THO LE-DUC

Design and Control of Efficient Order Picking Processes

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Ontwerp en besturing van efficiënte orderverzamelingsprocessen

Thesis

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Tho Le-Duc – Lê Đức Thọ

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Introduction

Within a logistics chain, products (raw material, goods-in-process, finished goods) need to be physically moved from one location to another (i.e. at and between point origin and point of consumption, from manufacturers to end users). During this process, they may be buffered or stored at certain places (*warehouses*) for a certain period of time. Many activities are carried out in a warehouse. Among them, order picking (or order selection) - *the process of retrieving individual items (from storage locations) for the purpose of fulfilling an order for a customer*[1] - is the most critical one. It has long been identified as a very labor intensive operation in manual systems, and a very capital intensive operation in automated systems (Goetschalckx and Ashayeri, 1989). It may consume as much as 60% of all labor activities in the warehouse (Drury, 1988). And for a typical warehouse, the cost of order picking is estimated to be as much as 55% of the total warehouse operating expense (Tompkins et al., 2003). For these reasons, warehousing professionals consider order picking as the highest-priority activity for warehousing productivity improvements.

Since the last decade, electronic commerce, globalized economy and customers-oriented market have significantly changed the business environment. As a consequence, there have been several new trends in warehousing. Warehouses nowadays are more functional than before. The obvious role of warehousing is to store or buffer products, but warehouses nowadays provide other value-added activities or services as well. Example for these activities and services are product consolidating, cross-docking, quality checking, final assembling, packaging, refurbishing (reverse logistics), information services, etc. Warehouses are also becoming bigger. It is because of the fact that users are consolidating

[1] According to the Material Handling Institute of America

their distribution networks to reduce safety stock, to gain economics of scales, and to make the network easier to manage. It is also due to many manufacturers and whole sellers want to focus on their core business, and thus outsource entirely their warehousing activities. Consequently, products are often stored in central (often very large) warehouses of third-party logistics providers. Furthermore, with the growing success of e-commerce, warehouses nowadays often receive a large amount of small (i.e. few items) orders which have to be picked within tight time windows. Additionally, there are also other trends like small production lot-sizes, product customization, point-of-use delivery, revered logistics, and environmental protection. All in all, these new developments make warehouse operations in general and order picking in particular more complex and the study of warehousing becomes more vital for many companies nowadays.

As a united part of the logistics chain, order picking operations have an important impact on the chain performance. Any inefficiency in order picking can lead to unsatisfactory service and high operational cost for its warehouse, and consequently for the whole supply chain. In order to operate efficiently, the order process needs to be robustly designed and optimally controlled. The overall aim of the thesis is therefore to provide analytical models to support the design and control of efficient order picking processes. In particularly, the thesis addresses issues such as travel distance estimation, optimal layout design, order batching and zoning.

In this introductory chapter, we briefly highlight warehouse missions and functions in Section 1.1. We focus on order picking activities in Section 1.2, and review recent literature concerning the major issues in design and control of order picking processes in Section 1.3. Consequently, we introduce the research problems in Section 1.4. Finally, we give an outline of the thesis in Section 1.5.

1.1 Warehouse as an integral part of every logistics system

1.1.1 Missions of warehouses

Lambert et al. (1998) state that there are more than 750,000 warehouse facilities worldwide, including state-of-art, professionally managed warehouses, as well as company stockrooms and self-store facilities. Warehouses often involve large investments and operating costs (e.g. cost of land, facility equipment, labor). So, why do warehouses exist? They do exist to carry on one or more of the following missions (Lambert et al., 1998):

1. Achieve transportation economies (e.g. combine shipment, full-container load).
2. Achieve production economies (e.g. make-to-stock production policy).

3. Take advantage of quantity purchase discounts and forward buys.
4. Maintain a source of supply.
5. Support the firm's customer service policies.
6. Meet changing market conditions and again uncertainties (e.g. seasonality, demand fluctuations, competition).
7. Overcome the time and space differences that exist between producers and customers.
8. Accomplish least total cost logistics commensurate with a desired level of customer service.
9. Support the just-in-time programs of suppliers and customers.
10. Provide customers with a mix of products instead of a single product on each order (i.e. consolidation).
11. Provide temporary storage of material to be disposed or recycled (i.e. reverse logistics).
12. Provide a buffer location for trans-shipments (i.e. direct delivery, cross-docking).

Indeed, in some special situations (e.g. lean manufacturing, 'virtual' inventory), storage functions in a supply chain can be reduced. But, in almost all supply chains, raw materials, parts, and product inventories still need to be stored or buffered, implying that warehouses are needed and play a critical role in the companies' logistics success.

1.1.2 Warehouse operations

Figure 1.1 shows the typical functional areas and flows within warehouses. Three main functions are movement, storage, and information transfer.

- The *movement function* can be further divided into several activities: receiving, transfer and put away, order picking/selection, accumulation/sortation, cross-docking, shipping. The *receiving* activity includes the unloading of products from the transport carrier, updating the inventory record, inspection to find if there is any quantity or quality inconsistency. The *transfer and put away* involves the transfer of incoming products to storage locations. It may also include repackaging (e.g. full pallets to cases, standardized containers), and physical movements (from the receiving docks to different functional areas, between these areas, from these areas to the shipping docks). The *order picking/selection* involves the process of obtaining a right amount of the right products for a set of customer orders. It is the major activity in most warehouses. The *accumulation/sortation* of picked orders into individual (customer) orders is a necessary activity if the orders have been picked in batches. The *cross-docking* activity is performed when the received products are transferred directly to the shipping docks (short stays or services may be required but no order picking is needed).

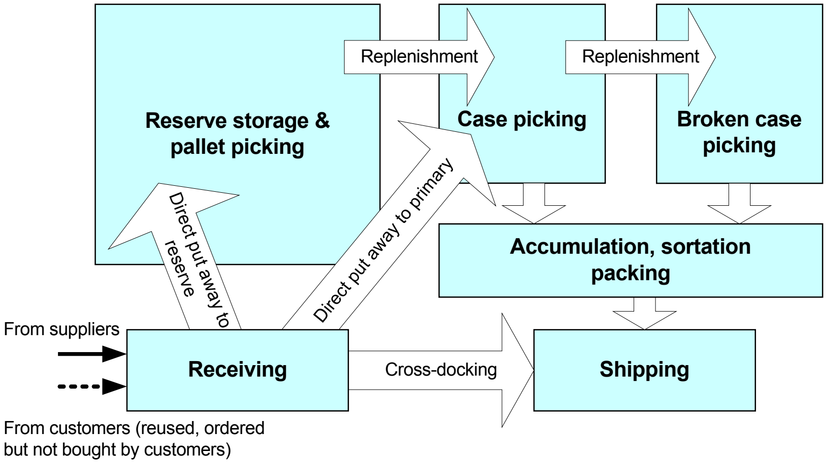

Figure 1.1 Typical warehouse functions and flows (*Tompkins et al., 2003*)

- The *storage function* is the physical containment of products while they are awaiting customer demands. The form of storage will depend on the size, quantity of the products stored, and the handling characteristic of products or their product carriers (Tompkins et al., 2003).
- The *information transfer* is the third function of warehousing; it occurs simultaneously with the movement and storage functions. Warehousing information (inventory level, stock-keeping locations, customer data, inbound, outbound shipments, etc.) is not only important for administering the warehouse operations itself but also for the efficiency of the whole supply chain.

1.2 Order picking

1.2.1 Order picking systems

As previously mentioned, order picking involves the process of clustering and scheduling the customer orders, releasing them to the floor, the picking of the items from storage locations and the disposal of the picked items. Many different order picking (OP) system types can be found in warehouses (often multiple OP systems are employed within one warehouse). Figure 1.2 distinguishes OP systems according to whether humans or automated machines are used. The majority of warehouses employ humans for order picking. Among these, the *picker-to-parts* system, where the order picker walks or drives along the aisles to pick items, is most common.

