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Maarten C.W. Janssen
Vladimir A. Karamychev

Department of General Economics, Faculty of Economics, Erasmus University Rotterdam, and Tinbergen Institute

Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the
Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and
Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

Dynamic Insurance and Adverse Selection

Maarten C. W. Janssen

(Erasmus University Rotterdam)

and

Vladimir A. Karamychev

(Tinbergen Institute Rotterdam)

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Abstract: We take a dynamic perspective on insurance markets under adverse selection and study a generalized Rothschild and Stiglitz model where agents may differ with respect to the accidental probability *and* their expenditure levels in case an accident occurs. We investigate the nature of dynamic insurance contracts by considering both conditional and unconditional dynamic contracts. An unconditional dynamic contract has insurance companies offering contracts where the terms of the contract depend on time, but not on the occurrence of past accidents. Conditional dynamic contracts make the actual contract also depend on individual past performance (like in car insurances). We investigate whether allowing insurance companies to offer dynamic insurance contracts results in Pareto-improvements over static contracts. Our main results are as follows. When agents only differ in their accidental expenditures, then dynamic insurance contracts yield a welfare improvement only if dynamic contracts are conditional on past performance. When, however, agents' expenditures differ just a little bit dynamic insurance contracts are strictly Pareto improving even for unconditional dynamic contracts.

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Address for Correspondence: Maarten Janssen, Department of Economics, Erasmus University Rotterdam, H7-22, Burg. Oudlaan 50, 3062 PA Rotterdam, The Netherlands; tel.: +31-(10)-4082341, fax: +31-(10)-4089149, E-mail: janssen@few.eur.nl.

1. Introduction

Adverse selection is potentially a serious problem in any type of insurance market (see, e.g. the seminal paper by Akerlof (1970)).[1] If agents have different risks and if insurance companies are not able or are not allowed to distinguish between different risk categories, "low-risk" agents may find the insurance premium too costly and will not fully insure themselves, or in the extreme, will not take insurance at all. This mechanism of adverse selection generally leads to welfare losses, as potential benefits from trade are not fully realized by the market participants. One way to overcome the adverse selection problem in insurance markets is through screening (Rothschild and Stiglitz, 1976): insurance companies offer a variety of insurance contracts, each with a different premium and coverage, and agents select the insurance contract that they like best. By employing screening mechanisms, the market is able to re-gain part of the welfare loss due to asymmetric information. Screening equilibria in competitive insurance markets may not exist, however, and there is still a welfare loss associated with them (see Riley, 2001, for an overview of the literature).[2]

Even though the probability of an accident is a recurrent one in most insurance markets (with life insurance as an exception), the typical model of insurance markets considers a static environment where agents incur a loss only once. This modeling assumption may be justified if we want to explain the behavior of insurance companies as quite a few insurance contracts are essentially static (with car insurance as a notable exception): the terms of the insurance contract are independent of the time period and past history. In this paper, we ask a normative question, namely whether Pareto-improvements can be achieved if some kind of dynamic insurance would be provided.

We consider two types of dynamic insurance contracts. The first type, which we call *conditional* dynamic contracts, allows insurance conditions in future periods to depend on an agent's accidental history. In such contracts, agents that from an *ex ante* point of view take identical contracts may view different insurance terms in later periods when their accidental history differs. The second type of dynamic contract is *unconditional*, as an insurer is not able or not allowed to use an agent's past accidental history. Unconditional contract can still have a dynamic nature as the terms of the contract may depend on the time period.

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[1] A recent empirical confirmation can be found in Oosterbeek et al. (2001).

[2] For alternative equilibrium definitions see the papers by Wilson (1977) and Riley (1979).

We consider these two types of dynamic contracts for the following reasons. Conditional dynamic contracts are observed in the car insurance market with the infamous bonus/malus rules. It is important to understand the welfare implications of such contracts. We do not know of markets where unconditional dynamic contracts are offered, but they may be considered in markets where conditional contracts are politically not viable, like in some health insurance markets. In such markets it may be considered unfair if someone has to pay a very high premium because she simply had bad luck and got many health accidents in a row. In some of these markets (e.g., the Dutch market for dental insurance; see Oosterbeek et.al., 2001) there is a clear indication of adverse selection and one may wonder whether unconditional dynamic contracts may help to overcome (partially) the adverse selection problem and improve welfare.

The model we consider is a generalized version of the well-known Rothschild-Stiglitz (1976) world, where insurance contracts last for some finite number of periods. Agents discount future utility and profit levels at a given discount rate. There are two types of risk-averse agents: low-risk and high-risk. The probability that an accident happens to an individual is constant and the same in every time period. This means that we abstract from moral hazard issues. Low-risk agents have lower accidental probability than high-risk agents and their expenditures in case of an accident are also not higher (and in most cases lower). Although the formal model treats these expenditures as certain numbers, we like to think of them in terms of expected values so that insurance companies cannot discriminate between the two types of agents on the basis of the differences in expenditures. *Unconditional dynamic contracts* only condition the terms of the insurance on the time period. *Conditional dynamic contracts* can, in addition, condition the terms of the contract on the accidental history.

Apart from allowing insurance companies to change the terms of the insurance contract over time, a dynamic analysis may introduce also other complications. In particular, agents may shift wealth from one period to another: insurance companies may shift profits between different time periods so that competition doesn't need to result in zero profits in every time period, and consumers may save or borrow. In the main body of the paper we abstract from these complications and concentrate only on the effect of dynamic contracts on welfare. We do this by considering competitive Nash equilibria in which insurance companies offer a set of dynamic contracts such that each type of agent chooses an optimal contract from this set and no insurance company can unilaterally benefit by adding contracts to this set. We analyze the properties of these equilibria in three different settings: a "static" setting where insurance companies offer the same terms of the contract in every period, and the two dynamic settings.

We have several results. First, in all three settings, competitive Nash equilibria only exist for a relatively small fraction of low-risk agents in the population. Generally speaking, competitive Nash equilibria with unconditional dynamic contracts exist for larger fractions of low-risk agents than those equilibria with "static" contracts. Existence conditions in the other settings cannot be easily compared. Second, high-risk agents get full insurance, in all the equilibria in all three settings. Third, when they do exist, equilibria under conditional dynamic contracts yield a Pareto-improvement over static equilibrium contracts and the optimal contract charges lower premiums to agents with better accidental histories. The main reason is that the probability of having a better accidental history is larger for low-risk agents than for high-risk agents allowing insurance companies to screen the two types of agents more easily. For a certain class of utility functions when the number of periods gets large, the welfare achieved through conditional dynamic contracts approaches first-best welfare levels even if agents discount the future. Fourth, unconditional dynamic contracts only provide a welfare improvement over static contracts when low-risk agents have lower expenditures than high-risk agents. When this is so, optimal unconditional contracts have some periods without insurance and much better insurance conditions in the remaining periods. As expenditures differ, high-risk agents are hurt more in periods without insurance than low-risk agents. This allows unconditional dynamic contracts to better screen the different types of agents. Finally, by means of simulations we show that the welfare improvements of using dynamic insurance contracts can be considerable. Depending on the context and on the parameter values, dynamic contracts can reduce the welfare loss for low-risk agents between the first-best solution and the static equilibrium outcome by more than 60%.

The paper is related to different branches of literature (apart from the seminal paper by Rothschild and Stiglitz, 1976). First, the paper is closely related to the literature on the use of experience ratings in multi-period self-selection models, see, e.g., Dionne and Laserre (1985) and Cooper and Hayes (1987). The idea in this literature is that the terms of future coverage may depend on previous loss experience as, for example, in car insurances. This is the setting we study when considering *conditional* dynamic contracts. Dionne and Laserre (1985) study infinite horizon contracts where agents maximize *average* per period utility. They show that in such a world, insurance companies can screen agents in such a way that the first-best outcome is achieved. Cooper and Hayes (1987) study a similar problem in a two-period model. Their main focus is on the differences in equilibrium outcomes under monopoly and perfect competition. Our main focus in this paper is different. We want to understand why and under what conditions dynamic contracts are welfare improving vis-à-vis static contracts: is it because of the state-dependent nature of conditional contracts or is it because of the time (and not state) dependency that is also present in unconditional contracts. In so far as our paper deals with conditional dynamic contracts we analyze the intermediate case of finite horizon contracts where, in addition, agents discount future utility. We show that contrary to what is argued by Cooper and Hayes (1987) in order to get close to the first-best, it is not necessary that agents do not discount future payoffs. Moreover, by means of simulations we provide insight in the question by how much welfare can be improved.

Part of the insurance literature studies the way probationary periods can be used to separate agents with different risk profiles (see, e.g., Eeckhoudt et al. (1988) and Fluet (1992) among others). The basic idea of a probationary period is that prior to the reimbursement of losses incurred, the insurance company pays no indemnity. A probationary period is one of the possibilities in our framework and we show that the optimal *unconditional* contract has a probationary period. The literature on probationary periods considers, however, a situation where agents incur only one loss over a certain time period where the timing of the loss may be different for different types of agents. This situation is relevant in life insurance markets. In contrast, our model considers situations in which in any given period, agents have a certain probability of getting an accident independent of previous accidents. Hence, our model does not cover life insurance markets, but is more relevant in situations where agents may incur many losses at different moments in time.

Finally, there is a series of articles (Janssen and Roy 1999*a*, 1999*b*, Janssen and Karamychev, 2000) showing that through dynamic trading the competitive market mechanism allows high quality sellers of a durable good to trade even in the presence of asymmetric information. Dynamic equilibria typically involve increasing prices over time and higher quality sellers waiting to sell in later periods. In other words, waiting time before selling can act as a screening device in dynamic competitive markets with adverse selection. Our analysis in the context of unconditional dynamic contracts has a similar flavor: low-risk agents (i.e., "high quality" agents) incur an initial loss of not being insured in order to get much better insurance conditions later on.

The rest of the paper is organized as follows. Section 2 discusses definitions and notations that we will use in the rest of the paper. Section 3 briefly analyzes the static model for reference purposes. Sections 4 and 5 consider the analysis of the dynamic world of conditional and unconditional contracts respectively. Section 6 concludes with a discussion of the results. Some of the more elaborate proofs are contained in the appendix.

2. Preliminaries

The environment studied here is a generalization of the model first described by Rothschild and Stiglitz (1976). Individual agents come in two types, high-risk agents "*H*" and low-risk agents "*L*". Everyone is endowed with some income level in every period, which is normalized to be equal to 1. Each type $i \in \{H, L\}$ is characterized by a level of (expected) expenditure e_i in case of an accident, where $0 < e_L \leq e_H < 1$, and a probability of an accident q_i, $0 < q_L \leq q_H < 1$.[3] The probability of an accident and the related expenditures are private knowledge and constant through time. All agents are risk averse, they have the same state independent strictly concave and increasing utility function *u* and for the sake of convenience we assume that $u(1) = 0$. Let $\alpha \in (0,1)$ denote the share of low-risk agents within the population.

On the supply side of the market there are a number of risk neutral insurance companies competing with each other. These companies are not able to discriminate between the different types. In what follows we will use the superscripts "*S*" and "*D*" to refer to static and dynamic variables, respectively, and we will compare the welfare implications of two types of insurance contracts: static and dynamic. A *static* insurance contract $\Theta^S = (P^S, D^S)$ consists of a constant premium P^S and a constant deductible D^S such that in case of an accident an insured individual receives $\max\{e_i - D^S, 0\}$ from the insurance company. By $\Theta_0^S = (0, \infty)$ we denote an artificial contract, which gives no insurance at all. The expected utility of type *i* under contract Θ^S is $U_i^S(\Theta^S) \equiv q_i u(1 - P^S - D^S) + (1 - q_i) u(1 - P^S)$.

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[3] Although formally, we treat the level of expenditures to be fixed numbers, we do not allow insurance companies to offer insurance contracts that are able to discriminate between different types only because of the differences in fixed expenditures. For example, we do not allow to condition future terms of an insurance contract on observed expenditure levels. One way to think of these expenditure levels is, therefore, as expected values so that differences in types cannot be based on different realizations of expenditure levels.

A dynamic contract Θ^D lasts T time periods and consists of T parts, each part specifying the terms of the contract in that time period. Unlike a static contract, dynamic contracts may offer different insurance conditions for an agent in time periods $t = 2,\dots,T$ depending on her previous accidental history h_t. Thus, a dynamic contract's term in time period t is a set of 2^{t-1} insurance policies that correspond to every $h_t \in H_t$, where H_t is a set of all possible history realizations up to period *t*. For example, in period 1 a dynamic contract Θ^D offers a simple static insurance policy $\Theta_1 = (P_1, D_1)$, in period 2 a (static) policy $\Theta_2^{(1)} = \left(P_2^{(1)}, D_2^{(1)}\right)$ applies if there was an accident and $\Theta_2^{(0)} = \left(P_2^{(0)}, D_2^{(0)}\right)$ applies if there was no accident. Hence, $\Theta_2 = \left\{\Theta_2^{(0)}, \Theta_2^{(1)}\right\}$. In a similar fashion $\Theta_3 = \left\{\Theta_3^{(0,0)}, \Theta_3^{(0,1)}, \Theta_3^{(1,0)}, \Theta_3^{(1,1)}\right\}$ and so on. We will call such a contract $\Theta^D = (\Theta_1, \Theta_2, \dots, \Theta_T)$.

The *ex ante* expected utility of type *i* under a contract Θ^D is

$$\sum_{t=1}^{T}\left\{\delta^{t-1}\sum_{h_t\in H_t}\Pr_i(h_t)\left(q_i u\left(1-P_t^{(h_t)}-D_t^{(h_t)}\right)+(1-q_i)u\left(1-P_t^{(h_t)}\right)\right)\right\},$$

and her expected *per period* utility is

$$U_i^D\left(\Theta^D\right)\equiv \tfrac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\left\{\delta^{t-1}\sum_{h_t\in H_t}\Pr_i(h_t)\left(q_i u\left(1-P_t^{(h_t)}-D_t^{(h_t)}\right)+(1-q_i)u\left(1-P_t^{(h_t)}\right)\right)\right\},$$

where $\delta \in (0,1)$ is the common discount factor and $\Pr_i(h_t)$ is an i agent's probability to end up with a history h_t at time period *t*. For example, for $h_4 = (0,1,0)$, i.e., no accidents in time periods 1 and 3 and an accident in time period 2, $\Pr_i(h_4) = q_i(1-q_i)^2$.

One can see that for a dynamic contract with constant insurance conditions, i.e., for $\Theta_t^{(h_t)} = \Theta^*$, $U_i^D\left(\Theta^D\right) = U_i^S\left(\Theta^*\right)$. This allows us to make welfare comparisons between static and dynamic contracts.

