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Maintenance Costs, Obsolescence, and Endogenous Growth

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Tinbergen Institute

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Tinbergen Institute Working Paper

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Koen G. Berden and Charles van Marrewijk*

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Abstract

We analyze the impact of obsolescence of economic inventions by incorporating maintenance costs in the endogenous growth model of expanding product varieties. This contrasts with the existing literature, which ignores maintenance costs and uses the model of quality improvements to describe obsolescence. If the maintenance costs become too high, the operating profits become negative and the firm stops producing the variety. This diminishes the life span of innovations, thus reducing the return on investment in research and development and the growth rate of the economy.

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Maintenance costs, obsolescence, and endogenous growth

1 Introduction

Large resources are spent worldwide on research and development (R&D) to invent and introduce new types of goods and services to satisfy customer and client needs. Innovation is undoubtedly very important in today's world. However, one may wonder how many of the inventions and discoveries done at the time of Napoleon do we still cherish and witness today? New management techniques appear, for example, to support the organizational structure and management information processes of firms, and disappear again once they are replaced by even more up-to-date techniques.

Endogenous growth models – both the AK-type of models of Romer (1986), Lucas (1988) and Rebelo (1991), and the R&D-type models of Romer (1990), Grossman & Helpman (1991), and Aghion and Howitt (1992) – investigate the relationships between innovative behavior and economic growth. In contrast to exogenous growth models, inventions are not a function of elapsed calendar time, but the result of conscious decisions to invest in R&D, arising from people's inspiration and perspiration. Within the widely used framework of expanding product variety, the phenomenon of obsolescence is disregarded. Aghion and Howitt (1998, p.39) even argue that "in order to formalize the notion of (technical or product) obsolescence, one needs to move away from horizontal models of product development à la Dixit and Stiglitz (1977) into vertical models of quality improvements." Although vertical models of quality improvements are constructed to deal with the obsolescence phenomenon, we disagree with Aghion and Howitt's statement as such, by analyzing the role of obsolescence if we incorporate maintenance costs in the canonical model of horizontal product differentiation (see Grossman and Helpman, 1991, ch. 3).

Evidently, new products, that is goods, services, or production processes, become obsolete over time. The early maritime industry in New England, for example, which had nothing much useful to do in the winter time, used to cut ice from frozen rivers and lakes, store it underground, and ship it to India. It has now been replaced by refrigerators. Other examples of once useful but now obsolete items in advanced societies are buggy whips, slide rules, oil lamps, and the telegraph. We argue that the rate at which inventions become obsolete over time is influenced by the degree of maintenance costs. The term maintenance costs should be interpreted in a broad sense and can refer to both technical and economic obsolescence. Some examples of maintenance costs are:

- Costs of *preventive* maintenance. To avoid machinery from breaking down too frequently, preventive maintenance is carried out. The most important costs of preventive maintenance is usually not the cost of labor involved in the maintenance process, nor the parts that need to be replaced, but the fact that the machinery is not productive during the maintenance process. Over time, as the machine-park is getting older, preventive maintenance will be carried out more often.
- Costs of (emergency) *repair* maintenance. Despite the fact that preventive maintenance is carried out more frequently as the production process ages, every now and then a machine will break down and has to be fixed again. Again, the fact that the production process is stopped represents the highest costs. In most cases, non-scheduled repair maintenance is more costly than preventive maintenance. Moreover, the older the production process, the higher the breakdown frequency.
- Costs of *updating* the production process. The introduction of new production techniques or a different marketing strategy, frequently requires changes or adjustments in the production process. Such changes are more likely to occur if the production process has been operative for some time, as new production techniques become available and changes in consumers' preferences and demands require an adjustment of the marketing strategy.
- Cost of *replacing* part of the production process. In many cases, only part of a production line, rather than the entire production process, is replaced. Nonetheless, this frequently means that the whole production process is stopped. The older the structure of the production process, the larger the possibility that part of the line will have to be replaced, and thus the larger the fraction of time the machinery is not productive.
- Costs of *better alternatives*. A clear example of economic maintenance costs is represented by the arrival of better alternative ways of production or organizing the production process, which makes the old production technique more expensive in terms of income foregone. The more alternatives arise, the higher the likelihood that a production process is replaced by a better one.

2 The model

We extend the Grossman and Helpman (1991, ch. 3) model of horizontal product differentiation to incorporate maintenance costs. Labor, the only factor of production, is used for maintenance, to produce goods, and for R&D. The returns to R&D arise from monopoly rents in imperfectly competitive product markets.

Consumer behavior

The representative consumer maximizes utility U over an infinite time horizon, using preferences as given in equation (1). The term $D(t)$ represents an index of consumption at time t, and r is the discount rate.

$$U(t) = \int_t^{\infty} e^{-rt} \log\big(D(t)\big) dt \tag{1}$$

The index D reflects a taste for diversity in consumption, based on the Dixit-Stiglitz (1977) approach of horizontal product differentiation. We take the product space to be continuous. Preferences are defined over an infinite set of products using the index j. At any moment, only a subset of these varieties is available, identified by $A(t)$, which indicates the set of firms active in period t. The set of available products will expand as a result of innovation, and contract as a result of obsolescence. The households can purchase at time t all products of active firms at time t. Using the Dixit-Stiglitz specification, we let $x(j;t)$ denote the consumption of brand j at time t and define the elasticity of substitution between two products $e \equiv 1/(1-a) > 1$, to define the index D as:[1]

$$D(t) = \left[\int_{A(t)} x(j;t)^a \, dj \right]^{1/a} \tag{2}$$

A household spending an amount $E(t)$ at time t maximizes instantaneous utility by purchasing the number of units of brand j given in equation (3), where $p(j;t)$ is the price charged by firm j at time t.

$$x(j;t) = \frac{E(t)p(j;t)^{-e}}{\int_{A(t)} p(j';t)^{1-e} \, dj'} \tag{3}$$

The demand for a variety features a constant price elasticity of demand e and unitary expenditure elasticity. It can thus be aggregated across consumers to arrive at aggregate demand, where E represents aggregate spending. Defining an exact price index (see the appendix), the consumer's intertemporal optimization problem given in equation (1), under a budget constraint that allows

[1] An alternative interpretation, in which the index D is production and the varieties x are intermediate goods, is provided by Ethier (1982).

borrowing and lending at the interest rate $r(t)$, implies that the growth rate of spending is equal to the difference between the interest rate $r(t)$ and the discount rate r, that is $\dot{E}(t)/E(t) = r(t) - r$, where an overdot indicates the rate of change over time. Following Grossman and Helpman by normalizing aggregate spending to unity, that is $E(t) = 1$ for all t, implies that the interest rate is equal to the discount rate, that is $r(t) = r$ for all t.

Producer behavior

As indicated above, producers participate in three types of activities. First, they manufacture the varieties that have been developed in the past. Second, they spend resources on R&D in order to invent and introduce new varieties. Third, and most important for obsolescence, they have to maintain the production process in working condition.

