight and volume capacity of 10000 kg and 75 m^3 respectively. These are the capacities available for the spot sales after correcting the total capacity of the plane for the space taken up by the long-term cargo contracts. The booking period is made up of T discrete time intervals of length one. Booking requests come in according to a Poisson arrival process with an arrival rate λ. This means that with probability λ a booking request is made in a time period and with probability 1-λ there is not. The total expected number of booking requests is therefore λT and the maximum number of requests is T.

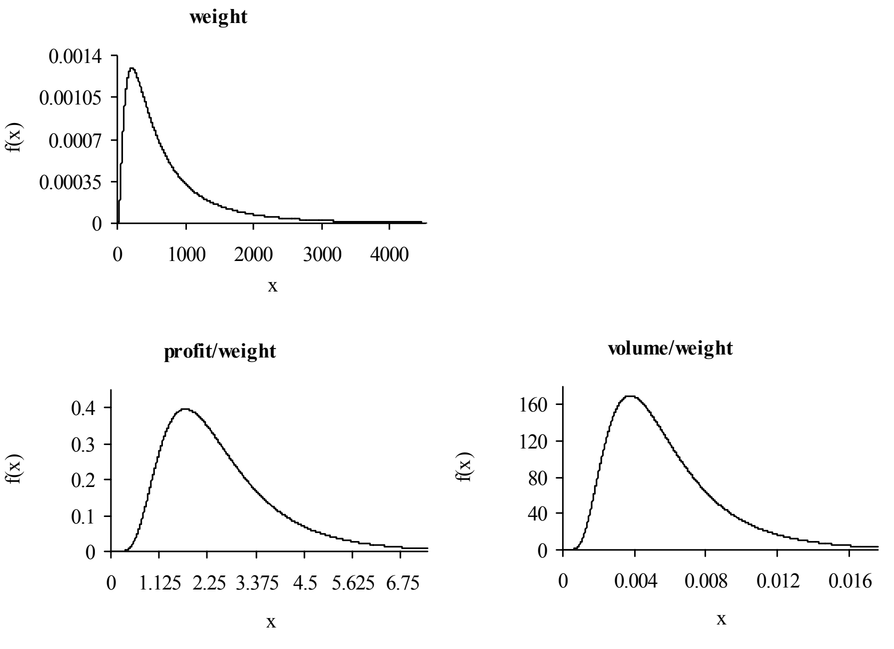
Each booking request has a unique profit, weight and volume that we denote by r, w and v respectively. The profit, weight and volume are related to each other since they all reflect the size of the shipment. To get around this problem, we define the profit and volume relative to the weight of the shipment and assume the variables w, r/w and v/w to be independent random variables. We assume all booking requests to be independent identically distributed and consider w, r/w and v/w to follow a log-normal distribution. The log-normal distribution generates values that resemble the data that we encountered in practice. We note, however, that we do not try to model the large amount of very small shipments, of for example 1, 2, 10 or 20 kg that an airline company usually has to deal with. These small shipments are generally highly profitable per kg and take up very little capacity. This means that they are almost always accepted and in practice are often not even subjected to the revenue management decision rule. More than anything, these shipments are used to fill up the wholes in the capacity that are left by the bigger shipments. We choose to exclude the small shipments from the test case such that the algorithm that sets the bid prices is not influenced by these relatively insignificant shipments. Moreover, note also that it does not seem worthwhile to accurately model shipments of 2 kg next to shipments of 1000 kg. A small error in the latter can easily be greater than the first shipment as a whole.

The parameters of the simulation are given in Table 5.1, whereas a graphical presentation of the probability density functions of w, r/w and v/w up until the 99$^{\text{th}}$ percentile are given in Figure 5.3. We note that the capacities and the demand are chosen to reflect a real life situation and correspond to capacity/demand ratios of 0.56 and 0.72 for the weight and volume respectively. However, in Section 5.5.3 we present computational results for various levels of the capacity/demand ratios. Finally, we mention that we do not consider cancellations, no-shows or deviations from the requested capacity requirements. Neither do we allow overbooking.

Table 5.1: Parameters for the demand simulation.

Parameter	Value	
T	10000	
λ	0.00225	
	Mean	Std. Dev.
W	793.474	942.370
r/w	2.55885	1.39501
v/w	0.00581	0.00338

Figure 5.3: Probability density functions for *w*, *r/w* and *v/w*.