As explained in the introduction, in certain cases an insurer is not able, or not allowed, to use the information, which is obviously available to him, about an agent's past accidents. In this case the contract terms Θ_t are no longer sets of policies but simply a sequence of static contracts $\Theta_t = (P_t, D_t)$ and the expression for the expected per period utility simplifies to

$$U_i^D\left(\Theta^D\right) = \tfrac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\left\{\delta^{t-1}\left(q_i u\left(1-P_t-D_t\right)+(1-q_i)u\left(1-P_t\right)\right)\right\}.$$

We will call such a contract an *unconditional* dynamic contract. The difference between these contracts and the *conditional* dynamic contracts described above is that unconditional contracts make the terms of the insurance contract in period t unconditional on the accidental history.

Let Σ_T^D be the set of all T-period dynamic (conditional or unconditional, depending on the context) insurance contracts. Then, the set Σ^S, which is the set of all static insurance contracts, coincides with Σ_1^D and, therefore, any static contract Θ^S can be treated as a 1-period dynamic contract. What we will do then in this paper is to describe welfare properties and existence conditions of a competitive Nash equilibrium over the set Σ_T^D for an arbitrary but fixed $T \geq 1$.

All insurance companies offer T-periods insurance contracts to the agents. Because of competition insurance companies do not make any profit in equilibrium. Every agent chooses the contract, possibly Θ_0^S, that maximizes her expected per period utility. The formal definition of a (competitive Nash) equilibrium is as follows.

Definition 1. A T-period competitive Nash equilibrium is a subset of T-period insurance contracts, $\Psi_T \subset \Sigma_T^D$, present in the market satisfying the following conditions:

a) Each agent chooses an insurance contract that maximizes her per period utility, i.e., every type $i \in \{H, L\}$ chooses the contract $\Theta_i \in \arg\max_{\Theta \in \Psi_T} U_i(\Theta)$.

b) Any equilibrium contract is bought by at least one type, i.e., for any $\Theta' \in \Psi_T$ $\exists i \in \{H, L\}$ such that $\Theta' = \Theta_i$.

c) Any equilibrium contract yields nonnegative profit to an insurer.

d) No insurance company can benefit by unilaterally offering a different insurance contract, i.e., any insurance company offering a contract $\Theta' \in \Sigma_T^D \setminus \Psi_T$ such that for some $i \in \{H, L\}$ $U_i(\Theta') > \max_{\Theta \in \Psi_T} U_i(\Theta)$ makes strictly negative profit.

Standard arguments rule out any *pooling* insurance contract Θ_P to be a Nash equilibrium. For static contracts, the argument is given by Rothschild and Stiglitz (1976). In a dynamic world a similar argument holds true: for any (partial) pooling contract there exists a contract that differs from it in only one time period in such a way that only low-risk agents prefer the latter contract. This implies that the deviation yields strictly positive profit.

On the other hand, a *separating* Nash equilibrium (static or dynamic), which involves two contracts Θ_H and Θ_L, may not exist if there exists a *profitable* pooling contract Θ_P that gives a higher utility level to the low-risk agent than Θ_L. Hence, the existence of a separating Nash equilibrium is guaranteed if any pooling contract yielding nonnegative profit, Θ_P, gives less utility to low-risk type agents than Θ_L, i.e., $U_L(\Theta_P) \leq U_L(\Theta_L)$.

Throughout the following three sections we assume that an insurance company is forced to price its contract in such a way that it yields zero profit in *every* time period and that agents are also not allowed to transfer wealth between periods.

3. Static Insurance Contracts

In this section we start off by briefly generalizing the standard results of Rothschild and Stiglitz (1976) to the case where types of agents differ not only in accidental probabilities but also in their expenditures in case of an accident. Equilibria under static contracts, which are considered here, are a benchmark for further analysis.

A competitive Nash equilibrium, if it exists, involves two contracts Θ_H^S and Θ_L^S such that they generate zero profit for the insurer. This implies that $P_H^S = q_H\left(e_H - D_H^S\right)$ and $P_L^S = q_L\left(e_L - D_L^S\right)$. Moreover, it follows that high-risk agents take full insurance, i.e., $\Theta_H^S = \left(P_H^S, D_H^S\right) = \left(q_H e_H, 0\right)$. Low-risk agents get at most partial insurance according to the contract Θ_L^S. This contract is such that high-risk agents are either indifferent between Θ_H^S and Θ_L^S, i.e., $U_H^S\left(\Theta_H^S\right) = U_H^S\left(\Theta_L^S\right)$, or strictly prefer Θ_H^S. Partial, or even no insurance, is the price low-risk agents have to pay in order to be separated from high-risk agents. Existence of equilibrium is guaranteed only, as is well known, for relatively small values of α. The following proposition formally states this standard result.

Proposition 1. Let $e_L^S = 1 - m\left(\frac{1}{q_H} u\left(1 - q_H e_H\right)\right)$ where *m* is the inverse of the utility function *u*. Then, there exists an $\alpha^S \in (0,1)$ such that:

a) For all $\alpha \in \left(0, \alpha^S\right)$ there exists a unique separating competitive Nash equilibrium $\Psi_1 = \left\{\Theta_H^S, \Theta_L^S\right\}$. High-risk agents get full insurance $\Theta_H^S = \left(P_H^S, D_H^S\right) = \left(q_H e_H, 0\right)$ while low-risk agents get partial insurance, i.e., $\Theta_L^S = \left(P_L^S, D_L^S\right)$ and $D_L^S \in \left(0, e_L\right)$, if $e_L \in (e_L^S, e_H]$ and no insurance, i.e., $\Theta_L^S = \Theta_0^S$, if $e_L \in [0, e_L^S]$.

b) For all $\alpha \in \left(\alpha^S, 1\right)$ a separating competitive Nash equilibrium Ψ_1 does not exist.

Proof. The utility low-risk agents get under Θ_L^S does not depend on α, i.e., $U_L^S\left(\Theta_L^S\right)$ is a constant determined by the incentives compatibility constraint $U_H^S\left(\Theta_H^S\right) \geq U_H^S\left(\Theta_L^S\right)$. Given $P^S = q_L\left(e_L - D^S\right)$, $U_H^S\left(\Theta^S\right)$ becomes a decreasing function of $D^S \in [0, e_L]$:

$$\frac{d}{dD^S}U_H^S\left(\Theta^S\right) = \frac{d}{dD^S}\left(q_H u\left(1 - P^S - D^S\right) + \left(1 - q_H\right)u\left(1 - P^S\right)\right) =$$

$$= -\left(q_H\left(1 - q_L\right)u'\left(1 - P^S - D^S\right) - q_L\left(1 - q_H\right)u'\left(1 - P^S\right)\right) < 0.$$

It takes its minimum value of $q_H u\left(1 - e_L\right)$ at $D^S = e_L$. Hence, if $U_H^S\left(\Theta_H^S\right) \equiv u\left(1 - q_H e_H\right) \leq q_H u\left(1 - e_L\right)$ then any competitive contract providing partial insurance to the low-risk agents is more attractive for the high-risk agents than $U_H^S\left(\Theta_H^S\right)$ and, therefore, $\Theta_L^S = \Theta_0^S$. This happens if $e_L \leq 1 - m\left(\frac{1}{q_H}u\left(1 - q_H e_H\right)\right) \equiv e_L^S$. If, on the other hand, $e_L > e_L^S$ then the incentives compatibility constraint becomes binding that determines $D_L^S \in \left(0, e_L\right)$ in such a way that $U_H^S\left(\Theta_H^S\right) = U_H^S\left(\Theta_L^S\right)$.

While $U_L^S\left(\Theta_L^S\right)$ is independent on α the maximum utility low-risk agents may ever obtain from a competitive pooling contract Θ_P^S, i.e., $U_L^S\left(\hat{\Theta}_P^S\right) = \max_{\Theta_P^S} U_L^S\left(\Theta_P^S\right)$, does depend on α as the "pooling price" P_P^S, which is defined to be equal to $P_P^S = \alpha q_L\left(e_L - D_P^S\right) + \left(1 - \alpha\right)q_H\left(e_H - D_P^S\right)$, depends on it. Solving the maximization problem

$$\begin{aligned}&\max\left(q_L u\left(1 - P_P^S - D_P^S\right) + \left(1 - q_L\right)u\left(1 - P_P^S\right)\right)\\&\text{s.t.}: P_P^S = \alpha q_L\left(e_L - D_P^S\right) + \left(1 - \alpha\right)q_H\left(e_H - D_P^S\right)\end{aligned}$$

yields the first order condition

$$\frac{q_L}{1 - q_L}u'\left(1 - \hat{P}_P^S\left(\hat{D}_P^S\right) - \hat{D}_P^S\right) = \frac{\alpha q_L + \left(1 - \alpha\right)q_H}{1 - \left(\alpha q_L + \left(1 - \alpha\right)q_H\right)}u'\left(1 - \hat{P}_P^S\left(\hat{D}_P^S\right)\right),$$

which implicitly defines $\hat{D}_P^S$. Now, the first order derivative of $U_L^S\left(\hat{\Theta}_P^S\right)$ with respect to α becomes

$$\frac{dU_L^S\left(\hat{\Theta}_P^S\right)}{d\alpha} = \frac{\partial U_L^S\left(\hat{\Theta}_P^S\right)}{\partial\alpha} = \left(q_L u'\left(1 - \hat{P}_P^S - \hat{D}_P^S\right) + \left(1 - q_L\right)u'\left(1 - \hat{P}_P^S\right)\right)\left(q_H\left(e_H - \hat{D}_P^S\right) - q_L\left(e_L - \hat{D}_P^S\right)\right).$$

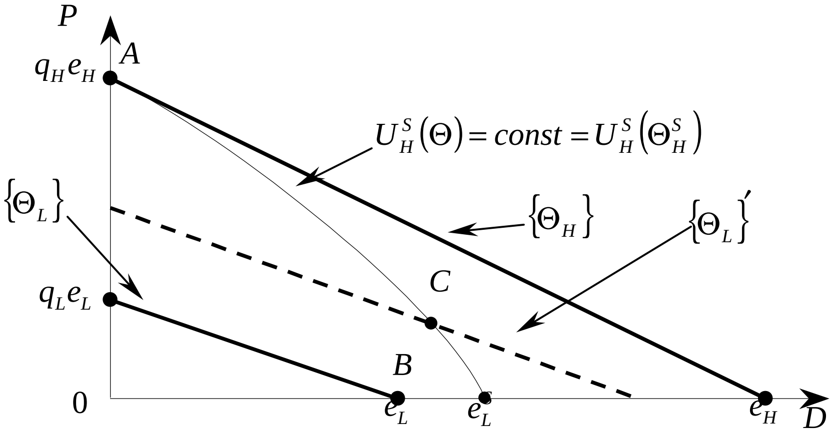

Figure 1. Separating static insurance contracts.

One may easily see that $\frac{d}{d\alpha} U_L^S\left(\hat{\Theta}_P^S\right) > 0$ for all $0 \le \hat{D}_P^S \le e_H$. Finally, note that $U_L^S\left(\hat{\Theta}_P^S\right)\Big|_{\alpha=0} < U_L^S\left(\Theta_L^S\right) < U_L^S\left(\hat{\Theta}_P^S\right)\Big|_{\alpha=1}$. This implies that there exists a unique $\alpha^S \in (0,1)$ such that $U_L^S\left(\Theta_L^S\right) = U_L^S\left(\hat{\Theta}_P^S\right)\Big|_{\alpha=\alpha^S}$ and the result follows. ■

Figure 1 presents the main idea. It shows that there is no insurance for low-risk agents if their expenditures are relatively small. In the figure $\{\Theta_H\}$ and $\{\Theta_L\}$ are the sets of competitive contracts designated for high- and low-risk agents respectively. These contracts satisfy the zero-profit conditions $P = q_H(e_H - D)$ and $P = q_L(e_L - D)$, respectively. Point *A* denotes the optimal contract for high-risk agents.

One may see that if the set of competitive contracts that can be offered to low-risk agents only lies entirely below the indifference curve that passes through point *A*, as depicted in Figure 1, i.e., when $e_L \le e_L^S$, then any contract from the set $\{\Theta_L\}$ is more attractive for high-risk agents than Θ_H^S. Even the worst contract *B*, which gives zero coverage, i.e., when $D = e_L$, gives a higher utility level to high-risk agents than Θ_H^S. Hence, in the separating equilibrium there is no insurance for low-risk agents and we are in a case of pure adverse selection. If, on the other hand, $e_L > e_L^S$, such that the set $\{\Theta_L\}'$ denoted by the dashed line intersects with the indifference curve that passes through point *A*, then in equilibrium the low-risk agents get a contract *C* which gives partial insurance.

Since the model described here involves four parameters, namely q_H, e_H, q_L and e_L, for presentational purposes in what follows we fix q_L and e_H at arbitrary levels and consider the parameter space $(q_H, e_L) = [q_L, 1) \text{x} [0, e_H]$. For any fixed level of q_L and e_H,

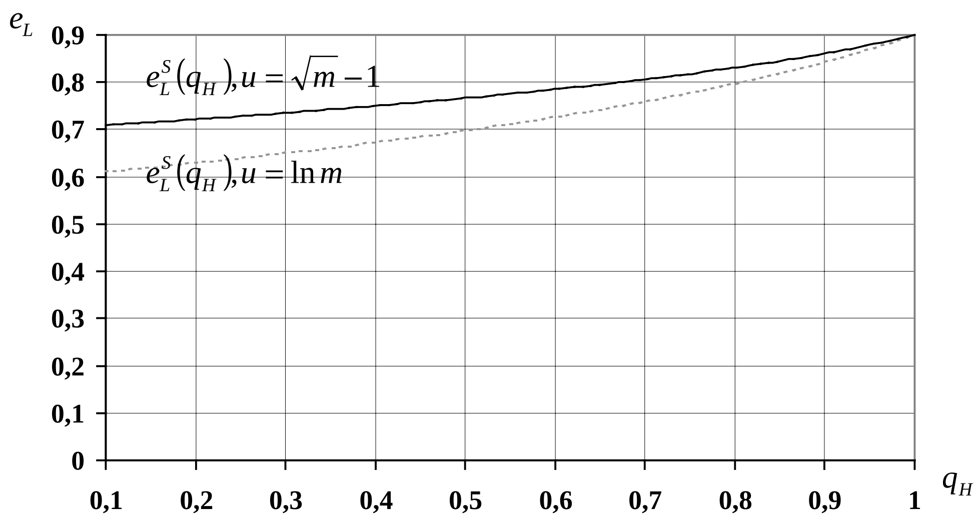

Figure 2. Regions of parameter values where low-risk agents do and do not get positive insurance.

e_L^S can be written as a strictly increasing function of q_H, $e_L^S(q_H) = 1 - m\left(\frac{1}{q_H} u(1 - q_H e_H)\right)$, and $e_L^S(1) = e_H$.

To get an idea about the relative importance of dynamic insurance contract, we have done several simulations. In the context of static insurance contracts, the following example shows for a particular choice of utility functions the region of the parameter values where low-risk agents are partially insured.