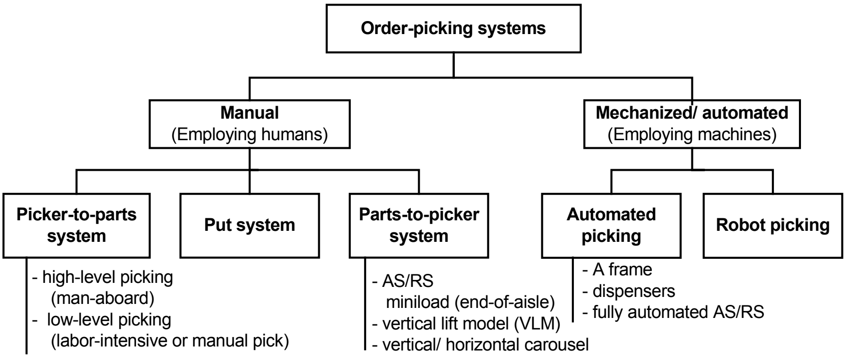

Figure 1.2 Classification of order picking systems (*based on De Koster, 2004*)

We can distinguish two types of picker-to-parts systems: *low-level* picking and *high-level* picking. In low-level OP systems, the order picker picks requested items from storage racks or bins (bin-shelving storage). It is similar to a shopper traveling up and down in a grocery store to fill a cart with one or several products. Because of the labor intensity, low-level OP systems sometimes are called *manual-pick* OP systems. Some other OP systems have high storage racks; order pickers travel to the pick locations on board of a lifting order-pick truck or crane. The crane automatically stops in front of the appropriate pick location and waits for the order picker to perform the pick. This type of system is called a high-level or a *man-aboard* OP system. *Parts-to-picker* systems include automated storage and retrieval systems (AS/RS), using mostly aisle-bound cranes that retrieve one or more unit loads (bins: mini-load system, or pallets) and bring the loads to a pick position (i.e. I/O point). At this position the order picker takes the number of pieces required by the customer order, after which the remaining load is stored again. This type of system is also called a *unit-load* or *end-of-aisle* OP system. The automated crane (also: *storage and retrieval (S/R) machine*) can work under different operating modes: *single, dual and multiple command cycles*. The single-command cycle means that either a load is moved from the I/O point to a rack location or from a rack location to the I/O point. In the dual-command mode, first a load is moved from the I/O point to the rack location and next another load is retrieved from the rack. In multiple command cycles, the S/R machines have more than one shuttle and can pick up several loads in one cycle, at the I/O point or retrieve them from rack locations. For example, in a four-command cycle (described in Sarker et al., 1994), the S/R machine leaves the I/O point with two storage loads, travels to the first storage location to store the first load. Then it proceeds to a retrieval location to retrieve a load by the recently-emptied shuttle, and travels to the next storage location to unload the remains storage load. And then it proceeds to a pick location to retrieve the second load. Finally it returns to the I/O point, after two storages and two retrievals. Other systems use *modular vertical lift modules (VLM), or carousels* that also offer unit loads to the order picker, who is responsible for taking the right quantity. There exist systems which combine the principles of parts-to-picker and picker-to-parts OP systems (referred as *put systems* in Figure 1.2). First, items have to be retrieved, which can be done in a parts-to-picker or picker-to-parts manner. Second, the carrier (usually a bin) with these 'parts' is offered to an order picker who distributes the parts over customer orders. Put systems are particularly popular in case a large number of customer order lines have to be picked in a short time window (for example at the Amazon Germany warehouse) and can result in about 500 picks on average per order picker hour (for small items) in well-managed systems (De Koster, 2004).

Manual-pick picker-to-parts systems are the most common (De Koster, 2004). The basic variants include picking by article (*batch picking*) or pick by order (*discrete picking*). In the case of picking by article, multiple customer orders (the batch) are picked simultaneously by an order picker. Many in-between variants exist, such as picking multiple orders followed by immediate sorting (on the pick cart) by the order picker (*sort-while-pick*), or the sorting takes place after the pick process has finished (*pick-and-sort*). Another basic variant is *zoning*, which means that a logical storage area (this might be a pallet storage area, but also the entire warehouse) is split in multiple parts, each with different order pickers. Depending on the picking strategy, zoning may be further classified into two types: progressive zoning and synchronized zoning. Under the *progressive* (or *sequential*) *zoning* strategy, each batch (possibly of one order) is processed only in one zone at a time; at any particular point in time each zone processes a batch that is different from the others. Hence, the batch is finished only after it sequentially visits all the zones containing its line items. Under the *synchronized zoning* strategy, all zone pickers can work on the same batch at the same time. There may be some idle time of zone pickers waiting until all other zone pickers finish the current batch. This synchronization of pickers intends to keep the batches from being mixed, and so to lessen the complexity of the following stages such as the accumulation and sortation. The term *wave picking* is used if orders for a common destination (for example, departure at a fixed time with a certain carrier) are released simultaneously for picking in multiple warehouse areas. Usually (but not necessarily) it is combined with batch picking. The batch size is determined based on the required time to pick the whole batch completely, often between 30 minutes to 2 hours (see Petersen, 2000). Order pickers pick continuously the requested items in their zones, and a next *picking-wave* can only start when the previous one is completed.

The design of real OP systems is often complicated, due a wide spectrum of external and internal factors which impact design choices. According to Goetschalckx and Ashayeri (1989) external factors that influence the OP choices include marketing channels, customer demand pattern, supplier replenishment pattern and inventory levels, the overall demand for a product, and the state of economy. Internal factors include system characteristics, organization and operational policies of OP systems. System characteristics consist of mechanization level, information availability and warehouse dimensionality (see Figure 1.3). Decision problems related to these factors are often concerned at the design stage. The organization and operational policies include mainly five factors: routing, storage, batching, zoning and order release mode. Figure 1.3 also shows the level of difficulty of OP systems; it is proportional to the distance of the representation of this problem in the axis system to the origin. In other words, the farther a system is located from the origin, the harder the system is to design and control.

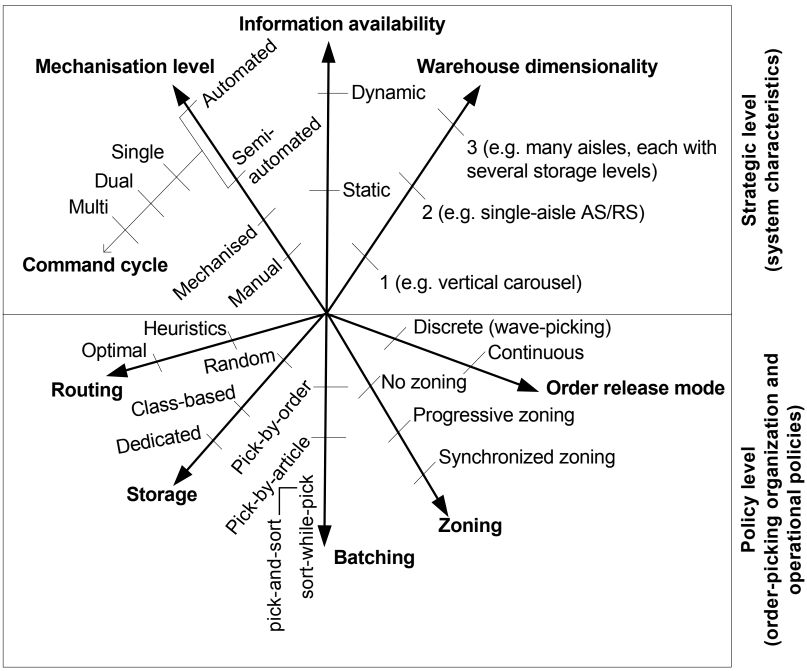

Figure 1.3 Complexity of order picking systems (*based on Goetschalckx and Ashayeri, 1989*)

In this thesis we limit ourselves to OP systems employing humans. More specifically, we will consider (low-level) manual-pick OP in Chapters 2, 3, 5 and 6, and AS/RS in Chapter

4. Manual-pick systems and AS/RS are present in most warehouses. Automated and robotized picking is only used in special cases (e.g. valuable, small and delicate items).

1.2.2 Order picking planning goals

The most common objective of OP systems is to maximize the service level subject to resource constraints such as labor, machines, and capital (Goetschalckx and Ashayeri, 1989). The service level is composed of a variety of factors such as average and variation of order delivery time, order integrity, and accuracy. A crucial link between order picking and service level is that the faster an order can be retrieved, the sooner it is available for shipping to the customer. If an order misses its shipping due time, it may have to wait until the next shipping period or is subject to expedition cost. Minimizing the order retrieval time (or picking time) is, therefore, a need for any OP system. Figure 1.4 shows the OP time components in a typical distribution centre: about 50% of the OP time is the travel time. The travel time to retrieve an order is a direct expense, but does not add value (Bartholdi and Hackman, 2003). For these reasons, in many OP situations, minimizing travel time is chosen as an objective for the improvement.