Manufacturing

Each variety is produced by a single atomistic firm[2] under constant returns to scale. By choice of units, it requires one unit of labor to produce one unit of good x. To maintain the production process in working condition, each active firm has to incur a fixed labor cost. As explained in the introduction, the maintenance costs arise as a result of preventive maintenance, repair maintenance, updating, replacement, and the arrival of better alternatives. Following Romer (1990) and Grossman and Helpman (1991), we assume that part of the knowledge created in the economy, as measured by the range of active firms, results in non-appropriable benefits in other sectors of the economy. In particular, there are positive knowledge spill-overs for maintaining the production process at the time of invention and introduction of a new variety. As a result, the fixed maintenance costs in terms of labor, which depend on a parameter b, are inversely related to the range of active firms at the time of invention of the good. If we let $w(t)$ be the wage rate at time t and $m(.)$ denote the Lebesgue measure, such that $m(A(t))$ measures the range of active firms at time t, then the operating profits $p_j(t;t)$ for firm j at time t producing a variety invented at time t is given by:

$$p_j(t;t) = p(j;t)x(j;t) - w(t)x(j;t) - \frac{bw(t)}{m(A(t))} \tag{4}$$

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[2] This assumption can be justified in two ways. First, one could argue that inventions are protected by infinitely lived patents. Second, if imitation is costly and firms engage in ex post price competition, the imitator would earn no profits and consequently would not be able to recuperate its costs made.

Profit maximization and obsolescence

The monopolistic producer maximizes the operating profits, given the demand for its variety as derived in equation (3). Since the price elasticity of demand e is constant, this results in the well-known constant mark-up over marginal cost:

$$(1-1/e)\,p(j;t) = w(t), \qquad or \qquad p(j;t) = w(t)/a \equiv p(t) \tag{5}$$

Note that the optimal pricing rule is the same for all active firms at time t, and independent of the time t of invention of the variety. All firms active at time t will therefore sell an equal quantity of goods, and receive the same revenue. In view of the normalization of expenditure, we can therefore calculate the operating profits for all firms active at time t with a variety invented at time t:

$$p(t,t) = \frac{1-a}{m(A(t))} - \frac{bw(t)}{m(A(t))} \tag{6}$$

Naturally, the firm will only produce its variety invented at time t if the operating profits at time t are positive. Equivalently, the firm will stop production if the operating profits become negative. This allows us to determine the range of active firms at time t using the indicator function $I_A(t,s)$, defined to be equal to 1 if a firm producing a variety invented at time $t-s$ is still active at time t, and 0 otherwise.[3]

$$I_A(t,s) = \begin{cases} 1, & if \quad \dfrac{1-a}{m(A(t))} - \dfrac{bw(t)}{m(A(t-s))} > 0 \\ 0, & otherwise \end{cases} \tag{7}$$

Note that a firm with a variety invented at time t seizes to be active if the measure of active firms relative to the time of its invention exceeds a threshold level. If the range of active firms is non-decreasing and the wage rate is constant, as will be the case below, then the flow of firms from active to obsolete is on a first-in-first-out basis (FIFO). Equation (7) is called the *obsolescence criterion*.

The capital market

The profits generated in equation (6) go to the shareholders of a firm (for example in the form of dividends). If the stock markets correctly price the firms, the stock value $v(t,s)$ at time t of a firm

[3] Obviously, operating profits $p(t,s)$ in equation (6) are defined to be 0 if the firm is not active.

producing a variety invented at time s equals the present discounted value of its future stream of profits.[4] In view of our normalization, which implies $r(t) = r$, it is equal to:

$$v(t,s) = \int_{t}^{\infty} e^{-r(t-t)} p\,(t,s)\,dt \tag{8}$$

Recall that an overdot indicates the rate of change over time of a variable. If there are two time indices, as occurs frequently in the presentation since we have to distinguish between the time at which a firm is active and the time of invention of the variety, we let a subindex denote the time index. Differentiating equation (8) with respect to time t gives

$$\dot{v}_t(t,s) = rv(t,s) - p\,(t,s) \tag{9}$$

This represents a 'no-arbitrage condition' on the capital market, since the sum of the profits plus the capital gains are equal to the yield on a riskless loan.

Research and development

An entrepreneur can add to the range of active firms by inventing a new variety, which requires a finite amount of labor invested for a brief period of time into R&D. There is free entry and exit of entrepreneurs into the R&D sector. Following, for example, Romer (1990) and Grossman and Helpman (1991), R&D generates not only new varieties, the revenues of which are appropriated by the entrepreneur through claims on the future stream of profits generated by the firm, but also positive knowledge spill-overs in the form of increases in the general stock of knowledge. In our specification, these knowledge spill-overs reduce the amount of labor required for developing new varieties and for the maintenance of new varieties. It is well-known that the growth rate of the economy would stop without such beneficial knowledge spill-overs. See Van Marrewijk (1999) and Funke and Strulik (2000) for a general discussion of the literature. If we let $N(t)$ denote the range of all varieties invented up to time t, we assume therefore that an entrepreneur denoting $L_n(t)$ laborers to R&D for a time period dt develops $dN = [m(A(t))L_n(t)/a]dt$ new products. The costs of a new blueprint at time t are therefore equal to $aw(t)/m(A(t))$. Given free entry and exit in the entrepreneurial market at time t, these costs must be at least as high as the value $v(t,t)$ at time t of developing a new variety:

[4] As Grossman and Helpman (1991, p. 50) note, this is not an assumption but an equilibrium condition in a perfect foresight model with infinite lived households maximizing lifetime utility, since speculative bubbles cannot arise. The presentation in the text is somewhat simpler.

$$\frac{w(t)a}{m(A(t))} \geq v(t,t), \qquad with\, equality\ if\ \ \dot{N}(t) > 0 \tag{10}$$

Labor market equilibrium

Finally, we turn to the labor market equilibrium. The labor force is active in three types of activities. There is labor demand L_n to develop new varieties in the R&D sector, labor demand L_x for the production of goods, and labor demand L_m for the maintenance costs. The constant labor supply L is provided perfectly inelastically. Equilibrium in the labor market therefore requires

$$L_n + L_x + L_m = L \tag{11}$$

First, note that the required number of R&D laborers depends on the speed $\dot{N}/N$ with which new products are developed: $L_n = a(\dot{N}/N)(N/m(A))$. Second, note that each firm sells $1/pm(A)$ units of goods. Since *m(A)* firms are active, they need *1/p* units of production labor. Third, note that if a firm with a variety invented at time $t - t$ is still active at time *t*, the maintenance labor requirement for that firm equals $b/m(A(t-t)$. Since the number of such firms depends on the speed at which new varieties were developed at time $t - t$, there are $L_n(t-t)m(A(t-t))/a$ such firms. The total maintenance labor required for firms still active at time t with a variety invented at time $t - t$ is therefore $L_n(t-t)b/a$ units. Using the indicator function $I_A(t,t)$ defined in equation (7), it follows that the total maintenance labor requirement at time t is given in equation (12). The labor market clearing condition is therefore given in equation (11')

$$L_m(t) = \int_0^{\infty} L_n(t-t)(b/a)I_A(t,t)\,dt = \int_0^{\infty} \frac{bN(t-t)}{m(A(t-t))}\frac{\dot{N}(t-t)}{N(t-t)} I_A(t,t)\,dt \tag{12}$$

$$\frac{aN(t)}{m(A(t))}\frac{\dot{N}(t)}{N(t)} + \frac{1}{p(t)} + \int_0^{\infty} \frac{bN(t-t)}{m(A(t-t))}\frac{\dot{N}(t-t)}{N(t-t)} I_A(t,t)\,dt = L \tag{11'}$$

This completes the description of the model.