5.5.2 Computational Results

We compare the performance of the various booking control policies when they are applied to the simulated test case described in the previous section. In order to obtain values for the bid prices, we simulate 100 demand sequences before the booking process starts. For each demand sequence, we obtain the optimal knapsack bid prices (KSBP) by applying the algorithm discussed in Section 5.4, and LP bid prices (LPBP) by taking the dual prices of the LP-relaxation of the problem. Taking the average bid prices over the 100 simulated demand sequences gives the bid prices that we use for the on-line decision problem. This has to be done only once before the actual booking process starts. Also for the dynamic approximation scheme (DAS) we generate 100 simulated demand sequences to base its decision on. However, this has to be done every time a booking request comes in. This means that the policy needs on-line computation time every time a request comes in. A booking control policy that needs on-line computation time is generally not considered to be useful in practice. However, the dynamic approximation scheme can still provide useful insights as an approximation of the optimal on-line booking control policy.

We simulate 100 demand sequences and report the average performance of the LPBP, KSBP and DAS policies in Table 5.2. We also include the ex-post optimal results. This is a natural upper bound for any booking control policy. The computations are performed on a Pentium III 550 MHz personal computer (256 MB RAM), using CPlex 7.1 to optimize the mathematical programming models. All computation times are reported in seconds.

Table 5.2: Performance of the LPBP, KSBP, DAS and ex-post optimal booking control policies.

	LPBP	KSBP	DAS	Ex-post Optimal
Profit				
Average	26491	28725	30047	33555
Std. Dev.	6293	6803	7747	8146
Minimum	15955	16767	14155	19723
Maximum	63136	64004	65993	66579
% of Ex-post Optimal				
Average	80.45	86.58	89.71	100
Std. Dev.	13.00	11.11	9.20	0
Minimum	33.28	45.59	55.09	100
Maximum	100	99.65	100	100
Weight				
Load Factor	0.978	0.951	0.915	0.996
Bid Price	0.190	0.878	-	-
Volume				
Load Factor	0.739	0.651	0.688	0.751
Bid Price	0.868	112.882	-	-
Average Comp. Time (sec.)	0	0	205.09	-

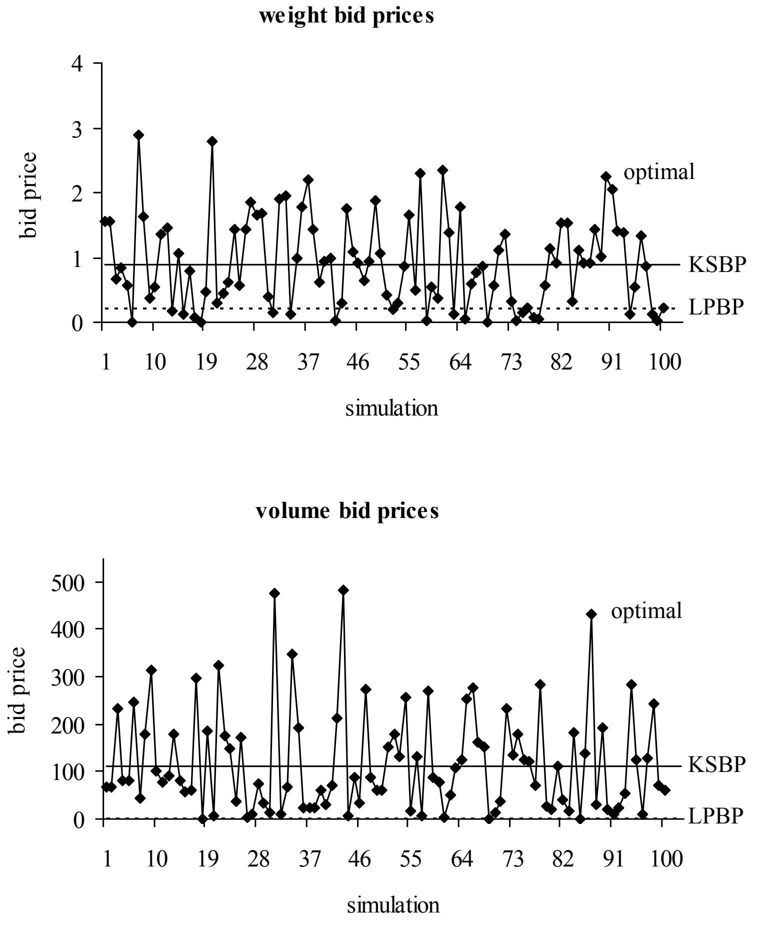
Table 5.2 shows that the knapsack formulation of the problem indeed produces better bid prices than the dual prices of the LP-relaxation. On average the knapsack bid prices produce € 2234 more profit than the LP bid prices. This test case is largely modelled after a daily cargo flight, for which this would add up to more than € 800,000 additional profit per year. Further we see that on average the LP bid prices obtain 80.45% of the optimal profit that could have been generated with perfect information. The

knapsack bid prices reach up to 86.58% of the optimal profit and have a smaller deviation. The dynamic approximation scheme obtains 89.71% of the optimal profit with an even smaller deviation. This means that the knapsack bid prices reduce the gap between the average performance of the LP bid prices and the dynamic approximation scheme by more than 66%. Here we note that the LP bid-price policy is a well-known booking control policy and that the dynamic approximation scheme can be seen as an indication of what an on-line policy can optimally perform. Also observe that the two bid-price policies have no computation time worth mentioning, whereas the dynamic approximation scheme needs an average of 205.09 seconds to handle a demand sequence.