For manual-pick OP systems it is usually realistic to assume that the travel time is an increasing function of the travel distance (see for example: Jarvis and McDowell, 1991, Hall, 1993, Petersen, 1999, Roodbergen and De Koster, 2001b, and Petersen and Aase, 2003). Consequently, the travel distance is often considered as a primary objective in warehouse (layout) design and optimization. Two types of travel distance are widely used in the OP literature: the average travel distance of a picking tour (or *average tour length*) and the total travel distance. However, it should be noted that, for a given pick load (a set of orders), minimizing the average tour length is equivalent to minimizing the total travel distance.

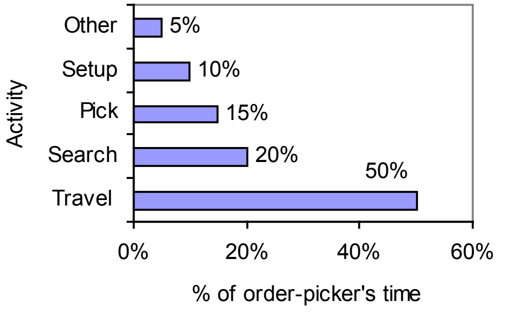

Figure 1.4 Typical distribution of an order picker's time (*Tompkins et al., 2003*)

Clearly, minimizing the average travel distance (or, equivalently, total travel distance) is only one of many possibilities. Another important objective would be minimizing the total cost (that may include both investment and operational costs). Other considerations which are often taken into consideration in warehouse design and optimization are:

- minimizing the throughput time of an order
- minimizing the overall throughput time (e.g. to complete a batch of orders)
- maximize the use of space
- maximize the use of equipment
- maximize the use of labor
- maximize the accessibility to all items

These objectives should be chosen according to their relevance to a certain situation. As we consider different planning and control problems, different objectives are concerned in this thesis. However, they are all closely related to the order throughput time. In Chapter 3, we consider minimizing the average pick tour length as the objective function for the problem of determining the optimal layout of the picking area (i.e. number of aisles, aisle's length). In Chapter 4, we use minimizing the average cycle time (of the S/R machine) by optimizing the rack's dimensions. In Chapter 5, we select minimizing the average throughput time of an order as the objective in order to determine the optimal picking batch size where orders arrive online and need to be picked in a short time. Finally in Chapter 6, we choose minimizing the overall system throughput time in order to determine the optimal number of zones in a pick-and-sort OP system.

1.3 Issues in planning and control of order picking processes

As shown in Rouwenhorst et al. (2000), issues in planning and control of OP processes can be on either tactical or operational level. From the organization perspective, common decisions at these levels are:

- layout design and dimensioning of storage system
- storage assignment
- batching and zoning
- routing
- order accumulation/sorting

In this section, we first give an introduction to the above decision issues and then mention briefly the related literature concerning these decisions. Issues in design and planning of warehousing systems have been reviewed and discussed in Ashayeri and Gelders (1985), Cormier and Gunn (1992), Cormier (1997), Van den Berg (1999), Van den Berg and Zijm (1999), and Rouwenhorst et al. (2000). Issues in design and control of OP processes in particularly are mentioned in Goetschalckx and Ashayeri (1989), Choe and Sharp (1991), Roodbergen (2001), and Wäscher (2004). An extensive bibliography on OP systems is gathered in Goetschalckx and Wei (1994), Roodbergen (1999, 2001) and Le-Duc and De Koster (2005d). Bellow we call for these publications and update them whenever applicable.

1.3.1 Layout design problem

In the context of OP, the layout design concerns two sub-problems: the layout of the facility containing the OP system and the layout within the OP system. The first problem is usually called the *facility layout problem*; it concerns the decision of where to locate various departments (receiving, picking, sorting, and shipping, etc.). It is often carried out by taking into account the activity relationship between the departments. The common objective is minimizing the handling cost, which in many cases is represented by a linear function of the travel distance. We refer to Tompkins et al. (2003) for a description of several efficient layout design procedures and to Meller and Gau (1996) for an overview on this subject. In this thesis, we focus on the second sub-problem, which can also be called the *internal layout design* or *aisle configuration problem*. It concerns the determination of the number of blocks, and the number, length and width of aisles in each block of a picking area (or department). The common goal is to find a 'best' warehouse layout with respect to a certain objective function among the layouts which fit a given set of constraints and requirements. Again, the most common objective function is the travel distance. For example, in Chapter 3 of this thesis, we consider the problem of determining the number of aisles and the aisle length such that the average tour length is minimized.

An early publication concerning layout design for a manual OP system is by Bassan et al. (1980). They present several deterministic models for determining the warehouse's dimensions such that the handling distance, handling time, space utilization, or costs are minimized. Rosenblatt and Roll (1984), using both analytical and simulation methods, study the effect of storage policy (i.e. how to assign products to storage locations) on the internal layout of warehouse. Rosenblatt and Roll (1988) examine the effect of stochastic demands and different service levels on the warehouse layout and storage capacity. Recently, Roodbergen (2001) proposes a non-linear objective function (i.e. average travel time in terms of number of picks per route and pick aisles) for determining the aisle configuration for random storage strategy warehouses (including single and multiple blocks) that minimizes the average tour length. Also considering minimization the average tour length as the major objective, Caron et al. (2000) consider 2-block warehouses (i.e., one middle cross aisle) under the cube-order-index (COI)-based storage assignment (see Heskett (1963) for the definition and Section 1.3.2 for a discussion of storage assignment methods), while Le-Duc and De Koster (2005a,b) focus on the class-based storage assignment. For both random and volume-based storage assignment methods, Petersen (2002) shows, by using simulation, the effect of the aisle length and number of blocks on the total travel time.

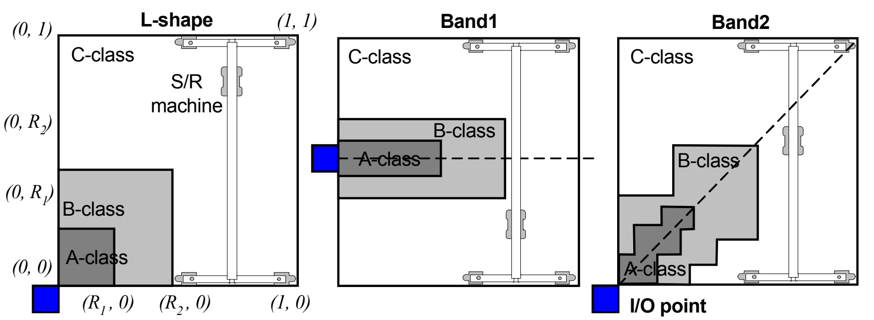

Figure 1.5 A size view of zone configurations in a rack with three product storage classes

Compared to the manual-pick OP systems, the layout design problem for AS/RS has received much attention. Most of the studies first develop a throughput or travel time model and then find the optimal rack dimensions such that the travel time (of the S/R machine) is minimized. For a literature review on the throughput and travel time models, we refer to Section 4.2 of this thesis. We will briefly mention here the literature particularly in designing the picking face. For random storage assignment, Bozer and White (1984) show that a square-in-time (SIT) rack (i.e. a rack with a ratio of height to length equals the ratio of the S/R machine vertical to horizontal velocity) is optimal for single and dual-command cycles. Hausman et al. (1976) consider the problem of finding class regions for an AS/RS using the class-based storage assignment method and the single-command operating mode (see Section 1.2.1 for the definitions of the S/R machine operating modes). The authors prove that L-shaped class regions where the boundaries of zones accommodating the corresponding classes are SIT (see Figure 1.5) are optimal with respect to minimizing the mean single-command travel time. They also analytically determine optimal storage class-sizes for two product classes in a SIT rack. Rosenblatt and Eynan (1989) extend Hausman et al. method to establish optimal class boundaries for any given number of classes in a SIT rack. Eynan and Rosenblatt (1994) extend this method further to any rectangular rack. For S/R machines with dual-command cycles and class-based storage racks, Graves et al. (1977) show by simulation that the L-shaped regions with SIT boundaries are not necessarily optimal. However, an L-shaped class allocation will in general be no more than 3% above the optimum. For multi-command cycles with class-based storage, Guenov and Raeside (1992) compare three zone shapes (L-shape, Band1, and Band2, see Figure 1.5) in an AS/RS. They conclude that L-shape and Band2 give best performance for the I/O point location located at the bottom left corner of the rack. Band2 appears to improve its performance when the number of picks per cycle increases. This means that the L-shaped zone boundaries should not be considered as the global optimal for multi-command cycles. Band1 may outperform the others in the case that the I/O point location is a half way between the two left corners of the rack.

1.3.2 Storage assignment problem

Items (or *stock keeping units - SKUs*) need to be put into storage locations before they can be picked to fill customer orders. A storage assignment method is a set of rules which are used to assign items to storage locations. The following storage assignment methods are mentioned in the literature.