3 Derivation of balanced growth equilibrium

We want to discuss some aspects of the model by analyzing a balanced growth equilibrium in which the measure of active firms grows at a constant rate *g*, that is $m(A(t)) = m_0 e^{gt}$. The distribution of labor over the three types of activities, production, maintenance, and R&D, will be constant in the balanced growth equilibrium. This implies, as the appendix shows, that the wage rate w is constant over time, which implies in turn, using the mark-up pricing rule, that the price p charged for a variety of a good is constant as well.

Obsolescence and active production

Combining the constant growth rate g of the number of active firms and the constant wage rate w with the obsolescence criterion derived in the previous section allows us to explicitly calculate how long a variety invented at time t will be actively and profitably used. Recall equation (6) on the operating profits for all firms active at time t with a variety invented at time t (using the fact that the wage rate w will be constant):

$$p(t,t) = \frac{1-a}{m(A(t))} - \frac{bw}{m(A(t))} \tag{6'}$$

Clearly, the first part of the operating profits on the right-hand-side of equation (6') will decrease slowly over time as the number of active firms on the market is expanding. In contrast, the second term on the right-hand-side of equation (6'), representing the costs of maintenance, is constant. The value of this constant depends on the number of active firms on the market at the time of the invention of the variety. These costs are therefore lower the newer the production process. As described in the introduction, the maintenance costs are therefore higher for older production processes. As soon as the first part of the operating profits is not high enough to recuperate the maintenance costs, the firm will stop the production process. If the growth rate of the number of active firms is g, it is straightforward to calculate the number of time periods f in which the firm will actively produce a new variety using equation (6'), which gives

$$e^{fg} = \frac{1-a}{bw} \quad \Rightarrow \quad f = \frac{\ln[(1-a)/bw]}{g} \equiv f(g,w) \tag{13}$$

The explicit definition in equation (13) of the time period f as a function of the growth rate g and the wage rate w serves as a reminder that we still have to (endogenously) determine the value of these variables. Note also from equation (13) that, other things equal, the period of active production f is longer:

- The lower the growth rate g. If the growth rate g of the number of active firm falls, the firm's profits are less rapidly eroded, which means that the firm can stay in business for a longer period of time.
- The lower the maintenance cost parameter b. The firm is ultimately driven out of business because the maintenance costs become too high relative to the revenue generated by the mark-up over marginal costs. Clearly, therefore, if the maintenance cost parameter b falls, the firm can stay in business for a longer time period. In the limit, as b approaches 0, the firm can stay in business indefinitely.
- The lower the wage rate w. The maintenance costs are directly influenced by the wage rate. A fall in the wage rate therefore allows the firm to stay in business for a longer time period by reducing the maintenance costs.
- The lower the elasticity of substitution parameter a (equivalently, the lower the price elasticity of demand e). If the different varieties are less perfect substitutes for one another, that is if the elasticity of substitution falls, the firm is able to charge a higher mark-up over marginal costs, which increases its operating profits. Again, this allows the firm to stay in business for a longer time period.

LE line (Labor market Equilibrium)

The labor market equilibrium is already given in equation (11'). We can simplify this equation considerably along a balanced growth path in which the growth rate g of the number of varieties N ever invented is equal to the growth rate of obsolete varieties and the growth rate of the number of active firms. Since $L_n = a(\dot{N}/N)(N/m(A))$, this implies that the labor input in the R&D sector is constant because $\dot{N}/N = g$ and the ratio $N/m(A)$ does not change. Let $s_0 \equiv m(A(0))/N(0)$ be the initial share of active firms, then it follows that $L_n = ag/s_0$. Determining the number of production workers is trivial since the wage rate is constant, such that $L_x = 1/p = a/w$. As for the demand for maintenance workers, we first note that as a result of the first-in-first-out nature of the number of actively produced varieties, the indicator function defined in equation (7) simplifies to:

$$I_A(t,t) = \begin{cases} 1, & 0 \le t \le f(g,w) \\ 0, & otherwise \end{cases} \tag{7'}$$

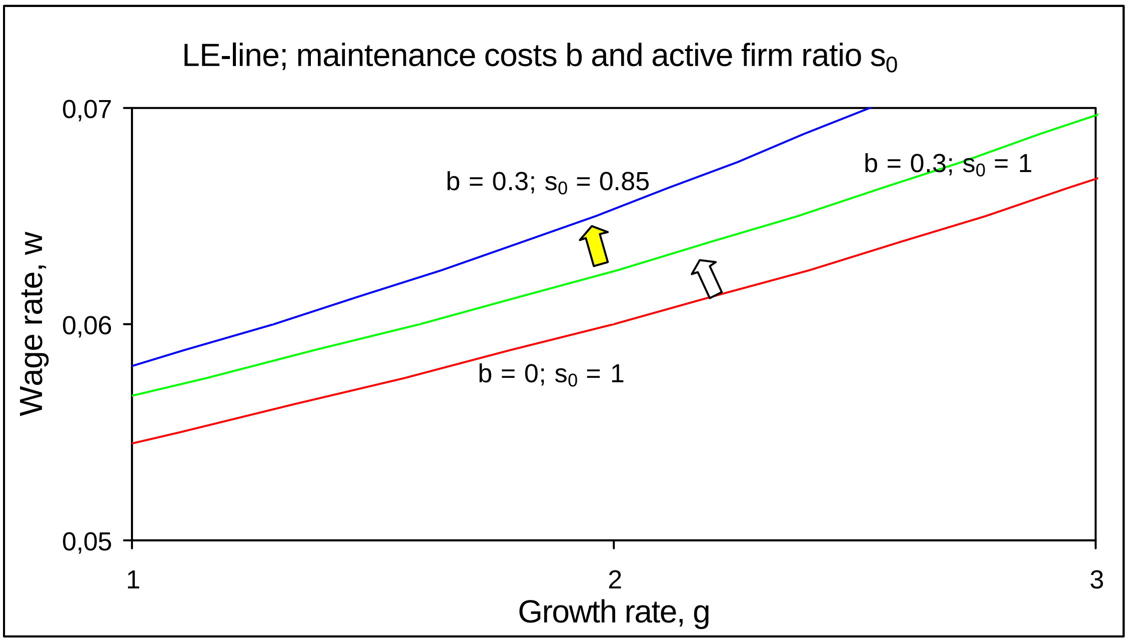
Using this in equation (12) and recalling that the number of workers in the R&D sector is constant gives the number of maintenance workers:

$$L_m = \int_0^\infty L_n(t-t)(b/a)I_A(t,t)\,dt = f(g,w)bL_n/a \tag{12'}$$

Using the demand for L_n derived above and the definition of $f(g,w)$ given in equation (13), it follows that $L_m = (b/s_0)\ln[(1-a)/bw]$. Equating these demands for labor to the supply of labor gives the *Labor Equilibrium line*:

$$\underbrace{\frac{ag}{s_0}}_{L_n} + \underbrace{\frac{a}{w}}_{L_x} + \underbrace{\frac{b}{s_0}\ln\left(\frac{1-a}{bw}\right)}_{L_m} = L \tag{14}$$

*Figure 1 Labor market Equilibrium (LE line)**



*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$

The labor market equilibrium is illustrated in (g,w)-space in Figure 1. As is evident from equation (14), if the wage rate rises fewer production workers are needed, since (using the mark-up pricing rule) the price charged by the firm for its variety rises, thus lowering the demand. These extra production workers can be used in the R&D sector to produce new varieties. This increases the growth rate g, such that the labor market equilibrium is upward sloping in (g,w)-space. Figure 1 shows three different LE lines. The first line, labeled "b = 0; s_0 = 1" displays the labor market equilibrium if there are no maintenance costs and all firms remain active indefinitely. This line therefore corresponds to the Grossman and Helpman (1991, ch. 3) model. The second line, labeled 'b = 0.3;

$s_0 = 1$" shows that, other things equal, the growth rate of the economy will fall if part of the work force is devoted to maintaining the production process, as indicated by the open arrow in Figure 1. The third line, labeled 'b = 0.3; $s_0 = 0.85$" shows that if, in addition, only a share of the firms remains active after *f* periods, this further reduces the growth rate of the economy, since this effectively reduces the productivity of the maintenance and R&D work force, see equation (14), as indicated by the shaded arrow in Figure 1.