When we use a t-test to test the hypothesis that the average performance of two booking control policies is equal, this is rejected with a p-value of 0.000 for the two bid-price policies. When we compare the knapsack bid prices with the dynamic approximation scheme, the t-test yields a p-value of 0.002. A sign test that counts the number of times one policy outperforms another and tests whether the outcome corresponds to a Binomial distribution with parameter 0.5, is rejected with a p-value of 0.000 in all cases.

Remarkable in Table 5.2, are the differences between the knapsack and LP bid prices. The LP bid prices are much smaller. With hindsight, we can compute an optimal set of bid prices for each of the 100 simulated demand sequences by applying the algorithm presented Section 5.4. This gives us average values of 0.916 and 119.24 for the weight and volume bid prices. These values are very different from the LP bid prices of 0.190 and 0.868, but close to the knapsack bid prices that take on values of 0.878 and 112.88. Note that the latter is not surprising since the knapsack bid prices are itself defined as the average values of the optimal bid prices for a set of 100 simulations. It does indicate that the algorithm to obtain the knapsack bid prices is reasonably robust when we use 100 simulations. In Figures 5.4 we visualize the LP and knapsack bid prices together with the optimal bid-price values for all simulations. The figure shows that the LP bid prices are situated far below most optimal bid prices. In fact, the LP bid prices are almost non-restrictive. This means that the policy almost reduces to a simple first come first serve policy for this test case, which shows the inefficiency of traditional LP bid prices for cargo revenue management.

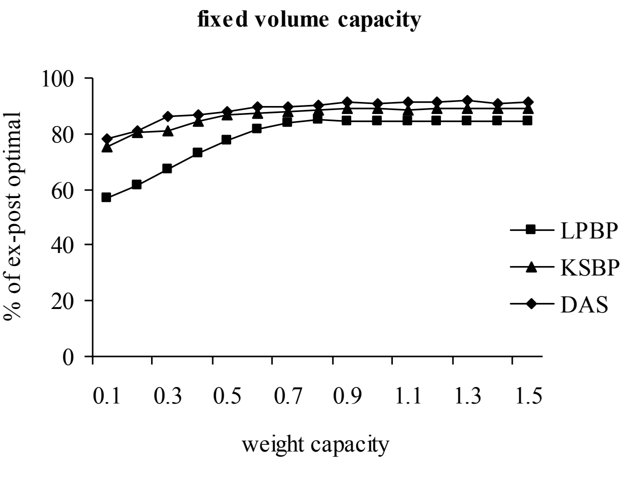
Figure 5.4: Optimal and policy bid prices for the weight and volume capacities.



5.5.3 Results for Different Capacity/Demand Ratios

The results discussed in the previous section describe one specific test case. It is interesting to see, however, to what extend the performance of the booking control policies is influenced by the set-up of the test case. In this section we show results for some different values for the capacity/demand ratios. We do this by varying the capacity levels while keeping the demand as it is. We define the weight and volume capacities as a percentage of the average demand and shift them from 0.1 to 1.5. We note that, in this section, we only use 10 demand realizations for each decision of the dynamic approximation scheme as opposed to 100 in the previous section. This reduces the computation time of the many results that we generate in this section. In Chapter 3 we have seen that this did not reduce the performance of the randomization technique much for the test case considered there. Tests show that this also holds in this test case. Figure 5.5 shows the profit of the two bid-price policies and the dynamic approximation scheme relative to the ex-post optimal profit for varying values for the weight capacity while keeping the volume capacity fixed at 0.7 and for varying values of the volume capacity while keeping the weight capacity fixed at 0.6. Each point reflects the average performance of the policy over 100 simulated demand sequences.

Figure 5.5: Performance of the booking control policies while varying one of the capacity/demand ratios.