Random storage assignment

This storage assignment method allocates items randomly over the available storage locations. This method is considered widely in the literature; in many studies, it is used as a benchmark for the improvement by using other storage assignment methods.

Closest-open-location storage assignment

In practice, incoming items (e.g. on a pallet) are often allocated to the closest empty location. ‘Closeness’ here is defined by the travel distance from the input/output (I/O) point (or depot) to the storage location. This is probably the simplest method and often used when order pickers have to choose storage locations themselves. As a result, items do not have a fixed location and, in the long run, their locations are scattered over the picking area. In some studies, it is showed that the random and closest-open-location method are converged in a long run (see, for example, Schwarz et al., 1978).

Dedicated storage assignment

With this storage assignment type, each item has its own storage location. To minimize the travel distance, the closest-to-depot storage locations are commonly reserved for items with a high turnover and little storage space occupation. An early type of this storage assignment method is the *COI-based* storage assignment, where the COI of an item is defined as the ratio of the required storage space to the order frequency of the item (see for example Heskett, 1963, Heskett, 1964, Kallina and Lynn, 1976, Malmborg and Bhaskaran, 1987, 1989, 1990, and Malmborg, 1995, 1996). The COI-based method sorts items by increasing COI ratio and locations on increasing distance from the I/O point. Next, items are assigned one by one to locations in this sequence (items with the next lowest COI ratio to next quickest-to-access locations). *Volume-based* (also: *frequency-based* or *turnover-based*) storage assignment is a different type of dedicated storage assignment method. It is studied by, for example, Petersen (1997), Petersen (1999), Petersen and Schmenner (1999), Petersen (2000), Petersen et al. (2004), and Petersen and Aase (2003). This method assigns items to storage locations according to their (expected) pick volume; usually high pick items are located closest to the I/O point. The pick volume of an item can be expressed in number of units or pick lines during a certain time horizon. The difference between this method and COI-based storage is that the volume-based assignment only considers the popularity of items, not their space occupation.

Class-based storage assignment (also: ABC-storage, group-based storage)

This method assigns items to storage locations on a group basis. It divides both items and storage locations into an identical number of classes. Item classes are based on turnover frequency (like pick lines per time unit, or product units picked per time unit). Figure 1.6 shows an example of a division of the items in three classes. The item classes are sorted on decreasing turnover frequency and the storage location classes on increasing travel distance from the I/O point. Next, the item classes are assigned to the storage location classes (which should be large enough to contain the SKUs) in this sequence. Within a storage class, items are randomly stored. The major difference between this method and the volume-based assignment method is that this method assigns items to storage locations based on a group basis, while the volume-based method uses an individual basis. Figure 1.7 shows some examples of allocating items in a warehouse by using the class-based storage assignment method. This method can be considered as a combination of the volume-based and randomized storage assignment method. However, compared to random storage, it provides a saving on travel distance. A drawback of this method is that it involves several issues that are not trivial to solve. The first issue is the problem of drawing the borders between product classes. In inventory control, a classical way for dividing items into classes based on popularity is Pareto's method. The idea is to group items into classes in such a way that the fastest moving class (A-class) contains only about 15% of the items stored but contributes to about 85% of the turnover[2]. In the literature, there is no firm rule to define a class partition strategy (number of classes, percentage of items per class, and percentage of the total pick volume per class). Usually, the number of item classes is restricted to 3 and item classes are named A, B and C (for fastest, medium and slowest moving items), that is why this method is also called the ABC-storage assignment. However, more classes are also possible and may reduce the travel distance further. For a (low-level) manual-pick OP system, Petersen et al. (2004) recommend that the number of classes should be between 2 and 4. While for AS/RS, Yang (1988) and Van

[2] It is based on an observation of the Italian sociologist and economist Vilfredo Pareto: *"85% of the wealth of the world is held by 15% of the people"*.

den Berg and Gademann (2000) found that (in their studies) 6-class is the best among other options. After the item classes have been clearly defined, for each item class, an appropriate amount of storage space and storage locations have to be determined. The amount of storage space per item depends on the item size, quantity stored and product carrier on which the item is stored. ABC-storage can be applied for the total stored quantity of the items, or for the quantity to be stored in a forward (pick) area only (meaning the bulk quantity is stored in a reserve storage area). In order to decide the ABC-location divisions, the closeness (to the I/O point) can be used. Since usually multiple aisles are present, the closeness depends on the routing method used. We devote Chapter 3 of this thesis to investigate this problem in-depth.

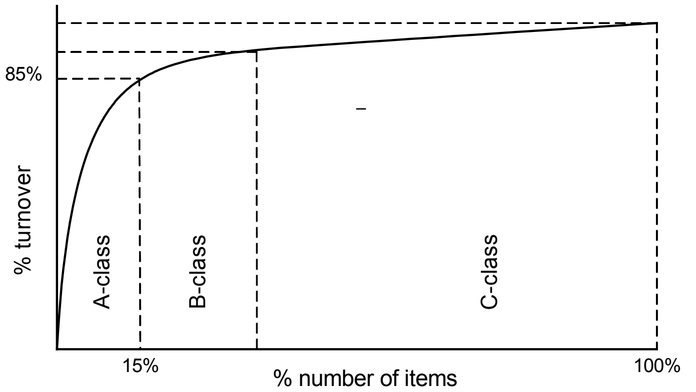

Figure 1.6 An example of the class partition strategy

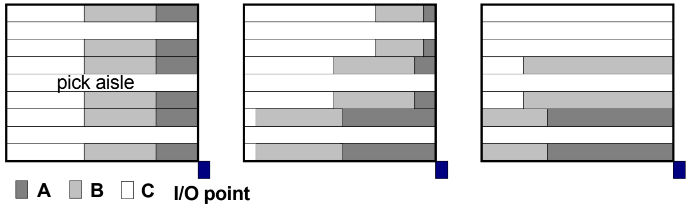

Figure 1.7 Examples of locating item classes in a warehouse

Besides the publications above-mentioned, the class-based storage assignment method is also considered in: Hausman et al. (1976), Graves et al. (1977), Schwarz et al. (1978), Park and Webster (1989), Rosenblatt and Eynan (1989), Goetschalckx and Ratliff (1990), Guenov and Raeside (1992), Eynan and Rosenblatt (1994), Kouvelis and Papanicolaou (1995), Malmborg (1996), Larson et al. (1997), Ashayeri et al. (2002, 2003), and Park et al. (2005) for parts-to-picker OP systems, and in Lee (1992), Jarvis and McDowell (1991), Tang and Chew (1997), Chew and Tang (1999), Le-Duc and De Koster (2005a, b, 2004a), and Roodbergen (2005) for picker-to-parts OP systems.

Family-grouping storage assignment

The idea of this type of storage assignment methods is that items that are likely to appear together on an order, or are likely to be picked in the same tour are stored close together, and by doing so the travel distance will be reduced. Another reason for items to be stored next to each other can also be: they are from the same supplier or same owner (for example, in the case of a service provider's warehouse). In order to group items the statistical correlation between items (e.g. frequency at which they appear together in an order, see Frazelle and Sharp, 1989, and Brynzér and Johansson, 1996) should be known or at least be predictable. This storage assignment method can also be used in combination with other methods. For example, we can group items into classes based on their statistical correlations, determine the turnover rate of each class, and then assign classes to storage locations based on their turnover rate. In the literature, two types of the family-grouping storage assignment are mentioned. The first method is called the *complimentary-based* method, which contains two major phases. In the first phase, it clusters the items into groups based on a measure of strength of joint demand ('complimentary'). In the second phase, it locates the items within one cluster as close to each other as possible (Wäscher, 2004). Rosenwein (1994) has shown that the clustering problem can be formulated as a *p-median* problem. For finding the position of clusters, Liu (1999) suggests that the item type with the largest demand should be assigned to the location closest to the I/O point (volume-based strategy), while Lee (1992) proposes to take into account also the space requirement (COI-based strategy). The second type of family-grouping method is called the *contact-based* method. This method is similar to the complimentary method, except it uses *contact frequencies* to cluster items into groups. For a given (optimal) routing solution, a contact frequency between item type i and item type j is defined as the number of times that an order picker picks either item type i directly after item type j, or item type j directly after item type i. However, the routing decision is dependent on the location of the item types, which demonstrates the strong interrelationship between item location and routing. Due to the fact that finding a joint optimal solution for both problems is not a realistic approach, at least not for problem instances of the size encountered in practice, contact-based solution methods alternate between the two problem types (Wäscher, 2004). The contact-based method is considered in, for example, Van Oudheusden et al. (1988), and Van Oudheusden and Zhu (1992). We refer to Wäscher (2004) for a thorough discussion of the complimentary-based and contact-based methods.