IE line (Innovation Equilibrium)

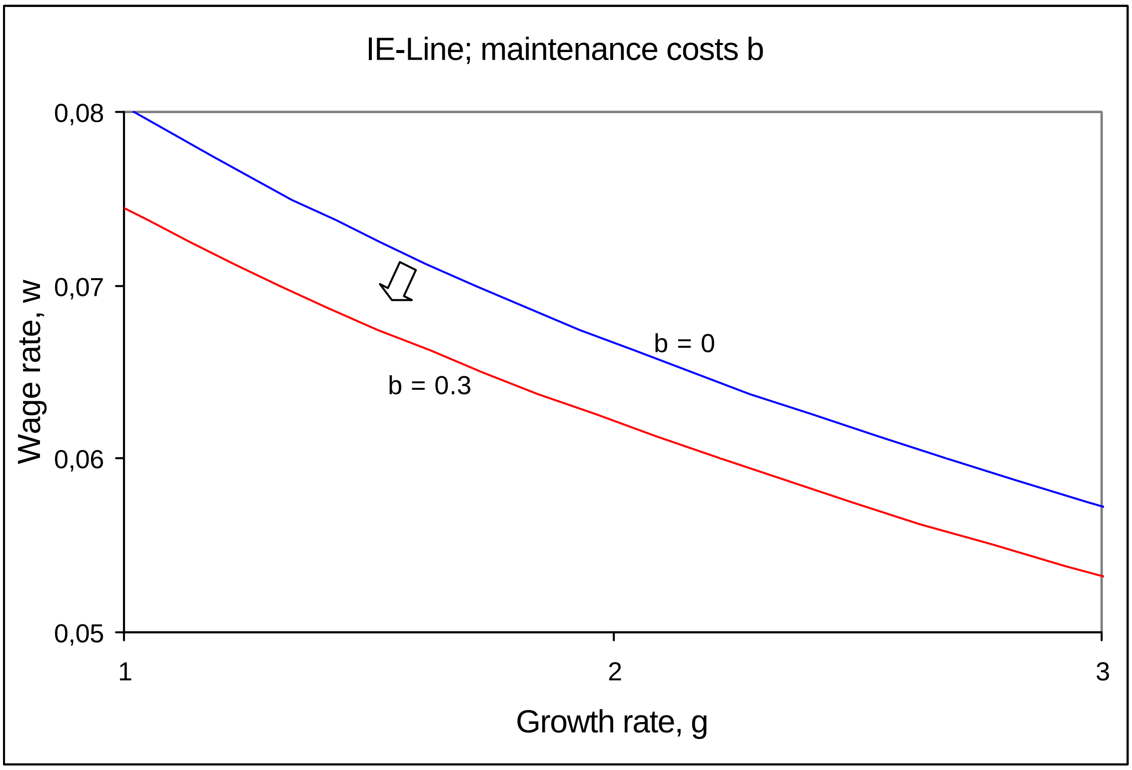
Now that we know from equation (13) the time period *f* during which the firm will be able to actively produce its goods and reap positive operating profits, we can also determine the present value of the stream of future profits, which determines the value of the firm for a variety invented at time *t*:

$$v(t,t) = \int_t^{t+f(g,w)} e^{-r(t-t)} \left[\frac{1-a}{m(A(t))} - \frac{wb}{m(A(t))} \right] dt = \tag{8'}$$
$$\frac{1}{m(A(t))} \left[\frac{1-a}{r+g} \left(1-e^{-(r+g)f(g,w)} \right) - \frac{wb}{r} \left(1-e^{-rf(g,w)} \right) \right] \equiv \frac{1}{m(A(t))} F(g,w)$$

For ease of reference we have defined the function F, which depends on the growth rate g and the wage rate w. Note that the value of the firm at the time a new variety is invented is inversely related to the number of active firms on the market at that time. Innovation takes place at time t if equation (10) holds with equality. Since the costs $aw/m(A)$ of inventing a new variety are also inversely related to the number of active firms at time t, this term drops out. Substituting equation (8'), in equation (10) gives the *Innovation Equilibrium line*:

$$wa = F(g,w) = \left[\frac{1-a}{r+g} \right] - bw \left[\frac{1}{r} - \frac{g}{r(r+g)} \left(\frac{wb}{1-a} \right)^{\frac{r}{g}} \right] \tag{15}$$

*Figure 2 Innovation Equilibrium (IE line)**



*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$

The innovation equilibrium is illustrated in (g,w)-space in Figure 2. Note that the innovation equilibrium line is more complicated than the labor market equilibrium line, since it can only be written as an implicit function (except when $b = 0$, see the next section). As is clear from the first part in square brackets on the right-hand-side of equation (15), an increase in the growth rate g erodes the operating profits more quickly, and thus reduces the profitability of new inventions. To restore the innovation equilibrium, the costs of inventing a new variety, as determined by the wage rate, will have to fall. Consequently, the innovation equilibrium is a downward sloping line in (g,w)-space. Figure 2 shows two IE lines. The first line, labeled "b = 0" displays the innovation equilibrium if there are no maintenance costs. This line therefore corresponds to the Grossman and Helpman (1991, ch. 3) model. The second line, labeled "b = 0.3" shows that, other things equal, the growth rate of the economy will fall if the profitability of R&D falls as a result of the costs of maintaining the production process, as indicated by the arrow in Figure 2.

Together, the labor market equilibrium and the innovation equilibrium, that is the LE line and the IE line, give two equations in the endogenous variables g and w and determine the balanced growth equilibrium. The next section analyzes this equilibrium.[5]

4 Maintenance costs and the balanced growth equilibrium

As derived in section 3, the balanced growth equilibrium is determined by the point of intersection of the labor market equilibrium and the innovation equilibrium, as given in equations (14) and (15), respectively. Obviously, an equilibrium is only economically useful if it is in the first quadrant, such that the wage rate and the growth rate of the economy are both positive. Otherwise, the equilibrium growth rate of the economy is zero, innovation does not take place, the share of active firms is constant, and firms produce forever.