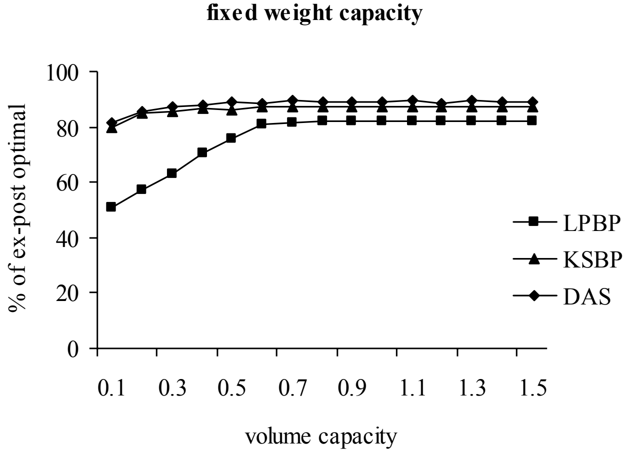
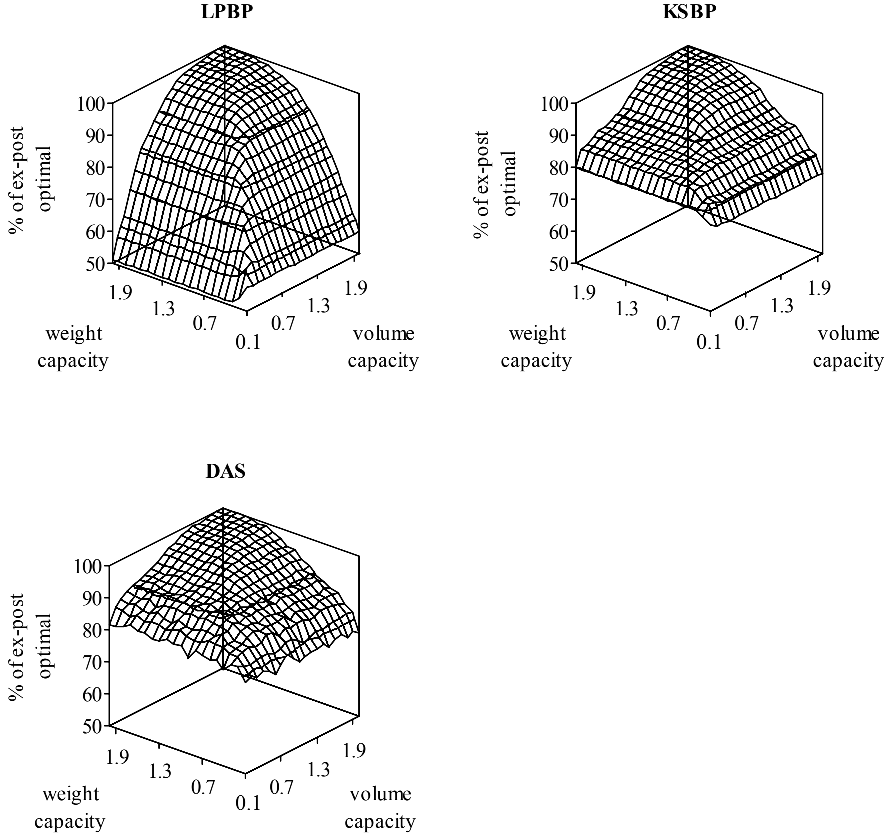


Figure 5.5 shows that the knapsack bid-price policy outperforms the LP bid-price policy for all cases. The average difference over all capacity combinations between the knapsack bid-price policy and the dynamic approximation scheme is 2% of the optimal profit. For the LP bid-price policy this is equal to 11.2%. Especially when one of the capacities is scarce the LP bid prices tend to perform badly. The performance of the knapsack bid prices, on the other hand, follows the performance of the dynamic approximation scheme consistently for all capacity combinations. For a full view of the performance of the booking control policies, we present 3-dimensional plots of the performance when we vary both capacity dimensions at the same time in Figure 5.6.

Figure 5.6: Performance of the booking control policies for different capacity/demand ratios.



In Figure 5.6 we see that all three booking control policies obtain 100% of the available profit when both capacities are large. This is easy to understand since all booking requests can be accepted in this case. By the steepness of the hillside presented in the figure, we see that the performance of the LP bid-price policy decreases more rapidly than that of the other two policies as the capacities reduce in size and the revenue management decision becomes more important. Note also that the performance of the dynamic approximation scheme is more volatile than those of the bid-price policies. The dynamic approximation scheme adapts itself to the situation at hand. This generally leads to higher profits but also to a more variable kind of decision making. The bid-price policies keep to a fixed policy that is set beforehand and that produces approximately the same results every time it is applied.

5.6 Summary and Conclusion

Cargo revenue management differs from passenger revenue management in a number of ways. Passengers generally belong to one of a limited number of booking classes and all take up one seat of the total seat capacity. Cargo shipments, on the other hand, are unique in profit, weight and volume. This means that traditional revenue management techniques originally constructed for the passenger problem cannot be used for the cargo revenue management problem. Instead, we formulate the problem as a 0-1 multi-dimensional on-line knapsack problem. This formulation is capable to include a network of flights. As for the passenger problem, we show that a bid-price acceptance policy is asymptotically optimal if demand and capacity increase proportionally and the bid prices are set correctly. We provide a polynomial time algorithm to obtain optimal bid prices for a given demand sequence. Bid prices for on-line use can be constructed by taking the average bid-price values over a number of simulated demand sequences.