Comparison between different storage assignment methods

The random assignment method results in a high space utilization (or low space requirement) at the expense of increased travel distance (Choe and Sharp, 1991). For unit-load AS/RS, Graves et al. (1977) observe that in order to enable an incoming load to be stored in its class region, the space requirements increase with the number of classes. Accordingly, class-based storage requires more rack space than randomized storage, and dedicated storage requires more rack space than class-based storage. Roll and Rosenblatt (1983) and Rosenblatt and Roll (1984) compare the space requirements for the random, volume-based and class-based storage assignment for a port warehouse by using simulation. Their results show that the class-based storage assignment can significantly reduce the space requirement compared to the volume-based assignment.

Dedicated storage assignment methods (i.e. COI-based and volume-based) have several advantages compared to the other methods. First, they yield the largest saving in travel distance. Second, if each item has its own location, it is easier for the order picker to remember the item location (thus the searching time is substantially reduced). Additionally, it is possible to take into account physical item properties (like heavy items should be put at the low levels, while lighter ones can be put on top). The main disadvantage of the method is that a storage space may even be reserved for an obsolescent item. Therefore, the space utilization is often low in the environments where the product assortment frequently changes over time. Furthermore, according to Caron et al. (1998), the adoption of COI-based storage assignment is generally a more 'information intensive' approach than random storage, since order and storage data must be processed in order to rank and assign items by a increasing COI. The availability of low cost computer systems operating on large data-bases makes the above requirement negligible, especially if the dramatic improvements in picking efficiency which stem from the adoption of advanced stock location assignment policies are taken into account. However, COI-based storage really requires item locations to be constantly reviewed in order to maintain storage strictly based on the ratio of required space to order frequency which is always changing in a highly dynamic environment. Since in practice only periodic reviews are possible, this method usually does not perform well if demand varies from day to day. The class-based method is somewhere between the dedicated and random method, depending on other parameters like skewness of the demand, partition of classes, and routing method used. Schwarz et al. (1978), Kim and Seidmann (1990), Petersen et al. (2004), Le-Duc and De Koster (2005a,b, 2004a) and many others show that class-based storage leads to a reduction in travel distance (in both automated and manual-pick warehouses) compared to the random method. Compared to COI-based methods, it may result in a longer travel distance. Based on simulation experimental results, Petersen et al. (2004) show that with regards to the travel distance, volume-based storage outperforms class-based storage assignment. The gap between two methods depends on the class partition strategy (i.e. number of classes, percentage of the total volume per class) and the routing method used. However, they suggest using the class-based method in practice as it is easier to implement than the volume-based method; it does not require a complete list of the items ranked by volume and it requires less time to administer than the other dedicated methods do. All above-mentioned papers treat the demand as deterministic (i.e. the probability to visit a storage location is known or can be determined exactly). Thonemann and Brandeu (1998) consider the AS/RS described by Hausman et al. (1976) with stochastic demands. They conclude that for a stochastic environment the volume-based and class-based storage assignment lead to a reduction in the expected single-command cycle time compared with random storage assignment, and the volume-based assignment performs best.

With regard to *traffic congestion* in the aisles, the random storage assignment generates a uniformly distributed activity over the picking area, while the COI-based storage assignment tends to concentrate picking operations in the areas dedicated to items with low COI. Therefore, traffic may become congested. The class-based method leads to moderate traffic congestion (higher than in the case of random assignment but lower than in the case of COI-based assignments).

1.3.3 Batching, zoning and bucket brigade

When orders are large, in relation to the capacity of the transportation device, each order can be picked independently from other orders (i.e. one order per picking tour). This way of picking is often referred as the *single order picking* policy (or discrete or pick-by-order, as mentioned in Section 1.2.1). However, when orders are small, we can reduce the travel distance (thus increase the productivity) by picking a set of orders in a single picking tour. *Order batching* is the method of grouping a set of orders into a number of sub-sets, each of which can then be retrieved by a single picking tour. According to Choe and Sharp (1991), there are basically two criteria for batching: the proximity of pick locations and time window batching.

Proximity order batching

Proximity batching assigns each order to a batch based on proximity of its storage location to those of other orders. The major issue in proximity batching is how to measure the proximities among orders, which implicitly assumes a pick sequencing rule to visit a set of locations. Gademann et al. (2001) consider the proximity order-batching problem in a manual-pick wave-picking warehouse. The objective is to minimize the maximum lead-time of any batch (this is known as a common objective in wave picking). They show that the order-batching in this case is an NP-hard problem. They propose a branch-and-bound algorithm to solve this problem exactly for small instances and a 2-opt heuristic procedure for large instances. Furthermore, they claim that the 2-opt heuristic provides very tight upper bounds and would suffice in practice. Also for a manual-pick OP system, Gademann and Van de Velde (2005) consider the order-batching problem with a more general objective: minimizing the total travel time. They show that the problem is still NP-hard in strong sense when the number of orders per batch is greater than 2. A branch-and-price algorithm is designed to solve instances of modest size to optimality. For larger instances, it is suggested to use an iterated descent approximation algorithm. Chen and Wu (2005) measure the proximity of orders by taking into account the level of overlapping (or *association*) between orders (orders having more similar items have a high association and may form a batch). They develop a clustering model based on 0-1 integer programming to maximize the total association of batches.

As order batching is an NP-hard problem, many studies focus on developing heuristic methods for solving it. For manual-pick OP systems, we can distinguish two types of order-batching heuristics: seed and savings algorithms. *Seed algorithms* construct batches in two phases: seed selection and order congruency. *Seed selection rules* define a seed order for each batch. Some examples of a seed selection rule are: (a) a random order; (b) an order with large number of positions; (c) an order with longest pick tour; (d) an order with most distantly-located (i.e. furthest from the I/O point); (f) an order with the largest difference between the aisle number of the right-most and the left-most aisle to be visited (see De Koster et al. 1999a for more seed selection rules). *Order congruency rules* determine which unassigned order should be added next into the current batch. Usually, an order is selected, to be included in a batch, based on a measure of the 'distance' from the order to the seed order of the batch. Examples are: (a) the number of additional aisles which have to be visited if the order is added; (b) the difference between the gravity center of the order and the gravity center of the seed order; (c) the sum of the travel distances between every location of an item in the order and the closest location of item in the seed order (see more in De Koster et al., 1999a). The seed algorithms are considered in Elsayed (1981), Elsayed and Stern (1983), Hwang et al. (1988), Hwang and Lee (1988), Gibson and Sharp (1992), Pan and Liu (1995), and Ruben and Jacobs (1999) for AS/SR, and Rosenwein (1994), and De Koster et al. (1999a) for manual-pick OP systems. *Saving algorithms* are based on the well-known Clarke-and-Wright algorithm for the vehicle routing problem: a saving on travel distance is obtained by combining a set of small tours into a smaller set of larger tours. Elsayed and Unal (1989) propose four batching heuristics called EQUAL, SL, MAXSAV, CWright for an AS/RS. Among them, the SL algorithm (combine Small with Large orders), which classifies orders as 'large' or 'small' ones before assigning them to different batches, generates minimal travel distances.

De Koster et al. (1999a) perform a comparative study for the seed and time savings heuristics mentioned above for multiple-aisle picker-to-parts OP systems. The performance of the algorithms is evaluated using two different routing heuristics: the S-shape and the largest gap (see Section 1.3.4 for a description of these routing methods). The batching heuristics are compared for travel time, number of batches formed and also for the applicability in practice. They conclude that: (a) even simple order batching methods lead to significant improvement compared to the first-come first-serve batching rule; (b) the seed algorithms are best in conjunction with the S-shape routing method and a large capacity of the pick device, while the time savings algorithms perform best in conjunction with the largest gap routing method and a small pick-device capacity.

Time window order batching

Under *time window batching*, the orders arriving during the same time interval (fixed or variable length), called a time window, are grouped as a batch. These orders are then processed simultaneously in the following stages. If order splitting is not allowed (thus each order picker picks a group of complete orders in one picking tour), it is possible to sort items by order during the picking process. This picking strategy is often referred as the sort-while-pick picking strategy (see also Section 1.2.1). If order splitting is allowed, a further effort is needed to sort the picked items (the pick-and-sort picking strategy). Tang and Chew (1997), Chew and Tang (1999), Le-Duc and De Koster (2003, 2004b) consider variable time window order batching (i.e. number of items per batch is 'fixed') with stochastic order arrivals for manual-pick OP systems. They model the problem as a batch service queue (this approach will be discussed in-depth in Chapter 5 of this thesis).