No maintenance costs (Grossman-Helpman model)

In the Grossman-Helpman model, there are no maintenance costs, such that $b=0$ and firms are active indefinitely ($s_0=1$). In that case, equations (14) and (15) simplify to:

$$ag+\frac{a}{w}=L \tag{14'}$$

$$wa=\frac{1-a}{r+g} \tag{15'}$$

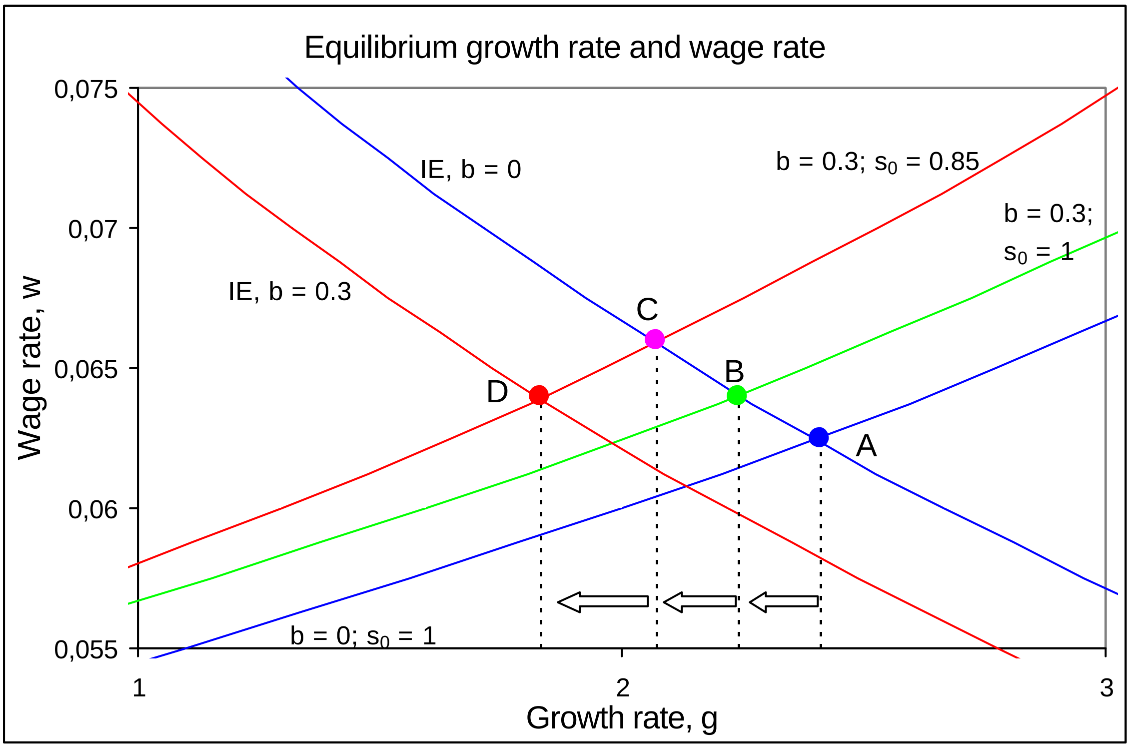
Both the labor market equilibrium and the innovation equilibrium can be written as explicit functions in (g,w)-space. It is straightforward to solve for the balanced growth equilibrium:

$$g\big|_{b=0}=(1-a)\frac{L}{a}-a\,r; \qquad w\big|_{b=0}=\frac{1}{ar+L} \tag{16}$$

We thus provide an alternative method to deriving the Grossman-Helpman equilibrium. Note that the rate of innovation is larger, the higher the effective labor force L/a and the lower the discount rate r.

[5] It should be pointed out that the analysis in the sequel ignores the transition dynamics by implicitly assuming, as is customary in this type of research, that the growth rate g determined by the intersection of the IE line and the LE line also held for the f periods prior to period 0. We leave this for future research.

*Figure 3 Determination of the balanced growth equilibrium**



*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$

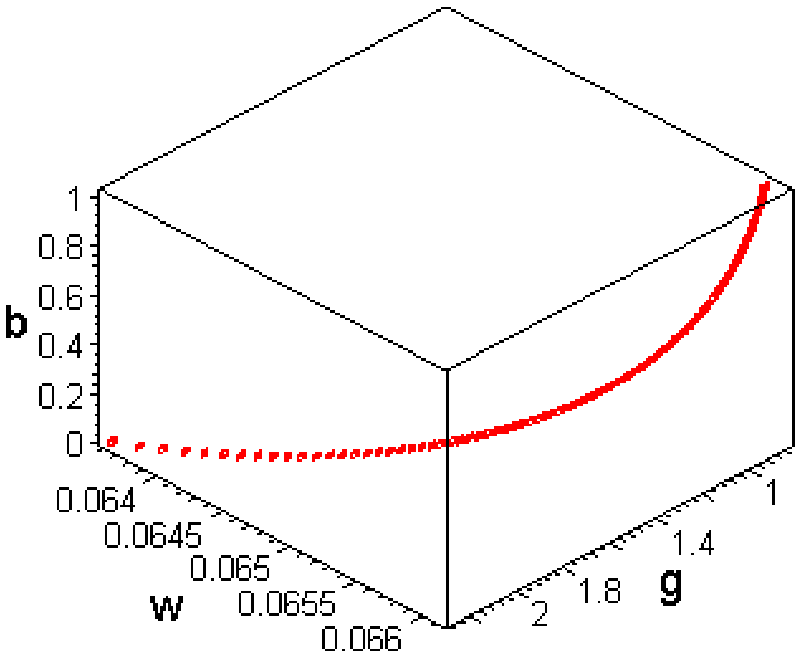
Impact of maintenance costs and obsolescence

To discuss the impact of positive maintenance costs and obsolescence, we compare the balanced growth equilibrium of equations (14) and (15) with the equilibrium of equation (16), the point of intersection of the lines 'b = 0; s_0 =1" and 'IE, b = 0' as illustrated by point *A* in Figure 3. In the presence of maintenance costs, we can distinguish between three different effects. First, as discussed in section 3, an increase in the maintenance costs *b* implies that more workers have to maintain the production processes in working condition, such that fewer workers are available for research to develop new varieties. This shifts the labor market equilibrium line up from 'b = 0; s_0 =1" to 'b = 0.3; s_0 =1", such that the equilibrium moves from point *A* to point *B*, with a lower growth rate and a higher wage rate. Second, if after *f* time periods a share of the firms becomes obsolete, this reduces the productivity of the labor force for research and maintenance. This shifts the labor market equilibrium line up even further, from 'b = 0.3; s_0 =1" to 'b = 0.3; s_0 =0.85", such that the equilibrium moves from point *B* to point *C*, further lowering the growth rate and increasing the wage rate. Third, an increase in the maintenance costs reduces the firm's profitability, which shifts the innovation equilibrium line down from 'IE, b = 0' to 'IE, b = 0.3", moving the equilibrium from point *C* to point *D*, this time lowering the wage rate and further reducing the growth rate. The growth rate of the economy is therefore lower as a result of maintenance costs and obsolescence. The wage rate may either rise or fall.

The equilibrium as a function of maintenance costs

The discussion above, illustrated in Figure 3, gives only the result of one balanced growth equilibrium. We argued that an increase in the maintenance costs will decrease the growth rate of the economy, while the effect on the wage rate is ambivalent. To get a better view of this claim, we calculated the "point *D*" equilibrium of Figure 3 for many different values of the maintenance costs b.[6] The results are depicted in the three-dimensional Figure 4, with the maintenance costs, growth rates and wage rates on the axes.