Example 1. In order to get an idea of the range of parameter values that yields partial insurance to the low-risk agents we calculated $e_L^S(q_H)$ for $e_H = 0.9$ and $q_L = 0.1$. Figure 2 shows the functions $e_L^S(q_H)$ for two different utility functions: $u_1(m) = \ln m$ and $u_2(m) = \sqrt{m} - 1$. Below the curves, the expenditure of low-risk agents is too low to give them any insurance in equilibrium. //

4. Conditional Dynamic Contracts

We next study the properties and existence conditions of competitive Nash equilibria in a setting where insurance companies can offer conditional dynamic contracts. As explained in the Introduction, insurance conditions in this case may depend on the time period *and* on the accidental history of insured agents, as is the case with car insurances. Although insurance companies are not allowed to transfer profits between different periods, they may "transfer profits" from one accidental history to another, i.e., competition between insurance companies results in a zero-profit condition of the form

$$\sum_{h_t \in H_t} \Pr_i(h_t)\left(P_t^{h_t} - q_L\left(e_L - D_t^{h_t}\right)\right) = 0 \,,\; t = 1, \ldots, T \,.$$

This means that even though insurance companies know that only a certain type i of agents may decide to take a certain insurance contract, they may nevertheless find it optimal to distinguish between agents who (by pure chance) have a different accidental history. As we will see, they may do so in order to better screen high and low-risk agents.

The proposition below states the main result for conditional dynamic contracts. Wherever competitive Nash equilibria in this setting exist, they yield a Pareto-improvement over the static equilibrium contracts: high-risk agents also get full insurance in every period independent of their accidental history and low-risk agents get (at most) partial insurance in every period and the insurance premium they pay is lower, the better their accidental history. These equilibria exist wherever the fraction is small enough so that no company wants to deviate by offering a pooling contract. Finally, when the utility level associated with very low income levels falls dramatically, formally when $\lim_{m \to 0} u(m) = -\infty$, then when the time horizon is very large, it is possible to offer lower-risk agents almost full insurance even if the fraction of low-risk agents is high.

Proposition 2. For any T there exists an $\alpha_C^D(T) \in (0,1)$ such that

a) For all $\alpha \in \left(0, \alpha_C^D\right)$ there exist a separating competitive Nash equilibrium $\Psi_T = \left\{\Theta_H^D, \Theta_L^D\right\}$. The contract Θ_H^D is unique and coincides with Θ_H^S, Θ_L^D generally need not to be unique but all multiple contracts Θ_L^D yield the same utility.

b) For all $\alpha \in \left(\alpha_C^D, 1\right)$ a separating competitive Nash equilibrium Ψ_T does not exist.

c) If $\alpha < \min\left\{\alpha^S, \alpha_C^D\right\}$ then $U_L^D\left(\Theta_L^D\right) > U_L^S\left(\Theta_L^S\right)$.

d) For any given time period t, if low-risk agents get positive insurance then the optimal premium and deductible given a history of k accidents, $P_{L,t}^k$ and $D_{L,t}^k$, satisfy the following relation: $P_{L,t}^k = P_{L,t}^{k-1} + D_{L,t}^{k-1}$.

e) If $\lim_{m \to 0} u(m) = -\infty$ then $\lim_{T \to \infty} \alpha_C^D(T) = 1$ and for any fixed $\alpha \in (0,1)$ $\lim_{T \to \infty} U_L^D\left(\Theta_L^D\right) = u(1 - q_L e)$.

The proof of Proposition 2 is in the appendix. The result can be understood along the following lines. For any difference between q_L and q_H, it is more likely that high-risk agents get an accident than low-risk agents do. Accordingly, the terms of the insurance contract after a few accidents gets a relatively smaller weight in the overall evaluation of the insurance contract by a low-risk agent than by a high-risk agent. Hence, it is possible to design a dynamic contract that low-risk agents prefer to the best static contract insurance companies can offer, while high-risk agents still prefer the full insurance contract. Those contracts give worse insurance conditions to agents that (just by chance) have had many accidents (see property (d)). In expected terms, as the probability of an accident is constant over time, insurance companies make losses over those agents with better accidental histories, while they gain (in expected terms) on those agents with worse histories.

Even though low-risk agents get a higher utility in the separating equilibrium under conditional dynamic contracts than under static contracts, it is not guaranteed that this type of equilibrium exists for a wider range of parameter values of α. The reason is that low-risk agents also get a higher utility under possible conditional dynamic pooling contracts than under possible static pooling contracts.

Finally, the result that when the number of periods is large, low-risk agents can get a conditional dynamic contract that gives them a utility level almost equal to the utility of full insurance, is based on the following considerations. For any $T > 1$ there exist a contract $\hat{\Theta}_L^D$ with full insurance at a premium $q_L e_L$ in all periods $t = 1,\ldots,T-1$, and full insurance at a relatively high premium P^+ if the history was one with only accidents and a relatively lower premium P^- otherwise in the last period. The premiums P^+ and P^- at time period T are constructed such that the zero-profit condition holds as well as the incentive compatibility constraint, i.e., high-risk agents prefer to take the full insurance contract at a premium $q_H e_H$ in all periods. The proof shows that when T becomes large, P^- approaches $q_L e_L$, while P^+ approaches 1, the full income level, in such a way that, because of the difference between q_L and q_H, the expected utility of this event for the low-risk agent approaches 0, while it remains sufficiently negative for high-risk agents. As contract $\hat{\Theta}_L^D$ need not be the equilibrium contract, expected utility under the equilibrium contract is even higher. Note that the result does not depend on agents not discounting future utility.

It is important to understand the role of commitment on the part of the insurance companies. If they had not been committed to the contract, a company after one or more periods could have offered better insurance conditions to those low-risk agents that had been unlucky enough to get many accidents, e.g., they could have offered to start almost the same contract from the beginning (as if there had been no bad accidental history) and made a profit as high-risk agents had not been attracted *at that moment*. However, if high-risk agents anticipate this behavior on the part of the insurance companies, then they would not opt for the full insurance contract designed for them.

In this model, we have implicitly assumed that insurance companies *and* agents are committed to the contracts they have signed and that if an agent switches to another insurance company she will not start the contract from the beginning. In some markets, like the one for car insurance, commitment on the part of insurance companies is achieved as insurance companies share information about accidental history of their clients. It is more difficult, however, to ensure that agents are committed to the contract.

If this form of commitment is not achievable, additional constraints have to be imposed, namely after every history contract terms should be such that it is not possible to design a profitable contract that agents prefer to continuing the existing contract. Moreover, after every history we have to ensure that agents would like to stay insured instead of quitting the insurance market altogether. In a *T*-period world, it is very difficult to satisfy all those constraints.[4] This is another reason why we consider unconditional contracts in the next section.

Before we do so, we provide an estimate of the welfare improvements that are possible for certain specific cases.

Example 1 continued. To determine by how much welfare could be improved by conditional dynamic insurance contracts, we normalize the total low-risk welfare loss due to asymmetric information to be 100% so that the static contract Θ_L^S gets a score of 0% and full insurance under the full information gets a score of 100%. We then calculate the ratio $\frac{U_L^D\left(\hat{\Theta}_L^D\right)-U_L^S\left(\Theta_L^S\right)}{u(1-q_L e_L)-U_L^S\left(\Theta_L^S\right)}$ for fixed $e_H = 0.9$, $q_L = 0.1$ and different values of $q_H \in [q_L, 1)$ and $e_L \in [0, e_H]$, where Θ_L^S is the static

[4] See Cooper and Hayes (1987) for some of the relevant considerations when $T = 2$.

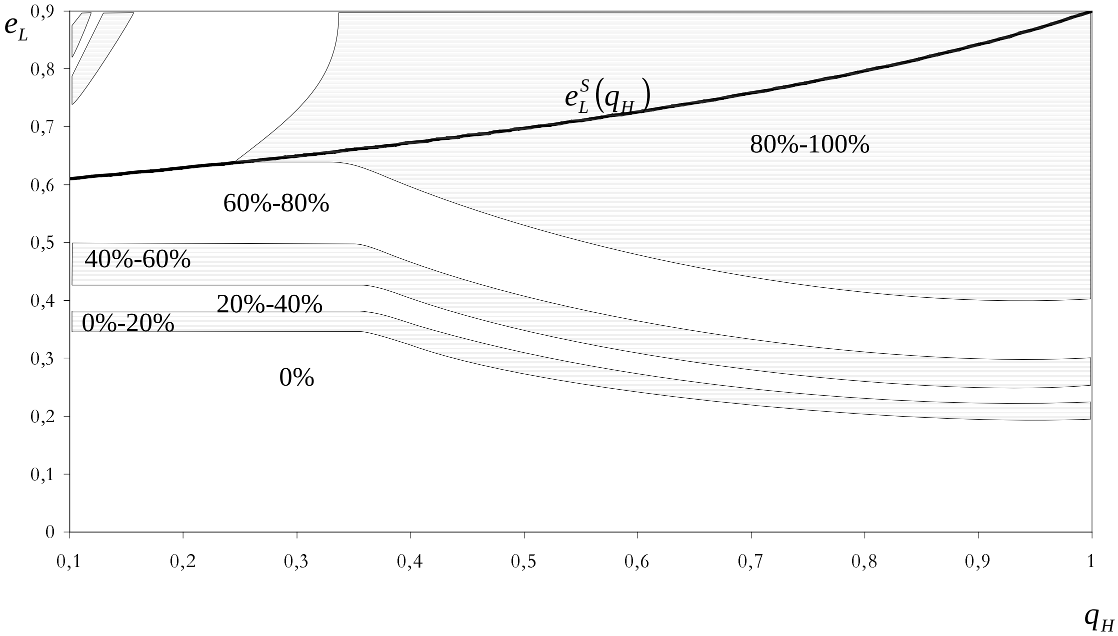

Figure 3. Welfare improvements under conditional contracts $\hat{\Theta}_L^D$, e_H=0.9, q_L=0.1, $u(m)$=ln m, T=4.

equilibrium contract and $\hat{\Theta}_L^D$ is the contract mentioned above. The contract $\hat{\Theta}_L^D$ is chosen, as is it difficult to calculate the equilibrium contract Θ_L^D itself.

Figure 3 and Figure 4 show parameter regions, which yield different levels of the welfare gain for low-risk agents for $u(m) = \ln m$, $\delta = 0.9$ and $T = 4$ and

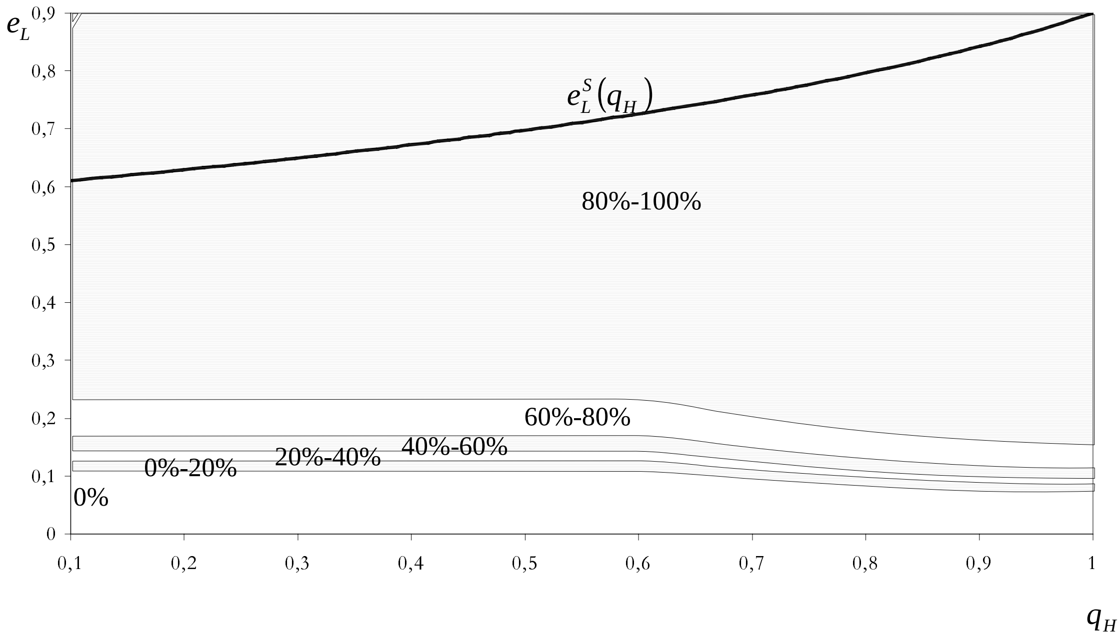

Figure 4. Welfare improvements under conditional contracts $\hat{\Theta}_L^D$, e_H=0.9, q_L=0.1, $u(m)$=ln m, T=5.

$T = 5$ correspondingly. For larger values of T almost all parameter combinations lead to almost 100% welfare improvement.

As the equilibrium conditional dynamic contract Θ_L^D gives an even higher utility level for the low-risk type agents than $\hat{\Theta}_L^D$, the potential welfare improvement and the region where it is possible is even larger than presented. //

5. Unconditional Dynamic Contracts

In the introduction we have explained that in certain markets, like health insurance markets, conditional dynamic contracts may be considered unfair or politically not viable. When, after a sequence of many accidents, car insurance becomes too expensive, a person may always decide not to drive a car anymore. This is not true for health insurance. For this reason we consider in this section whether unconditional dynamic contracts may recoup part of the welfare loss due to adverse selection in the static equilibrium outcome. Another advantage of unconditional contracts is that the commitment problem can be easily avoided here.

It is clear from the outset that wherever they both exist, unconditional contracts yield lower welfare than conditional contracts as the latter include the former. What is not clear from the outset, however, is whether the equilibrium existence conditions are stricter for the case of unconditional contracts. This is because the best unconditional pooling contract for low-risk agents also yields lower utility than the best pooling contract in the ease of conditional contracts. We start the analysis by considering the Rothschild-Stiglitz case in which $e_L = e_H$. In this case, we have a straightforward negative result, which is that the best unconditional contract is the repeated static contract. In other words, welfare gains are not possible using unconditional dynamic contracts in such a world.

Proposition 3. If $e_L = e_H = e$ then for all $\alpha \in (0, \alpha^S)$ there exists a unique separating competitive Nash equilibrium that has the static insurance policies Θ_H^S and Θ_L^S in any time period. For all $\alpha \in (\alpha^S, 1)$ a separating competitive Nash equilibrium does not exist.