All publications above-mentioned do not take into account the order due time and the penalty of violating the due time. Elsayed et al. (1993) and Elsayed and Lee (1996) consider the order-batching problem in a man-aboard OP system with minimizing of the penalties and tardiness as respective objectives. They propose a heuristic which first establishes batches and then determines the release times for the batches.

Zoning

Closely relating to order batching, *zoning* is the problem of dividing the whole pick area into a number of smaller areas (or zones); each zone is then assigned to one or more order pickers to pick requested items stored in the zone. The major advantages of zoning are reduction in the travel time (because of the smaller traversed area and also the familiarity of the order picker with the item locations in the zone) and of the traffic congestion. Depending on the picking strategy, zoning may be further classified into two types: *progressive zoning* and *synchronized zoning* (mentioned in Section 1.2.1). Compared to other planning issues, the zoning problem has received little attention despite its important impact on the performance of OP systems. Mellema and Smith (1988) examine the effects of the aisle configuration, stocking policy and batching and zoning rules by using simulation. They suggest that a combination of batching and zoning can significantly increase the productivity (pieces per man-hour). Also, using simulation, Petersen (2002) shows that the zone shape (number of aisles per zone, the aisle lengths), the number of items on the pick-list and the storage policy have a significant effect on the average travel distance within the zone. Choe et al. (1993) study the effects of three strategies in an aisle-based OP system: single-order-pick, sort-while-pick, and pick-and-sort. They propose analytical tools for a planner to quickly evaluate various alternatives without using simulation. Jane (2000) proposes several heuristic algorithms to balance the workloads among the order picker and to adjust the zone size for order volume fluctuation in a progressive zoning OP system. Jane and Laih (2005) consider the problem of heuristically assigning products to zones in a synchronized OP system. The method is based on *co-appearance* of items in the same order (i.e. items appear in the same order are stored in the same zone). Le-Duc and De Koster (2005c), consider the problem of determining the optimal number of zones (for a given picking area) in a pick-and-pack OP system. The objective is to minimize the throughput time of the system.

Bucket brigade is a way of coordinating workers who are progressively assembling product along flow line. If the workers are positioned from slowest to fastest along the line (with respect to the direction of product flow), then a balanced allocation of work will spontaneously emerge (see Bartholdi and Eisenstein, 1996a, 2002, 2005 and Bartholdi et al., 1999, 2001, 2005). In a progressive zoning system, order pickers can also work as a bucket brigade (Bartholdi and Eisenstein, 1996b). Each order picker follows the rule "Pick forward until someone takes over your work; then go back for more". When the last order picker completes an order, this order picker pushes it way (e.g., onto a conveyor) and then walks back to take over the order of the previous order picker, who in turn takes over the order of the previous order picker, and so on until the first order picker begins a new order. It is further required that the pickers be sequenced from slowest-to-fastest, so that the slowest picker is starting new orders and the fastest is finishing them. Bucket brigade can be seen as a version of zoning where the zone sizes are variable. Bartholdi and Eisenstein (1996b) implemented bucket brigades in the distribution center of Revco Drug Stores in North America and showed that bucket brigade increases the throughput rate and reduces management efforts.

1.3.4 Routing methods

For given item storage locations, the (order picker) routing problem is to determine a visit sequence to pick up all the items such that the travel distance is minimized. As noted by Ratliff and Rosenthal (1983), this is a special case of the well-known Traveling Salesman

Problem, and the *optimal route* for a rectangular, *narrow aisles*, *single-block* warehouse (i.e. no middle cross aisle, see Figure 1.8 for an example of a single-block warehouse) can be quickly found by using dynamic programming. The algorithm has running time linear in the number of aisles and the number of pick locations. (An aisle is called narrow if the order picker can simultaneously access storage locations on both sides of the aisle, thus there is no additional travel time when the order picker changes picking from one aisle-side to the other.) De Koster et al. (1998) extend the method for a warehouse where the I/O point location is decentralized, meaning that the order picker can deposit picked items at the head of every aisle (e.g. on a transportation conveyor). Roodbergen and De Koster (2001b) extend the method for warehouses with a middle-aisle (i.e. two 2-bock warehouses). For *wide-aisle* warehouses, the order picker needs to move (physically) from one side to the other in order to pick items on both sides of the aisle. Cleary, with a same amount of picks, the travel distance is longer in the case of wide-aisle warehouses. Hall (1993) addresses the problem of determining optimal route length when the aisle's width is non-negligible. Goetschalckx and Ratliff (1988a,b) deal with this problem in greater detail. They conclude that the problem of determining the optimal route in a wide aisle (with random storage assignment) can be solved very efficiently in a few seconds, and the optimal routes can yield up to 30% saving in travel distance compared to routes obtained from heuristic methods.

The disadvantages of the exact (optimal) method are as follows. First, it produces pick routes that may seem illogical or suboptimal to the order pickers who then, as a result, deviate from the specified routes (Gademann and Van de Velde 2005). Indeed, De Koster et al. (1999b) and Dekker et al. (2004) experienced this phenomenon in the warehouses of De Bijenkorf, a department store chain, and Ankor, a wholesaler of tools and garden equipment. Second, the exact method depends on the I/O location, whether the I/O point is fixed or not, number of blocks, and the layout shape (rectangular or not). Exact methods are only available for standard layouts (i.e. rectangular, single or two blocks). Third, the exact method has to be executed for every route. This can be a burden for the warehouse management information system. Fourth, the exact method does not take aisle congestion into account, while with heuristic methods it may be possible to avoid (or at least to reduce) the aisle congestion (i.e. the S-shape method has a single traffic direction if the pick density is sufficiently high). Finally, the exact method does not include the fact that aisle or direction changing may be time consuming in practice. In many such cases, order pickers leave their pick cart in the cross aisle. By using a heuristic method (e.g. S-shape method), the number of aisle changes can be reduced. Because of these reasons, usually a simple and standardized routing rule is preferable in practice. Furthermore, Hall (1993) notes that heuristic methods can develop near-optimal routes with less confusion. Petersen

(1997) and Roodbergen (2001) distinguish the following heuristic methods for routing order pickers in narrow-aisle, single-block warehouses.

S-shape (or traversal) heuristic

One of the simplest heuristics for routing order pickers is the S-shape method. Routing order pickers by using the S-shape method means that any aisle containing at least one pick is traversed entirely (except potentially the last visited aisle). Aisles without pick are not entered. From the last visited aisle, the order picker returns to the I/O point. An example of the S-shape route is shown in Figure 1.8.

Return heuristic

Another simple heuristic for routing order pickers is the return method, shown in Figure 1.8. With the return heuristic, an order picker enters and leaves an aisle from the same end. Only aisles with picks are visited.

Midpoint heuristic

A midpoint method essentially divides the warehouse into two sections (see Figure 1.8). Picks in the font half are accessed from the front cross aisle and picks in the back half are accessed from the back cross aisle. The order picker traverses to the back half by either the last or the first aisle to be visited. As we can see this method is similar to the return method, the only difference is that the warehouse is divided in two halves. According to Hall (1993), this method performs better than the S-shape method when the number of picks per aisle is small (i.e. one pick per aisle on average). See Figure 1.8 for an example route.

Largest gap heuristic

Figure 1.8 shows a largest gap route example. As described in Petersen (1997): the largest gap strategy is similar to the mid-point strategy except that an order picker enters an aisle as far as the largest gap within an aisle, instead of the midpoint. The gap represents the separation between any two adjacent picks, between the first pick and the front aisle, or between the last pick and the back aisle. If the largest gap is between two adjacent picks, the order picker performs a return route from both ends of the aisle. Otherwise, a return route from either the front or back aisle is used. The largest gap within an aisle is therefore the portion of the aisle that the order picker does not traverse. The back aisle can only be accessed through either the first or last aisle. The largest gap method outperforms the mid-point method (see Hall, 1993). However, from the implementation point of view, the mid-point method is simpler.

Composite heuristic

This method is proposed in Petersen (1995, 1997); it combines the best features of the return and traversal strategies. It minimizes the travel distance between the farthest picks in two adjacent aisles, and determines for each aisle whether it is shorter to travel the aisle entirely (S-shape strategy) or to make a turn in it (the return strategy).