*Figure 4 Balanced growth and maintenance costs I**



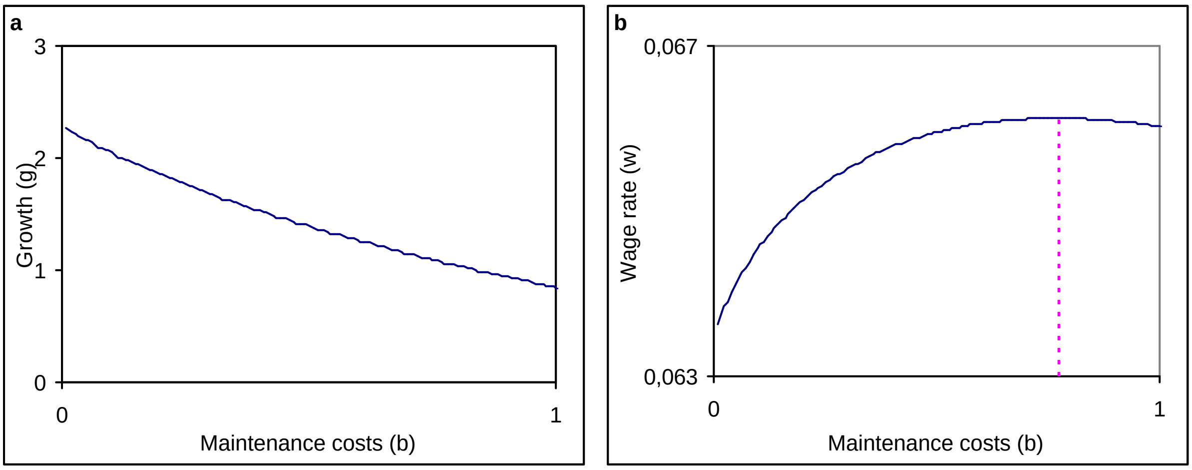
* Other parameters: $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$.

The information is summarized in two dimensions in Figure 5, depicting all equilibrium combinations in *(b,g)*-space and *(b,w)*-space. When maintenance costs are zero, economic growth is highest (in this setting around 2.3%). As the maintenance costs rise the rate of innovation decreases (in accordance

[6] We are grateful to Roel Stroeker for valuable assistance with this endeavor.

with the graphical results obtained in Figure 3), both because innovation becomes less profitable and because a larger share of the labor force is engaged in maintenance activities, and therefore no longer available for production or R&D. The wage rate increases for a large set of maintenance costs, as depicted in Figure 5b. Since $w = a / L_x$, this indicates that the share of the workforce engaged in production activities initially declines as maintenance costs increase until a minimum level is reached, after which they start to rise again (see also Figure 6).

*Figure 5 Balanced growth and maintenance costs II**

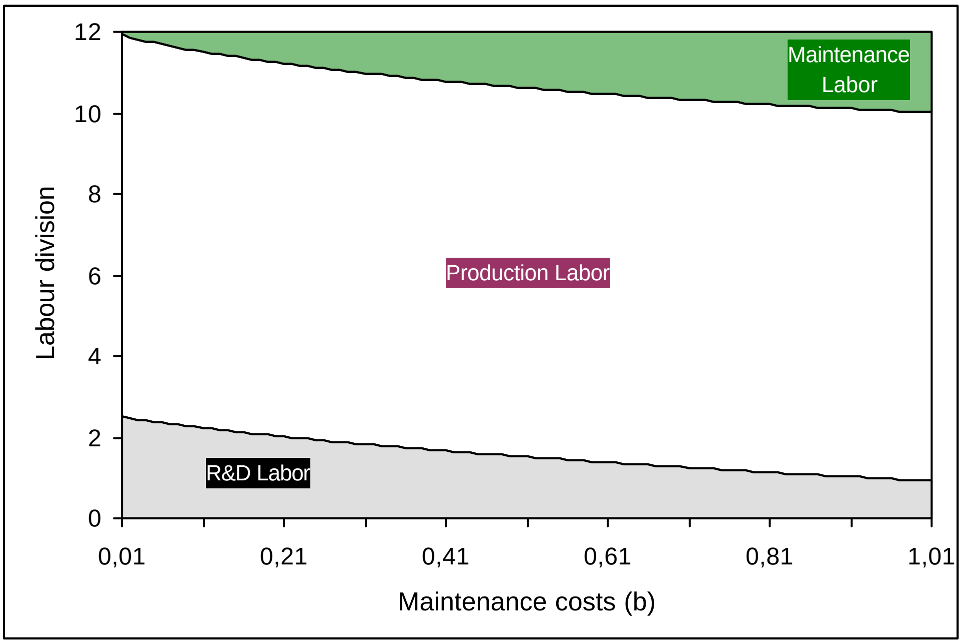


*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$. The dashed line indicates the maximum wage rate.

Maintenance costs and the division of labor

Higher maintenance costs have, of course, an impact on the division of labor between the three types of activities: production, maintenance, and R&D. This is illustrated in Figure 6. Obviously, as the costs of maintenance (measured in terms of the labor requirement) rises, the share of labor devoted to maintenance activities also rises. Simultaneously, the share of labor devoted to R&D activities (and the rate of innovation) falls, as R&D becomes less profitable. Both effects appear to be monotonic, that is we have not found counterexamples in the simulations we performed. As argued above, the effect on the share of labor devoted to production is ambivalent, although usually the increase in the share of labor devoted to maintenance dominates the decrease in the share of labor devoted to R&D, thus leading to a reduction in the share of labor devoted to production activities.

*Figure 6 Division of labor as a function of maintenance costs**



*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$.

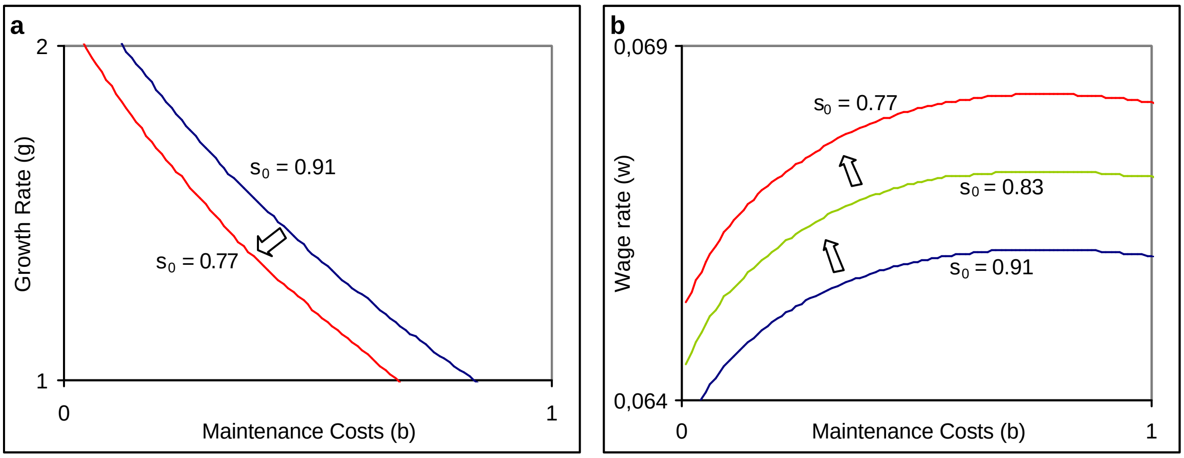
5 Impact of other parameters, obsolescence, and welfare

In this section we, first, discuss the impact of the share of active firms and the elasticity of substitution on the balanced growth equilibrium, and, second, the effect of maintenance costs on the speed of obsolescence of new goods and services and on the welfare level achieved in the economy.

Share of active firms

We argued in section 4 that a fall in the share of initially active firms s_0 is equivalent to a simultaneous fall in the productivity of the work force in the maintenance sector and the R&D sector. Clearly, then, a fall in the share of active firms will reduce the growth rate of the economy. This is illustrated in Figure 7a, indicating that for each value of maintenance costs, a decrease in the share of active firms reduces the growth rate of the economy. Since the total labor force does not change and productivity in the R&D sector and the maintenance sector falls if the share of active firms falls, this pulls away laborers from the production of final goods, thus increasing the wage rate (recall $w = a / L_x$), as illustrated in Figure 7b.

