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Complexity, Robustness, and Performance

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Complexity, Robustness, and Performance: Trade–Offs in Organizational Design

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Abstract

This paper analyses the relationship between organizational complexity (the degree of detail of information necessary to correctly assign agents to positions), robustness (the relative loss of performance due to mis–allocated agents), and performance. More complex structures are not necessarily more profitable, but are less robust. One of the least complex structures always performs worst. Superior organizational performance may vanish completely due to mis–allocated agents. Organizational performance can be enhanced through training agents;

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re–assigning them when adequate knowledge about their characteristics is obtained through monitoring; simplifying the organizational structure; and influencing the environment. The trade–offs involved are analysed.

1 Introduction

The notion of complexity figures prominently in the literature on organizational design. A structure is called complex if it is made up of a large number of divisions or hierarchical layers or if it contains many interdependent parts the individual functioning of which is of importance to the overall performance of the organization. The more complex an organization the heavier the demands on its information processing capacities.[1]

If the superiority of the information processing capacity of the overall organization is predicated on judiciously positioned employees within the organization, the knowledge requirements the organizational designer faces may well be challenging. The job of designing and implementing the optimal complex organization may grow too difficult, impractical, and too time consuming to be considered seriously. In fact, if information about relevant characteristics of the employees is missing or partial, simplicity and ease of optimization may outweigh the superiority of optimally structured, but more complex organizations. The balance may shift still further in favour of simple organizations, if the performance of complex organizations is very sensitive to small

[1] See for example Galbraith (1973, 1977), Huber and Daft (1987), Jablin (1987), Lawrence and Lorsch (1967), Scott (1998), and Thompson (1967).

deviations from whatever it takes to make them operate correctly.

Secondly, an important question is whether complex organizations are suited for many types of environments or only for a relatively small set. Are complex organizations more sensitive to changes in the environment than simple ones, or is the reverse the case?

Any answer to the above mentioned issues involves an assessment of the robustness of complex organizations. If complexity stimulates profitability but at the cost of diminishing robustness, organizational designers face an important trade-off.

This paper looks at the relationship between organizational complexity, robustness, and performance. It tries to answer the following questions: What does it mean for an organization to be complex? How can one measure degrees of complexity? How does one measure robustness? What does it mean for an organization to be less robust than some other? Are more complex organizations more profitable but less robust than less complex ones?

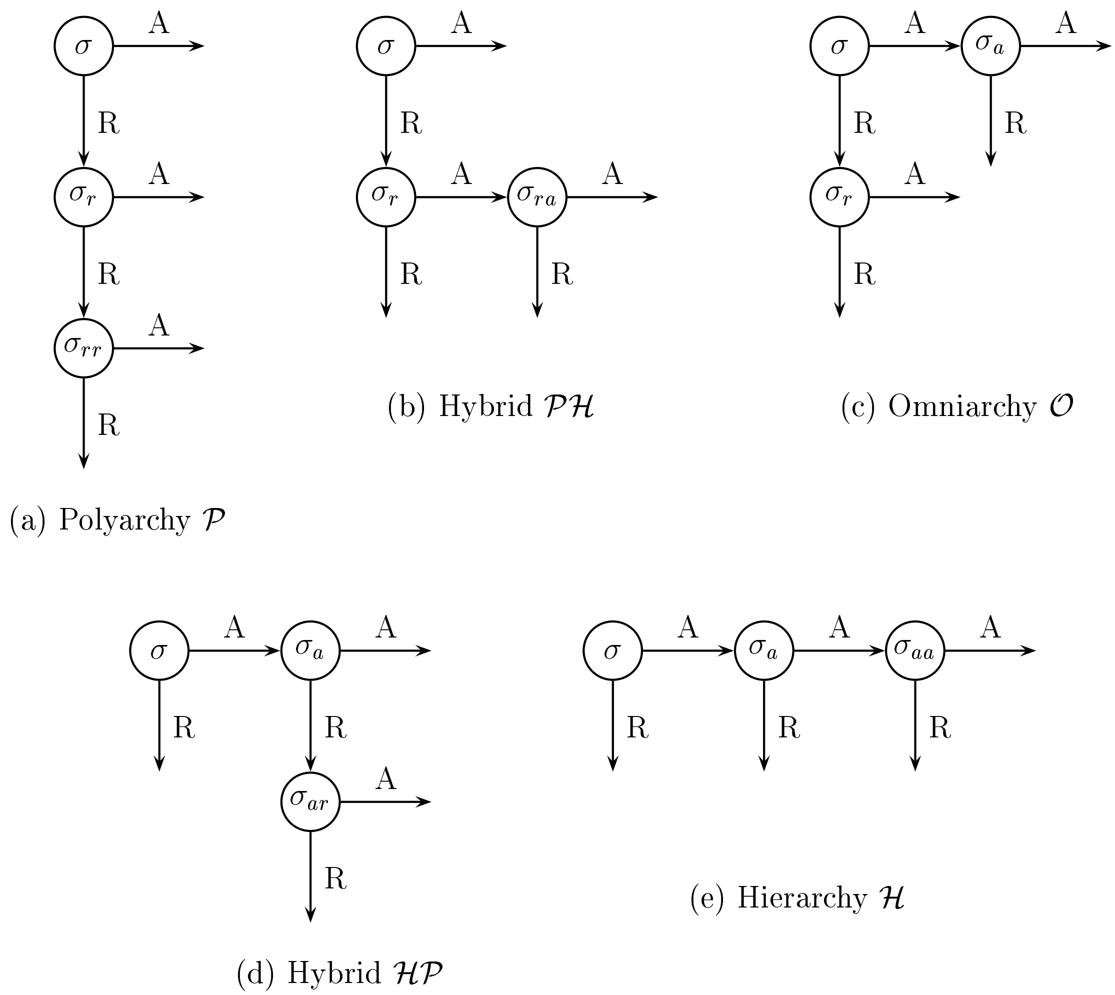
I use a very simple model introduced by Sah and Stiglitz (1985, 1986) to address these questions. Their model captures an important feature of real world organizations, project selection, in a simple fashion. Organizations are modelled as sequential decision structures. Projects, when implemented, can either lead to a profit, X or a loss, $-Y$. Screening takes place by error–prone agents: some good projects are rejected (R), while some bad projects are accepted (A).[2] In other words, agents are characterized

[2] Campbell (1958) and authors in the area of small group communication, *e.g.*, Hirokawa and Scheerhorn (1986) and Gouran and Hirokawa (1986), emphasize the persistence of fallible human decision making.

by a pair of probabilities of acceptance (p^g, p^b), where p^g (p^b) stands for the probability of acceptance of good (bad) projects. Sah and Stiglitz limit analysis to a hierarchy $\mathcal{H}$ and a polyarchy $\mathcal{P}$[3]. In the former structure, if the first agent rejects a project it is rejected by the organization, while if it is accepted it moves on to the next agent. The decision of the second agent is final. In the polyarchy, projects accepted by the first agent are implemented by the organization. Rejected projects are screened once more before a final decision (acceptance or rejection) is taken. Sah and Stiglitz show that the way in which an organization aggregates individual errors depends critically on the nature of the sequential screening process. Moreover, which organization is performing best depends on the type of environment in which it is operating. In 'friendly' environments, *i.e*, situations in which losses made due to erroneously accepted bad projects are small relative to the foregone profits stemming from the incorrect rejection of good projects, a polyarchy performs better than a hierarchy. The opposite holds for 'tough' environments where possible losses are substantial relative to foregone profits. The intuition is that hierarchies are 'tighter' than polyarchies, which is beneficial in case of tough environments, but an exaggeration in friendly ones.

I extend their model by introducing heterogeneous agents (some agents accept less bad projects and more good projects than others) and by studying three more screening structures, see Figure 1. The nodes stand for organizational departments, bureaus or

[3] A note on terminology. I use the terms hierarchy and polyarchy to be as explicit as possible on the relationship between the sequential decision structures I use and the ones used by Sah and Stiglitz. Casual evidence suggests that decision structures in real world hierarchical organizations can take on a number of forms, including the ones studied here.



(a) Polyarchy $\mathcal{P}$ (b) Hybrid $\mathcal{PH}$ (c) Omniarchy $\mathcal{O}$ (d) Hybrid $\mathcal{HP}$ (e) Hierarchy $\mathcal{H}$

Figure 1: The five organizational structures studied

desks and the directed edges represent the direction in which projects flow in the organizations. The label on an edge starting at a node is associated with the action taken at that node. A node is indexed by the sequence of actions necessary to reach the node. For example, in the polyarchy $\mathcal{P}$, Figure 1 (a), the project lands on desk σ_{rr} after a sequence of two rejections. I limit analysis to these five structures for various reasons. First of all, by limiting myself to structures with an equal number of nodes, and by assuming that agents with the same screening capabilities will be paid the same wage irrespective of the organization they work for, I can ignore the total wage bill in profit comparisons. Secondly, as will become clear from this paper, three agents is the minimal number required to make a discussion of complexity interesting. Thirdly, the three structures I add have in common with the hierarchy and polyarchy introduced by Sah and Stiglitz that decisions are taken sequentially, that one person is the first to analyse any project, and that an agent can either reject or accept a project. In the language of graph theory, analysis is limited to the class of finite binary directed rooted trees. I call the polyarchy and hierarchy 'pure' structures. Structures $\mathcal{PH}$ and $\mathcal{HP}$ are 'hybrid' structures, as they combine in some sense characteristics of both pure structures.

I define the *complexity of a structure* as the level of detail of information that is necessary and sufficient to correctly assign agents to positions in the structure. As I noted at the beginning of this section, the literature on organizational design views the complexity of an organization as stemming from the number of elements or subsystems or the type of linkages between organizational parts. This paper shows that differences in level of detail of information required in the optimisation problem parallel differences in type of structural connections between agents. This definition therefore provides a unequivocal measure of degrees of complexity. It has the added advantage of measuring the complexity of an organization not by a feature of its source—characteristics of the organizational structure—but by its effect on human cognition. A structure is not difficult, complicated or complex of itself, but in the eye of the designer or observer.

As agents are heterogeneous and as correctly assigning agents to organizational positions may require information about these agents, it will be useful to introduce a measure that gauges the extent to which the optimal performance of an organization depends on the correctness of the allocation of agents. If the organizational designer is in possession of limited information only, or is error–prone herself, some notion of organizational robustness to such limitations is key. The notion of robustness introduced in this paper measures the maximal extent to which the expected profits on implemented projects can fall short of the expected profits in the optimal case due to the erroneous assignment of agents to organizational positions (a worst case scenario). The smaller the maximal reduction relative to the expected profits in the optimal case, the more robust the structure will be called.

Turning to the results, I show that the hierarchy $\mathcal{H}$ and the polyarchy $\mathcal{P}$ are of least complexity as they do not require any information about the agents to position them correctly. Note that the probability with which a hierarchy accepts a project is the product of individual probabilities of acceptance. Clearly, who is first, second, or third is immaterial. The same holds for a polyarchy: observe that rejection by the polyarchy requires rejection by all members. The hybrid structures $\mathcal{HP}$ and $\mathcal{PH}$ require ordinal information and are therefore of intermediate complexity. The agent located at the first node, making as it were a pre–selection, should be the best. Once a project has passed the first desk, it moves on to a structure which is really a two person hierarchy or polyarchy. Swapping agents at these desks leaves the probability of acceptance unaffected. Finally, in the omniarchy, cardinal information is required. Whether the best agent should be located at σ_a or at σ_r depends crucially on the characteristics of all the agents. This structure is therefore the most complex of the five structures studied. As claimed above, differences in organizational complexity parallel differences in structural connections.

I show that an increase in the complexity of an organization does not necessarily lead to superior performance. First of all, and in line with the findings of Sah and Stiglitz, which structure performs best depends on the type of environment. The polyarchy performs best in friendly environments, the hybrid $\mathcal{PH}$ in moderately friendly environments, the $\mathcal{HP}$ in moderately tough environments, and finally the hierarchy in tough environments. That hybrid structures perform better for 'intermediate' environments stems from their combining features of both the hierarchy and the polyarchy. Secondly, the most complex structure, the omniarchy, is never the best organization. This is quite surprising as an omniarchy in some sense also combines traits of both a hierarchy and a polyarchy. The intuition is that for friendly environments, the polyarchy $\mathcal{P}$ and the $\mathcal{PH}$ outperform the omniarchy $\mathcal{O}$ as the former organizations accept more good projects than the latter. Conversely, for relatively unfriendly environments, the influence of the probability of acceptance of bad projects increases, making the hierarchy $\mathcal{H}$ and the $\mathcal{HP}$ more suitable. Even for intermediate environments, the interaction of the characteristics of the agents and of the environment is such that the omniarchy is never the best.

Interestingly, it is also shown that for any type of environment, and whatever the characteristics of the agents, the hybrid structures and the omniarchy are never the worst. It is always one of the *least* complex structures, the hierarchy or the polyarchy, that performs worst (the hierarchy when the environment is friendly; the polyarchy in case of a hostile environment). The reason is that the hybrid structures and the omniarchy combine structural features of both a hierarchy and a polyarchy. The former structures can therefore improve upon whatever is the worst performing pure structure. In other words, the hierarchy and the polyarchy are highly geared towards a special type of environment, and quickly loose their attractiveness in other environments.