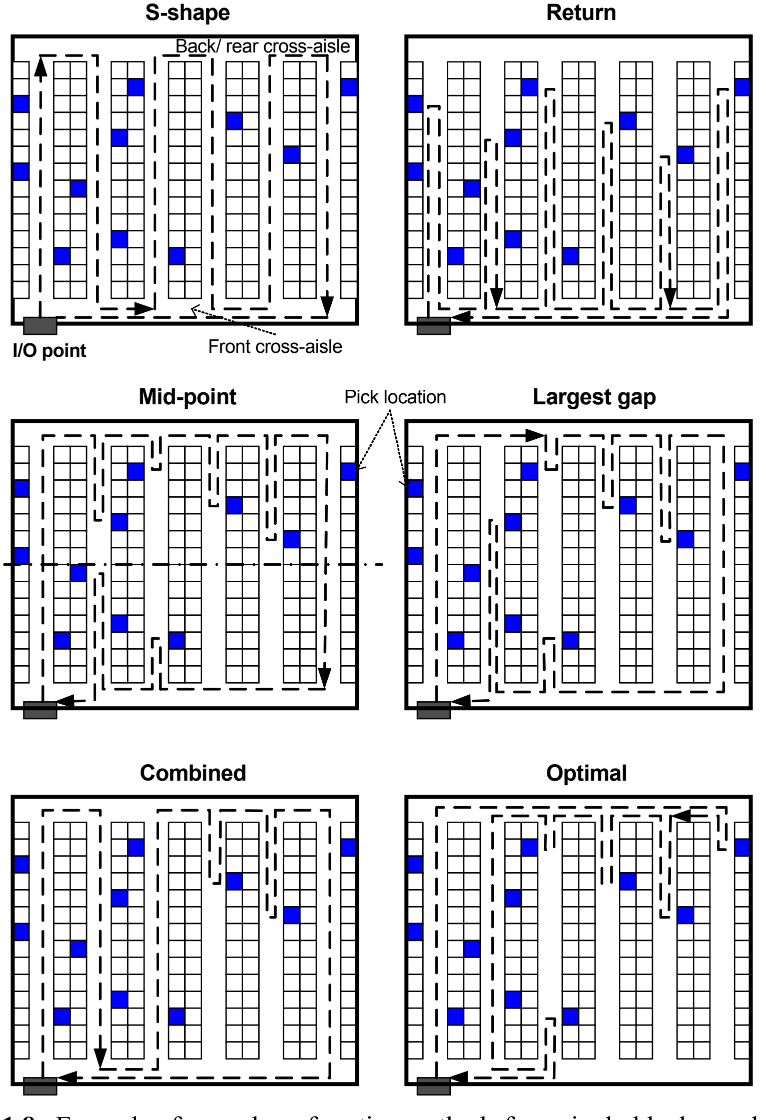

Figure 1.8 Example of a number of routing methods for a single-block warehouse (*Roodbergen, 2001*)

Combined heuristic

The idea of this method is similar to the composite method, and results in routes which are similar to the composite routes. For this method, aisles with picks are either entirely traversed or entered and left at the same end. However, for each visited aisle, the choice is made by using dynamic programming (see Roodbergen, 2001, and Roodbergen and De Koster, 2001a for a detailed description of the method). An example route is given in Figure 1.8.

Comparison between routing methods

In the literature, several studies compare these routing methods. Hall (1993) considers the largest gap and S-shape method for a random storage, single-block warehouse. His analysis shows that largest gap is better if the pick density (number of picks per aisle) is approximately less than 3.8, and the S-shape outperforms the largest gap method when the pick density is greater than 3.8. Also considering such warehouses, Petersen (1997) carries out a number of numerical experiments to compare six routing methods: the S-shape, return, largest gap, mid-point, composite and optimal. He concludes that a best heuristic solution is on average 5% over the optimal solution and the overall best heuristic procedures are the composite and largest gap methods, which were 9% to 10% over the optimum. De Koster and Van der Poort (1998), and De Koster et al. (1998) compare the optimal and S-shape method for several typical types of single-block random storage strategy warehouses. They find that the S-shape provides routes which are, on average, between 7.3% and 12.7% longer than the optimum solutions for the first warehouse, between 12.5% and 20.8% for the second, and between 30% and 32.4% for the third warehouse. The employed storage assignment may strongly influence the efficiency of the routing method used. For example, the S-shape method favors the storage assignment methods which locate the highest frequency demand items in one aisle and somewhat less frequently demanded items in the next aisle and so on (Roodbergen, 2001). Caron et al. (1998) consider the S-shape and return method in a COI-based storage assignment warehouse consisting of two blocks with an I/O point in between. They conclude that the return heuristic is only better than S-shape for a low number of average picks per aisle (i.e. < 1) and for skewed COI-based ABC curves (for instance 70/20, meaning that 20 percent of total number of items stored count for 70 percent of the total demand volume). For single-block and volume-based storage assignment warehouses, Petersen and Schmenner (1999) compare four routing methods: composite, largest gap, S-shape and optimal for a single-block warehouse. Their experimental results show an average solution gap of around 10% for the composite, largest gap, and mid-point method, and around 30% for the return and S-shape method. Overall, the composite method appears to perform consistently well. The largest gap method is better than the S-shape with low pick densities, and worse with high pick densities (i.e. greater than 28). For class-based storage, Le-Duc and De

Koster (2005a, b, 2004a) develop a travel distance model for estimating the average tour length in 2-block warehouse when either S-shape or return method is used. The numerical results show that the return method is only better than S-shape for relatively small pick-list size and very skewed storage assignments (ABC curves). This is similar to the finding in Caron et al. (1998) for the COI-based storage assignment.

All above-mentioned methods were originally developed for single-block warehouses, however they can be used for *multi-block* warehouses with some modifications (see Roodbergen and De Koster, 2001a). Besides that, Vaughan and Petersen (1999) present a method called *aisle-by-aisle* heuristic for a warehouse with multi-block aisles. For this method, every pick aisle is visited exactly once. A dynamic programming method is used to determine the best cross aisles to go from pick aisle to pick aisle. Roodbergen and De Koster (2001a) adapt the combined heuristic, in a method called *combined*$^+$ heuristic, for the case of multi-block warehouses. Roodbergen and De Koster (2001a) compare six routing methods (optimal, largest gap, S-shape, aisle-by-aisle, combined and combined$^+$), in 80 warehouse instances, with the number of aisles varying between 7 and 15, the number of cross aisle between 2 and 11 and the pick-list size between 10 and 30. They report that the combined$^+$ heuristic gives the best results in 74 of the 80 instances, with negligible computational times per route. The gaps between the results from the combined$^+$ and the optimal method are large in the case of many aisles and/or large pick-list sizes; they vary between 1% and 25%. The aisle-by-aisle, combined and combined$^+$ method are identical in the case of single-block warehouse. The combined heuristic provides results which are never worse than S-shape. However, the gap with S-shape reduces when the number of cross aisles or the pick density (i.e. average number of picks per aisle) is small. A clear point here is that, among the heuristic methods there is no robust heuristic that is good for all situations; a specific heuristic may be good for one situation but may perform poorly in other situations.

So far we have considered methods for routing order pickers in manual-pick OP systems. In the literature, the problem of determining the sequence of visits for the S/R machine in AS/RS or man-aboard systems (often called the *sequencing problem*) has also received considerable attention. It should be noted that the routing problems for order pickers and for the S/R machine are different. First, within an aisle, the travel distance is measured in Chebychev norm for the automated OP system and in rectilinear norm for manual-pick OP systems. Second, the number of picks is usually large in manual-pick OP systems (consisting of small items, cases or boxes …) and small in automated OP systems (pallets, large cases…). Furthermore, S/R machines are, in most cases, aisle-bound. Aisle-changing S/R machines are relatively rare in practice. They often take a considerable time to change from one pick aisle to another. Therefore, the S/R machine often picks all requested picks in an aisle before moving to the next aisle. Because of these differences, the routing methods developed for order pickers are, in general, not applicable for the sequencing problem in AS/RS systems. Studies on sequencing S/R machine's visits have been carried out by Murty (1968), Hausman et al. (1976), Barrett (1977), Graves et al. (1977), Schwarz et al. (1978), Linn and Wysk (1987, 1990), Han et al. (1987), Seidmann (1988), Bozer et al. (1990), Linn and Xie (1993), Sarker et al. (1994), Keserla and Peters (1994), Lee and Kim (1995), Lee and Schaefer (1996, 1997), Van den Berg and Gademann (1999), and Kim et al. (2003). Sequencing problems in man-aboard OP systems have been studied in Goetschalckx and Ratliff (1988c), Van Oudheusden et al. (1988), Hwang and Song (1993), and Daniels et al. (1998). For a complete review, we refer to Sarker and Babu (1995), and Van den Berg (1999).