Proof. We first show that $\Theta_H^D \equiv (\Theta_{1,H}, \ldots, \Theta_{T,H}) = (\Theta_H^S, \ldots, \Theta_H^S)$. Maximizing $U_H^D(\Theta_H^D) = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \delta^{t-1} U_H^S(\Theta_{t,H})$ with respect to all $D_{t,H} \in [0,e]$ and subject to zero profit condition $P_{t,H} = q_H(e - D_{t,H})$ yields $D_{t,H} = 0$ and $P_{t,H} = q_H e$ for all $t = 1, \ldots, T$, hence,

$\Theta_{t,H} = \Theta_H^S$ and $U_H^D\left(\Theta_H^D\right) = U_H^S\left(\Theta_H^S\right)$. On the other hand, maximizing $U_L^D\left(\Theta_L^D\right) = \frac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\delta^{t-1}U_L^S\left(\Theta_{t,L}\right)$ with respect to all $D_{L,H} \in [0,e]$ and subject to $P_{t,L} = q_L\left(e - D_{t,L}\right)$ and the incentive compatibility constraint $U_H^D\left(\Theta_L^D\right) \le U_H^S\left(\Theta_H^S\right)$ yields the following Lagrangian:

$$
\begin{aligned}
L = &\tfrac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\delta^{t-1}\left(q_L u\left(1 - P_{t,L} - D_{t,L}\right) + \left(1 - q_L\right)u\left(1 - P_{t,L}\right)\right) + \\
&+ \lambda\left(U_H^S\left(\Theta_H^S\right) - \tfrac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\delta^{t-1}\left(q_H u\left(1 - P_{t,L} - D_{t,L}\right) + \left(1 - q_H\right)u\left(1 - P_{t,L}\right)\right)\right),
\end{aligned}
$$

and the first order conditions for $t = 1,\ldots,T$ are:

$$
q_L\left(1 - q_L\right)\left(u'\left(1 - P_{t,L} - D_{t,L}\right) - u'\left(1 - P_{t,L}\right)\right) = \lambda\left(q_H\left(1 - q_L\right)u'\left(1 - P_{t,L} - D_{t,L}\right) - \left(1 - q_H\right)q_L u'\left(1 - P_{t,L}\right)\right).
$$

It immediately follows that the constraint is binding and the first order conditions become:

$$
\lambda = \frac{q_L\left(1 - q_L\right)u'\left(1 - P_{t,L} - D_{t,L}\right) - q_L\left(1 - q_L\right)u'\left(1 - P_{t,L}\right)}{q_H\left(1 - q_L\right)u'\left(1 - P_{t,L} - D_{t,L}\right) - q_L\left(1 - q_H\right)u'\left(1 - P_{t,L}\right)} \equiv \varphi\left(D_{t,L}\right),\ t = 1,\ldots,T\,.
$$

As $\varphi'\left(D_{t,L}\right) > 0$ for all $D_{t,L} \ge 0$, all $D_{t,L}$ have to be equal to each other, i.e., $D_{t,L} = D_{1,L}$ for all t and, therefore, Θ_L^D is just a repetition of a static contract. But we know that the best contract for the low-risk type is Θ_L^S. Finally, Θ_L^S exists if and only if $\alpha \in \left(0, \alpha^S\right)$. ■

It is interesting to better understand the reason for this result. A first reason is that we require the zero-profit condition to hold in every period. This together with the fact that the utility function is time-separable yields a set of first-order conditions, which are the same for every period. The second reason is the concavity of the utility function, which makes sure that less-risky outcomes with the same expected expenditures are preferred to more risky outcomes.

This result also sheds another light on the positive result obtained for conditional dynamic contracts. There are two important differences between the two settings when $e_L = e_H$. First, with conditional contracts insurance companies are able to shift profits between different accidental histories for every given time period. Second, even if expected profits are zero after every history, insurance companies may give agents with better histories contracts with more insurance (lower deductible and higher premium). In these two ways insurance companies are able to relax the static incentives compatibility constraint from the perspective of the low-risk agents. As the insurance company is risk-neutral, it is indifferent between (i) a contract giving constant insurance conditions with zero expected profit in each state, (ii) a contract making zero expected profits in each state (at different terms) or (iii) a contract making zero expected profits in every period (but not in every state). To the contrary, the low-risk agents make a distinction between these cases.

Another point in the above intuitive explanation of Proposition 3 is that we have not considered corner solutions of type $D_{t,L} = e$ where no insurance is offered in certain periods. In the case where $e_L = e_H$ this is also not really necessary as both types of agents have the same evaluation (utility) of no insurance: $u(1-e_H) = u(1-e_L)$. When $e_L < e_H$, this is no longer the case and we may use "no insurance in certain periods" (a probationary period) as a way to screen agents in order to reach welfare improvements. The next proposition summarizes our results for this case.

Proposition 4. There exists an $\alpha^D \in [\alpha^S, 1)$ and $e_L^D \in (0, e_H)$ such that:

a) For all $\alpha \in (0, \alpha^D)$ there exists a T^* such that for all $T > T^*$ there exists a separating competitive Nash equilibrium $\Psi_T = \{\Theta_H^D, \Theta_L^D\}$. High-risk agents get full insurance, i.e., $\Theta_H^D = (\Theta_H^S, \ldots, \Theta_H^S)$. The low-risk agents get a contract Θ_L^D such that

$$\Theta_{L,t}^D = \begin{cases} \Theta_{L,t}^D = \Theta_0^S, \text{for } t \in N_{sep} \\ \Theta_{L,t}^D = \Theta_L = (P_L, D_L), \text{for } t \in N_T \setminus N_{sep} \end{cases},$$

where N_{sep} is the separation phase of the contract. If $e_L^D \in [e_L^D, e_H]$ then $N_{sep} = \varnothing$, $D_L = D_L^S$ and $P_L = P_L^S$, i.e., low-risk agents get static insurance $\Theta_L^D = (\Theta_L^S, \cdot\cdot\cdot, \Theta_L^S)$. If, on the other hand, $e_L^D \in (0, e_L^D)$ then $N_{sep} \neq \varnothing$, $0 \le D_L < D_L^S$ and $P_L > P_L^S$. In this case $U_L^D(\Theta_L^D) > U_L^D(\Theta_L^S)$.

b) For any $\alpha \in (\alpha^D, 1)$ a separating competitive Nash equilibrium does not exist.

c) For all $q_H \in (q_L, 1)$ $e_L^D(q_H) \in (e_L^S(q_H), e_H)$ and $e_L^D(q_L) = \lim_{q_H \to 1} e_L^D(q_H) = e_H$.

The proof of Proposition 4 is in the appendix. Proposition 4 tells us that the results of Proposition 3 are robust only in a (possibly small) neighborhood of $e_L = e_H$, i.e., when e_L is close enough to e_H. When e_L falls outside this neighborhood, i.e., when $e_L < e_L^D$, then a Pareto-improvement is possible vis-à-vis the static outcome. The best screening contract for low-risk agents involves a "separation phase" with no insurance and an "insurance phase" with better (and constant) insurance conditions than in the static contract.[5] The range of fractions of low-risk agents in the population for which such a separating equilibrium exists is also larger than in the case of a static separating equilibrium. Finally, we are able to show that the neighborhood around e_H for which no welfare improvements vis-à-vis the static equilibrium are possible becomes very small when q_H is close to q_L or close to 1 (part (c) of Proposition 4).

In order to better understand the reason for the "large *T* assumption", we have to explain a part of the more formal proof given in the Appendix. In the proof we write the overall utility level low-risk agents get as a convex combination of the utility in the separation and the insurance phase. The weights are expressed in terms of the discount factor *δ*, the number of insurance periods *T* and the set of time periods in the separation phase N_{sep} . We show that in order to have a welfare improving contracts that satisfies the incentive compatibility constraint this weight has to be in a certain interval. As *T* is a finite number, the weights can only take on a finite number of values. For any relatively small value of *T*, it may happen that by none of the possible choices for the length of the separation phase the weight of utility function falls in the required interval. When *T* is large enough but still finite, this is no longer the case. In summary, the requirement that *T* be large enough has to do with the assumption that time is measured discretely, rather than that we need the contract to last for a very long period of time.

We next show by means of an example for which region of parameter values welfare can be improved and by how much it can be improved.

Example 1 continued. Example 1 showed parameter regions where low-risk agents will (not) have some insurance contract under the static equilibrium. Figure 5 shows the functions $e_L^D(q_H)$ for the same two utility functions as studied in Example 1 for the limit case when $T \to \infty$ and $\delta \geq \frac{1}{2}$. For all the parameter values below the graph dynamic contracts allow for welfare improvements. Parameter values above the graph are such that the dynamic and static equilibrium contracts coincide, hence welfare improvement is not possible. One

[5] We implicitly assume that low-risk agents have to register with an insurance company even though they don't get any insurance in the initial separation phase, i.e., before they are able to get to the good insurance phase they already have to be known to the insurance company. At the same time, they can not buy an insurance from another company. This is possible when insurance companies share information, a practice that is common, for example, in the car insurance market.

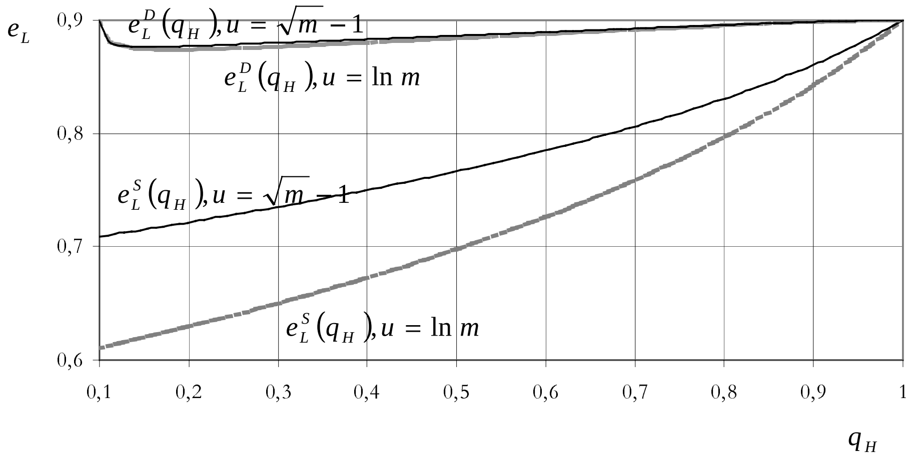

Figure 5. Region of parameter values where welfare can be improved using unconditional dynamic contracts.

can see that for the given utility functions the regions where welfare improvements are possible are quite large. In particular, e_L^D is quite close to e_H.

To determine by how much welfare could be improved by dynamic insurance contracts, we follow the same procedure as in the example of Section 4. Figure 6 and Figure 7 show parameter regions, which yield different levels of the welfare gain for low-risk agents, for $u(m) = \ln m$ and $u(m) = \sqrt{m} - 1$, respectively.

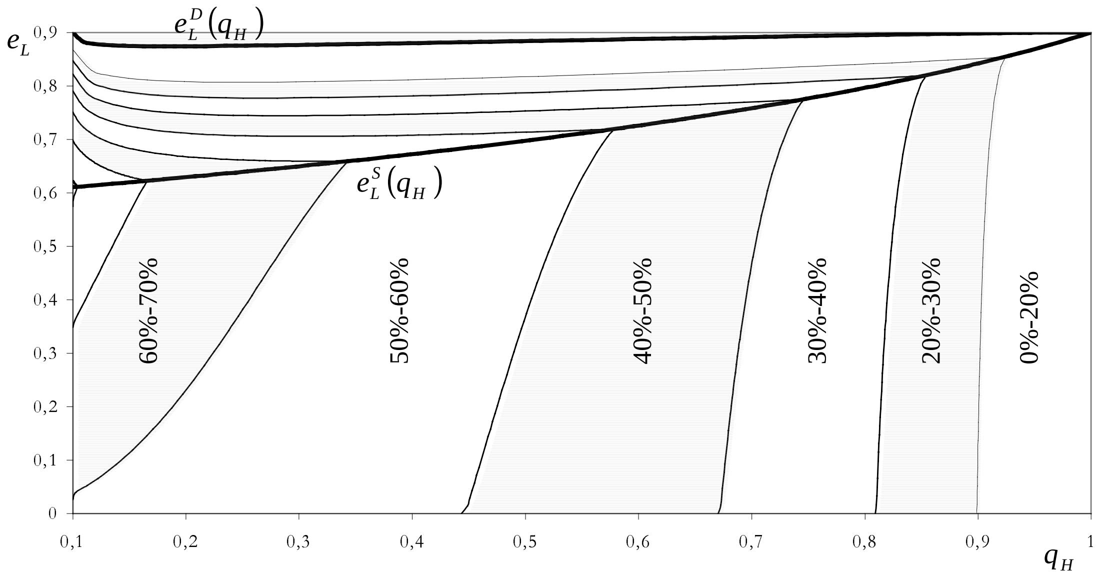

Figure 6. Welfare improvements under unconditional contract Θ_L^D, e_H=0.9, q_L=0.1, $u(m)$=ln m, $T \to \infty$ and $\delta \geq \frac{1}{2}$.

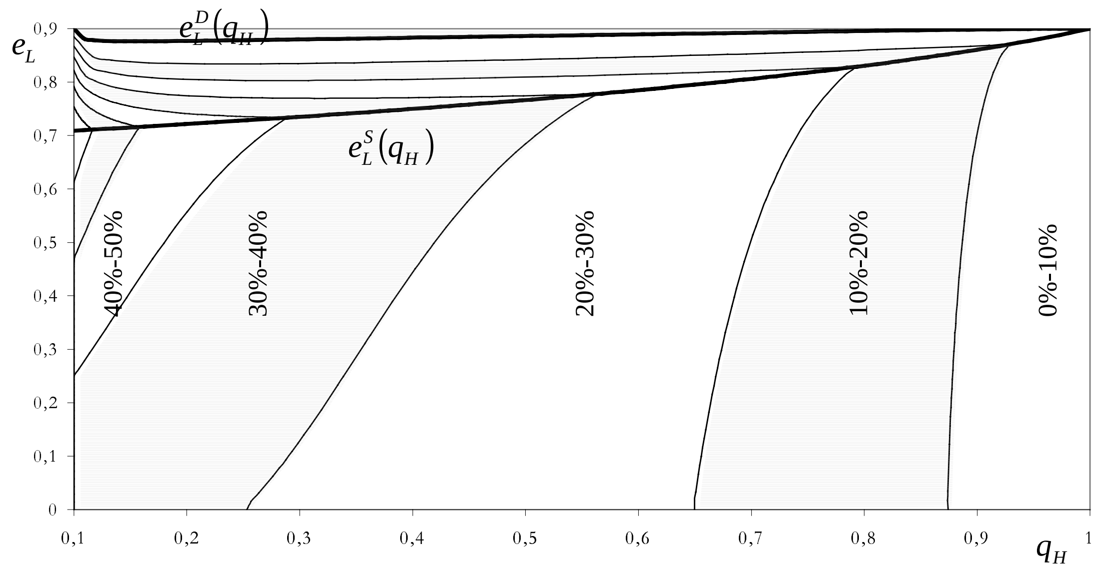

Figure 7. Welfare improvements under unconditional contract Θ_L^D, e_H=0.9, q_L=0.1, $u(m)=\sqrt{m}-1$, $T \to \infty$ and $\delta \geq \frac{1}{2}$.