Turning to the results on robustness, I show that in virtually all instances the maximal drop in expected profits ensues from *(a)* interchanging the optimal position of the best and the worst agent and *(b)* agents that are maximally heterogeneous. This is quite intuitive if one recalls the definition of robustness as the maximal drop in performance due to erroneously assigned agents. Any drop in performance is absent if all agents are equal, and is likely to be larger *(a)* the more the precise assignment affects overall performance and *(b)* the larger the differences in individual qualities. As a consequence, the most complex structure, the omniarchy, is uniformly less robust than both hybrid structures. That is, for all environments the drop in relative performance is largest for the omniarchy. That this drop can be substantial is illustrated by a comparison of the hybrid structure and the omniarchy on the one hand, and the pure structures on the other. Remember that in the latter structures, no mis–allocation can occur as the ordering of heterogeneous agents is immaterial. I show that the maximal reduction in performance is such as to eliminate any advantage the hybrid structures and the omniarchy have over pure structures.

Which of the hybrid structures is most robust depends on the type of environment. The $\mathcal{PH}$ is more robust in relatively friendly environments, while the $\mathcal{HP}$ is less sensitive to errors when environments are relatively hostile. This is an important observation as it means that the superior performance of the $\mathcal{PH}$ in terms of expected profits in friendly environments does not come at the cost of relatively low robustness when compared to the $\mathcal{HP}$. The same applies for the $\mathcal{HP}$ in case of tough environments.

These results suggest a number of means an organizational designer has at her disposition when intending to improve the organization's performance: one can improve the quality of the agents; reduce the gap between information required and information actually possessed either by monitoring the existing agents or simplifying the organizational structure; and influence the environment. These means and the concomitant trade–offs are discussed in section 4 in an informal way.

The Sah and Stiglitz model I use in this paper is simple in many respects. It should be interesting to study the interplay of organizational complexity, robustness, and performance in different decision structures. Possible extensions include the introduction of authority, departmental specialisation, and incentives. In Visser (2000) I study hierarchies and polyarchies with agents that are fully rational. Their decision rules reflect the position in the organization they occupy and the information revealed by the action taken by any preceding agent. In that paper I show that although agents may be heterogeneous, switching their position in the organization leaves the expected profit unaffected. That is, the result that polyarchies and hierarchies are of least complexity carries over to a model with fully rational agents.

Although I am not aware of other studies formally analysing the relationship between robustness, complexity, and performance, two papers are clearly related to the present study. Ioannides (1987) applies findings from information theory to sequential decision structures to show that one can increase the performance of some organizations by a special type of replication called composition. Composition means that one replaces an individual agent by a replica of the entire organization. In this way, one can at the same time arbitrarily increase the probability of acceptance of good projects and, at the same time, reduce the probability of acceptance of bad projects. Such organizations will be very complex. Within the realm of qualified majority decision rules, Ben–Yashar and Nitzan (2001) study the robustness of optimal decision rules. Robustness is measured by the maximal change in the total number of agents that does not alter the optimal qualified majority. They establish that, in general, such decision rules are not very robust. In particular, neither the hierarchy and the polyarchy are very robust according to this measure.[4]

In the next section, I introduce the model and the main concepts used. Section 3 presents the main results. The section that follows discusses the effectiveness of various means of improving organizational performance. Section 5 concludes. Proofs can be found in the Appendix.

2 The Model and Concepts Used

2.1 The Project Environment

There exists a pool of projects of size 1. Projects can be either of good quality, $q = g$ (which is the case with probability α) or of bad quality, $q = b$. An implemented, good project gives rise to a profit X, while an implemented, bad project leads to a loss equal to $-Y$. It will be useful to summarise the state of the environment by $\beta := \frac{1-\alpha}{\alpha}\frac{Y}{X}$.

[4]Note that a hierarchy implements projects with the same probability as a majority voting rule requiring acceptance by all, whereas polyarchies behave like a voting rule requiring acceptance by just one agent for a project to be implemented.

The higher β, the tougher the environment. This means that either possible losses rise relative to potential profits, or that bad projects become predominant.

2.2 The Agents

There are three agents $i \in \{1, 2, 3\}$. Each agent can either accept, A, or reject, R, a project. Ideally, one would like the agents to accept all good projects and to reject all bad projects. Let p_i^b (p_i^g) stand for the probability with which agent i accepts bad (good) projects. I assume that every agent is fallible: $0 < p_i^b < 1/2 < p_i^g < 1$. In words, agents accept bad projects, reject good ones, but do so less frequently than a randomizing device that accepts one out of two projects. Agent i is more skilled than j if the former accepts more good projects, $p_i^g > p_j^g$, and less bad projects, $p_i^b < p_j^b$. This will be denoted by $i \succ j$. I assume that $1 \succ 2 \succ 3$. The characteristics of the agents can be denoted by the ordered pair of vectors $(\overline{p}^b, \overline{p}^g) = (p_1^b, p_2^b, p_3^b, p_3^g, p_2^g, p_1^g)$.

As the characteristics of the agents differ, it will be useful to introduce a measure H of heterogeneity of agents. A useful measure is the spread of the probabilities of acceptance of either good or bad projects:

$$H^q = \sum_i^3 \left(p_i^q - \frac{\sum_i^3 p_i^q}{3} \right)^2, \quad q = b, g \tag{1}$$

This is similar to the variance of a random variable. Note that I measure the heterogeneity per type of project separately, without establishing a measure of overall heterogeneity.

2.3 Organizations

An organization (Σ, ϕ) consists of a structure Σ and an allocation ϕ of agents to positions in this structure. As explained in the introduction, I limit attention to the five structures, $\Sigma \in \{\mathcal{H}, \mathcal{HP}, \mathcal{O}, \mathcal{PH}, \mathcal{P}\}$ depicted in Figure 1. A structure fixes the flow of projects. The nodes in the structure are indexed by the sequence of decisions necessary to reach the node, *e.g.*, σ_{ar} is reached after first an acceptance at the root and a rejection at node σ_a. For every structure Σ, an allocation ϕ places agents 1, 2 and 3 at a desk.

An organization (Σ, ϕ) accepts projects of quality q with probability $p^q(\Sigma, \phi)$. The organizational structure fixes the functional *form* of $p(\Sigma, \phi)$, the same for both good and bad projects. Its precise *value* depends on the allocation ϕ, the characteristics of the agents, and the type of project q. The functional forms of $p(\Sigma, \phi)$ for $\Sigma \in \{\mathcal{H}, \mathcal{HP}, \mathcal{O}, \mathcal{PH}, \mathcal{P}\}$ are as follows:

$$\begin{array}{lll}
p(\sigma) + (1 - p(\sigma))[p(\sigma_r) + (1 - p(\sigma_r))p(\sigma_{rr})] & \text{if} & \Sigma = \mathcal{P} \\
p(\sigma) + (1 - p(\sigma))p(\sigma_r)p(\sigma_{ra}) & \text{if} & \Sigma = \mathcal{PH} \\
p(\sigma)p(\sigma_a) + (1 - p(\sigma))p(\sigma_r) & \text{if} & \Sigma = \mathcal{O} \\
p(\sigma)[p(\sigma_a) + (1 - p(\sigma_a))p(\sigma_{ar})] & \text{if} & \Sigma = \mathcal{HP} \\
p(\sigma)p(\sigma_a)p(\sigma_{aa}) & \text{if} & \Sigma = \mathcal{H}
\end{array} \tag{2}$$

Note that I have dropped the reference q to a type of project as the functional form does not depend on the type of project.

2.4 Organizational Performance

The overall goal of the organizational designer is to maximize the expected value of the implemented projects. This is determined by characteristics of the environment α, X and Y; the characteristics of the agents $(\overline{p}^b, \overline{p}^g)$; and by the structure Σ and the allocation ϕ. That is, $W(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \alpha, X, Y) = \alpha X p^g(\Sigma, \phi) - (1-\alpha) Y p^b(\Sigma, \phi)$. It will be useful to work with a monotone transformation of W:

$$V(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \beta) = p^g(\Sigma, \phi) - \beta p^b(\Sigma, \phi) \tag{3}$$

An allocation ϕ such that, say, $(\phi(1), \phi(2), \phi(3)) = (\sigma_a, \sigma, \sigma_r)$ in the omniarchy leads to a profit of $V(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \beta) = p_2^g p_1^g + (1 - p_2^g) p_3^g - \beta \left[p_2^b p_1^b + (1 - p_2^b) p_3^b \right]$.

For a given structure Σ and agents with characteristics $(\overline{p}^b, \overline{p}^g)$, the designer is interested in finding an optimal allocation ϕ^* that maximizes Equation 3:

$$\phi^*(\Sigma, (\overline{p}^b, \overline{p}^g); \beta) = \arg\max_{\phi} V(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \beta) \tag{4}$$

Equation 4 makes clear that the optimal allocation may depend on the organizational structure, on the characteristics of the agents, and on the environment.

2.5 Knowledge

Although the optimal allocation may depend on the characteristics of the agents, the organizational designer may not know these characteristics. Indeed, I distinguish three types of information the organizational designer may have concerning the screening capabilities of her agents. She may have no information at all about the screening capabilities of the agents, ordinal information about the agents' characteristics, or cardinal information. This is made precise in the three definitions that follow.

Definition 1 *There is* **no information** *about the screening capabilities of agents 1, 2, and 3, if the organizational designer does not know the pair* (p_i^b, p_i^g) *of any of these agents, nor can she order the agents. Indeed, she only knows that agents are fallible.*

The other extreme in terms of richness of information about the agents is cardinal information:

Definition 2 *The organizational designer has* **cardinal information** *about the screening capabilities of the agents if she knows the vector* $(\overline{p}^b, \overline{p}^g)$.

In between no information at all and cardinal information about all the agents, there is the situation of ordinal information.

Definition 3 *The organizational designer is in possession of* **ordinal information** *about the screening capabilities of the agents if she knows* $1 \succ 2 \succ 3$, *and if agents are known to be fallible.*

2.6 Complexity

One of the main tasks of the designer is to assign agents to positions within a given structure. Ideally, she would like to find the best allocation given the characteristics of the agents. To do so, she may have to use information about the (relative) qualities of the agents. The more information is required, the more demanding the structure. The notion of complexity I use here captures differences in demands placed on the organizational designer.

Definition 4 *A structure* Σ_1 *is called* **more complex** *than* Σ_2*, if finding the optimal allocation of agents to positions requires more detailed knowledge about these agents in* Σ_1 *than in* Σ_2*.*

In fact, I will call structures of least complexity if no information is required to assign agents correctly. If ordinal information is necessary and sufficient, structures will be called of intermediate complexity. Structures requiring cardinal information are most complex.

2.7 Organizational Robustness

If the designer does not possess the information needed to correctly allocate agents to positions in the organization, errors can be made. In an $\mathcal{HP}$ or a $\mathcal{PH}$, an error may occur if the designer misses ordinal information. In an omniarchy, additional errors can arise if information about the precise qualities, or cardinal information, is lacking.

An erroneous allocation reduces the performance of a structure: too many good projects are rejected, and too few bad ones are rejected. Is there something the designer can do about this? She could consider to improve her knowledge of the agents, in line with the requirements of the structure. Agents could then be re–allocated within the given structure. Alternatively, she could simplify the structure in line with the knowledge she has. Agents could then be correctly allocated within the simpler structure. The dilemma she faces is that the more complex structure may be performing better than the simpler structure if she possesses the required information, but worse in case an error is made. However, a badly organized but more complex structure may still outperform a well organized, but less complex structure.

In any event, it is important to evaluate the impact a mis–allocation may have on the performance of an organization. The smaller the drop in performance, the more robust the structure will be called. For the purpose of this paper I measure the robustness of an organization in the following way.

Definition 5 *Consider a structure Σ, and let $\phi^* = \phi^*(\Sigma, (\overline{p}^b, \overline{p}^g); \beta)$ be an optimal allocation and ϕ any allocation. Then the* **robustness** *of a structure Σ in the environment β is defined as*

$$R(\Sigma; \beta) = \min_{\phi, (\overline{p}^b, \overline{p}^g)} \frac{V(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \beta)}{V(\Sigma, \phi^*, (\overline{p}^b, \overline{p}^g); \beta)} \tag{5}$$

That is, $R(\Sigma; \beta)$ measures the maximal extent to which an erroneous allocation in conjunction with characteristics of agents may lead to a reduction in performance. The larger the value of the ratio in Equation 5, *i.e.,* the smaller the relative drop in expected profits, the more robust the structure. $R(\Sigma; \beta)$ is typically a function of the environment, β. In fact, the allocation $\tilde{\phi}$ and the vector of characteristics $(\overline{p}^b, \overline{p}^g)$ that minimize $\frac{V(\Sigma, \phi, (\overline{p}^b, \overline{p}^g); \beta)}{V(\Sigma, \phi^*, (\overline{p}^b, \overline{p}^g); \beta)}$ may themselves vary with β.

The measure $R(\Sigma; \beta)$ can be used to compare the robustness of different organizations, for specific values of β or for all values. A definition that will prove useful is the following.

Definition 6 *A structure Σ_1 will be called* **uniformly more robust** *than Σ_2 if for all β*

$$R(\Sigma_1; \beta) > R(\Sigma_2; \beta) \tag{6}$$

Of course, one may not be able to order organizations using this strong definition: Σ_1 could be more robust than Σ_2 in specific environments, while the opposite holds in other environments.

Nothing in the definition of robustness excludes the possibility that although Σ_1 is uniformly more robust than Σ_2, yet Σ_2 attains higher profits even when it performs at its worst. The robustness of an organization measures the variability of its performance, not its absolute level of performance. The robustness of an organization, although of interest in itself, should also be studied in conjunction with the organization's expected profits.

3 Results

I now classify the structures $\mathcal{H}, \mathcal{HP}, \mathcal{O}, \mathcal{PH}$ and $\mathcal{P}$ in terms of their complexity. I also provide the optimal allocations of heterogeneous agents in each of these structures and compare their performance of the organizational structures. Finally, I compute their robustness.

3.1 Organizational Complexity

I now apply the definition of organizational complexity to the five structures presented in Figure 1.