Figure 7 Balanced growth, maintenance costs, and the share of active firms

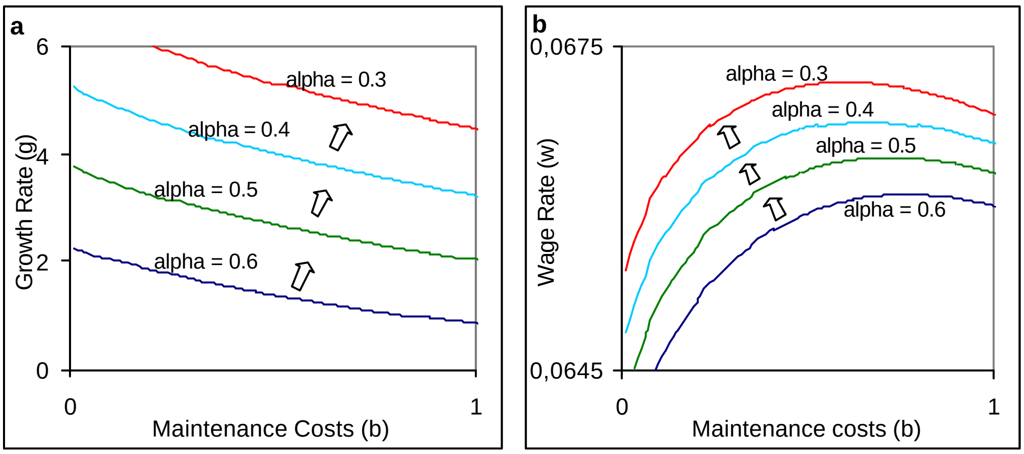


*Other parameters: $a = 0.6$; $L = 12$; $r = 4$; $a = 1$.

Elasticity of substitution $e = 1/(1-a)$

Another important parameter in this framework is the elasticity of substitution between different varieties, as measured by the parameter a. Its impact is straightforward to understand. The lower a, the harder it is to substitute between different varieties of goods and services and the lower the price elasticity of demand. This allows the firms to charge a higher price relative to marginal costs (see the mark-up pricing rule), thus increasing the profitability of inventing and introducing a new variety. The increase in profitability implies that more resources will be shifted into the R&D sector, thus increasing the rate of innovation, as illustrated in Figure 8a. Since $w = a / L_x$ and a lower value of a increases the rate of innovation and thus, other things equal, decreases the share of production labor (L_x) the effect on the wage rate is a priori ambivalent. Figure 8b shows that the latter effect dominates.

Figure 8 Balanced growth, maintenance costs, and the elasticity of substitution

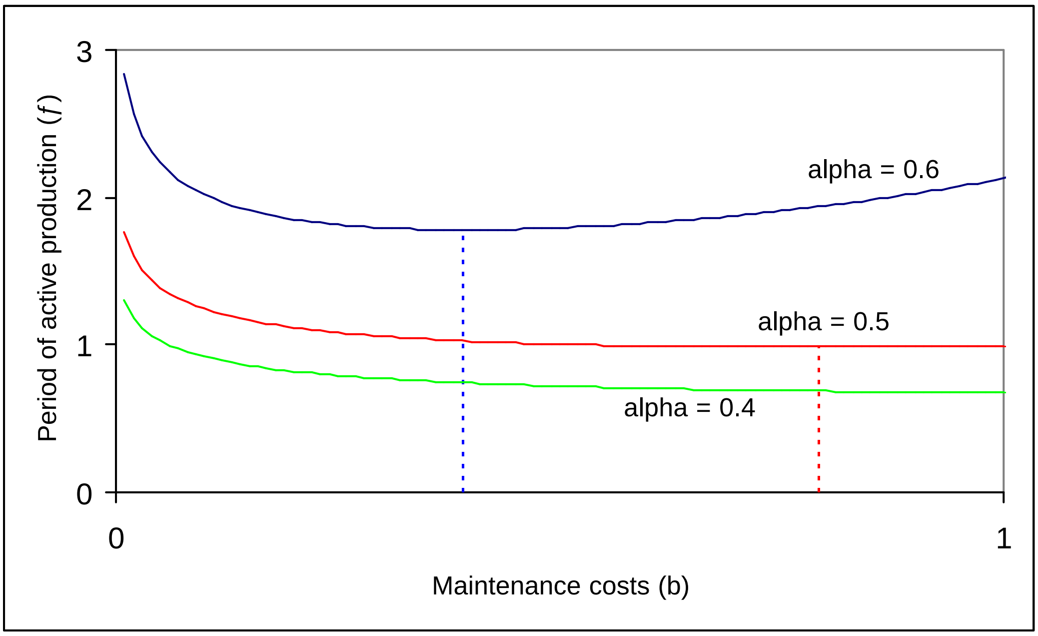


*Other parameters: $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$.

Obsolescence

One of the main implications of incorporating maintenance costs in the expanding variety endogenous growth model is the fact that newly developed goods and services eventually become obsolete and are no longer produced. It was shown in a partial equilibrium setting in section 3, that is keeping other things equal, that the period of active production f is longer if (i) the rate of innovation is lower, (ii) the maintenance costs are lower, (iii) the wage rate is lower, and (iv) the elasticity of substitution is lower. Two of these variables, namely the rate of innovation and the wage rate, are determined within the balanced growth equilibrium of the model, such that it is time to investigate the obsolescence criterion in a general equilibrium setting for the two remaining variables, that is the maintenance costs and the elasticity of substitution. This is illustrated in Figure 9.

*Figure 9 Period of active production as a function of maintenance costs**

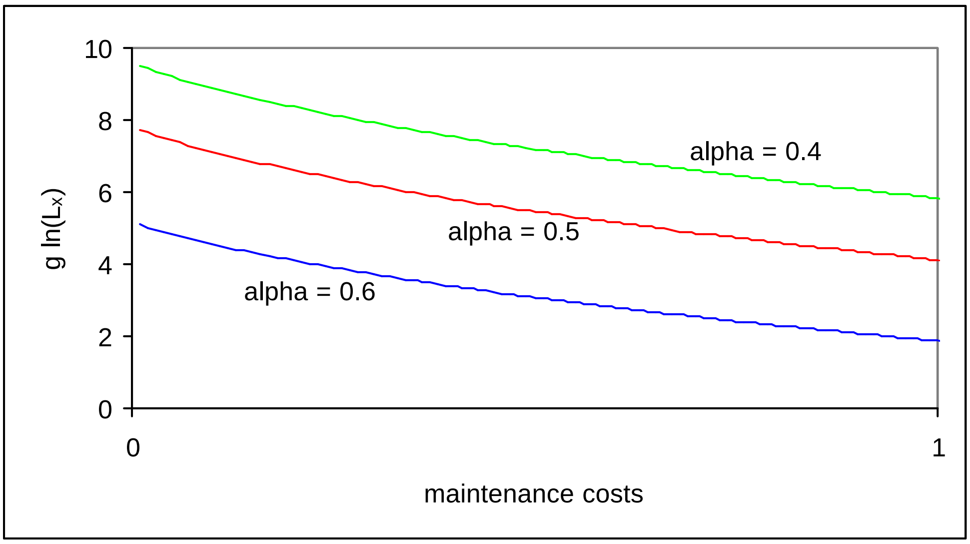


* Other parameters: $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$. The dashed line indicates where f reaches a minimum.