One can see that if $e_L < e_L^S$ and low-risk type gets no insurance in a static equilibrium, then the welfare improvement dynamic insurance yields is very sensitive to q_H while if $e_L > e_L^S$ then the difference $e_H - e_L$ plays a crucial role. //

6. Discussion and Conclusion

In this paper we studied a generalization of the Rothschild and Stiglitz model of a competitive insurance market affected by adverse selection. We allowed agents to have different expenditures and investigated the nature of dynamic contracts. We showed that in the multi-period dynamic model a competitive Nash equilibrium exists as long as the share of low-risk agents is sufficiently small. If such an equilibrium exists, it is Pareto-superior to the static equilibrium if *conditional* contracts are allowed.

When contracts are unconditional, welfare improvements are only possible if expenditures of the two groups are different. If this is so, these equilibria exist for a larger fraction of low-risk agents than static equilibria. The optimal contract has a separation phase offering no insurance and insurance phase offering much better insurance conditions.

Both conditional and unconditional dynamic contracts have been derived under the assumption that they yield zero profit in *every period* and that agents are not allowed to shift wealth between periods. Here we discuss at a more informal level how these assumptions can be relaxed.

We begin with the discussion of unconditional contracts. It is not difficult to see that as before high-risk agents will get a purely static full insurance contract in equilibrium. The equilibrium low-risk contract cannot be obtained explicitly, however. What can easily be shown is that an optimal contract that is Pareto-superior to the static equilibrium contract exists. When the static equilibrium contract gives no insurance to low-risk agents, i.e., $e_L \le e_L^S(q_H)$, then, like in the basic model, an insurer is able to separate the types by offering a dynamic contract with a sufficiently long separation phase. Indeed, when the length of the separating phase increases the incentive compatibility constraint can be easily satisfied. On the other hand, as the dynamic contract is not worse than the static contract Θ_L^S during the whole term and is strictly better in the insurance phase, the contract is Pareto-superior as well.

Hence, the set of Pareto-superior separating contracts is not empty and the equilibrium dynamic contract is the one that maximizes the utility of low-risk agents. The existence of such a contract is guaranteed by the continuity of both the objective function and the incentive compatibility constraint and by the compactness of the feasible parameter set.

Therefore, for all parameter combinations, which lie below the curve $e_L = e_L^S(q_H)$ in Figure 2, Pareto improvement by means of dynamic insurance is always possible. In the example below we calculate the highest low-risk expenditure $\hat{e}_L^D$ such that the welfare improvement is possible in the whole interval $\left(0, \hat{e}_L^S\right)$. Thus, the difference between $\hat{e}_L^D$ and e_L^D reflects the sensitivity of the model with respect to savings.

Example 1 continued. Apart from the functions $e_L^D(q_H)$ that were already presented in Figure 2 for two specific utility functions, Figure 8 presents functions $\hat{e}_L^D(q_H)$ for the same example. For all the model's parameters below the graphs $\hat{e}_L^D(q_H)$ dynamic contracts allow welfare improvements to be made.

The figure shows that savings do not change the outcome significantly and just change the set of parameters allowing for welfare improvement a little bit. //

Given this result for unconditional contracts, we will be brief about conditional contracts. As dynamic conditional contracts yield weakly higher utility for the low-risk agents the region of possible welfare improvement is even wider. But, again, the pooling low-risk utility maximizing conditional dynamic contract provides higher utility than the

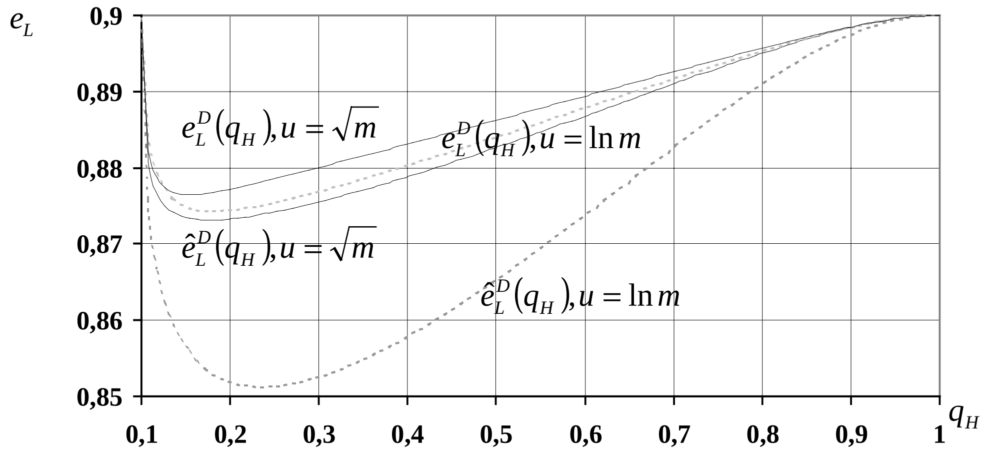

Figure 8. Region of parameter values where welfare can be improved by unconditional contracts with and without savings.

static pooling contracts. Hence, it is not guaranteed that this type of equilibrium exists for a wider range of parameter values of α.

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Appendix

Proof of Proposition 2. We begin by deriving a set of competitive contracts $\{\Theta_H^D, \Theta_L^D\}$ satisfying the incentives compatibility constraints and maximizing $U_L^D(\Theta_L^D)$. Then we derive a competitive pooling contract $\hat{\Theta}_{P,t}$ maximizing the low-risk utility. Finally, we show that there exist an $\alpha_C^D \in (0,1)$ such that for all $\alpha < \alpha_C^D$ ($\alpha > \alpha_C^D$) $\hat{\Theta}_{P,t}$ gives a lower (higher) utility for the low-risk type than Θ_L^D.

Contract $\Theta_H^D = (\Theta_{H,1}, \ldots, \Theta_{H,T})$ with $\Theta_{H,t} = \{\Theta_{H,t}^{h_t}\}_{h_t \in H_t}$ and $\Theta_{H,t}^{h_t} = (P_{H,t}^{h_t}, D_{H,t}^{h_t})$, maximizes $U_H^D(\Theta^D)$, which is

$$U_H^D(\Theta^D) = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^T \left(\delta^{t-1} \sum_{h_t \in H_t} \Pr_H(h_t) U_H^S(\Theta_t^{h_t}) \right),$$

subject to zero profit constraints

$$\sum_{h_t \in H_t} \Pr_H(h_t)\left(P_t^{h_t} - q_H\left(e_H - D_t^{h_t}\right)\right) = 0, \; t = 1, \ldots, T.$$

The Lagrange function and the first order conditions for the interior solution are:

$$L = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^T \left(\delta^{t-1} \sum_{h_t \in H_t} \Pr_H(h_t)\left(q_H u\left(1 - P_t^{h_t} - D_t^{h_t}\right) + (1-q_H) u\left(1 - P_t^{h_t}\right)\right) \right) + \\ + \sum_{t=1}^T \left(\lambda_t \sum_{h_t \in H_t} \Pr_H(h_t)\left(P_t^{h_t} - q_H\left(e_H - D_t^{h_t}\right)\right) \right),$$

$$\begin{cases} \dfrac{\partial L}{\partial P_t^{h_t}} = -\dfrac{1-\delta}{1-\delta^T} \delta^{t-1} \Pr_H(h_t)\left(q_H u'\left(1 - P_{H,t}^{h_t} - D_{H,t}^{h_t}\right) + (1-q_H) u'\left(1 - P_{H,t}^{h_t}\right)\right) + \lambda_t \Pr_H(h_t) = 0 \\ \dfrac{\partial L}{\partial D_t^{h_t}} = -\dfrac{1-\delta}{1-\delta^T} \delta^{t-1} \Pr_H(h_t) q_H u'\left(1 - P_{H,t}^{h_t} - D_{H,t}^{h_t}\right) + \lambda_t \Pr_H(h_t) q_H = 0 \end{cases}.$$

Solving them together with zero profit conditions yields $D_{H,t}^{h_t} = 0$ and $P_{H,t}^{h_t} = q_H e_H$ for all t and h_t, in other words, high-risk agents always get full insurance in a separating equilibrium, $\Theta_H^D = (\Theta_H^S, \ldots, \Theta_H^S)$. This solution is unique due to the global concavity of the objective function and we do not need to look at corner solutions with some $\Theta_t^{h_t} = \Theta_0^S$.

Contract $\Theta_L^D = (\Theta_{L,1}, \ldots, \Theta_{L,T})$ with $\Theta_{L,t} = \{\Theta_{L,t}^{h_t}\}_{h_t \in H_t}$ and $\Theta_{L,t}^{h_t} = (P_{L,t}^{h_t}, D_{L,t}^{h_t})$, maximizes $U_L^D(\Theta^D)$, which is

$$U_L^D\left(\Theta^D\right)=\frac{1-\delta}{1-\delta^T}\sum\nolimits_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t\in H_t}\Pr_L\left(h_t\right)U_L^S\left(\Theta_t^{h_t}\right)\right),$$

subject to zero profit condition $\sum_{h_t\in H_t}\Pr_L\left(h_t\right)\left(P_t^{h_t}-q_L\left(e_L-D_t^{h_t}\right)\right)=0$, $t=1,\ldots,T$ and incentives compatibility constraint $U_H^D\left(\Theta^D\right)\le U_H^D\left(\Theta_H^D\right)=u\left(1-q_He_H\right)$.

The Lagrange function for this problem is

$$L=\frac{1-\delta}{1-\delta^T}\sum\nolimits_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t\in H_t}\Pr_L\left(h_t\right)\left(q_Lu\left(1-P_t^{h_t}-D_t^{h_t}\right)+\left(1-q_L\right)u\left(1-P_t^{h_t}\right)\right)\right)+$$
$$+\sum\nolimits_{t=1}^{T}\left(\lambda_t\sum_{h_t\in H_t}\Pr_L\left(h_t\right)\left(P_t^{h_t}-q_L\left(e_L-D_t^{h_t}\right)\right)\right)+$$
$$+\mu\left(U_H^D\left(\Theta_H^D\right)-\frac{1-\delta}{1-\delta^T}\sum\nolimits_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t\in H_t}\Pr_H\left(h_t\right)\left(q_Hu\left(1-P_t^{h_t}-D_t^{h_t}\right)+\left(1-q_H\right)u\left(1-P_t^{h_t}\right)\right)\right)\right)$$

The first order conditions for an interior solution are:

$$\begin{cases}\dfrac{\partial L}{\partial P_t^{h_t}}=-\dfrac{1-\delta}{1-\delta^T}\delta^{t-1}\Pr_L\left(h_t\right)\left(q_Lu'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)+\left(1-q_L\right)u'\left(1-P_{L,t}^{h_t}\right)\right)+\\ \qquad+\lambda_t\Pr_L\left(h_t\right)+\mu\dfrac{1-\delta}{1-\delta^T}\delta^{t-1}\Pr_H\left(h_t\right)\left(q_Hu'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)+\left(1-q_H\right)u'\left(1-P_{L,t}^{h_t}\right)\right)=0\\ \dfrac{\partial L}{\partial D_t^{h_t}}=-\dfrac{1-\delta}{1-\delta^T}\delta^{t-1}\Pr_L\left(h_t\right)q_Lu'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)+\\ \qquad+\lambda_t\Pr_L\left(h_t\right)q_L+\mu\dfrac{1-\delta}{1-\delta^T}\delta^{t-1}\Pr_H\left(h_t\right)q_Hu'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)=0\end{cases},$$

which can be rewritten as follows

$$\begin{cases}\mu=\dfrac{\Pr_L\left(h_t\right)}{\Pr_H\left(h_t\right)}\dfrac{q_L\left(1-q_L\right)\left(u'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)-u'\left(1-P_{L,t}^{h_t}\right)\right)}{q_H\left(1-q_L\right)u'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)-q_L\left(1-q_H\right)u'\left(1-P_{L,t}^{h_t}\right)}\\ \lambda_t=\dfrac{1-\delta}{1-\delta^T}\delta^{t-1}\dfrac{\left(q_H-q_L\right)u'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)u'\left(1-P_{L,t}^{h_t}\right)}{q_H\left(1-q_L\right)u'\left(1-P_{L,t}^{h_t}-D_{L,t}^{h_t}\right)-q_L\left(1-q_H\right)u'\left(1-P_{L,t}^{h_t}\right)}\end{cases}.$$

Then, it follows that the incentive compatibility constraint is binding, otherwise it would have been $\mu=0$, $D_{L,t}^{h_t}=0$ for all t and h_t, and finally, $P_{L,t}^{h_t}=q_Le_L$ that yields $U_H^D\left(\Theta_L^D\right)>U_H^D\left(\Theta_H^D\right)$, a contradiction. Hence, $D_{L,t}^{h_t}>0$ for all t and h_t.