A test case based on insights obtained from actual flight data, shows that the knapsack bid prices outperform the commonly used LP bid prices for every situation that we consider. Further, the performance of the knapsack bid prices closely follows the performance of a dynamic approximation scheme that we formulate. Such a dynamic approximation scheme is generally not considered to be useful in practice, but can give an indication of how well an on-line policy can optimally perform. Bid prices, on the other hand, are very practical and are already widely used for both passenger and cargo revenue management.

The cargo revenue management problem as we formulate it in this study is largely how we encountered it in practice. However, some extensions to the problem that would be worthwhile to examine still remain. First of all, we excluded the overbooking problem from our study. Overbooking plays an important role in cargo transportation. For cargo, the question is not so much whether or not a booked shipment shows up, but how much the actual weight and volume will deviate from the requested quantities. Overbooking decisions are usually made alongside revenue management decision but it would be interesting to combine the two. Further, as Kasilingam (1996) points out, cargo can be shipped among different routes as long as it arrives at its destination on time. Extending the model to take into account the routing of a cargo shipment raises a lot of opportunities for the company. Finally, the allocation of the cargo capacity over the long-term contracts and the spot sales would also make an interesting topic for further research. Until now this has largely been a managerial decision. Simulation studies similar to those presented in this paper can be used to estimate the profit that can be generated when a certain amount of capacity is made available for the spot sales. This way, the certainty of a long-term contract can be better evaluated against the additional profit that can possibly be generated by the spot sales.

Chapter 6

Summary and Conclusion

In this thesis we studied new features and models in revenue management. Revenue management has grown into one of the most successful OR applications to date. Although multiple definitions of revenue management exist, we mainly focus on the problem for which it is commonly known. This is to sell a fixed and perishable capacity as profitable as possible by selecting the right customers to sell to. As discussed in the introduction, this can be done by either quantity-based controlling of the sales or price-based controlling of the sales. In this thesis we use quantity-based techniques. By doing so, we stay close to the techniques used in practice for the problems that we consider. Price-based methods would result in different data requirements and more complicated models for these specific problems.

Each chapter in this thesis studies a different revenue management application. In Chapter 2 we extensively discuss the well-known airline revenue management problem. Revenue management originates from the airline industry and the airline revenue management problem still is the prototype for which a revenue management problem is known. In fact, most other revenue management applications make use of the solution techniques developed for the airline industry. By discussing the airline problem, we provide an overview of the general problem with its assumptions, difficulties and solution techniques for most revenue management problems. In Chapter 2 we introduce and discuss the two most popular booking control policies currently used in revenue management: bid prices and booking limits. Both policies have their advantages and disadvantages. However, both policies can be shown to be asymptotically optimal and are easy to use in practice. Further, we distinguish solution methods for a single flight and a network of flights. The latter are preferred by most airlines since they take into account the combinatorial effects that occur when passengers make use of multiple connecting flights to reach their destination. However, we also show that doing so drastically complicates the model.

A number of the solution techniques discussed in Chapter 2 are applied to the hotel industry in Chapter 3. Hotel revenue management resembles the airline revenue management problem. The main difference between the two problems is the fact that the combinatorial effects differ. In the airline problem the combinatorial effects are caused by the different flights that a passenger can use to reach his/her destination. In the hotel problem the combinatorial effects are caused by the overlapping stays when guests stay in the hotel for multiple days. This means that the overlap faced by a hotel does not end at a pre-specified point in time, but goes on without end. Our contribution in Chapter 3 is twofold. First, we provide a means to capture the effect of the overlapping stays by a rolling horizon of overlapping decision periods. Second, unlike previous studies we apply models to the hotel revenue management problem that take into account the stochastic nature of demand. We provide two methods to do this. The first is to apply a randomization technique to the bid-price policy. The second is to formulate a stochastic mathematical programming model to construct bid prices and booking limits. The booking control policies are tested in a realistic test case. The results show that the randomized bid prices outperform all other policies. However, this policy can be computationally cumbersome. Booking limits provide better results than bid prices when both policies are based on a deterministic mathematical programming model. The performance of the booking-limit policy is not improved when the stochastic model is used. The bid prices, on the other hand, greatly benefit from the use of the stochastic model and are able to obtain better results than both the deterministic and stochastic booking limits. However, the performance of the stochastic bid prices depends greatly on the demand discretization used in the underlying mathematical programming model, which adds an unwanted risk to applying the booking control policy in practice.