Proposition 1 *$\mathcal{H}$ and $\mathcal{P}$ are structurally of least complexity as no information is required to find the optimal assignment of agents. $\mathcal{HP}$ and $\mathcal{PH}$ are of intermediate complexity. Ordinal information is necessary and sufficient: the best agent should be* *located at the root. The allocation of agent 2 and 3 to the remaining nodes is immaterial. The structure $\mathcal{O}$ is the most complex: cardinal information is necessary. The worst agent should be located at the root. Agent 1 should be at σ_a and agent 2 at σ_r if and only if*

$$\beta < \frac{(2p_3^g - 1)(p_2^g - p_1^g)}{(2p_3^b - 1)(p_2^b - p_1^b)} \tag{7}$$

I provide the proof here. Note that in the hierarchy implementation requires acceptance by all three agents, with the probability of implementation equal to the product of the probability of acceptance by the individual agents. Obviously, the precise allocation of agents to these three nodes is immaterial. In the polyarchy, the probability of final rejection equals the product of the individual probabilities of rejection. This is independent of the way agents are assigned to nodes. As the probability of implementation equals one minus the probability of final rejection, the polyarchy does not require information about the agents either. Hence, the hierarchy and the polyarchy are of least complexity.

Now consider $\mathcal{HP}$. Switching the agents located at σ_a and σ_{ar} leaves the probability of implementation unchanged: the 'substructure' that starts at σ_a is a two–node polyarchy in which the allocation is immaterial. To understand why the best agent should be positioned at the root, observe that the expected profits stem from projects implemented by *(i)* both the first two agents, with probability $p(\sigma)p(\sigma_a)$, or *(ii)* by the third agent after acceptance by the first and rejection by the second, with probability $p(\sigma)(1 - p(\sigma_a))p(\sigma_{ar})$. In *(i)*, who is the first or the second is immaterial for expected profits. In *(ii)*, however, the ordering matters. This can easily be seen from a hypothetical switch of agents. Suppose initially the best agent were located at σ_a and the second or third best agent at σ. Switching these agents leads to an increase in the probability of acceptance of good projects at σ and an increase of rejected good projects at σ_a. Hence, the probability of good projects reaching σ_{ar} increases, and therefore the probability of implementation of good projects. By the same token this switch leads to a reduction in bad projects being accepted at σ and rejected at σ_a, and therefore to a lower probability of bad projects reaching σ_{ar}. That is, both assignments $\phi^* \in \{(\sigma, \sigma_a, \sigma_{ar}), (\sigma, \sigma_{ar}, \sigma_a)\}$ are optimal. The same line of reasoning holds in case of $\mathcal{PH}$. Ordinal information is necessary and sufficient in both $\mathcal{HP}$ and $\mathcal{PH}$. These structures are therefore of intermediate complexity.

In the omniarchy, the optimal allocation depends on the values of the characteristics of the agents and the type of environment. First of all, agent 3, the worst agent, should be located at σ. The intuition is that the agent at σ merely delegates the decision to implement to the agents at σ_a and σ_r. Hence, the agents located at the latter two nodes should be better than the one at the former.

Formally, consider the two possible allocations with agent 3 at the root. As I leave undefined the allocation of agents 1 and 2 to nodes σ_a and σ_r, the probability of implementation can be written as $p_3 p(\sigma_a) + (1 - p_3) p(\sigma_r)$. Switching agent 3 and whoever was initially located at σ_a leads to a probability of implementation of $p(\sigma_a)p_3 + (1 - p(\sigma_a))p(\sigma_r)$. The difference in probability ensuing from this switch equals $p(\sigma_r)(p_3 - p(\sigma_a))$. Now observe that, irrespective of whether agent 1 or 2 was initially assigned to σ_a, $p_3^g - p^g(\sigma_a) < 0$ and $p_3^b - p^b(\sigma_a) > 0$. That is, starting with agent 3 at the root, the switch leads to fewer good projects being accepted, and less bad projects being rejected. This has an unambiguous negative effect on expected profits. Similarly, it can be shown that switching agent 3 at σ and agent 1 or 2 at σ_r leads to a reduction in expected profits. This establishes that in any optimal allocation agent 3 should be assigned to the root.

The question that remains is the allocation of agent 1 and 2. Consider allocation $\phi^1 = (\sigma_a, \sigma_r, \sigma)$ and compare its expected profits with those of allocation $\phi^2 = (\sigma_r, \sigma_a, \sigma)$. That is, agents 1 and 2 are being switched.

$$V(\mathcal{O}, \phi^1, (\overline{p}^b, \overline{p}^g); \beta) - V(\mathcal{O}, \phi^2, (\overline{p}^b, \overline{p}^g); \beta) = (2p_3^g - 1)(p_2^g - p_1^g) - \beta(2p_3^b - 1)(p_2^b - p_1^b) \quad (8)$$

That is, agent 1 should be located at σ_a and agent 2 at σ_r (or, ϕ^1 is better than ϕ^2) if and only if

$$\beta < \frac{(2p_3^g - 1)(p_2^g - p_1^g)}{(2p_3^b - 1)(p_2^b - p_1^b)} \quad (9)$$

This shows that cardinal information is required in an omniarchy. It is therefore the most complex organization studied. This completes the proof.

Equation 9 says that, for sufficiently friendly environments, the best agent should be located at σ_a, whereas for relatively tough environments this agents is best located at σ_r. The reason is as follows. Most good projects land on σ_a (as $p_3^g > 1/2$), while most bad projects go to σ_r (as $1 - p_3^b > 1/2$). Therefore, if there are relatively many good projects, or the profit X is high relative to the possible loss Y, that is, if β is relatively low, it is more important to have the best agent at σ_a. The opposite holds for high values of β.

3.2 Comparing Performance

In the preceding subsection, I determined the correct ordering of agents for different structures. In this subsection I will assume agents have been correctly assigned, and compare the ensuing expected profits of the five structures. Before presenting some general results formally, I use Figure 2 to illustrate a few characteristics that generally hold.

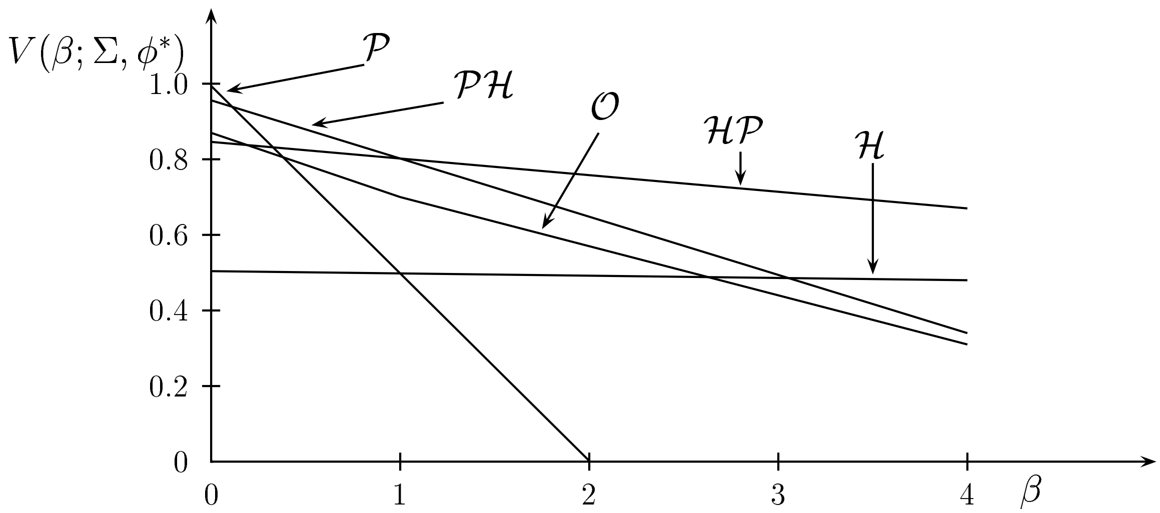


Figure 2: The expected pay–offs of the five organizational structures studied, with $(\overline{p}^b, \overline{p}^g) = (0.1, 0.2, 0.3, 0.7, 0.8, 0.9)$

Although Figure 2 is based on specific values of $(\overline{p}^b, \overline{p}^g)$, it illustrates a few important general features of the profit comparison that hold for all possible vectors of agents' capabilities.

Note first of all that by moving from left to right, from friendly to hostile environments, first the polyarchy, then the hybrid $\mathcal{PH}$, then $\mathcal{HP}$, and finally the hierarchy is the best structure. This follows from the progressively growing demands imposed on project implementation experienced when moving from a polyarchy to a hierarchy.

The second observation is that the omniarchy is never the best structure. The intuition is as follows. The performance of $\mathcal{O}$ is a convex combination of the performance of agents 1 and 2, where the optimal weights depend on condition 7. Hence, $\mathcal{O}$ behaves approximately as a single agent organization. In friendly environments, the polyarchy $\mathcal{P}$ and the $\mathcal{PH}$ outperform the omniarchy $\mathcal{O}$ as the former organizations accept more good projects than the latter. Conversely, for rising values of β, the influence of the probability of acceptance of bad projects increases, making the hierarchy $\mathcal{H}$ and the $\mathcal{HP}$ more suitable. The formal proof establishes that the interaction of the various dimensions is such that even for intermediate environments the omniarchy is never the best.

Thirdly, the worst performing structure is either the polyarchy or the hierarchy. Their superior performance in extremely friendly or extremely hostile environments, respectively, comes at the cost of quickly loosing performance outside these environments.

Formally, these three observations amount to:

Result 1 *For every vector $(\overline{p}^b, \overline{p}^g)$, there are values $\beta_1 < \beta_2 < \beta_3$, such that $\mathcal{P}$ attains higher profits than any other organization for $\beta < \beta_1$; $\mathcal{PH}$ for $\beta_1 < \beta < \beta_2$; $\mathcal{HP}$ for $\beta_2 < \beta < \beta_3$; and $\mathcal{H}$ for $\beta > \beta_3$. This implies that for every vector $(\overline{p}^b, \overline{p}^g)$ and for any β, there is always an organization (Σ, ϕ^*) with $\Sigma \in \{\mathcal{H}, \mathcal{HP}, \mathcal{PH}, \mathcal{P}\}$ that attains higher expected profits than $(\mathcal{O}, \phi^*)$.*

Result 2 *For every vector* $(\overline{p}^b, \overline{p}^g)$ *and for any* β*, either the hierarchy* $\mathcal{H}$ *or the polyarchy* $\mathcal{P}$ *attains the lowest expected profits. For low values of* β*, the hierarchy performs the worst, whereas for high values of* β*, the polyarchy does.*

What do these results mean in terms of the relationship between complexity and expected profits? The first result indicates that complexity is not beneficial per se: only for intermediate environments does the extra complexity that comes with a hybrid structure pay off. The first result also implies that the most complex structure is never the best. Hence, in the present model and with definition of complexity used here, complexity is not unequivocally beneficial: only for specific environments in combination with particular values of the characteristics of the agent do structures of intermediate complexity outperform ones of least complexity. The second result can be restated as follows. It is a structure of least complexity that attains the lowest expected profits for any given environment.

Jointly, these results suggest an important trade–off between complexity and performance, especially in case of uncertainty about the type of environment. Hierarchies and polyarchies require little information and they perform best in specific environments. This is a plus. However, they perform worse than other structures virtually as soon as they are not the best anymore. Hybrid structures require more information, which may be costly to acquire, but their performance is not as susceptible to changes in the environment as is the performance of a hierarchy or a polyarchy. Moreover, they are best for some types of environment. There is no trade–off at the most advanced level of complexity: the omniarchy is never better than all other organizations, yet is requires the highest level of detail of information.

3.3 Robustness

The robustness of an organization measures the maximal drop in expected profits due to an erroneous allocation of agents. The smaller this drop, the less sensitive the organization is to errors, the higher is the organization's robustness.

Before discussing the findings, let me explain the method used to establish the results. Remember that to find $R(\Sigma;\beta)$ one has to minimise over both the admissible values of the characteristics of the agents and any possible allocation. This space is seven dimensional. Fortunately, the minimisation problem can be split up in two parts.

First of all, for every structure I determine whether some mis–allocation would lead to uniformly larger errors, *i.e.*, for all values of $(\overline{p}^b, \overline{p}^g)$, than other mis–allocations. Both in case of $\mathcal{HP}$ and $\mathcal{PH}$, the allocation with the worst agent instead of the best at node σ creates uniformly the largest relative drop in performance. The allocation of agent 1 and 2 to the remaining positions is immaterial. In the omniarchy, locating the best agent instead of the worst agent at σ leads uniformly to the largest error. Where agent 2 and 3 should be located, at σ_a or at σ_r, depends on the characteristics of the agents.

Result 3 *For both $\Sigma = \mathcal{HP}$ and $\Sigma = \mathcal{PH}$ and for any characteristics of the agents $(\overline{p}^b, \overline{p}^g)$, switching the best and the worst agents leads to the maximal drop in expected profits. For $\Sigma = \mathcal{O}$, assigning the best agent to the optimal position of the worst agent is necessary to create the largest fall in profits.*

This comes close to saying that the maximum drop in profitability is always a consequence of assigning the best and the worst agents to *each other's* correct position. This statement is correct for the hybrid structures, but not quite for the omniarchy: although the best agent should be located at node σ, the position of the *worst* agent depends on the agents' characteristics and the type of environment.

Then I determine for each structure and the selected mis–allocation(s) the characteristics of the agents that maximize the drop in performance. Here I allow for the possibility of boundary cases, *i.e.*, $p_i^b \in [0, 1/2]$ or $p_i^g \in [1/2, 1]$. If maximising the relative error actually requires a boundary value, this should be interpreted as an interior solution arbitrarily close to the boundary. The continuity of $V(\cdot)$ in the characteristics of the agents ensures the correctness of such an interpretation.