One may note here that all $P_{L,t}^{h_t}$ and $D_{L,t}^{h_t}$ depend only on $\Pr_i\left(h_t\right)$ but not on h_t itself. Hence, if, for instance, $t=4$ then $h_4'=\left(0,0,1\right)$ and $h_4''=\left(0,1,0\right)$ correspond to different states of the world but $\Pr_i\left(h_4'\right)=q_i\left(1-q_i\right)^2=\Pr_i\left(h_4''\right)$ and, therefore, $P_{L,4}^{h_4'}=P_{L,4}^{h_4''}$ and $D_{L,4}^{h_4'}=D_{L,4}^{h_4''}$, that allows us to change notations: by $\hat{h}_t=k$ we will denote all states of the world where there were exactly k accidents in time periods from 1 up to $t-1$. Then it follows that $\Pr_i(k) = C_{t-1}^k {q_i}^k (1-q_i)^{t-k-1}$, where $C_{t-1}^k = \frac{k!(t-1-k)!}{(t-1)!}$ are binomial coefficients. Plugging it into the above system yields

$$\begin{cases} \mu = \dfrac{{q_L}^k (1-q_L)^{t-k-1}}{{q_H}^k (1-q_H)^{t-k-1}} \dfrac{q_L(1-q_L)\left(u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - u'\left(1-P_{L,t}^k\right)\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)} \\ \lambda_t = \dfrac{1-\delta}{1-\delta^T}\delta^{t-1} \dfrac{(q_H - q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right)u'\left(1-P_{L,t}^k\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)} \end{cases}.$$

Getting rid of λ_t and μ leads to

$$\begin{cases} \dfrac{{q_L}^k (1-q_L)^{t-k-1}}{{q_H}^k (1-q_H)^{t-k-1}} \dfrac{u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - u'\left(1-P_{L,t}^k\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)} = \\ \qquad = \dfrac{{q_L}^{k-1} (1-q_L)^{t-k}}{{q_H}^{k-1} (1-q_H)^{t-k}} \dfrac{u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) - u'\left(1-P_{L,t}^{k-1}\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) - q_L(1-q_H)u'\left(1-P_{L,t}^{k-1}\right)} \\ \dfrac{u'\left(1-P_{L,t}^k - D_{L,t}^k\right)u'\left(1-P_{L,t}^k\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)} = \\ \qquad = \dfrac{u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right)u'\left(1-P_{L,t}^{k-1}\right)}{q_H(1-q_L)u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) - q_L(1-q_H)u'\left(1-P_{L,t}^{k-1}\right)} \end{cases}.$$

Both equations can be written as

$$\begin{cases} \left(q_H(1-q_L)u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) - q_L(1-q_H)u'\left(1-P_{L,t}^{k-1}\right)\right)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) = \\ \qquad = \left(q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)\right)u'\left(1-P_{L,t}^{k-1}\right) \\ \left(q_H(1-q_L)u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) - q_L(1-q_H)u'\left(1-P_{L,t}^{k-1}\right)\right)u'\left(1-P_{L,t}^k - D_{L,t}^k\right)u'\left(1-P_{L,t}^k\right) = \\ \qquad = \left(q_H(1-q_L)u'\left(1-P_{L,t}^k - D_{L,t}^k\right) - q_L(1-q_H)u'\left(1-P_{L,t}^k\right)\right)u'\left(1-P_{L,t}^{k-1}\right)u'\left(1-P_{L,t}^{k-1} - D_{L,t}^{k-1}\right) \end{cases}.$$

Dividing the second equation by the first one we obtain $P_{L,t}^k = P_{L,t}^{k-1} + D_{L,t}^{k-1}$, which together with the zero profit condition yields

$$\begin{cases} P_{L,t}^0 = q_L e_L - \sum_{k=0}^{t-1} \Pr_L(k) \left(\sum_{i=0}^{k-1} D_{L,t}^i + q_L D_{L,t}^k \right), \\ P_{L,t}^k = P_{L,t}^{k-1} + D_{L,t}^{k-1} \end{cases}$$

that defines $P_{L,t}^k$ (hence, $P_{L,t}^{h_t}$) in terms of $D_{L,t}^k$. The profit an insurer gets at time t from a low-risk agent with a history k is

$$\begin{aligned} \pi_{L,t}^k &= P_{L,t}^k - q_L\left(e_L - D_{L,t}^k\right) = P_{L,t}^{k-1} + D_{L,t}^{k-1} - q_L\left(e_L - D_{L,t}^k\right) = \\ &= P_{L,t}^{k-1} - q_L\left(e_L - D_{L,t}^{k-1}\right) + (1-q_L)D_{L,t}^{k-1} + q_L D_{L,t}^k = \pi_{L,t}^{k-1} + (1-q_L)D_{L,t}^{k-1} + q_L D_{L,t}^k, \end{aligned}$$

hence, $\pi_{L,t}^{t-1} > \ldots > \pi_{L,t}^{k} > \pi_{L,t}^{k-1} > \ldots > \pi_{L,t}^{0}$. Therefore, in accordance with the zero profit condition $\sum_{k=1}^{t-1} \Pr_L(k)\pi_{L,t}^{k} = 0$, $\pi_{L,t}^{t-1} > 0$ and $\pi_{L,t}^{0} < 0$, in other words, an insurer makes losses over those agents with better accidental histories, while they gain (in expected terms) on those agents with worse histories.

It might happen that the solution $\Theta_L^D = (\Theta_{L,1}, \ldots, \Theta_{L,T})$ is just a local but not global maximum. Hence, we may have to find all corner solutions imposing $\Theta_{L,t}^{h_t} = \Theta_0^S$ for some set of states of the world $H_t^0 \subset H_t$. The Lagrange function in this case becomes

$$L^{H_t^0} = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \left(\delta^{t-1} \sum_{h_t \in H_t \setminus H_t^0} \Pr_L(h_t)\left(q_L u\left(1 - P_t^{h_t} - D_t^{h_t}\right) + (1-q_L) u\left(1 - P_t^{h_t}\right)\right) \right) +$$

$$+ \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \left(\delta^{t-1} \sum_{h_t \in H_t^0} \Pr_L(h_t) q_L u(1 - e_L) \right) + \sum_{t=1}^{T} \left(\lambda_t \sum_{h_t \in \setminus H_t^0} \Pr_L(h_t)\left(P_t^{h_t} - q_L\left(e_L - D_t^{h_t}\right)\right) \right) +$$

$$+ \mu \left(U_H^D(\Theta_H^D) - \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \left(\delta^{t-1} \sum_{h_t \in H_t \setminus H_t^0} \Pr_H(h_t)\left(q_H u\left(1 - P_t^{h_t} - D_t^{h_t}\right) + (1-q_H) u\left(1 - P_t^{h_t}\right)\right) \right) \right) -$$

$$- \mu \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \left(\delta^{t-1} \sum_{h_t \in H_t} \Pr_H(h_t) q_H u(1 - e_H) \right).$$

Consequently, the only difference from the previous analysis here is that the first order conditions will involve the summation over the subset $h_t \in H_t \setminus H_t^0$ instead of the whole set $h_t \in H_t$. Having been calculated for all $H_t^0 \subset H_t$ a contract Θ_L^D is chosen in such a way that, first, it satisfies $D_{L,t}^{h_t} < e_L$ for all $h_t \in H_t \setminus H_t^0$, and, second, it maximizes $U_L^D(\Theta_L^D)$.

Thus, we have described a set of competitive contracts $\{\Theta_H^D, \Theta_L^D\}$ satisfying the incentive compatibility constraint and maximizing $U_L^D(\Theta_L^D)$. This set becomes a competitive Nash equilibrium if no competitive pooling contract gives a higher utility level for the low-risk agents. We will prove that for small enough values of α this is the case.

The utility low-risk agents get under Θ_L^D does not depend on α while the utility they get under a pooling contract $\Theta_P^D = (\Theta_{P,1}, \ldots \Theta_{P,T})$ with $\Theta_{P,t} = \{\Theta_{P,t}^{h_t}\}_{h_t \in H_t}$ and $\Theta_{P,t}^{h_t} = (P_{P,t}^{h_t}, D_{P,t}^{h_t})$ depends on it. As the low-risk agents' utility

$$U_L^D(\Theta_P^D) = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \left(\delta^{t-1} \sum_{h_t \in H_t} \Pr_L(h_t) U_L^S\left(\Theta_{P,t}^{h_t}\right) \right)$$

is time-separable as well as the zero profit conditions, which are

$$\sum_{h_t\in H_t}\left(\alpha \Pr_L(h_t)\left(P_{P,t}^{h_t}-q_L\left(e_L-D_{P,t}^{h_t}\right)\right)+(1-\alpha)\Pr_H(h_t)\left(P_{P,t}^{h_t}-q_H\left(e_H-D_{P,t}^{h_t}\right)\right)\right)=0\,,\ t=1,\dots,T\,,$$

maximization of $U_L^D\left(\Theta_P^D\right)$ over $D_{P,t}^{h_t}\in[0,e_L]$ splits into T parts:

$$\max \sum\nolimits_{h_t\in H_t}\Pr_L(h_t)U_L^S\left(\Theta_{P,t}^{h_t}\right),$$

$$\text{s.t.}\ \sum_{h_t\in H_t}\left(\alpha \Pr_L(h_t)\left(P_{P,t}^{h_t}-q_L\left(e_L-D_{P,t}^{h_t}\right)\right)+(1-\alpha)\Pr_H(h_t)\left(P_{P,t}^{h_t}-q_H\left(e_H-D_{P,t}^{h_t}\right)\right)\right)=0$$

and it has a unique solution due to the global concavity of the objective function and linear constraints. The Lagrange function and the first order conditions are:

$$L_t=\sum\nolimits_{h_t\in H_t}\Pr_L(h_t)\left(q_L u\left(1-P_{P,t}^{h_t}-D_{P,t}^{h_t}\right)+(1-q_L)u\left(1-P_{P,t}^{h_t}\right)\right)+$$
$$+\lambda_t\sum_{h_t\in H_t}\left(\alpha \Pr_L(h_t)\left(P_{P,t}^{h_t}-q_L\left(e_L-D_{P,t}^{h_t}\right)\right)+(1-\alpha)\Pr_H(h_t)\left(P_{P,t}^{h_t}-q_H\left(e_H-D_{P,t}^{h_t}\right)\right)\right),$$

and

$$\begin{cases}\Pr_L(h_t)\left(q_L u'\left(1-\hat{P}_{P,t}^{h_t}-\hat{D}_{P,t}^{h_t}\right)+(1-q_L)u'\left(1-\hat{P}_{P,t}^{h_t}\right)\right)=\lambda_t\left(\alpha \Pr_L(h_t)+(1-\alpha)\Pr_H(h_t)\right)\\ \Pr_L(h_t)q_L u'\left(1-\hat{P}_{P,t}^{h_t}-\hat{D}_{P,t}^{h_t}\right)=\lambda_t\left(\alpha \Pr_L(h_t)q_L+(1-\alpha)\Pr_H(h_t)q_H\right)\end{cases}.$$

Solving them yields

$$\begin{cases}u'\left(1-\hat{P}_{P,t}^{h_t}\right)=\lambda_t\left(\alpha+(1-\alpha)\frac{\Pr_H(h_t)(1-q_H)}{\Pr_L(h_t)(1-q_L)}\right)\\ u'\left(1-\hat{P}_{P,t}^{h_t}-\hat{D}_{P,t}^{h_t}\right)=\lambda_t\left(\alpha+(1-\alpha)\frac{\Pr_H(h_t)q_H}{\Pr_L(h_t)q_L}\right)\end{cases}.$$

One may see that

$$\frac{u'\left(1-\hat{P}_{P,t}^{h_t}\right)}{u'\left(1-\hat{P}_{P,t}^{h_t}-\hat{D}_{P,t}^{h_t}\right)}=\frac{\alpha+(1-\alpha)\frac{\Pr_H(h_t)(1-q_H)}{\Pr_L(h_t)(1-q_L)}}{\alpha+(1-\alpha)\frac{\Pr_H(h_t)q_H}{\Pr_L(h_t)q_L}}<1\,,$$

hence $u'\left(1-\hat{P}_{P,t}^{h_t}-\hat{D}_{P,t}^{h_t}\right)>u'\left(1-\hat{P}_{P,t}^{h_t}\right)$ and, therefore, $\hat{D}_{P,t}^{h_t}>0$.

If such an interior solution has $D_{P,t}^{h_t}>e_L$ then we have to look at the corner solutions, where $\hat{\Theta}_{P,t}^{h_t}=\Theta_0^S$ for some set of states of the world $H_t^0\subset H_t$. In this case the Lagrange function becomes:

$$L_t=\sum_{h_t\in H_t\setminus H_t^0}\Pr_L(h_t)\left(q_L u\left(1-P_{P,t}^{h_t}-D_{P,t}^{h_t}\right)+(1-q_L)u\left(1-P_{P,t}^{h_t}\right)\right)+\sum_{h_t\in H_t^0}\Pr_L(h_t)q_L u(1-e_L)+$$
$$+\lambda_t\sum_{h_t\in H_t\setminus H_t^0}\left(\alpha \Pr_L(h_t)\left(P_{P,t}^{h_t}-q_L\left(e_L-D_{P,t}^{h_t}\right)\right)+(1-\alpha)\Pr_H(h_t)\left(P_{P,t}^{h_t}-q_H\left(e_H-D_{P,t}^{h_t}\right)\right)\right),$$

hence, the first order conditions remain the same but now only for $h_t\in H_t\setminus H_t^0$. Solving them for $\hat{\Theta}_{P,t}$ for all $H_t^0\subset H_t$ and taking one that maximizes $U_L^D\left(\Theta_P^D\right)$ gives us needed contract.

The contract conditions, hence, $U_L^D\left(\hat{\Theta}_P^D\right)$, now become functions of α. Taking the first order derivative and using the envelope theorem and zero profit conditions in a form $\alpha \sum_{h_t\in H_t\backslash H_t^0} \Pr_L(h_t)\left(P_{P,t}^{h_t} - q_L\left(e_L - D_{P,t}^{h_t}\right)\right) = -(1-\alpha) \sum_{h_t\in H_t\backslash H_t^0} \Pr_H(h_t)\left(P_{P,t}^{h_t} - q_H\left(e_H - D_{P,t}^{h_t}\right)\right)$ yields:

$$\frac{d}{d\alpha} L_t\left(\hat{\Theta}_P^D\right) = \frac{\partial}{\partial\alpha} L_t\left(\hat{\Theta}_P^D\right) =$$

$$= \lambda_t\left(\sum_{h_t\in H_t\backslash H_t^0} \Pr_L(h_t)\left(P_{P,t}^{h_t} - q_L\left(e_L - D_{P,t}^{h_t}\right)\right) - \sum_{h_t\in H_t\backslash H_t^0} \Pr_H(h_t)\left(P_{P,t}^{h_t} - q_H\left(e_H - D_{P,t}^{h_t}\right)\right)\right) =$$

$$= \frac{\lambda_t}{1-\alpha}\left(\delta^{t-1} \sum_{h_t\in H_t\backslash H_t^0} \Pr_L(h_t)\left(\hat{P}_{P,t}^{h_t} - q_L\left(e_L - \hat{D}_{P,t}^{h_t}\right)\right)\right),$$

and, finally,

$$\frac{d}{d\alpha} U_L^D\left(\hat{\Theta}_P^D\right) = \frac{1-\delta}{1-\delta^T} \sum_{t=1}^T \left(\delta^{t-1} \frac{d}{d\alpha} L_t\left(\hat{\Theta}_P^D\right)\right) =$$

$$= \frac{\lambda_t}{1-\alpha}\frac{1-\delta}{1-\delta^T} \sum_{t=1}^T \left(\delta^{t-1} \sum_{h_t\in H_t\backslash H_t^0} \Pr_L(h_t)\left(\hat{P}_{P,t}^{h_t} - q_L\left(e_L - \hat{D}_{P,t}^{h_t}\right)\right)\right) =$$

$$= -\frac{\lambda_t}{\alpha}\frac{1-\delta}{1-\delta^T} \sum_{t=1}^T \left(\delta^{t-1} \sum_{h_t\in H_t\backslash H_t^0} \Pr_H(h_t)\left(\hat{P}_{P,t}^{h_t} - q_H\left(e_H - \hat{D}_{P,t}^{h_t}\right)\right)\right).$$

It is easily seen that $\sum_{h_t\in H_t\backslash H_t^0} \Pr_H(h_t)\left(\hat{P}_{P,t}^{h_t} - q_H\left(e_H - \hat{D}_{P,t}^{h_t}\right)\right) < 0 < \sum_{h_t\in H_t\backslash H_t^0} \Pr_L(h_t)\left(\hat{P}_{P,t}^{h_t} - q_L\left(e_L - \hat{D}_{P,t}^{h_t}\right)\right)$. In other words, an insurer gets a positive profit from the low-risk type and a negative profit from the high-risk type. Therefore, $\frac{d}{d\alpha} U_L^D\left(\hat{\Theta}_P^D\right) > 0$ as $\lambda_t > 0$.