In Chapter 4 we return to the airline revenue management problem. In this chapter we relax the fixed capacity assumption often thought essential for revenue management. We do this by considering planes with convertible seats. A row of these seats can be converted from five business seats to six economy class seats and vice versa. This offers the airline company the opportunity to adjust the configuration of the plane to the demand at hand. We formulate a mathematical programming model to take this shifting of the capacity into account. This model is also extended to include cancellations and overbooking. We present a dynamic approximation scheme for the optimal control policy based on the mathematical programming model. The stochastic nature of demand is accounted for by randomizing the acceptance scheme over a number of simulated demand realizations. In order to evaluate the revenue potential of the shifting capacity opportunity, we construct a realistic test case in which three consecutive flights with different demand patterns are flown by the same plane. The results show that the revenue potential created by the convertible seats is considerable even when the capacity configuration of the plane is adjusted only once before the booking period starts. When the shifting capacity opportunity is taken into account continuously during the booking process the revenues can be further improved, especially when the dynamic approximation scheme is improved by randomization. The same results are obtained when cancellations and overbooking are taken into account. It is interesting to note, however, that one passenger can block a whole row from converting from the business to the economy class or vice versa. Therefore, when the shifting capacity opportunity is considered continuously, booking requests can be turned into denied boardings intentionally even when the booking process has not ended yet.

Finally, in Chapter 5 we consider revenue management for cargo transportation. The fundamental difference between cargo and passenger revenue management, is that cargo shipments are uniquely defined by their profit, weight and volume whereas passengers generally belong to one of a limited number of price classes. In other studies on cargo revenue management (see: Kasilingam (1996) and Karaesmen (2001)) the cargo shipments are categorized into classes such that standard revenue management techniques developed for passenger revenue management can be applied. Unlike these studies, we treat the cargo shipments as the unique items they are. This results in a 0-1 decision problem. Since a cargo shipment can use multiple connecting flights to reach its destination and the accept/deny decision has to be made as the booking request occurs, we model the problem as a 0-1 multi-dimensional on-line knapsack problem. We show that a bid-price acceptance policy is asymptotically optimal if demand and capacity increase proportionally and the bid prices are set correctly. We present a polynomial time algorithm to obtain optimal bid prices for a given set of booking requests and use this to construct a booking control policy. A realistic test case shows that our knapsack bid prices perform better than traditional LP-based bid prices as known for passenger revenue management. Further, the performance of the knapsack bid prices closely follows the performance of a computational cumbersome dynamic approximation scheme that we formulate.

In this thesis we have extended hotel, airline and cargo revenue management problems for new features such as a rolling horizon, convertible seats and unique booking requests. We provide mathematical models that incorporate these features and derive booking control policies based on those models. We test the performance of the booking control policies in a simulated environment. Unlike most other revenue management studies, we put a lot of effort in testing the ways to account for the stochastic nature of demand in the models. We do this by formulating stochastic mathematical programming models or by randomization techniques. Especially the latter provides an improvement of the performance of the policies. However, it also increases the computation times. In some studies, we also develop dynamic approximation schemes for the optimal control policy. These dynamic approximation schemes are generally not considered to be useful in practice because of the on-line computation time involved. However, they do provide an approximation of the optimal performance of an on-line policy. The bid-price and booking-limit booking control policies that we present, however, are very well suited for practical use.

Many of the studies performed in this thesis come forth from questions raised by revenue management practitioners. Also the test cases that we construct are chosen to reflect realistic environments. This means that the results presented in this thesis provide useful insights for practitioners and can be used to further develop and extend revenue management techniques in practice.

Nederlandse Samenvatting (Summary in Dutch)

Bedrijven die bederfelijke goederen verkopen, zijn vaak genoodzaakt om een vaste capaciteit van hun product te verkopen binnen een gegeven tijdsperiode. Indien de markt gekarakteriseerd wordt door klanten die bereid zijn om verschillende prijzen te betalen voor het product, is het vaak mogelijk om met behulp van product differentiatie verschillende klantsegmenten tegelijkertijd te bedienen. Dit heeft als gevolg dat hetzelfde product tegen verschillende prijzen kan worden verkocht. Voorbeelden hiervan zijn het hanteren van verschillende prijzen op verschillende tijdstippen of voor verschillende service niveaus. Wanneer er verschillende prijzen worden gehanteerd voor hetzelfde product moet echter voorkomen worden dat dit ten koste gaat van de opbrengst doordat te veel klanten ervoor kiezen om het product voor een lage prijs te kopen. Het is derhalve belangrijk om de klantsegmenten voldoende te differentiëren en beslissingen te maken omtrent welk gedeelte van de capaciteit wordt verkocht in elk prijssegment. Hierbij is het zaak om niet te veel producten te verkopen voor een lage prijs, maar tegelijkertijd ook te voorkomen dat producten die tegen een hogere prijs worden aangeboden, onverkocht blijven. Hoe de capaciteit te verdelen over de verschillende prijssegmenten zodat de opbrengst maximaal is, staat algemeen bekend als het revenue management probleem.