Result 4 summarizes for which values of $(\overline{p}^b, \overline{p}^g)$ and for which mis–allocation the drop in performance is maximal.

Result 4 *For every structure $\Sigma \in \{\mathcal{HP}, \mathcal{PH}, \mathcal{O}\}$, Table 1 reports the correct allocation ϕ^*, the one that maximizes the drop in relative performance $\tilde{\phi}$, and the values of $(\overline{p}^b, \overline{p}^g)$ that minimize the relative performance.*

Clearly, in virtually all cases the drop in relative performance is largest when the characteristics of the agents take on extreme values, like 0, $\frac{1}{2}$, or 1. The only exceptions are the hybrid structures in case of specific environments: for $\Sigma = \mathcal{HP}$ and $\beta = \frac{2}{3}$, and for $\Sigma = \mathcal{PH}$ and $\beta = \frac{5}{6}$, the values of p_1^b and p_1^g, respectively, can be freely chosen. As these are non–generic cases, I do not discuss them.

The most important observation to make, however, concerns the relationship be-

Σ	$(\phi^*(1), \phi^*(2), \phi^*(3))$	$(\tilde{\phi}(1), \tilde{\phi}(2), \tilde{\phi}(3))$	$(\overline{p}^b, \overline{p}^g)$
$\mathcal{HP}$	$(\sigma, \sigma_a, \sigma_{ar})$	$(\sigma_{ar}, \sigma_a, \sigma)$	$\begin{cases} (0, 1/2, 1/2, 1/2, 1, 1) & \text{if } \beta \in [0, 2/3) \\ ([0, \frac{1}{2}], 1/2, 1/2, 1/2, 1, 1) & \text{if } \beta = 2/3 \\ (1/2, 1/2, 1/2, 1/2, 1, 1) & \text{if } \beta \in (\frac{2}{3}, \frac{4}{3}] \end{cases}$
$\mathcal{PH}$	$(\sigma, \sigma_r, \sigma_{ra})$	$(\sigma_{ra}, \sigma_r, \sigma)$	$\begin{cases} (0, 0, 1/2, 1/2, 1/2, 1) & \text{if } \beta \in [0, \frac{5}{6}) \\ (0, 0, 1/2, 1/2, 1/2, [\frac{1}{2}, 1]) & \text{if } \beta = \frac{5}{6} \\ (0, 0, 1/2, 1/2, 1/2, 1/2) & \text{if } \beta \in (\frac{5}{6}, \frac{5}{4}] \end{cases}$
$\mathcal{O}$	$(\sigma_a, \sigma_r, \sigma)$	$(\sigma, \sigma_r, \sigma_a)$	$(0, 1/2, 1/2, 1/2, 1, 1)$

Table 1: Characteristics of the agents and allocations that jointly maximize the relative drop in performance per structure.

tween these extreme values and the heterogeneity of agents: in virtually all instances the maximal drop in performance is the result of having agents that are maximally heterogeneous. In case of three agents, maximal heterogeneity is characterized by Lemma 1:

Lemma 1 *Suppose one wants to maximize the heterogeneity H of agents as measured by*

$$H = \sum_{i}^{3} \left(p_i^q - \frac{\sum_i^3 p_i^q}{3} \right)^2 \tag{10}$$

for $q = b$ and $q = g$, subject to $0 \leq p_1^b \leq p_2^b \leq p_3^b \leq 1/2$ and $1/2 \leq p_3^g \leq p_2^g \leq p_1^g \leq 1$, respectively. The values that solve these problems are $\left(p_1^b, p_2^b, p_3^b\right) = (0, 0, 1/2)$ or $(0, 1/2, 1/2)$ in case of bad projects, and $(p_3^g, p_2^g, p_1^g) = (1/2, 1/2, 1)$ or $(1/2, 1, 1)$ for good projects.

One directly observes, by comparing the values of $(\overline{p}^b, \overline{p}^g)$ in Table 1 with Lemma 1, that in case of the omniarchy $\mathcal{O}$, the $\mathcal{HP}$ for $\beta < 2/3$, and the $\mathcal{PH}$ for $\beta < 5/6$, the maximal drop in profitability stems from maximal heterogeneity of the agents, both as far as good projects and bad projects are concerned. In case of $\mathcal{HP}$ or $\mathcal{PH}$ and high values of β, maximal heterogeneity is limited to one dimension of the agents' characteristics (probability of acceptance of good projects in case of $\mathcal{HP}$, and of bad projects for $\mathcal{PH}$). It is quite intuitive that the maximal drop in profitability stems from agents that are as heterogeneous as possible. Any change in assignment will then have a maximal impact.

Result 5 *For all structures $\Sigma \in \{\mathcal{HP}, \mathcal{PH}, \mathcal{O}\}$ and for virtually all environments β, a maximal reduction in expected profits requires agents that are maximally heterogeneous.*

Roughly speaking, the implication of Results 3 and 5 is that the maximal drop in performance stems from, firstly, switching the best and the worst agent, and secondly making agents maximally different.

Complementing the information in Table 1 is information about expected profits associated with the allocations and characteristics reported in that Table. This information, together with the ensuing robustness of each structure, $R(\Sigma;\beta)$, can be found in Table 2. That is, the information presented in Tables 1 and 2 corresponds line by line. Take for example the line concerning $\mathcal{HP}$. The vector ϕ^* given in Table 1, $\phi^* = (\sigma, \sigma_a, \sigma_{ar})$, is the one used in the calculation of $V(\Sigma, \phi^*; \beta)$ for $\Sigma = \mathcal{HP}$ in Table 2. Similarly for $\tilde{\phi}$ and $V(\Sigma, \tilde{\phi}; \beta)$. Of course, $R(\Sigma;\beta) = V(\Sigma, \tilde{\phi}; \beta)/V(\Sigma, \phi^*; \beta)$. Figure 3 shows the graphs of $R(\Sigma, \beta)$. It reveals that $\mathcal{O}$ is uniformly the least robust

Σ	$V(\Sigma, \phi^*; \beta)$	$V(\Sigma, \tilde{\phi}; \beta)$	$R(\Sigma; \beta)$
$\mathcal{HP}$	1 for $\beta \in [0, \frac{2}{3})$	$0.5 - 0.25\beta$	$0.5 - 0.25\beta$
	$1 - \frac{1}{2}p_1^b$ for $\beta = \frac{2}{3}$	$\frac{1}{3} - \frac{1}{6}p_1^b$	$\frac{1}{3}$
	$1 - 0.375\beta$ for $\beta \in (\frac{2}{3}, \frac{4}{3}]$	$0.5 - 0.375\beta$	$\frac{0.5 - 0.375\beta}{1 - 0.375\beta}$
$\mathcal{PH}$	1 for $\beta \in [0, \frac{5}{6})$	$0.75 - 0.5\beta$	$0.75 - 0.5\beta$
	$\frac{3}{4}p_1^g + \frac{1}{4}$ for $\beta = \frac{5}{6}$	$\frac{1}{4}p_1^g + \frac{1}{12}$	$\frac{1}{3}$
	0.625 for $\beta \in (\frac{5}{6}, \frac{5}{4}]$	$0.625 - 0.5\beta$	$1 - 0.8\beta$
$\mathcal{O}$	$1 - 0.25\beta$	$0.5 - 0.5\beta$	$\frac{0.5 - 0.5\beta}{1 - 0.25\beta}$

Table 2: Expected profits with correctly allocated agents, with mis–allocated agents, and the robustness per structure.

structure. The $\mathcal{PH}$ is the most robust structure in relatively friendly environments, as is $\mathcal{HP}$ in tougher environments. Roughly speaking, when $\mathcal{PH}$ outperforms $\mathcal{HP}$ in terms of expected pay–offs $V(\Sigma, \phi^*, \beta)$ it is also more robust than $\mathcal{HP}$[5]. This is an interesting result as it shows that good performance of a hybrid structure does not come at the cost of increased sensitivity to mis–allocations.

Proposition 2 *$\mathcal{O}$ is uniformly the least robust structure. The $\mathcal{PH}$ is the most robust structure in relatively friendly environments, as is $\mathcal{HP}$ in tougher environments.*

This proposition and Figure 3 capture to what extent the performance of a given structure is maximally reduced due to a misallocation. Given that the pure hierarchy and polyarchy are fully error–proof, it is interesting to see to what extent superior

[5]Roughly speaking, since an exact comparison is impossible as the calculations of $R(\mathcal{HP}, \beta)$ and $R(\mathcal{PH}, \beta)$ are based on different values of the characteristics of the agents.

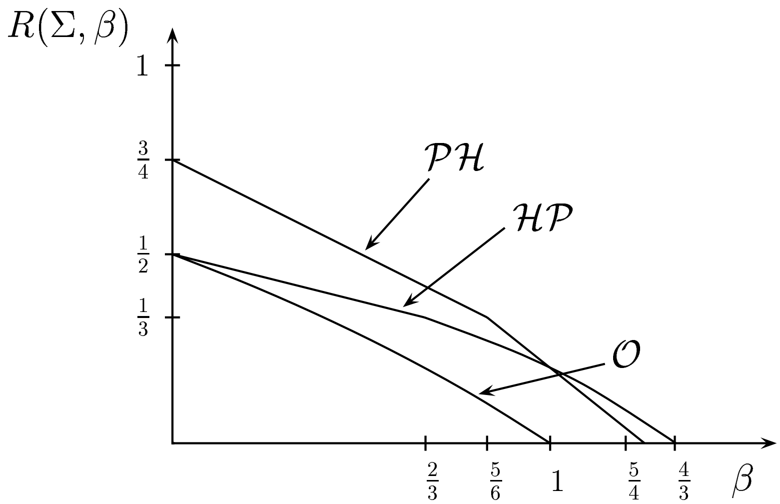


Figure 3: The robustness of $\mathcal{PH}$, $\mathcal{HP}$ and $\mathcal{O}$.

performance of $\mathcal{HP}$, $\mathcal{PH}$, and $\mathcal{O}$ is predicated on the optimal allocation. Figure 4 shows for every value of β and for the characteristics $(\overline{p}^b, \overline{p}^g)$ reported in Table 1 how well these structures—$\mathcal{HP}$ to the left, $\mathcal{PH}$ in the middle, $\mathcal{O}$ to the right—fare relative to the hierarchy and the polyarchy, both for the correct allocation, $\phi^*(\Sigma)$ in the upper part of the graph, and for $\tilde{\phi}(\Sigma)$, the mis–allocation maximising the drop in performance, in the lower part. For example, for $\Sigma = \mathcal{HP}$, if all agents have been correctly allocated $\mathcal{HP}$ performs better than $\mathcal{H}$ for $\beta < 1$, whereas the opposite holds for $\beta > 1$. $\mathcal{HP}$ performs as well as $\mathcal{P}$ for $\beta = 0$. If agents have been erroneously assigned, the lower part shows that either $\mathcal{P}$ or $\mathcal{H}$ perform better than $\mathcal{HP}$ for all β.

The three graphs show that the adverse effect on performance can be dramatic indeed: in the situations depicted the comparative advantage of $\mathcal{HP}$, $\mathcal{PH}$ or $\mathcal{O}$ completely vanishes due to allocating agents erroneously.

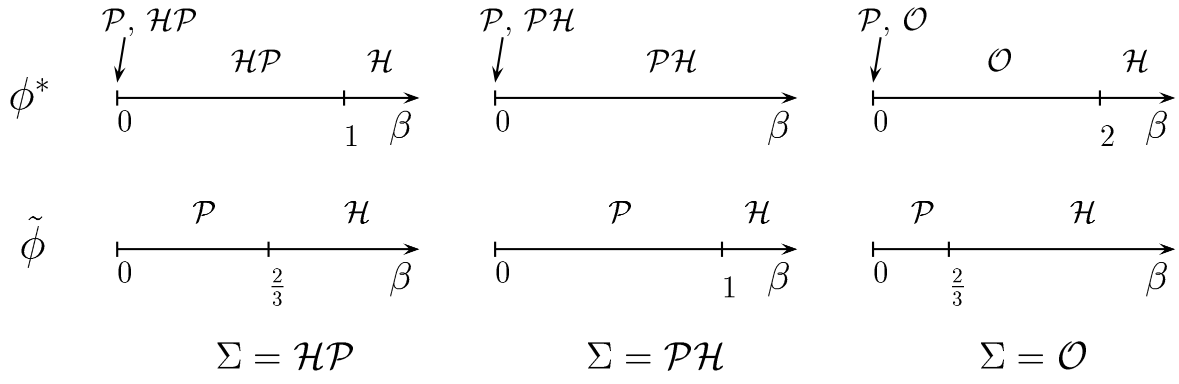


Figure 4: Area of superior performance of $\mathcal{HP}$, $\mathcal{PH}$ and $\mathcal{O}$ when compared with the least complex structures $\mathcal{H}$ and $\mathcal{P}$, when agents have been correctly assigned (ϕ^*, or top part) and when the relative error is maximized ($\tilde{\phi}$, or lower part).

4 Trade–offs in organizational design

Various ways are open to an organizational designer who wishes to improve the performance of her organization, and who possesses only limited knowledge on the qualities of her employees and of the environment in which she operates.

First of all, she could invest in training that enhances the screening capacity of her employees. Training leads to an unambiguous rise in the expected value of implemented projects, whether employees are correctly allocated or not. This is clear from Equation 2. Of course, there are costs associated with training. The effectiveness of money spent on training should be compared with that of other means of generating additional revenue. Replacing existing employees by more skilled ones is another possibility. This assumes the possibility of comparing the quality of existing and new employees.