Now, we will show that $U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=0} < U_L^D\left(\Theta_L^D\right) < U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=1}$ and, therefore, there exists an $\alpha_C^D \in (0,1)$ such that $U_L^D\left(\hat{\Theta}_P^D\right)\Big|_{\alpha=\alpha_C^D} = U_L^D\left(\Theta_L^D\right)$ and the results (a) and (b) of the proposition follow.

If $\alpha = 1$ then $\hat{\Theta}_P^D$ gives always the full insurance that leads to the first best outcome $U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=1} = u(1 - q_L e_L)$, hence, $U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=1} > U_L^D\left(\Theta_L^D\right)$. What we will show is that $U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=0} > U_L^D\left(\Theta_L^D\right)$. To this end we construct a competitive contract $\tilde{\Theta}_L^D$ such that $U_L^D\left(\Theta_L^D\right) \geq U_L^D\left(\tilde{\Theta}_L^D\right) > U_L^D\left(\hat{\Theta}_P^D\right)\big|_{\alpha=0}$. Obviously, $U_L^D\left(\Theta_L^D\right) \geq U_L^D\left(\Theta^D\right)$ for any competitive contract Θ^D by the construction of Θ_L^D.

As an example of such a contract $\tilde{\Theta}_L^D$ we take a contract that coincides with $\hat{\Theta}_P^D$ for all t and h_t except one, i.e., we put $\tilde{\Theta}_{L,t}^{h_t} = \hat{\Theta}_{P,t}^{h_t}$ for all t and $h_t \neq h^*$, and $\tilde{\Theta}_{L,t}^{h^*} \neq \hat{\Theta}_{P,t}^{h^*}$. In this

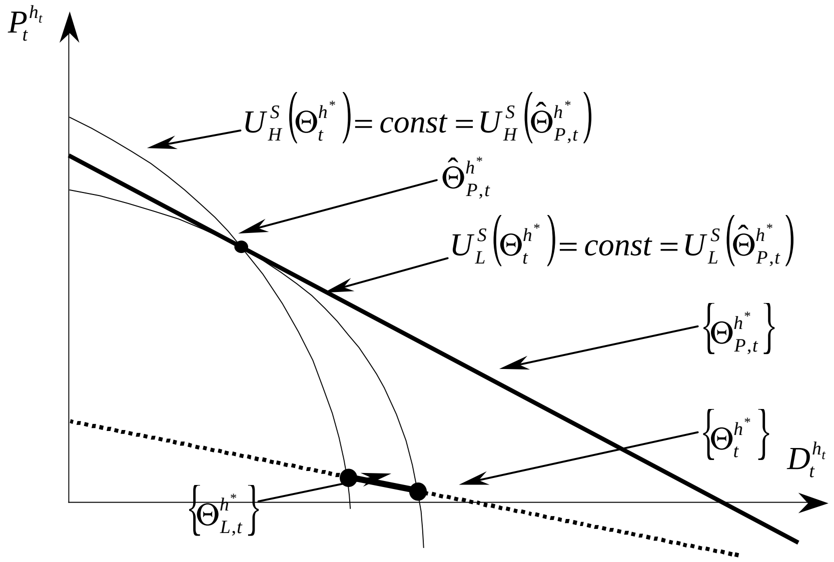

Figure A.1.

state of the world h^* a policy $\tilde{\Theta}_{L,t}^{h^*}$ can offer a much better insurance for the low-risk type than $\hat{\Theta}_{P,t}^{h^*}$ which is calculated for the whole population that consists of high-risk agents only. The following Figure A.1 represents the arguments. The downward-sloping line is a set of policies in the state h^* making the whole contract $\hat{\Theta}_P^D$ competitive. One point on the line is the contract $\hat{\Theta}_{P,t}^{h^*}$ that maximizes $U_L^D\left(\Theta_P^D\right)$ and, therefore, $U_L^S\left(\Theta_t^{h^*}\right)$. Two curves represent high- and low-type indifference curves where the latter is tangent to the set $\left\{\Theta_{P,t}^{h^*}\right\}$ at $\hat{\Theta}_{P,t}^{h^*}$. The set of contracts $\left\{\Theta_t^{h^*}\right\}$ satisfying zero profit condition for the low-risk type is denoted by the dotted line. This set lies below the former set as all the profit obtained in all the other time periods and states is transferred here.

One may easily verify that the contract $\tilde{\Theta}_L^D$ can be chosen as any point from the set $\left\{\Theta_{L,t}^{h^*}\right\}$ that lies between the low- and high-risk indifference curve. Hence, $U_L^D\left(\Theta_L^D\right) > U_L^D\left(\hat{\Theta}_P^D\right)\Big|_{\alpha=0}$.

Part (c) of the proposition is trivial as the contract $\Theta^D = \left(\Theta_L^S, \ldots \Theta_L^S\right)$ was available during the optimization procedure of searching Θ_L^D, hence, $U_L^D\left(\Theta_L^D\right) \geq U_L^S\left(\Theta_L^S\right)$, and the contract $\Theta^D = \left(\Theta_L^S, \ldots \Theta_L^S\right)$ does not satisfies the first order conditions for Θ_L^D, those are $P_{L,t}^k = P_{L,t}^{k-1} + D_{L,t}^{k-1}$, therefore, $U_L^D\left(\Theta_L^D\right) > U_L^S\left(\Theta_L^S\right)$ unless those first order conditions degenerate in a global corner solution $\Theta_L^D = \left(\Theta_0^S, \ldots \Theta_0^S\right)$.

Finally, in order to prove that $\lim_{T\to\infty}\alpha_C^D(T)=1$ we will show that for any *T* there exists a contract $\hat{\Theta}_L^D$ satisfying both zero profit and the incentive compatibility constraints an such that $\lim_{T\to\infty}U_L^D(\hat{\Theta}_L^D)=u(1-q_Le_L)$. As $U_L^D(\Theta_L^D)\geq U_L^D(\hat{\Theta}_L^D)$, $\lim_{T\to\infty}U_L^D(\Theta_L^D)=u(1-q_Le_L)$ holds as well. Then, as $U_L^D(\hat{\Theta}_P^D)<u(1-q_Le_L)$ for any $a<1$, this implies that $\lim_{T\to\infty}\alpha_C^D(T)=1$.

Let us consider a contract $\hat{\Theta}_L^D=(\hat{\Theta}_{L,1},\ldots,\hat{\Theta}_{L,T})$, where $\hat{\Theta}_{L,t}^k=(q_Le_L,0)$ for all $t=1,\ldots,T-1$, $\hat{\Theta}_{L,T}^k\equiv(\hat{P}_T^k,\hat{D}_T^k)=(P^-,0)$ for $k=0,\ldots,T-2$ and $\hat{\Theta}_{L,T}^{T-1}=(P^+,0)$ for some values of P^- and P^+. The zero profit condition for the time period *T* requires that $\sum_{k=0}^{T-1}\mathrm{Pr}_L(k)(\hat{P}_{T-1}^k-q_Le_L)=0$, that makes P^- dependent on P^+:

$$P^-(P^+)=\frac{q_Le_L-q_L{}^{T-1}P^+}{1-q_L{}^{T-1}} \tag{A.1}$$

hence $P^-<q_Le_L$ for all $P^+>q_Le_L$. On the other hand, the incentives compatibility constraint requires that $U_H^D(\Theta_H^D)=U_H^D(\hat{\Theta}_L^D)$, therefore,

$$u(1-q_He_H)=\sum_{t=1}^T\left(\frac{\delta^{t-1}}{\sum_{t=1}^T\delta^{t-1}}\sum_{k=0}^{t-1}\mathrm{Pr}_H(k)U_L^S(\Theta_t^k)\right)=$$

$$=u(1-q_Le_L)+\frac{(1-\delta)\delta^{T-1}}{(1-\delta^T)}\left(u(1-P^-)-u(1-q_Le_L)+q_H{}^{T-1}u(1-P^+)-q_H{}^{T-1}u(1-P^-)\right),$$

$$u(1-P^-)(1-q_H{}^{T-1})+q_H{}^{T-1}u(1-P^+)=u(1-q_Le_L)-(u(1-q_Le_L)-u(1-q_He_H))\frac{(1-\delta^T)}{(1-\delta)\delta^{T-1}}.$$

Together with (A.1) this equation defines unique values of $P^-<q_Le_L$ and $P^+>q_Le_L$. To see this we plug the (A.1) into the last equation:

$$u(1-P^-(P^+))(1-q_H{}^{T-1})+q_H{}^{T-1}u(1-P^+)=u(1-q_Le_L)-(u(1-q_Le_L)-u(1-q_He_H))\frac{(1-\delta^T)}{(1-\delta)\delta^{T-1}}.$$

The left hand side, being a function of P^+ has the following properties:

a) $\left.LHS\right|_{P^+=q_Le_L}=u(1-q_Le_L)(1-q_H{}^{T-1})+q_H{}^{T-1}u(1-q_Le_L)=u(1-q_Le_L)>RHS$.

b) If $\lim_{m\to+0}u(m)=-\infty$ then $\lim_{P^+\to1}LHS=(1-q_H{}^{T-1})u\left(\frac{1-q_Le_L}{1-q_L{}^{T-1}}\right)+q_H{}^{T-1}\lim_{P^+\to1}u(1-P^+)=-\infty.$

c) $\frac{dLHS}{dP^+}=q_H{}^{T-1}\left(\frac{q_L{}^{T-1}(1-q_H{}^{T-1})}{q_H{}^{T-1}(1-q_L{}^{T-1})}u'(1-P^-)-u'(1-P^+)\right)<0$.

Hence, P^+ and P^- are uniquely defined. Now we take limits

$$\lim_{T\to\infty} P^- = \lim_{T\to\infty} \frac{q_L e_L - q_L{}^{T-1}P^+}{1-q_L{}^{T-1}} = q_L e_L,$$

and

$$\lim_{T\to\infty} \delta^{T-1} q_H{}^{T-1} u\left(1-P^+\right) =$$
$$= \lim_{T\to\infty} \delta^{T-1}\left(u(1-q_L e_L) - \left(u(1-q_L e_L) - u(1-q_H e_H)\right)\frac{\left(1-\delta^T\right)}{(1-\delta)\delta^{T-1}} - u\left(1-P^-\right)\left(1-q_H{}^{T-1}\right)\right) =$$
$$= \lim_{T\to\infty}\left(-\left(u(1-q_L e_L) - u(1-q_H e_H)\right)\frac{\left(1-\delta^T\right)}{(1-\delta)} - \delta^{T-1} u(1-q_L e_L)\left(1-q_H{}^{T-1}\right)\right) =$$
$$= -\frac{1}{(1-\delta)}\left(u(1-q_L e_L) - u(1-q_H e_H)\right).$$

Therefore,

$$\lim_{T\to\infty} U_L^D\left(\hat{\Theta}_L^D\right) = \lim_{T\to\infty} \sum\nolimits_{t=1}^{T}\left(\frac{\delta^{t-1}}{\sum_{t=1}^{T}\delta^{t-1}}\sum_{k=0}^{t-1}\Pr_L(k)U_L^S\left(\Theta_t^k\right)\right) =$$
$$= \lim_{T\to\infty}\left(u(1-q_L e_L) + \frac{(1-\delta)\delta^{T-1}}{\left(1-\delta^T\right)}\left(u\left(1-P^-\right) - u(1-q_L e_L) + q_L{}^{T-1}u\left(1-P^+\right) - q_L{}^{T-1}u\left(1-P^-\right)\right)\right) =$$
$$= u(1-q_L e_L) + \lim_{T\to\infty}\left((1-\delta)\delta^{T-1} q_H{}^{T-1} u\left(1-P^+\right)\frac{q_L{}^{T-1}}{q_H{}^{T-1}} - (1-\delta)\delta^{T-1} q_L{}^{T-1} u(1-q_L e_L)\right) =$$
$$= u(1-q_L e_L) - \lim_{T\to\infty}\left(u(1-q_L e_L) - u(1-q_H e_H)\right)\frac{q_L{}^{T-1}}{q_H{}^{T-1}} = u(1-q_L e_L),$$

that ends the proof. ■

Proof of Proposition 4. As we have already established in Proposition 3, the dynamic contract maximizing the low-risk type utility and providing strictly positive insurance for *every period* coincides with the static equilibrium contract Θ_L^S.

Now we will search for the best dynamic contract, which gives no insurance in a separation phase $N_{sep} \subseteq N_T$, which is a subset of all time periods $N_T = \{j\}_{j=1}^{T}$, and strictly positive insurance in an insurance phase $N_T \setminus N_{sep}$. The same arguments as in the proof of Proposition 3 leads to the insurance conditions in the insurance phase are constant, i.e., $\Theta_t^D = \Theta = (P, D)$ for $t \in N_T \setminus N_{sep}$. Hence,

$$U_L^D\left(\Theta^D\right) = \frac{\sum_{t\in N_{sep}}\delta^{t-1}}{\sum_{t\in N_T}\delta^{t-1}} U_L^S\left(\Theta_0^S\right) + \frac{\sum_{t\in N_T\setminus N_{sep}}\delta^{t-1}}{\sum_{t\in N_T}\delta^{t-1}} U_L^S(\Theta),$$

which can be rewritten as

$$U_L^D\left(\Theta^D\right)=(1-w)U_L^S\left(\Theta_0^S\right)+wU_L^S(\Theta),$$

where $w\equiv\sum_{t\in N_T\setminus N_{sep}}\delta^{t-1}\left(\sum_{t\in N_T}\delta^{t-1}\right)^{-1}$ denotes the relative weight of the insurance phase of the whole dynamic contract. In this notations a dynamic contract Θ^D is defined by P_L, D_L and w^D.