Revenue management komt voort uit de luchtvaart industrie, waar al lange tijd verschillende prijsklassen worden aangeboden op een vlucht. De prijs die wordt betaald voor een ticket hangt veelal af van eigenschappen zoals: het tijdstip van boeken, de ticket opties of het opnemen van een zaterdag overnachting in het verblijf. Dit zijn geen eigenschappen die de prijs van een boeking hoeven te beïnvloeden, maar die wel worden aangegrepen door de luchtvaartmaatschappij om het product te differentiëren en derhalve verschillende prijsklassen te kunnen hanteren voor hetzelfde product. Andere bekende toepassingen van revenue management zijn onder andere te vinden bij hotels, spoorwegen, vracht vervoer en auto verhuur bedrijven. Dit proefschrift richt zich op de wiskundige modellen die het maken van revenue management beslissingen ondersteunen. Deze modellen zijn erop gericht de aanwezige capaciteit op de juiste manier beschikbaar te stellen aan de verschillende prijssegmenten zodat de verwachte opbrengst maximaal is. Elk hoofdstuk behandeld een andere toepassing van revenue management en geeft een analyse van de beslissingsondersteunende modellen die daarvoor gebruikt kunnen worden.

In hoofdstuk 2 bespreken we het revenue management probleem zoals bekend in de luchtvaart industrie. Het luchtvaart probleem staat bekend als het prototype revenue management probleem. Vele andere revenue management toepassingen zijn afgeleid van het luchtvaart probleem en met het bespreken van deze toepassing geven we derhalve ook een overzicht van de aannamen, moeilijkheden en oplossingsmethoden voor vele andere toepassingen. In dit hoofdstuk introduceren we ook de twee meest populaire technieken om boekingen te controleren: ‘bid prices’ en ‘booking limits’. Een bid price kan worden geïnterpreteerd als een minimaal te accepteren prijs voor een product. Door deze over de tijd te fluctueren kunnen op verschillende tijdstippen verschillende prijsklassen open en dicht worden verklaart. Booking limits daarentegen beperken het aantal boekingen dat geaccepteerd kunnen worden in elke prijsklasse. Beide technieken hebben zo hun voor- en nadelen. Echter, beide technieken zijn zeer praktisch in het gebruik en zijn asymptotisch optimaal. Verder maken we onderscheid tussen technieken voor enkele vluchten en voor een netwerk van vluchten. De laatstgenoemde technieken houden rekening met de combinatorische aspecten die geïntroduceerd worden in het probleem wanneer passagiers gebruik maken van meerdere vluchten om tot hun bestemming te geraken. Dit is wenselijk, maar compliceert het onderliggende model aanzienlijk.

Een aantal van de wiskundige modellen die we bespreken in hoofdstuk 2 voor de luchtvaart, passen we in hoofdstuk 3 toe in de hotel industrie. Hotel revenue management heeft veel weg van het bekende luchtvaart revenue management probleem. Het voornaamste verschil is dat de combinatorische effecten verschillen. In het luchtvaart probleem worden de combinatorische effecten veroorzaakt doordat passagiers meerdere vluchten kunnen gebruiken om tot hun bestemming te geraken. In het hotel probleem worden de combinatorische effecten veroorzaakt door de overlapping die kan bestaan tussen de verblijven van gasten die meerdere dagen in het hotel verblijven. Waar de combinatorische effecten in de luchtvaart beperkt zijn tot een gelimiteerd aantal vluchten, kunnen de combinatorische effecten voor een hotel tot in het oneindige doorlopen. In hoofdstuk 3 leveren we twee bijdragen aan de bestaande literatuur. Ten eerste tonen we aan hoe de effecten van de overlappende verblijven op te nemen in een model met behulp van een rollende horizon van beslissingsperioden. Ten tweede passen we, in tegenstelling tot andere studies in de hotel industrie, modellen toe die de stochasticiteit van de vraag in ogenschouw nemen. Dit laatste doen we op twee manieren. In de eerste plaats door gemiddelde bid-price waarden te nemen over meerdere willekeurige trekkingen uit de vraagverdeling. En in de tweede plaats door een stochastisch mathematisch programmerings model te definiëren op basis waarvan bid prices en booking limits kunnen worden bepaald. We testen de verschillende methoden in een realistische test omgeving en vergelijken de resultaten met die van deterministische bid prices en booking limits. Daaruit blijkt dat de eerste methode om de bid prices te verbeteren tot een sterke verbetering van de prestaties leidt. Echter, deze methode is rekenkundig zeer intensief. Het stochastische programmerings model leidt ook tot een verbetering van de bid-price methode alhoewel deze prestaties erg afhankelijk zijn van de benadering van de vraagverdeling in het model. Het model leidt niet tot een verbetering van de booking limits methode.