Secondly, she could invest in monitoring her present employees to get a better view of their skills. She may then be able to re–position her employees thanks to the additional information so obtained. How much she is maximally willing to spend on monitoring is related to the robustness of a structure, since the notion of robustness is based on a comparison of the expected profits of an optimally organized structure with the worst performance of the same structure when agents are incorrectly positioned.

Instead of monitoring, the organizational designer could decide to bridge the gap between information she possesses and information which is required by simplifying the present organizational structure in line with the information she currently has. The cost of a re–organization should be compared with the increase in revenues. Absent cardinal information, she could decide to content herself with hybrid or pure structures. Similarly, ordinal information lacking, she could settle with a hierarchy or a polyarchy. Although this prohibits mis–allocations from occurring, it does not necessarily guarantee an increase in profitability. The one situation in which it does, is when the omniarchy is replaced by a hybrid structure and ordinal information is present. This follows from Result 1. However, if any information about the qualities of the employees is missing, a hybrid structure in which agents are potentially mis–allocated may still perform better than a pure structure in which no information is required. Moreover, if the designer is uncertain about the exact type of environment, the pure structures expose her to a much larger variation in possible pay–offs, including to the lowest pay–offs possible, than do the hybrid structures. This is clear from Figure 2, and Result 2.

Yet another way of improving organizational performance is collecting information on the state of the environment with a view to adjusting the internal structure or influencing the environment to the benefit of the present organization. The costs of influencing may well be lower than those associated with measures meant to improve the adequacy of the organization for a *given* environment. It seems plausible to assume that the larger the organization, the lower the costs of influencing the environment when compared with the expenses incurred to improve the internal organization.

The means chosen to improve performance will depend on their relative cost effectiveness and on the degree of risk aversion of the organizational designer.

5 Conclusion

This paper illustrates how one could approach the relationship between organizational complexity, robustness, and performance. Complexity, defined as the level of detail of information needed to correctly allocate agents within an organizational structure, and robustness, defined as the degree to which organizations do not suffer from mis–allocations, prove useful categories to distinguish organizational structures: one can sensibly talk about different degrees of complexity, and one can compare organizational structures in terms of their robustness.

One of the results of this paper is that increasing the complexity of an organizational amounts to reducing its robustness. This may annihilate the superior performance of more complex organizations if the organizational designer does not possess the required detail of information. The superior performance of the hybrid structures $\mathcal{HP}$ and $\mathcal{PH}$ was shown to rely in a crucial way on ordinal information about the agents working in the organization. This information being absent, their superiority can vanish completely. The designer faces a difficult choice: improve her knowledge of the agents, improve the quality of the agents, simplify the structure of the organization, or influence the environment?

The conclusion of this paper concerning the omniarchy is quite negative. Reminiscent of commonly used delegation structures, it is more complex than the other structures and less robust. In theory, these disadvantages could be offset by superior performance. However, it was also shown that the omniarchy always performs worse than some other organization. This should not be taken as the final verdict on delegation structures. As I observed in the introduction, the approach taken in this paper should be considered a first step towards an understanding of the relationship between organizational performance, complexity, and robustness. I therefore excluded aspects of organizational design and behaviour, such as specialization, agency, and conflict of interests, the inclusion of which may have changed the evaluation of an omniarchy.

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Appendix for Referees

Proof of Result 1 As this result involves a general statement about a seven dimensional parameter–space ($(\overline{p}^b, \overline{p}^g)$ and β) involving five different structures providing an analytical proof is hard for the following reason. Fix values for $(\overline{p}^b, \overline{p}^g)$ and calculate for every pair of organizations the 'pivotal' value of β at which this pair performs equally well. In the statement of the result I mention three such values, β_i, $i = 1, 2, 3$. The 'less strict' organization always performs better for values of β lower than the pivotal value, and vice versa. These pivotal values are ordered along the positive line–segment. The problem is that this ordering may change with certain changes in $(\overline{p}^b, \overline{p}^g)$. This makes it difficult to establish analytically whether $\beta_1 < \beta_2 < \beta_3$ holds for all $(\overline{p}^b, \overline{p}^g)$, and whether the omniarchy is better than any other structure for some values of the parameters. I have therefore taken recourse to numerical methods. There was no instance among the 27 million randomly chosen vectors such that the omniarchy performed better than all other structures. □

Proof of Result 2 This Result is based on the same 27 million randomly chosen vectors. □

Proof of Result 3 I discuss the three structures $\Sigma \in \{\mathcal{HP}, \mathcal{PH}, \mathcal{O}\}$ in turn.

When $\Sigma = \mathcal{HP}$, a sufficient condition for optimality is to have agent 1 at node σ; the allocation of agents 2 and 3 to σ_a and σ_{ar} is immaterial (see Proposition 1). Any mis–allocation involves either agent 2 or 3 at σ. If, say, agent 2 is at the root, the precise allocation of agents 1 and 3 is immaterial. Similarly if agent 3 is at the root. There are therefore two generic mis–allocations: (i) agent 2 at node 1, or (ii) agent 3 at node 1.

In case (i) the probability of acceptance of a project of type q equals $p_2^q(p_1^q+(1-p_1^q)p_3^q)$, whereas in (ii) the probability of acceptance equals $p_3^q(p_1^q+(1-p_1^q)p_2^q)$. The *difference* in probability of acceptance between (i) and (ii) therefore amounts to $p_1^q(p_2^q-p_3^q)$, which is larger than zero for $q=g$, and smaller than zero for $q=b$ for all $(\overline{p}^b, \overline{p}^g)$. That is, allocation (i) accepts more good and less bad projects than allocation (ii). Expected profits are lowest in case (ii), or when the best and the worst agents have switched position.

An analogous proof can be provided for $\Sigma = \mathcal{PH}$.

Finally $\Sigma = \mathcal{O}$. Consider the case where $\beta \geq \frac{(2p_3^g-1)(p_2^g-p_1^g)}{(2p_3^b-1)(p_2^b-p_1^b)}$, or agent 1 should be located at σ_r and agent 2 at σ_a (see Proposition 1). Here, one has to compare the five possible mis–allocations: $\phi^1=(\sigma_r,\sigma,\sigma_a)$, $\phi^2=(\sigma_a,\sigma_r,\sigma)$, $\phi^3=(\sigma_a,\sigma,\sigma_r)$, $\phi^4=(\sigma,\sigma_a,\sigma_r)$ and $\phi^5=(\sigma,\sigma_r,\sigma_a)$. Just as in case of $\Sigma=\mathcal{HP}$, the analysis is based on comparing probabilities of acceptance. For convenience sake, let me denote the difference in probability of acceptance of a project of type q between allocation ϕ^i and ϕ^j by $\Delta(\phi^i,\phi^j;q)$. Thus, $\Delta(\phi^3,\phi^4;q)=p_3^q(p_1^q-p_2^q)$. This expression is positive for good projects, but negative for bad projects. That is, ϕ^4 generates uniformly smaller profits. Similarly, $\Delta(\phi^1,\phi^5;q)=(p_1^q-p_2^q)(1-p_3^q)$, which is positive for good projects, and negative for bad ones: ϕ^5 generates uniformly smaller profits. A comparison of ϕ^2 with ϕ^5 shows that $\Delta(\phi^2,\phi^5;q)=p_2^q(p_1^q-p_3^q)$. This expression is positive for $q=g$, and smaller than zero if $q=b$, meaning that ϕ^5 gives rise to larger reductions in profits than ϕ^2. That is, either ϕ^5 or ϕ^4 leads to the largest drop in profits. In either case, agent 1 is located at node σ. Whether ϕ^5 or ϕ^4 leads to a larger drop depends on

$(\overline{p}^b, \overline{p}^g)$ and β. A similar proof shows that also in case of $\beta < \frac{(2p_3^g-1)(p_2^g-p_1^g)}{(2p_3^b-1)(p_2^b-p_1^b)}$ the largest drop in profits results from assigning agent 1 to σ. $\square$

Proof of Result 4 The correct allocations ϕ^* follow from Proposition 1. The incorrect allocations $\tilde{\phi}$ for $\Sigma = \mathcal{HP}$ and $\Sigma = \mathcal{PH}$ follow from Result 3. The latter result also established that in the omniarchy the best agent should be located at σ if the drop in performance is to be maximal. In this proof, I determine the vector $(\overline{p}^b, \overline{p}^g)$ that maximizes the drop in performance when (i) $\Sigma = \mathcal{HP}$, (ii) $\Sigma = \mathcal{PH}$, and (iii) $\Sigma = \mathcal{O}$. In any event, this vector $(\overline{p}^b, \overline{p}^g)$ can be found by maximizing the Lagrangian:

$$L((\overline{p}^b, \overline{p}^g), \underline{\mu}; \Sigma, \beta) = -\frac{V((\overline{p}^b, \overline{p}^g); \Sigma, \tilde{\phi}, \beta)}{V((\overline{p}^b, \overline{p}^g); \Sigma, \phi^*, \beta)} - \sum_{i=1}^{8} \mu_i h_i(\overline{p}^b, \overline{p}^g) \tag{A.1}$$

with respect to the probabilities $(\overline{p}^b, \overline{p}^g)$ under the restrictions $0 \leq p_1^b \leq p_2^b \leq p_3^b \leq \frac{1}{2} \leq p_3^g \leq p_2^g \leq p_1^g \leq 1$ rewritten as $h_i(\overline{p}^b, \overline{p}^g) \leq c_i$ for $i = 1, \ldots, 8$, where $h_1 = -p_1^b \leq 0$, $h_2 = p_1^b - p_2^b \leq 0$, $h_3 = p_2^b - p_3^b \leq 0$, $h_4 = p_3^b \leq \frac{1}{2}$, $h_5 = -p_3^g \leq -\frac{1}{2}$, $h_6 = p_3^g - p_2^g \leq 0$, $h_7 = p_2^g - p_1^g \leq 0$, and $h_8 = p_1^g \leq 1$. Note that I have written $V((\overline{p}^b, \overline{p}^g); \Sigma, \phi, \beta)$ instead of $V(\Sigma, \phi(\overline{p}^b, \overline{p}^g); \beta)$ to highlight that the optimization leaves the structure, the allocations, and the value of β unchanged. In what follows it will be convenient to write the short–hand form $V(\phi)$ instead of $V((\overline{p}^b, \overline{p}^g); \Sigma, \phi, \beta)$. Moreover, the derivative of $p^q(\Sigma, \phi)$ with respect to p_i^q will be written as $p_i^{q\prime}(\Sigma, \phi)$ The necessary conditions for optimality are

$$\frac{p_1^{g\prime}(\Sigma, \phi^*)V(\tilde{\phi}) - p_1^{g\prime}(\Sigma, \tilde{\phi})V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.2}$$

$$\frac{p_2^{g\prime}(\Sigma, \phi^*)V(\tilde{\phi}) - p_2^{g\prime}(\Sigma, \tilde{\phi})V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.3}$$

$$\frac{p_3^{g\prime}(\Sigma,\phi^*)V(\tilde{\phi}) - p_3^{g\prime}(\Sigma,\tilde{\phi})V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.4}$$

$$\frac{p_1^{b\prime}(\Sigma,\tilde{\phi})V(\phi^*) - p_1^{b\prime}(\Sigma,\phi^*)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.5}$$

$$\frac{p_2^{b\prime}(\Sigma,\tilde{\phi})V(\phi^*) - p_2^{b\prime}(\Sigma,\phi^*)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.6}$$

$$\frac{p_3^{b\prime}(\Sigma,\tilde{\phi})V(\phi^*) - p_3^{b\prime}(\Sigma,\phi^*)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.7}$$

and $\mu_i \geq 0$, $h_i(\overline{p}^b, \overline{p}^g) \leq c_i$, and $\mu_i\big(h_i(\overline{p}^b, \overline{p}^g) - c_i\big) = 0$, for $i = 1, \ldots, 8$. Note that I have divided the conditions A.5–A.7 by β. The case $\beta = 0$ implies that bad projects are irrelevant. In that case, the relevant first order conditions become the first three with $V(\tilde{\phi})$ and $V(\phi^*)$ replaced by $p^g(\Sigma, \tilde{\phi})$ and $p^g(\Sigma, \phi^*)$, respectively. I come back to this case once I have discussed $\beta > 0$. In what follows, I limit attention to situations where $V(\tilde{\phi}) > 0$.

These conditions are necessary for a solution. If I were to find various values of the characteristics of the agents consistent with these conditions, I can easily identify the correct solution as I explicitly calculate the value of the optimization function as a function of β.

In case (i), $\Sigma = \mathcal{HP}$, $p^q(\mathcal{HP}, \phi^*) = p_1^q(p_2^q + (1-p_2^q)p_3^q)$ and $p^q(\mathcal{HP}, \tilde{\phi}) = p_3^q(p_2^q + (1-p_2^q)p_1^q)$. The array of equations A.2–A.7 becomes

$$\frac{(p_2^g + (1-p_3^g))V(\tilde{\phi}) - p_3^g(1-p_2^g)V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.8}$$

$$\frac{p_1^g(1-p_3^g)V(\tilde{\phi}) - p_3^g(1-p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.9}$$

$$\frac{p_1^g(1-p_2^g)V(\tilde{\phi}) - (p_2^g + (1-p_2^g)p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.10}$$

$$\frac{p_3^b(1-p_2^b)V(\phi^*) - (p_2^b + (1-p_2^b)p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.11}$$

$$\frac{p_3^b(1-p_1^b)V(\phi^*) - p_1^b(1-p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.12}$$

$$\frac{(p_2^b + (1-p_2^b)p_1^b)V(\phi^*) - p_1^b(1-p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.13}$$

As $p_1^g(1-p_2^g) < p_2^g + (1-p_2^g)p_1^g$ and $V(\tilde{\phi}) < V(\phi^*)$, it follows from Equation A.10 that $\mu_6 - \mu_5 < 0$, or $\mu_5 > 0$. That is, $p_3^g = \frac{1}{2}$.