The direct effect of an increase in the maintenance costs is to reduce the period f of active production. There are two indirect effects of a change in the maintenance costs, namely through the rate of innovation and the wage rate. Section 4 showed that an increase in the maintenance costs will reduce the rate of innovation and generally results in a rise in the wage rate, at least up to a certain point. From the partial equilibrium analysis of section 3, the former indirect effect will increase the period of active production while the latter indirect effect will decrease it. The total effect is therefore ambivalent. Figure 9 shows that the direct effect of an increase in the maintenance costs usually dominates, thus reducing the period of active production f and increasing the speed of obsolescence. However, at high levels of maintenance costs, the indirect effect of the reduction in the rate of innovation starts to dominate, thus increasing the period of active production f (in the range of maintenance costs depicted this holds for $a = 0.6$ and $a = 0.5$, but not for $a = 0.4$).

Recall that the partial equilibrium effect of a reduction in the elasticity of substitution is to increase the period of active production f. Figure 9 displays the relationship between the maintenance costs and the period of active production for three separate values of the elasticity of substitution, clearly demonstrating a reduction in the period of active production if the elasticity of substitution falls, in contrast to the partial equilibrium effect. This can be understood from the analysis at the beginning of this section, showing that a reduction in the elasticity of substitution increases both the rate of innovation and the wage rate, both of which will reduce the period of active production. In this case, the indirect effects of a change in the elasticity of substitution dominate.

*Figure 10 Balanced growth, maintenance costs, elasticity of substitution, and welfare**



* Other parameters: $L = 12$; $r = 4$; $a = 1$; $s_0 = 0.91$.

Welfare

The final issue to address is the impact of incorporating maintenance costs on the welfare level achieved by the economy in the balanced growth equilibrium. Here we did not find any surprises. As shown in the appendix, for a given level of the elasticity of substitution, the welfare level achieved by the economy in the balanced growth equilibrium is proportional to $g\ln(L_x)$, which is illustrated in Figure 10 for three different values of the elasticity of substitution.[7] An increase in maintenance costs, which reduces the share of workers available for production and R&D and reduces the profitability of R&D, leads to a reduction in the rate of innovation, and thus to a reduction in the welfare level of the economy. The fact that, for high levels of maintenance costs, the share of the workforce L_x engaged in production may rise a little bit if the maintenance costs increase (and the degree of obsolescence falls) is never powerful enough to lead to an increase in welfare in any of our simulations.

6 Conclusion

We analyze the impact of obsolescence of economic inventions by incorporating maintenance costs in the endogenous growth model of expanding product varieties. This contrasts with the existing literature, which ignores maintenance costs and uses the model of quality improvements to describe obsolescence. Firms invest funds in R&D to invent and introduce new products continuously. The profitability of these new products diminishes over time as a result of the invention and introduction of even newer products, and as a result of the ever higher costs of maintaining the production process in working condition. If the maintenance costs become too high, the operating profits become negative and the firm stops producing the older varieties. We show that in a partial equilibrium framework, that is other things equal, the economic life span of innovations, that is the period during which a new variety is actually produced before the product becomes obsolete and is replaced by even newer varieties, is longer (i) the lower the growth rate of the economy, (ii) the lower the maintenance costs, (iii) the lower the wage rate, and (iv) the lower the elasticity of substitution between different varieties.

[7] Note that differences in the levels achieved in Figure 10 for different values of the elasticity of substitution cannot be interpreted as differences in the welfare level as changes in the elasticity of substitution directly affect the utility function.

"Other things" are, however, not equal. The rate of innovation and the wage rate are determined within the general equilibrium structure of the model, thereby affecting the speed of obsolescence indirectly. We derive and analyze a balanced growth equilibrium, in which the rate of innovation and obsolescence and the share of active firms is constant. We show that an increase in maintenance costs (i) reduces the rate of innovation, (ii) increases the wage rate up to a critical level (after which the wage starts to decline), (iii) reduces the period of active production of a newly invented variety (i.e. increases obsolescence) up to a critical level (after which the period of active production starts to rise), and (iv) reduces the welfare level. Initially, therefore, the direct effect of an increase in maintenance costs (speeding up obsolescence) dominates, while eventually the indirect effect of an increase in maintenance costs, which reduces the rate of innovation (and reduces obsolescence), dominates.

We also analyze the impact of a change in the elasticity of substitution between varieties on the balanced growth equilibrium. Since a reduction in the elasticity of substitution allows for higher mark-ups over marginal costs and higher operating profits, this increases the rate of innovation and the wage rate in the economy. Moreover, despite the fact that the direct effect of a reduction of the elasticity of substitution is an increase in the period of active production (reducing obsolescence) the two indirect effects (the increase in innovation and the wage rate) work in the other direction and reverse the direct effect, thus speeding up obsolescence, rather than reducing it.

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Appendix

Constant wage rate

Using the optimal pricing rule, we can simplify the exact price index p_d for the consumption of varieties, as defined in equation (A.1).

$$\text{(A.1)}\quad p_d(t) \equiv \left[\int_{A(t)} p(j';t)^{1-e}\, dj' \right]^{1/1-e} = m(A(t))^{1/(1-e)} w(t)/a$$

Writing this in relative changes gives: $\tilde{p}_d = \tilde{w} + \tilde{m}(A(t))/(1-e) = \tilde{w} + g/(1-e)$, where we used the assumption that the measure of active firms is growing at a constant rate g. If the amount of labor used in the production sector is constant, it follows from equation (2) that the consumption index D rises according to $\tilde{D} = g(1-a)/a$. Using the normalization of expenditure $E = p_d D = 1$, it follows that $\tilde{p}_d + \tilde{D} = 0$. Combining this information and using the fact that $-(1-a)/a = 1/(1-e)$, implies that $\tilde{w} = 0$.

Welfare

In the balanced growth equilibrium the measure m of active firms grows at the constant rate g. Since it requires 1 unit of labor to produce 1 unit of a variety, the labor force engaged in the production of final goods L_x is constant, and each variety actually produced at any point in time is produced at an equal quantity, we get $L_x = mx$. Using this in equation (2) gives:

$$\text{(A.2)}\quad D(t) = m(t)^{(1/a)-1}[m(t)x(t)] = \left(m(0)e^{gt}\right)^{1/(e-1)} L_x$$

To determine the welfare level, we substitute this information in equation (1):

$$U = \int_0^\infty e^{-rt} \ln(D(t))\, dt = \int_0^\infty e^{-rt} \ln\left[m(0)^{1/(e-1)} L_x e^{gt/(e-1)}\right] dt =$$

$$\text{(A.3)}\quad = \ln\left[m(0)^{1/(e-1)} L_x\right] \int_0^\infty e^{-rt} \ln(e^{gt/(e-1)})\, dt = \ln\left[m(0)^{1/(e-1)} L_x\right] \int_0^\infty e^{-rt} \frac{gt}{e-1} dt =$$

$$= \ln\left[m(0)^{1/(e-1)} L_x\right] \left[-\left(\frac{ge^{-rt}t}{(e-1)r} + \frac{ge^{-rt}}{(e-1)r^2} \right)\Bigg|_0^\infty \right] = \frac{g \ln\left[m(0)^{1/(e-1)} L_x\right]}{(e-1)r^2}$$

For given levels of ρ and ε, the welfare level is therefore proportional to $g \ln(L_x)$.