In hoofdstuk 4 keren we terug naar het luchtvaart revenue management probleem. In dit hoofdstuk laten we echter de aanname van een vaste capaciteit los. We doen dit door vliegtuigen te beschouwen die voorzien zijn van zogeheten converteerbare stoelen. Een rij van deze stoelen kan omgezet worden van vijf business klasse stoelen naar zes economy klasse stoelen en andersom. Dit bied de luchtvaart maatschappij de mogelijkheid om de configuratie van het vliegtuig aan te passen aan de vraag van het moment. We formuleren een mathematisch programmerings model om het verschuiven van de capaciteit op te nemen in het beslissingsprobleem. We breiden dit model tevens uit om annuleringen en overboeking in ogenschouw te nemen. In dit hoofdstuk gebruiken we geen bid prices of booking limits, maar een dynamische benaderingsmethode van de optimale strategie. Om te corrigeren voor de stochasticiteit van de vraag, wordt de methode verbeterd aan de hand van willekeurige trekkingen uit de vraagverdeling. Om na te gaan wat de potentiële extra opbrengst is van de converteerbare stoelen, definiëren we een realistische test case waarin drie opeenvolgende vluchten, die elk een ander vraagpatroon hebben, worden gevlogen door hetzelfde vliegtuig. De resultaten geven aan dat de potentiële extra opbrengst van de converteerbare stoelen aanzienlijk is, zelfs wanneer de configuratie van het vliegtuig voor elke vlucht voor het begin van het boekingsproces wordt bepaald. Wanner de configuratie gedurende het boekingsproces kan worden aangepast, is de potentiële extra opbrengst nog hoger.

Tenslotte beschouwen we in hoofdstuk 5 revenue management voor vrachtvervoer. Het fundamentele verschil tussen revenue management voor vracht en passagiers is dat een vracht uniek gedefinieerd wordt door zijn winstgevendheid, gewicht en omvang. Een passagier behoort daarentegen tot één van een gelimiteerd aantal prijsklassen, waarin elke passagier gelijk is. In andere studies op het gebied van revenue management voor vrachtvervoer, worden de vrachten onderverdeeld in categorieën om zodoende de standaard revenue management technieken, ontwikkeld voor passagiers, toe te kunnen passen. In tegenstelling tot deze studies beschouwen wij de vrachten daadwerkelijk als uniek geïdentificeerd. Dit heeft een 0-1 beslissingsprobleem als gevolg. Omdat ook vrachten meerdere vluchten kunnen gebruiken om tot hun bestemming te komen, formuleren we het probleem als een 0-1 multi-dimensionaal on-line knapzak probleem. We tonen aan dat een bid-price strategie ook voor dit probleem asymptotisch optimaal is wanneer de capaciteit en de vraag proportioneel toenemen en de bid prices juist zijn gekozen. We leveren een algoritme dat in polynomische tijd een optimale set van bid prices construeert voor een gegeven set van vrachten. Tevens geven we aan hoe dit algoritme kan worden gebruikt om bid prices te creëren die gebruikt kunnen worden gedurende het boekingsproces. Een realistische test case wijst uit dat onze knapzak bid prices beter presteren dan bid prices zoals traditioneel berekend voor passagiers revenue management.

In dit proefschrift breiden we hotel, luchtvaart en vrachtvervoer revenue management problemen uit met nieuwe eigenschappen zoals een rollende horizon, converteerbare stoelen en unieke boekingen. We geven wiskundige modellen die deze eigenschappen opnemen en leiden van deze modellen beslissingsmethoden af die gebruikt kunnen worden om het accepteren van de boekingen te beheersen. We testen de prestaties van de verschillende beslissingsmethoden in een gesimuleerde omgeving. In tegenstelling tot de meeste andere revenue management studies, besteden we veel aandacht aan het opnemen van de stochasticiteit van de vraag in de modellen. Ook ontwikkelen we in enkele gevallen dynamische benaderingsmethoden voor de optimale strategie. De meeste studies in dit proefschrift zijn probleemstellingen die direct voortkomen uit de praktijk. Tevens zijn de test cases dusdanig gekozen dat ze de werkelijkheid weergeven. Dit houdt in dat de resultaten die we presenteren in dit proefschrift nuttige inzichten verschaffen voor de praktijk en gebruikt kunnen worden om revenue management modellen en technieken verder te ontwikkelen en uitbreiden.

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Revenue Management: New Features and Models

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Kevin Pak (1977) obtained his Master's degree in Econometrics and Operations Research from the Erasmus University Rotterdam in 2000. In the same year he joined ERIM in order to carry out his doctoral research on the subject of revenue management. Throughout the years his work has been published in and presented at a number of international journals and conferences. Currently, he applies his knowledge of operations research into practice as a consultant at ORTEC bv.

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