Similarly, if $p_1^b < p_3^b$, then $\mu_3 > 0$ or $p_3^b = p_2^b$ from Equation A.12. In Equation A.13, if $p_2^b > 0$, then $p_2^b + (1-p_2^b)p_1^b > p_1^b(1-p_3^b)$, and therefore $\mu_4 > 0$ or $p_3^b = \frac{1}{2}$. That is, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$.

The Lagrangian for this case can now be rewritten as

$$L(p_1^b, p_1^g, p_2^g, \underline{\mu}') = -\frac{\frac{1}{2}(p_2^g + (1-p_2^g)p_1^g) - \frac{\beta}{4}(1+p_1^b)}{\frac{p_1^g}{2}(p_2^g+1) - \beta\frac{3}{4}p_1^b} + \sum_{1=i}^{5}\mu_i'(h_i' - c_i') \tag{A.14}$$

with $h_1' = -p_1^b \leq 0$, $h_2' = p_1^b \leq \frac{1}{2}$, $h_3' = -p_2^g \leq -\frac{1}{2}$, $h_4' = p_2^g - p_1^g \leq 0$, and $h_5' = p_1^g \leq 1$.

If, however, in Equation A.12 $p_1^b = p_3^b$, in place of $p_1^b < p_3^b$ (and hence $p_2^b = p_2^b = p_1^b = p^b$, because $p_1^b \leq p_2^b \leq p_3^b$ by assumption), then $\mu_3 = \mu_2 \geq 0$. One needs to distinguish between $p^b = 0$ and $p^b > 0$. The case $p^b = 0$ is mathematically identical to the case $\beta = 0$. If $p^b > 0$, then $\mu_4 > 0$, or $p^b = p_3^b = \frac{1}{2}$. That is, $p_1^b = p_2^b = p_3^b = \frac{1}{2}$ and $\mu_4 > \mu_3 = \mu_2 \geq 0$. The associated maximization problem is a special case of Equation A.14 (substituting $p_1^b = \frac{1}{2}$).

If, in Equation A.13, $p_2^b = 0$ *and* $p_3^b = 0$ then $p_1^b = p_2^b = p_3^b = 0$, and the maximization problem is identical to problem when $\beta = 0$. If $p_2^b = 0$ but $p_3^b > 0$, then $p_1^b = 0$, and $\mu_4 = \mu_3 = 0$ (note that $\mu_4 = \mu_3 > 0$ is impossible as this would imply $p_3^b = p_2^b = \frac{1}{2}$, contradicting $p_2^b = 0$). Substituting $p_1^b = 0$ into Equation A.12 and using the condition

$\mu_3 = 0$, this equation becomes $p_3^b \frac{V(\phi^*)}{V(\phi^*)^2} = -\mu_2$. As both p_3^b, $V(\phi^*) > 0$ and $\mu_2 \geq 0$ this implies a contradiction.

The necessary conditions for Equation A.14 are

$$\frac{1}{2} \frac{(1+p_2^g)V(\tilde{\phi}) - (1-p_2^g)V(\phi^*)}{V(\phi^*)^2} = \mu_5' - \mu_4' \tag{A.15}$$

$$\frac{1}{2} \frac{p_1^g V(\tilde{\phi}) - (1-p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_4' - \mu_3' \tag{A.16}$$

$$\frac{\beta}{4} \frac{V(\phi^*) - 3V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2' - \mu_1' \tag{A.17}$$

It is useful to regard these expressions as functions of $V(\tilde{\phi})/V(\phi^*)$, p_1^g and p_2^g.

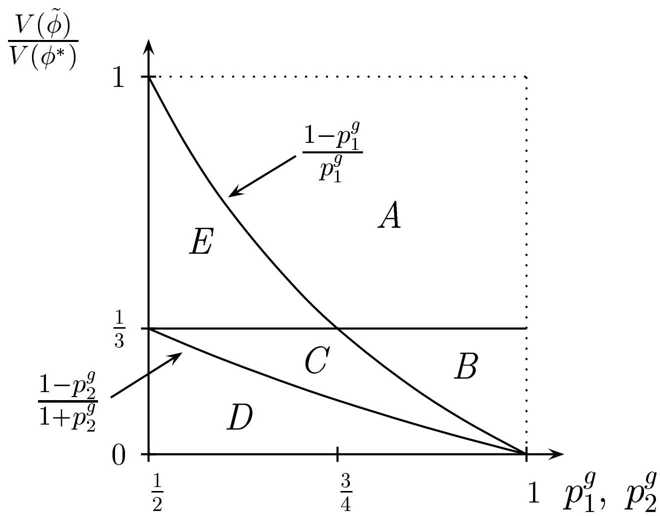

Figure 5: Different areas imply different signs for Equations A.15–A.17 (case of $\mathcal{HP}$).

Figure 5 depicts five areas, five line–segments and three points, each implying different signs for Equations A.15–A.17, as reported in Table 3:

Areas like A–B stand for the line–segments between areas A and B. E–C–D refers to the point of intersection of E, C and D. Note that if $\beta = 0$, Equation A.17 and therefore the line $\frac{V(\tilde{\phi})}{V(\phi^*)} = \frac{1}{3}$ in Figure 5 are irrelevant.

The analysis that follows is based on the following steps. First, the table indicates per area the signs of the Equations A.15–A.17. These signs determine which restrictions

Area	Eqn A.15	Eqn A.16	Eqn A.17
A	+	+	–
B	+	+	+
C	+	–	+
D	–	–	+
E	+	–	–
A–B	+	+	0
A–E	+	0	–

Area	Eqn A.15	Eqn A.16	Eqn A.17
B–C	+	0	+
C–D	0	–	+
C–E	+	–	0
E–C–D	0	–	0
B–C–D	0	0	+
A–B–C–E	+	0	0

Table 3: Sign of Equations A.15–A.17 for different areas, line–segments and points in Figure 5.

bind and which do not. This fixes some (or all) values of p_1^b, p_2^b and p_1^g. These values are then plugged into the LHS of Equations A.15–A.17 to check whether they can still satisfy the sign requirements and have values that are mutually consistent.

In A, the complementary slackness variables should satisfy $\mu_5' > 0$, $\mu_4' > 0$ and $\mu_1' > 0$. That is, $p_1^g = 1$, $p_2^g = p_1^g$, and $p_1^b = 0$ must hold. Substituting these values into the Equations A.15–A.17 on the LHS, one obtains $V(\tilde{\phi}) > 0$, $V(\tilde{\phi}) > 0$, and $V(\tilde{\phi})/V(\phi^*) > \frac{1}{3}$, respectively. The latter inequality is sufficient for the former two to hold. It amounts to $3V(\tilde{\phi}) > V(\phi^*)$ or $3[\frac{1}{2}(1) - \frac{\beta}{4}(1)] > \frac{1}{2}(1+1) - \beta\frac{3}{4}(0)$, or $\beta < \frac{2}{3}$. That is, $p_1^b = 0$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$ is a solution for $\beta < \frac{2}{3}$. That is, $R(\mathcal{HP}, \beta) = \frac{1}{2} - \frac{1}{4}\beta$.

In B, the implication is that $p_1^g = 1$, $p_2^g = p_1^g$, and $p_1^b = \frac{1}{2}$ must hold. Substituting these values into the LHS of Equations A.15–A.17, these conditions amount to $V(\tilde{\phi}) > 0$, $V(\tilde{\phi}) > 0$, and $V(\tilde{\phi})/V(\phi^*) < \frac{1}{3}$, respectively. Similar to the derivations in A, this reduces to $\beta \in (\frac{2}{3}, \frac{4}{3})$. That is, $p_1^b = \frac{1}{2}$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$ is a solution for $\beta \in (\frac{2}{3}, \frac{4}{3})$. Hence, $R(\mathcal{HP}, \beta) = \frac{\frac{1}{2} - \frac{3}{8}\beta}{1 - \frac{3}{8}\beta}$.

In C, the signs of the equations imply $p_1^g = 1$, $p_2^g = \frac{1}{2}$, and $p_1^b = \frac{1}{2}$. This gives rise to inconsistent conditions: Equation A.15 requires $\frac{3}{2}\,V(\tilde{\phi}) - \frac{1}{2}\,V(\phi^*) > 0$, whereas Equation A.16 reduces to $V(\tilde{\phi}) < 0$. As $V(\phi^*) > 0$ by assumption, these conditions are inconsistent. Therefore, there is no solution in C.

In D, the signs require $p_1^g = p_2^g$, $p_2^g = \frac{1}{2}$, and $p_1^b = \frac{1}{2}$. These values, together with the ones obtained from Equations A.8–A.13, imply $V(\tilde{\phi}) = V(\phi^*)$, whereas Equation A.17 requires $3\,V(\tilde{\phi}) > V(\phi^*)$. These conditions are inconsistent.

In E, the signs require $p_1^g = 1$, $p_2^g = \frac{1}{2}$, and $p_1^b = 0$. This gives rise to inconsistent conditions: Equation A.16 requires $V(\tilde{\phi}) < 0$, whereas Equation A.17 reduces to $V(\phi^*) - 3V(\tilde{\phi}) < 0$, which implies $V(\tilde{\phi}) > 0$ (as $V(\phi^*) > 0$), a contradiction. Therefore, there is no solution in E.

On the line–segment A–B, $p_1^g = 1$, $p_2^g = p_1^g$, and $p_1^b \in [0, \frac{1}{2}]$, must hold. For these values, $V(\phi^*) = 1 - \beta\frac{3}{4}p_1^b$ and $V(\tilde{\phi}) = \frac{1}{2} - \beta(\frac{1}{4} + \frac{1}{4}p_1^b)$, and, from Equation A.17, $V(\phi^*) = 3\,V(\tilde{\phi})$. This amounts to $\beta = \frac{2}{3}$. Therefore a solution is $p_1^b \in [0, \frac{1}{2}]$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$ for $\beta = \frac{2}{3}$. Robustness equals $R(\mathcal{HP}, \frac{2}{3}) = \frac{1}{3}$.

On the line–segment A–E, $p_1^g = 1$, $p_2^g \in [\frac{1}{2}, 1]$ and $p_1^b = 0$ must hold. Substituting $p_1^g = 1$ into Equation A.16 shows that $V(\tilde{\phi}) = 0$ should hold. This, however, is in conflict with Equation A.17, which requires $V(\tilde{\phi}) > 0$.

On the line–segments B–C, C–D and C–E conflicting conditions are imposed by

Equations A.15 and A.16, A.15 and A.17, and A.16 and A.17, respectively.

In the point E–C–D, $p_1^g = \frac{1}{2}$, $p_2^g = \frac{1}{2}$, and $p_1^b \in [0, \frac{1}{2}]$, must hold (that $p_1^g = \frac{1}{2}$ holds can be seen from Figure 5). For the values of $(\overline{p}^b, \overline{p}^g)$ derived here $V(\phi^*) = \frac{3}{8} - \beta\frac{3}{4}p_1^b$ and $V(\tilde{\phi}) = \frac{3}{8} - \beta\frac{1}{4}(1 + p_1^b)$. Equation A.16 amounts to $V(\phi^*) = 3\,V(\tilde{\phi})$. This requires $\beta = 1$. However, at $\beta = 1$, $\frac{1}{3}$ is not the minimum value of $V(\tilde{\phi})/V(\phi^*)$: this is attained in region B, where $R(\mathcal{HP}, 1) = \frac{1}{5}$.

In the point B–C–D, $p_1^g = 1$ and $p_2^g = 1$ (both from Figure 5), and $p_1^b = \frac{1}{2}$ (from Equation A.17) must hold. With these values of $(\overline{p}^b, \overline{p}^g)$, $V(\tilde{\phi}) = \frac{1}{2} - \beta\frac{3}{8}$. As $V(\tilde{\phi}) = 0$ from Equations A.15 and A.16, this amounts to $\beta = \frac{4}{3}$. That is, a solution is $p_1^b = \frac{1}{2}$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$ for $\beta = \frac{4}{3}$. Of course, $R(\mathcal{HP}, \frac{4}{3}) = 0$.

Finally, in the point A–B–C–E, $p_1^g = \frac{3}{4}$ from Figure 5, whereas Equation A.15 requires $p_1^g = 1$. These requirements are inconsistent.

What remains to be discussed is the case of $\beta = 0$. Now the differences between A and B, and between C and E respectively, are irrelevant. From the discussion of A, it is clear that $p_3^g = \frac{1}{2}$ and $p_2^g = p_1^g = 1$ is also the optimum for $\beta = 0$. Robustness equals $R = \frac{1}{2}$. In C and E, $R = \frac{2}{3}$, which is larger. In D, as $p_3^g = p_2^g = p_1^g$, $V(\tilde{\phi}) = V(\phi^*)$. Hence, $R = 1$. In other words, for $\beta = 0$, $R(\mathcal{HP}, 0) = \frac{1}{2}$.

In case (ii), or $\Sigma = \mathcal{PH}$, $p^q(\mathcal{PH}, \phi^*) = p_1^q + (1 - p_1^q)p_2^q p_3^q$ and $p^q(\mathcal{PH}, \tilde{\phi}) = p_3^q + (1 - p_3^q)p_1^q p_2^q$.