Obviously, Θ_L^D must have such P_L, D_L and w^D that maximize $U_L^D\left(\Theta^D\right)$ subject to zero profit condition $P=q_L(e_L-D)$ and the incentives compatibility constraint $U_H^D\left(\Theta^D\right)\le U_H^S\left(\Theta_H^S\right)$. We will look for such contracts that are also Pareto-superior to Θ_L^S, i.e., $U_L^D\left(\Theta^D\right)\ge U_L^S\left(\Theta_L^S\right)$. Those two constraints can be rewritten as follows:

$$w\le\frac{U_H^S\left(\Theta_H^S\right)-U_H^S\left(\Theta_0^S\right)}{U_H^S(\Theta)-U_H^S\left(\Theta_0^S\right)}\equiv\overline{w}(D), \text{ and}$$

$$w\ge\frac{U_L^S\left(\Theta_L^S\right)-U_L^S\left(\Theta_0^S\right)}{U_L^S(\Theta)-U_L^S\left(\Theta_0^S\right)}\equiv\underline{w}(D).$$

Now we will consider the cases $e_L\le e_L^S(q_H)$ and $e_L>e_L^S(q_H)$ separately.

d) If $e_L\le e_L^S(q_H)$, then $U_L^S\left(\Theta_L^S\right)=U_L^S\left(\Theta_0^S\right)$ and, therefore, $\underline{w}(D)=0$ while $U_H^S\left(\Theta_H^S\right)-U_H^S\left(\Theta_0^S\right)>0$ and, therefore, $\overline{w}(D)>0$, for all $D\in(0,e_L)$. Hence, any dynamic contract with $w\in\left(0,\overline{w}(D)\right)\subset(0,1)$ is strictly Pareto-superior to Θ_L^S.

e) If $e_L>e_L^S(q_H)$ then there exists a static contract Θ_L^S with $D_L^S<e_L$ and both $\overline{w}(D)$ and $\underline{w}(D)$ are strictly increasing functions as $\frac{dU_i^S(\Theta)}{dD}<0$, $i=H,L$ for $D\in[0,D_L^S]$. For all $D\in[0,D_L^S)$ they belong to the range $(0,1)$ and $\overline{w}\left(D_L^S\right)=\underline{w}\left(D_L^S\right)=1$. Hence, if there exists a $D\in[0,D_L^S)$ such that $\underline{w}(D)<\overline{w}(D)$ then any dynamic contract with $w\in\left(\underline{w},\overline{w}\right)$ is strictly Pareto-superior to the static contract Θ_L^S.

What we will show now is that the continuous function $F(D)$ being defined as $F(D)\equiv\overline{w}(D)-\underline{w}(D)$ is always negative over $D\in[0,D_L^S)$ if e_L exceeds a certain threshold level e_L^D, i.e., if $e_L\ge e_L^D$, and is strictly positive in some left neighborhood $D\in\left(\underline{D},D_L^S\right)\subset\left(0,D_L^S\right)$ otherwise. To this end, using the implicit definition of D_L^S, which is

$$U_H^S\left(\Theta_L^S\right)\equiv q_H u\left(1-P_L^S\left(D_L^S\right)-D_L^S\right)+\left(1-q_H\right)u\left(1-P_L^S\left(D_L^S\right)\right)=U_H^S\left(\Theta_H^S\right), \qquad \text{(A.2)}$$

we first rewrite $F(D)$ as $F(D)=\overline{w}(D)-\underline{w}(D)=F_1\cdot F_2$, where

$$F_1(D)\equiv\frac{1}{\left(U_H^S\left(\Theta_L^D\right)-U_H^S\left(\Theta_0^S\right)\right)\left(U_L^S\left(\Theta_L^D\right)-U_L^S\left(\Theta_0^S\right)\right)}>0, \text{ and}$$

$$F_2(D)\equiv\tfrac{q_H-q_L}{q_H}\left(\left(U_H^S\left(\Theta_H^S\right)-U_H^S\left(\Theta_0^S\right)\right)u(1-P)-\left(U_H^S\left(\Theta_L^D\right)-U_H^S\left(\Theta_0^S\right)\right)u\left(1-P_L^S\right)\right)+$$
$$+q_L\left(U_H^S\left(\Theta_L^D\right)-U_H^S\left(\Theta_H^S\right)\right)\left(u\left(1-e_L\right)-u\left(1-e_H\right)\right).$$

As $F_1>0$ over $D\in[0,D_L^S]$, we have to show that the function $F_2(D)$ has the same properties we require of the function $F(D)$.

Firstly, as $U_H^S\left(\Theta_L^S\right)=U_H^S\left(\Theta_H^S\right)$, it follows that $F_2\left(D_L^S\right)=0$. Then, the first and second order derivatives of $F_2(D)$ are

$$F_2'=\left(q_H-q_L\right)q_L\left(u\left(1-P_L^S-D_L^S\right)-u\left(1-e_H\right)\right)u'(1-P)+$$
$$+\left(q_H-q_L\right)\left(1-q_L\right)u\left(1-P_L^S\right)u'(1-P-D)+ \qquad \text{(A.3)}$$
$$+q_L\left(u\left(1-e_L\right)-u\left(1-e_H\right)\right)\left(q_L\left(1-q_H\right)u'(1-P)-q_H\left(1-q_L\right)u'(1-P-D)\right),$$

and

$$F_2''=q_L\left(u\left(1-e_L\right)-u\left(1-e_H\right)\right)\left(q_L^{\,2}\left(1-q_H\right)u''(1-P)+q_H\left(1-q_L\right)^2u''(1-P-D)\right)+$$
$$+\left(q_H-q_L\right)q_L^{\,2}\left(u\left(1-P_L^S-D_L^S\right)-u\left(1-e_H\right)\right)u''(1-P)+$$
$$+\left(q_L-q_H\right)\left(1-q_L\right)^2u\left(1-P_L^S\right)u''(1-P-D).$$

As all the three terms in the above expression are negative, $F_2''<0$.

Summarizing, F_2 is strictly concave over $D\in[0,D_L^S]$ and $F_2\left(D_L^S\right)=0$. Then it immediately follows that if $F_2'\left(D_L^S\right)\geq 0$ then $F_2(D)<0$ for all $D<D_L^S$, and $F_2(D)>0$ in some neighborhood $D\in\left(\underline{D},D_L^S\right)\subset[0,D_L^S]$ otherwise. Therefore, we have to investigate the sign of $F_2'\left(D_L^S\right)$, which now becomes a function of the model parameters and, in particular, a function of e_L, i.e., $F_3\left(e_L\right)\equiv F_2'\left(D_L^S\right)$.

Firstly, we note from (A.3) that $F_3\left(e_H\right)>0$. Indeed,

$$F_3\left(e_H\right)=\left(q_H-q_L\right)q_L\left(u\left(1-P_L^S-D_L^S\right)-u\left(1-e_H\right)\right)u'\left(1-P_L^S\right)-$$
$$-\left(q_H-q_L\right)\left(1-q_L\right)\left(u(1)-u\left(1-P_L^S\right)\right)u'\left(1-P_L^S-D_L^S\right).$$

Now, using the mean-value theorem we can write for some $x\in\left(1-e,1-P_L^S-D_L^S\right)$ and $y\in\left(1-P_L^S,1\right)$:

$$u'(x)=\frac{u\left(1-P_L^S-D_L^S\right)-u\left(1-e_H\right)}{\left(1-P_L^S-D_L^S\right)-\left(1-e_H\right)}=\frac{u\left(1-P_L^S-D_L^S\right)-u\left(1-e_H\right)}{\left(e_H-D_L^S\right)\left(1-q_L\right)}, \text{ and}$$

$$u'(y)=\frac{u(1)-u\left(1-P_L^S\right)}{1-\left(1-P_L^S\right)}=\frac{u(1)-u\left(1-P_L^S\right)}{\left(e_H-D_L^S\right)q_L}.$$

Plugging them into the last expression for $F_3(e_H)$ we finally obtain

$$F_3(e_H)=(q_H-q_L)(1-q_L)q_L\left(e-D_L^S\right)\left(u'(x)u'\left(1-P_L^S\right)-u'(y)u'\left(1-P_L^S-D_L^S\right)\right)>0,$$

as $u'(x)>u'\left(1-P_L^S-D_L^S\right)>0$ and $u'\left(1-P_L^S\right)>u'(y)>0$.

On the other hand, $F_3\left(e_L^S\right)<0$. Indeed, in this case $D_L^S\left(e_L^S\right)=e_L^S$, $P_L^S\left(e_L^S\right)=0$ and

$$F_3\left(e_L^S\right)=q_Lq_H(1-q_L)\left(u'(1)-u'\left(1-e_L^S\right)\right)\left(u\left(1-e_L^S\right)-u(1-e_H)\right)<0.$$

Hence, continuous function $F_3(e_H)$ takes the opposite signed values at the ends of the interval [e_L^S, e_H].

Secondly, differentiating (A.2) w.r.t. e_L yields

$$\frac{dD_L^S}{de_L}=-q_L\frac{q_Hu'\left(1-P_L^S-D_L^S\right)+(1-q_H)u'\left(1-P_L^S\right)}{q_H(1-q_L)u'\left(1-P_L^S-D_L^S\right)-(1-q_H)q_Lu'\left(1-P_L^S\right)}<0,$$

and, consequently, as $P_L^S=q_L\left(e_L-D_L^S\right)$, $\frac{d}{de_L}u'\left(1-P_L^S\right)=u''\left(1-P_L^S\right)q_L\frac{dD_L^S}{de_L}>0$ and $\frac{d}{de_L}u'\left(1-P_L^S-D_L^S\right)=-u''\left(1-P_L^S-D_L^S\right)(1-q_L)\frac{dD_L^S}{de_L}<0$. Using these expressions allows us to write the derivative F_3' as follows:

$$\begin{aligned}
F_3' &= q_L(q_H-q_L)\left(u\left(1-P_L^S-D_L^S\right)-u(1-e_L)\right)\tfrac{d}{de_L}u'\left(1-P_L^S\right)+\\
&+q_Lq_H(1-q_L)\left(u(1-e_L)-u(1-e_H)\right)\tfrac{d}{de_L}u'\left(1-P_L^S\right)+\\
&+(1-q_L)(q_H-q_L)u\left(1-P_L^S\right)\tfrac{d}{de_L}u'\left(1-P_L^S-D_L^S\right)+\\
&+(1-q_L)q_Lq_H\left(u(1-e_H)-u(1-e_L)\right)\tfrac{d}{de_L}u'\left(1-P_L^S-D_L^S\right)+\\
&+q_L\left(q_H(1-q_L)u'\left(1-P_L^S-D_L^S\right)-q_L(1-q_H)u'\left(1-P_L^S\right)\right)u'(1-e_L)-\\
&-q_L(q_H-q_L)u'\left(1-P_L^S\right)u'\left(1-P_L^S-D_L^S\right).
\end{aligned}$$

As the sum of the first four terms in the above expression is strictly positive we get:

$$\begin{aligned}
F_3' &> q_L\left(q_H(1-q_L)u'\left(1-P_L^S-D_L^S\right)-q_L(1-q_H)u'\left(1-P_L^S\right)\right)u'(1-e_L)-\\
&-q_L(q_H-q_L)u'\left(1-P_L^S\right)u'\left(1-P_L^S-D_L^S\right)\\
&=q_L{}^2(1-q_H)\left(u'\left(1-P_L^S-D_L^S\right)-u'\left(1-P_L^S\right)\right)u'(1-e_L)+\\
&+q_L(q_H-q_L)u'(1-e_L)-u'\left(1-P_L^S\right)u'\left(1-P_L^S-D_L^S\right)\\
&>0.
\end{aligned}$$

Hence, the function $F_3(e_L)$ has a unique null in the interval (e_L^S, e_H), which we denote as e_L^D, i.e., $F_3(e_L^D) = 0$. By construction of e_L^D, for all $e_L \in (e_L^S, e_L^D)$ there exists an interval $(\underline{D}, D_L^S) \subset (0, D_L^S)$ such that for all $D \in (\underline{D}, D_L^S)$ $0 < \underline{w}(D) < \overline{w}(D) < 1$ and, therefore, any dynamic contract with $w \in (\underline{w}, \overline{w})$ is strictly Pareto-superior to Θ_L^S. If, on the other hand, $e_L \in (e_L^D, e_H)$ then $\underline{w}(D) > \overline{w}(D)$ for all $D \in (0, D_L^S)$ and there is no dynamic contract which is Pareto-superior to Θ_L^S, that ends the case.

Hence, if and only if $e_L < e_L^D$ then the set of *D* and *w* that generate Pareto-superior contracts is not empty. Obviously, an insurer is able to choose *D* arbitrarily. As for *w*, it may only take discrete values:

$$w \in \left\{ \frac{1-\delta}{1-\delta^T} \sum_{t \in N_T \setminus N_{sep}} \delta^{t-1} \right\}_{N_{sep} \subset N_T} = W(\delta, T).$$

When $T \to \infty$ this set gets an *accumulation point* $w = 1$, in other words, the number of elements in any left neighborhood of $w = 1$ increases unboundedly when *T* becomes larger. This property allows an insurer to choose *D* sufficiently close but still smaller than D_L^S and find such a *w* that $w \in (\underline{w}(D), \overline{w}(D)) \bigcap W(\delta, T)$. Therefore, for all *T* sufficiently large the set of strictly Pareto-superior contracts is not empty and its closure, the set of weakly Pareto-superior contracts, contains a welfare-maximizing contract Θ_L^D.

Now we will show that $e_L^D(q_L) = \lim\limits_{q_H \to 1} e_L^D(q_H) = e_H$. As for any q_H $e_L^D(q_L) \in (e_L^S(q_L), e_H)$ and the function $e_L^S(q_L)$ is continuous and $e_L^S(q_H) = e_H$, it follows that $\lim\limits_{q_H \to 1} e_L^D(q_H) = e_H$. If, on the other hand, $q_H = q_L$ then

$$F_2'(D_L^S)\Big|_{q_H = q_L = q} = q^2(1-q)(u(1-e_L) - u(1-e_H))(u'(1-P_L^S) - u'(1-P_L^S - D_L^S)) < 0$$

for any e_L, so $e_L^D(q_L) = e_H$.

Finally, we define α^D. As Θ_L^D is the welfare maximizing dynamic contract, a competitive separating Nash equilibrium exists as long as $U_L^D(\Theta_L^D) \geq U_P^D(\alpha)$, where $U_P^D(\alpha)$ is the highest possible low-risk type utility under a pooling insurance contract. Similar argument to those in the beginning of the proof show that this pooling contract is purely static, i.e., $D_{P,t}^D = D_P^S$ and $P_{P,t}^D = P_P^S = \alpha q_L(e_L - D_P^S) + (1-\alpha) q_H (e_H - D_P^S)$. Then, that highest utility level $U_P^D(\alpha) \equiv U_L^D(\Theta_P^D)$ either is smaller than $U_L^S(\Theta_0^S)$ and there exists no competitive pooling contract that is better than Θ_0^S for the low-risk type, or it is an increasing function of α.[6] Together with $U_P^D(0) < U_L^D(\Theta_L^D)$ and $U_P^D(1) > U_L^D(\Theta_L^D)$ this implies that there exists a unique α^D such that $U_P^D(\alpha^D) > U_L^D(\Theta_L^D(\tau))$. The fact that $\alpha^D > \alpha^S$ follows from $U_L^D(\Theta_L^D) > U_L^S(\Theta_L^S)$. ■

[6] See proof of Proposition 1.