The array of equations A.2–A.7 becomes

$$\frac{(1 - p_2^g p_3^g)V(\tilde{\phi}) - (1 - p_3^g)p_2^g V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.18}$$

$$\frac{(1 - p_1^g)p_3^g V(\tilde{\phi}) - (1 - p_3^g)p_1^g V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.19}$$

$$\frac{(1-p_1^g)p_2^g V(\tilde{\phi}) - (1-p_1^g p_2^g)V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \quad \text{(A.20)}$$

$$\frac{(1-p_3^b)p_2^b V(\phi^*) - (1-p_2^b p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \quad \text{(A.21)}$$

$$\frac{(1-p_3^b)p_1^b V(\phi^*) - (1-p_1^b)p_3^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \quad \text{(A.22)}$$

$$\frac{(1-p_1^b p_2^b)V(\phi^*) - (1-p_1^b)p_2^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \quad \text{(A.23)}$$

From Equations A.23 one obtains $p_3^b = \frac{1}{2}$. When $p_2^g < 1$, Equation A.20 shows that $p_3^g = \frac{1}{2}$, and substituting this value into Equation A.19 we obtain $p_3^g = p_2^g$. With this information, the Lagrangian equals

$$L(p_1^b, p_2^b, p_1^g, \underline{\mu}') = -\frac{(\frac{1}{2} + \frac{1}{4}p_1^g) - \frac{\beta}{2}(1 + p_1^b p_2^b)}{\frac{1}{4}(3p_1^g + 1) - \beta(p_1^b + \frac{1}{2}(1 - p_1^b)p_2^b)} + \sum_{1=i}^{5} \mu_i'(h_i' - c_i') \quad \text{(A.24)}$$

with $h_1' = -p_1^b \leq 0$, $h_2' = p_1^b - p_2^b \leq 0$, $h_3' = p_2^b \leq \frac{1}{2}$, $h_4' = -p_1^g \leq -\frac{1}{2}$, and $h_5' = p_1^g \leq 1$.

If $p_2^g = 1$ but $p_3^g < 1$, then, from Equation A.20 $\mu_6 = \mu_5 = 0$ ($\mu_6 = \mu_5 > 0$ is impossible as this would imply $p_2^g = p_3^g = \frac{1}{2}$). Substituting $p_1^g = 1$, the implication of $1 \geq p_1^g \geq p_2^g = 1$, and $\mu_6 = 0$ into Equation A.19 leads to $-(1-p_3^g)\,\frac{V(\phi^*)}{V(\phi^*)^2} = \mu_7$. This cannot hold as $(1-p_3^g) > 0$, $V(\phi^*) > 0$ and $\mu_7 \geq 0$.

The first order conditions of Equation A.24 are

$$\frac{1}{4}\,\frac{3V(\tilde{\phi}) - V(\phi^*)}{V(\phi^*)^2} = \mu_5' - \mu_4' \quad \text{(A.25)}$$

$$\frac{\frac{1}{2}p_2^b V(\phi^*) - (1 - \frac{1}{2}p_2^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2' - \mu_1' \quad \text{(A.26)}$$

$$\frac{1}{2}\,\frac{p_1^b V(\phi^*) - (1-p_1^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3' - \mu_2' \quad \text{(A.27)}$$

As in case of $\Sigma = \mathcal{HP}$ above, one can usefully regard these expressions as functions of $V(\tilde{\phi})/V(\phi^*)$, p_1^b and p_2^b. Figure 6 depicts five areas, five line–segments and three points each implying different signs for Equations A.25–A.27, as reported in Table 4:

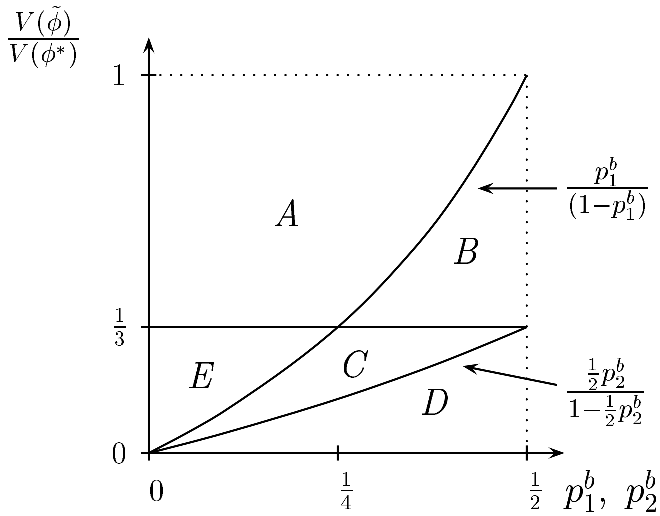

Figure 6: Different areas imply different signs for Equations A.25–A.27 (case of $\mathcal{PH}$).

In A, the complementary slackness variables of Equations A.25–A.27 are such that $p_1^g = 1$, $p_1^b = 0$ and $p_2^b = p_1^b$ must hold, respectively. With these values $V(\phi^*) = 1$ and $V(\tilde{\phi}) = \frac{3}{4} - \frac{1}{2}\beta$. In A, $V(\tilde{\phi})/V(\phi^*) > \frac{1}{3}$ should hold, or $\beta < \frac{5}{6}$. That is, $p_1^b = p_2^b = 0$, $p_3^b = p_3^g = p_2^g = \frac{1}{2}$, $p_1^g = 1$ solves the optimization problem for $\beta < \frac{5}{6}$. Therefore, $R(\mathcal{PH}, \beta) = \frac{3}{4} - \frac{1}{2}\beta$.

In B, $p_1^g = 1$, $p_1^b = 0$ and $p_2^b = \frac{1}{2}$ leads to inconsistent requirements. As $p_1^b = 0$, Equation A.27 reduces to $V(\tilde{\phi}) < 0$. However, this cannot be reconciled with $V(\tilde{\phi})/V(\phi^*) > \frac{1}{3}$ (from Equation A.25) since $V(\phi^*) > 0$.

In C, $p_1^g = \frac{1}{2}$, $p_1^b = 0$ and $p_2^b = \frac{1}{2}$. This amounts to requiring both $V(\tilde{\phi})/V(\phi^*) < \frac{1}{3}$ (because of Equation A.25) and $V(\tilde{\phi})/V(\phi^*) > \frac{1}{3}$ (Equation A.26). These conditions are inconsistent.

In D, because $p_1^g = p_1^b = p_2^b = \frac{1}{2}$, all agents are equal. Hence, $V(\tilde{\phi}) = V(\phi^*)$, which cannot be reconciled with the requirement $3V(\tilde{\phi}) < V(\phi^*)$ of Equation A.25.

In E, $p_1^g = \frac{1}{2}$, $p_1^b = 0$ and $p_2^b = p_1^b$. Hence, $V(\tilde{\phi}) = \frac{5}{8} - \frac{1}{2}\beta$ and $V(\phi^*) = \frac{5}{8}$. In E,

Area	Eqn A.25	Eqn A.26	Eqn A.27
A	+	–	–
B	+	–	+
C	–	–	+
D	–	+	+
E	–	–	–
A–B	+	–	0
A–E	0	–	–

Area	Eqn A.25	Eqn A.26	Eqn A.27
B–C	0	–	+
C–D	–	0	+
C–E	–	–	0
E–C–D	–	0	0
B–C–D	0	0	+
A–B–C–E	0	–	0

Table 4: Sign of Equations A.25–A.27 for different areas, line–segments and points in Figure 6.

$V(\tilde{\phi})/V(\phi^*) \in (0, \frac{1}{3})$. Therefore, if $\beta \in (\frac{5}{6}, \frac{5}{4})$, $p_1^b = p_2^b = 0$, $p_3^b = p_3^g = p_2^g = p_1^g = \frac{1}{2}$ solves the optimization problem. Robustness equals $R(\mathcal{PH}, \beta) = 1 - \frac{4}{5}\beta$.

On the line–segment A–B, $p_1^g = 1$, $p_1^b = 0$, and $p_2^b \in [0, \frac{1}{2}]$ must hold. This leads to inconsistent conditions: Equation A.25 requires $3\ V(\tilde{\phi}) > V(\phi^*)$, whereas Equation A.25 amounts to $V(\tilde{\phi}) = 0$. As $V(\phi^*) > 0$, these conditions cannot be reconciled.

Similar inconsistent conditions for $V(\phi^*)$ and $V(\tilde{\phi})$ are obtained on the line–segments B–C, C–D and C–E.

On the line–segment A–E, $p_1^g \in [\frac{1}{2}, 1]$, $p_1^b = 0$ and $p_1^b = p_2^b$ must hold. This amounts to $V(\tilde{\phi}) = \frac{1}{4}p_1^g + \frac{1}{2} - \frac{1}{2}\beta$ and $V(\phi^*) = \frac{3}{4}p_1^g + \frac{1}{4}$. From Equation A.25, or $V(\tilde{\phi})/V(\phi^*) = \frac{1}{3}$, one derives that $\beta = \frac{5}{6}$. In other words, a consistent solution is $p_1^b = p_2^b = 0$, $p_3^b = p_3^g = p_2^g = \frac{1}{2}$ and $p_1^g \in [\frac{1}{2}, 1]$ for $\beta = \frac{5}{6}$. Obviously, $R(\mathcal{PH}, \frac{5}{6}) = \frac{1}{3}$.

In the point E–C–D, $p_1^g = \frac{1}{2}$, $p_1^b = p_2^b = 0$ (from Figure 6). Equation A.25 amounts to $V(\tilde{\phi})/V(\phi^*) < \frac{1}{3}$, whereas both Equations A.26 and A.27 require $V(\tilde{\phi}) = 0$, where $V(\tilde{\phi}) = \frac{5}{8} - \beta\frac{1}{2}$ and $V(\phi^*) = \frac{5}{8}$. That is, $p_1^b = p_2^b = 0$, $p_3^b = p_3^g = p_2^g = p_1^g = \frac{1}{2}$ solves the optimization problem for $\beta = \frac{5}{4}$. Hence, $R(\mathcal{PH}, \frac{5}{4}) = 0$.

In the point B–C–D, $p_1^b = p_2^b = \frac{1}{2}$ (from Figure 6) and $p_1^g \in [\frac{1}{2}, 1]$. Both Equation A.25 and A.26 amount to $V(\tilde{\phi})/V(\phi^*) = \frac{1}{3}$, whereas Equation A.27 requires $V(\tilde{\phi}) < V(\phi^*)$. For these values of $(\overline{p}^b, \overline{p}^g)$, $V(\tilde{\phi}) = \frac{1}{2} + \frac{1}{4}p_1^g - \beta(\frac{5}{8})$ and $V(\phi^*) = \frac{1}{4} + \frac{3}{4}p_1^g - \beta(\frac{5}{8})$. These expressions can be made consistent with the condition $V(\tilde{\phi})/V(\phi^*) = \frac{1}{3}$ if and only if $\beta = 1$. However, for $\beta = 1$, $\frac{1}{3}$ is not the minimum value of $V(\tilde{\phi})/V(\phi^*)$. The minimum value is attained in region E: $R(\mathcal{PH}, 1) = \frac{1}{5}$.

Finally the point A–B–C–E. Here $p_1^b = 1/4$ from Figure 6, but $p_1^b = 0$ from Equation A.26. These requirements are inconsistent.

The case of $\beta = 0$ becomes particularly easy when $\Sigma = \mathcal{PH}$, as the first order conditions of Equation A.24 reduce to Equation A.25. When $V(\tilde{\phi})/V(\phi^*) < \frac{1}{3}$, $p_1^g = \frac{1}{2}$, and so $p_1^g = p_2^g = p_3^g$. That is, $V(\tilde{\phi}) = V(\phi^*)$. This violates $V(\tilde{\phi})/V(\phi^*) < \frac{1}{3}$. If $V(\tilde{\phi})/V(\phi^*) = \frac{1}{3}$, then $3\left[\frac{1}{2} + \frac{1}{4}p_1^g\right] = \frac{1}{2} + \frac{3}{4}p_1^g$ must hold. This is impossible for all p_1^g. Finally, if $V(\tilde{\phi})/V(\phi^*) > \frac{1}{3}$, then $p_1^g = 1$, and $V(\tilde{\phi})/V(\phi^*)$ equals $\frac{3}{4}$. That is, $R(\mathcal{PH}, 0) = \frac{3}{4}$.

In case (iii), or $\Sigma = \mathcal{O}$, the optimal allocation depends on the values of β and $(\overline{p}^b, \overline{p}^g)$ as specified in Equations 8 and 7. The case, $(iiiA)$, of $\phi^*(\mathcal{O}) = (\sigma_a, \sigma_r, \sigma)$ will be discussed first, followed by $(iiiB)$, $\phi^*(\mathcal{O}) = (\sigma_r, \sigma_a, \sigma)$. In any event, from Result 3, we know that either $\tilde{\phi}_1 = (\sigma, \sigma_r, \sigma_a)$ and $\tilde{\phi}_2 = (\sigma, \sigma_a, \sigma_r)$ maximizes the drop in relative performance.

In $(iiiA)$, with $\phi^*(\mathcal{O}_2) = (\sigma_a, \sigma_r, \sigma)$ and $\tilde{\phi} = \tilde{\phi}_1$, the first–order conditions Equations A.2–A.7 become

$$\frac{p_3^g V(\tilde{\phi}) - (p_3^g - p_2^g)V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.28}$$

$$\frac{(1-p_3^g)V(\tilde{\phi}) - (1-p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.29}$$

$$\frac{(p_1^g - p_2^g)V(\tilde{\phi}) - p_1^g V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.30}$$

$$\frac{(p_3^b - p_2^b)V(\phi^*) - p_3^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.31}$$

$$\frac{(1-p_1^b)V(\phi^*) - (1-p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.32}$$

$$\frac{p_1^b V(\phi^*) - (p_1^b - p_2^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.33}$$

In Equation A.28, $p_3^g - p_2^g \leq 0$, and so $\mu_8 - \mu_7 > 0$ or $\mu_8 > 0$, implying $p_1^g = 1$. Substituting $p_1^g = 1$ into Equation A.29, one observes that if $p_3^g < 1$, then $\mu_7 - \mu_6 > 0$ or $\mu_7 > 0$, which amounts to $p_2^g = p_1^g$. If, on the other hand, $p_3^g = 1$, then $p_1^g = p_2^g = 1$ by assumption. That is, Equation A.29 always requires $p_1^g = p_2^g = 1$. Equation A.30 implies $\mu_6 - \mu_5 < 0$ or $\mu_5 > 0$. That is, $p_3^g = \frac{1}{2}$. Hence, if $\beta = 0$, in which case one is only interested in the solution for p_1^g, p_2^g, and p_3^g, $R(\mathcal{O}_2, 0) = \frac{1}{2}$. In Equation A.32, $1 - p_1^b \geq 1 - p_3^b$, and therefore $\mu_3 - \mu_2 > 0$ or $\mu_3 > 0$, implying $p_2^b = p_3^b$. Using this equality, and assuming $p_3^b > 0$, one derives from Equation A.31 $\mu_2 - \mu_1 < 0$ or $\mu_1 > 0$. That is $p_1^b = 0$. If, on the other hand, $p_3^b = 0$, then $p_1^b = 0$ by assumption. That is, Equation A.31 always requires $p_1^b = 0$. Substituting this value into Equation A.33, one obtains $\mu_4 - \mu_3 > 0$ if $p_2^b > 0$, *i.e.*, $\mu_4 > 0$ or $p_3^b = \frac{1}{2}$. If, on the other hand, $p_2^b = 0$, then $\mu_4 - \mu_3 = 0$. In other words, there are two possible vectors. The first is $p_1^b = 0$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$. With these values of $(\overline{p}^b, \overline{p}^g)$,

$V(\tilde{\phi}) = \frac{1}{2} - \frac{1}{2}\beta$ and $V(\phi^*) = 1 - \frac{1}{4}\beta$, and therefore $R(\mathcal{O}_2, \beta) = \frac{\frac{1}{2}-\frac{1}{2}\beta}{1-\frac{1}{4}\beta}$. The second vector is $p_1^b = p_2^b = p_3^b = 0$, $p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$. Then $V(\tilde{\phi}) = \frac{1}{2}$ and $V(\phi^*) = 1$, and so $R(\mathcal{O}_2, \beta) = \frac{1}{2}$. Clearly, in the former case the drop in performance is larger than in the latter case. In other words, the vector that maximizes the drop in profitability equals $p_1^b = 0$, $p_2^b = p_3^b = p_3^g = \frac{1}{2}$, and $p_2^g = p_1^g = 1$. Hence, for $\tilde{\phi} = \tilde{\phi}_1$ $R(\mathcal{O}_2, \beta) = \frac{\frac{1}{2}-\frac{1}{2}\beta}{1-\frac{1}{4}\beta}$.

If $\phi^*(\mathcal{O}) = (\sigma_a, \sigma_r, \sigma)$, but $\tilde{\phi} = \tilde{\phi}_2$, the first–order conditions of Equations A.2–A.7 become

$$\frac{p_3^g V(\tilde{\phi}) - (p_2^g - p_3^g)V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.34}$$

$$\frac{(1 - p_3^g)V(\tilde{\phi}) - p_1^g V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.35}$$

$$\frac{(p_1^g - p_2^g)V(\tilde{\phi}) - (1 - p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.36}$$

$$\frac{(p_2^b - p_3^b)V(\phi^*) - p_3^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.37}$$

$$\frac{p_1^b V(\phi^*) - (1 - p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.38}$$

$$\frac{(1 - p_1^b)V(\phi^*) - (p_1^b - p_2^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.39}$$

From Equation A.35, $1 - p_3^g < p_1^g$, and therefore $\mu_7 - \mu_6 < 0$. That is, $\mu_6 > 0$ or $p_2^g = p_3^g$. Substituting this equality into Equation A.34, one notes that $\mu_8 > 0$ or $p_1^g = 1$. Equation A.36 does not provide any additional information at this stage. From Equation A.39, $1 - p_1^b > p_1^b - p_2^b$, and therefore $\mu_4 - \mu_3 > 0$. That is, $\mu_4 > 0$ or $p_3^b = \frac{1}{2}$.

Substituting this value into Equation A.38 and observing that $p_2^b - p_3^b \leq 0$ one obtains $\mu_1 > 0$ or $p_1^b = 0$. Substituting $p_1^b = 0$ into Equation A.39, one notes that $\mu_2 > 0$ or $p_1^b = p_2^b$. That is, $p_1^b = p_2^b = 0$, $p_3^b = \frac{1}{2}$, $p_3^g = p_2^g$ and $p_1^g = 1$. The value of p_3^g

remains to be determined. The function to be maximized reduces to

$$\frac{\frac{1}{2}\beta - p_3^g}{p_3^g(2 - p_3^g)} \tag{A.40}$$

subject to $\frac{1}{2} \leq p_3^g \leq 1$. For an interior solution p_3^g should solve $\frac{-{p_3^g}^2 + \beta p_3^g - \beta}{[p_3^g(2-p_3^g)]^2} = 0$. For the solution to this problem to be a real number $\beta > 4$ must hold. However, for such values $V(\tilde{\phi}) < 0$. Therefore, one has to look for a boundary solution. If $p_3^g = \frac{1}{2}$, $R(\mathcal{O}_2, \beta) = \frac{2}{3} - \frac{2}{3}\beta$, whereas for $p_3^g = 1$, $R(\mathcal{O}_2, \beta) = 1 - \frac{1}{2}\beta$. Since $1 - \frac{1}{2}\beta > \frac{2}{3} - \frac{2}{3}\beta$, the drop in relative performance is maximized when $p_3^g = \frac{1}{2}$. Hence, $R(\mathcal{O}_2, \beta) = \frac{2}{3} - \frac{2}{3}\beta$. One observes that $R(\mathcal{O}_2, \beta)$ is smaller if $\tilde{\phi}_1$ than in case of $\tilde{\phi}_2$. In other words, $\tilde{\phi}_1$ maximizes the drop in relative performance.

In case $(iiiB)$, $\phi^*(\mathcal{O}) = (\sigma_r, \sigma_a, \sigma)$. If $\tilde{\phi}_1 = (\sigma, \sigma_r, \sigma_a)$, the first–order conditions of Equations A.2–A.7 become

$$\frac{(1 - p_3^g)V(\tilde{\phi}) - (p_3^g - p_2^g)V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.41}$$

$$\frac{p_3^g V(\tilde{\phi}) - (1 - p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.42}$$

$$\frac{(p_2^g - p_1^g)V(\tilde{\phi}) - p_1^g V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.43}$$

$$\frac{(p_3^b - p_2^b)V(\phi^*) - (1 - p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.44}$$

$$\frac{(1 - p_1^b)V(\phi^*) - p_3^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.45}$$

$$\frac{p_1^b V(\phi^*) - (p_2^b - p_1^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.46}$$

Analysis similar to that conducted in case *(iiiA)* for $\tilde{\phi} = \tilde{\phi}_2$ shows that $p_1^b = 0$, $p_2^b = p_3^b$, $p_3^g = \frac{1}{2}$, $p_2^g = p_1^g = 1$. With this information, the function to be maximized reduces to

$$\frac{\beta p_3^b - \frac{1}{2}}{1 - \beta[p_3^g]^2} \tag{A.47}$$

subject to $0 \leq p_3^b \leq \frac{1}{2}$. For an interior solution p_3^b should solve $\frac{\beta {p_3^b}^2 - p_3^g + 1}{[1-\beta [p_3^g]^2]^2} = 0$. For the solution to this problem to be a real number $\beta < \frac{1}{4}$ must hold. For these admissible values of β, the solution $p_3^b \notin [0, \frac{1}{2}]$. Therefore, one has to look for a boundary solution. If $p_3^b = \frac{1}{2}$, $R(\mathcal{O}_1, \beta) = \frac{1}{2} \frac{1-\beta}{1-\frac{1}{4}\beta}$, whereas for $p_3^b = 0$, $R(\mathcal{O}_1, \beta) = \frac{1}{2}$. As $\frac{1-\beta}{1-\frac{1}{4}\beta} < 1$, the solution to the optimization problem is $p_3^b = \frac{1}{2}$, and therefore $R(\mathcal{O}_1, \beta) = \frac{1}{2} \frac{1-\beta}{1-\frac{1}{4}\beta}$ if $\tilde{\phi} = \tilde{\phi}_1$.

If $\tilde{\phi} = \tilde{\phi}_2$, however, the first–order conditions of Equations A.2–A.7 become

$$\frac{(1-p_3^g)V(\tilde{\phi}) - (p_2^g - p_3^g)V(\phi^*)}{V(\phi^*)^2} = \mu_8 - \mu_7 \tag{A.48}$$

$$\frac{p_3^g V(\tilde{\phi}) - p_1^g V(\phi^*)}{V(\phi^*)^2} = \mu_7 - \mu_6 \tag{A.49}$$

$$\frac{(p_2^g - p_1^g)V(\tilde{\phi}) - (1-p_1^g)V(\phi^*)}{V(\phi^*)^2} = \mu_6 - \mu_5 \tag{A.50}$$

$$\frac{(p_2^b - p_3^b)V(\phi^*) - (1-p_3^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_2 - \mu_1 \tag{A.51}$$

$$\frac{p_1^b V(\phi^*) - p_3^b V(\tilde{\phi})}{V(\phi^*)^2} = \mu_3 - \mu_2 \tag{A.52}$$

$$\frac{(1-p_1^b)V(\phi^*) - (p_2^b - p_1^b)V(\tilde{\phi})}{V(\phi^*)^2} = \mu_4 - \mu_3 \tag{A.53}$$

Analysis similar to that conducted in case *(iiiA)* for $\tilde{\phi} = \tilde{\phi}_1$ shows that $p_1^b = p_2^b = 0$, $p_3^b = p_3^g = p_2^g = \frac{1}{2}$ and $p_1^g = 1$. Therefore, $R(\mathcal{O}_1, \beta) = \frac{2}{3} - \frac{2}{3}\beta$. Note that $R(\mathcal{O}_1, \beta)$ is smaller for $\tilde{\phi}_1$ than for $\tilde{\phi}_2$. Hence, $\tilde{\phi}_1$ maximizes the drop in relative performance.

Summing up, in $(iiiA)$, $\phi^*(\mathcal{O}) = (\sigma_a, \sigma_r, \sigma)$ and in $(iiiB)$, $\phi^*(\mathcal{O}) = (\sigma_r, \sigma_a, \sigma)$. In either case, $\tilde{\phi}_1 = (\sigma, \sigma_r, \sigma_a)$ and $(\overline{p}^b, \overline{p}^g) = (0, 1/2, 1/2, 1/2, 1, 1)$ maximize the drop in relative performance. We now have to check for these values of $(\overline{p}^b, \overline{p}^g)$ whether $\phi^*(\mathcal{O}) = (\sigma_a, \sigma_r, \sigma)$ or $\phi^*(\mathcal{O}) = (\sigma_r, \sigma_a, \sigma)$ maximizes expected profits. Using Equation 8, with $\phi^1 = (\sigma_a, \sigma_r, \sigma)$ and $\phi^2 = (\sigma_r, \sigma_a, \sigma)$, one can easily see that $V(\phi^1, \beta; \mathcal{O}) - V(\phi^3, \beta; \mathcal{O}) =$

0 for the vector $(\overline{p}^b, \overline{p}^g)$ under consideration. $\square$

Proof of Lemma 1 Differentiating H with respect to p_1^g yields

$$\frac{\partial H}{\partial p_1^g} = \frac{4}{3}p_1^g - \frac{2}{3}[p_2^g + p_3^g] \overset{*}{\geq} \frac{4}{3}p_1^g - \frac{4}{3}p_2^g \tag{A.54}$$

where (*) uses $p_2^g \geq p_3^g$. That is, if $p_1^g > p_2^g$, then $\frac{\partial H}{\partial p_1^g} > 0$, and so $p_1^g = 1$. If $p_1^g = p_2^g$, obviously $p_3^g < p_2^g$ (otherwise agents would be identical as far as good projects are concerned, implying $H^g = 0$), and differentiating H with respect to p_2^g, and substituting $p_1^g = p_2^g$ yields $\frac{\partial H}{\partial p_2^g} = \frac{4}{3}(p_2^g - p_3^g) > 0$. That is, if $p_1^g = p_2^g$, then p_2^g as large as possible, or $p_1^g = p_2^g = 1$. In any event, $p_1^g = 1$. Similar analysis shows that in any event $p_3^g = \frac{1}{2}$. With this information $\frac{\partial H}{\partial p_2^g} = \frac{4}{3}p_2^g - \frac{2}{3}$. That is, $p_2^g \in \left\{\frac{1}{2}, 1\right\}$. In any event $H = \frac{1}{6}$. Clearly, heterogeneity is maximized when either (p_3^g, p_2^g, p_1^g) equals $(\frac{1}{2}, \frac{1}{2}, 1)$ or $(1, \frac{1}{2}, 1)$. By the same token, when projects are bad, the spread is maximized when (p_3^b, p_2^b, p_1^b) equals $(0, \frac{1}{2}, \frac{1}{2})$ or $(0, 0, \frac{1}{2})$. $\square$

Proof of Result 5 This follows directly from a comparison of the vectors $(\overline{p}^b, \overline{p}^g)$ reported in Table 1 and Lemma 1. $\square$

Proof of Proposition 2 This can be seen by applying Definition 6 to the fourth column of Table 2. $\square$