1.3.5 Order accumulation/sorting

When batching and/or zoning is applied, usually some additional effort is needed to split the batch and to consolidate the items per customer order or per destinations to which orders will be shipped. These processes are often called accumulation/sorting (A/S).

Figure 1.9 shows an example of a typical A/S system (mentioned in Meller, 1997, and Johnson, 1998). Items of a group of orders (a *pick-wave*) that are to be loaded onto a certain number of trucks are picked from the picking area. In general, items from the same order are assigned to multiple order pickers (to maintain high order picker efficiency) and the order pickers follow pre-specified routes to pick the items assigned to them. After picking, order pickers place their items on the transportation conveyor and the items are transported to the sorter. Owing to the assignment of orders to more than one order picker, the items of each order arrive at the sorter in a random sequence. Items are released onto the circulation conveyor of the sorter and enter the assigned shipping lane if all items of the preceding order assigned to that lane have already entered. If not, the items re-circulate around the circulation conveyor. Orders are released from shipping lanes as needed by the trucks and the lane capacity is made available for the next sort-group. The throughput of an A/S system depends not only on the equipment capacity (i.e. sorter capacity and conveyor speed) but also on operating policies like assignment of orders to shipping lanes (see Figure 1.9). The order-to-lane assignment problem is critical for most A/R systems as usually the number of shipping lanes is less than the number of orders, which may cause a blocking of orders at the entrance of the lanes.

The number of publications on A/S systems is limited. By simulation, Bozer and Sharp (1985) examine advantages of using a recirculation loop to avoid lane blocking in an A/R system when a shipping lane is full, assuming that each lane is assigned to one order. Considering A/S systems where multiple orders can be are assigned to one lane, Bozer et al. (1988) and Johnson (1998) recommend that assigning orders to shipping lanes just before the orders arrive at the circulation desk of the sorter is a better than any static fixed-assignment rule. Johnson and Lofgren (1994) describe an A/R system used at Hewlett-Packard. Meller (1997) proposes an integer formulation for the order-to-lane assignment problem in an A/S system. He claims that the problem can be solved efficiently for small instances (in terms of the number of lanes) by solving a number of minimum-cardinality sub-problems.

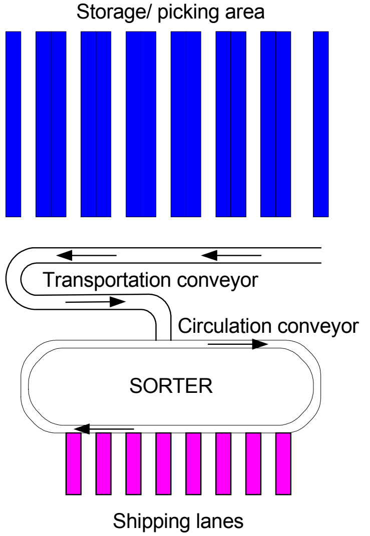

Figure 1.9 A typical accumulation/sorting (A/S) system

1.3.6 Other issues

We have mentioned five major issues in design and control of OP processes: layout design, storage assignment, batching and zoning, routing, and accumulation/sorting. There are several other issues that have received attention in the literature.

Forward-reserve storage

The forward-reserve problem considers (a) where to store items (i.e. which items are only stored in the *forward area* (the area for picking), in the *reserve area* (area for replenishing the forward area), or stored in both areas); (b) in which quantities; (c) frequency, timing and replenishment quantities. Literature on the forward-reserve problem can be found in Hackman and Rosenblatt (1990), Hackman and Platzman (1990), and Van den Berg et al. (1998).

Cross-docking

The second problem is optimally determining positions of receiving doors, (temporary) storage locations and shipping doors for *cross-docking operations*. Literature concerning cross-docking can be found in Tsui and Chang (1990, 1992), Witt (1992), Harrington (1993), Tompkins (1994), Andel (1994), Schwind (1995, 1996), Schaffer (1997, 1998), Witt (1998), Gue (1999), Richardson (1999), Apte and Viswathan (2000), Bartholdi and Gue (2000), Terreri (2001), and Vis (2005).

Dwell-point positioning

The third problem is the problem of determining the optimal position for an S/R machine when the system is idle (called *dwell-point positioning* problem). The dwell-point is often selected such that the expected travel time to the position of the first transaction after the idle period is minimized. The literature on this subject can be found in: Chang and Egbelu (1997a,b), and Hwang and Lim (1993), Egbelu and Wu (1993), Peters et al. (1996), Van den Berg (2002), and Meller and Mungwattana (2005).

Carousel

Under parts-to-picker OP systems, we have not considered carousel systems. For the literature related to carousel OP systems, see: Bartholdi and Platzman (1986), Han et al. (1988), Wen and Chang (1988), Hwang and Ha (1991), Ghosh and Wells (1992), Ha and Hwang (1994), Vickson and Fujimoto (1996), Van den Berg (1996), Vickson and Lu (1998), Su (1998), Hwang et al. (1999), Jacobs et al. (2000), Litvak et al. (2001), Litvak and Adan (2001a,b), Park et al. (2003b), Hassini and Vickson (2003), Litvak and Van Zwet (2004), Vlasiou and Adan (2004), Vlasiou et al. (2004), Wan and Wolff (2004), and Meller and Klote (2004). For literature on conveyors in general, see: Pritsker (1966), Gregory and Litton (1975), El Sayed et al. (1976), El Sayed and Proctor (1977), Proctor and El Sayed (1977), Muth and White (1979), Sonderman (1982), Xue and Proth (1987), Schmidt and Jacman (2000), and Bozer and Hsieh (2005).

1.3.7 Conclusions

We can draw the following conclusions from the literature review section. First, in spite of their dominance in practice, pickers-to-parts OP systems have received less research attention compared to parts-to-picker OP systems. Among more than 200 papers that we considered, there are only about 40 papers concerning pickers-to-part OP systems. It is because of the fact that parts-to-picker OP systems are often full or partly automated, and automation control systems often demand/ require much attention. Furthermore, picker-to-parts (or manual-pick) OP systems are often very complex and diverse. Second, existing studies in picker-to-parts OP systems mainly focus on random storage assignments.

Analytical models for optimizing dedicated and class-based storage assignment manual-pick OP systems are still lacking. Furthermore, storage assignment has an impact on the performance of the routing method. However, this effect seems to be neglected in the literature. Instead, many authors focus on random storage assignment to discus about the performance of routing methods. Finally, almost all research in order picking treats demand as given (or known in advance). Certainly, this is not true, especially in fast picking environments (e.g. small orders arrive on line and need to be shipped within a tight time window). These OP situations are becoming more and more daily practice, particularly for mail order companies which sell products online. Optimization problems arising from these OP systems, therefore, should be considered as stochastic optimization problems, not deterministic ones.

1.4 Research problems and contributions of the thesis

As shown, order picking is a subject that has been studied extensively in the literature to some extent. However, there are still several issues that have not been addressed adequately. This thesis enriches the current order picking literature by providing solution methods and insights for several new OP situations which arise from practice. More specifically, the thesis offers the following contributions:

- Develops probabilistic models for estimating average tour length in manual-pick class-based storage strategy warehouses. Although it focuses on specific routing methods (the S-shape and return routing method) and warehouse layouts (2-block warehouses), the models can be modified and applied to other routing methods and layouts.

- Explores the problems of finding the *optimal storage zones* (or storage class boundaries) and *layout optimization* (the number of aisles and aisle dimensions) with respect to minimizing the average tour length, for warehouses using class-based storage assignment. As the exact approach is intractable for practical conventional warehouse sizes, it presents an efficient heuristic procedure for solving the problems. Based on the numerical results, it highlights several layout design guidelines regarding the characteristics of the layout, demand skewness and routing methods.

- Extends the travel time models for 2-dimensional compact storage racks proposed in Bozer and White (1984) to a newly-designed 3-dimensional compact storage rack, using a gravity flow rack with conveyors working in pair. Based on that, the ratio between the rack dimensions minimizing the single-command cycle time can be determined.

- Studies the online order-batching problem in a dynamic picking environment. Although literature on order batching exists, it is not possible to use the existing methods for the new situations (e.g. many small orders need to be picked with tigh time windows). The thesis suggests a queuing-based approach to approximate the batch size which minimizes the throughput time of an order. The approach is simple with good quality. Therefore, it can be easily applied in practical situations.
- Finally, the thesis introduces the problem of determining the optimal number of zones such that the overall time to complete the entire batch of orders (throughput time) is minimized in a pick-and-pack OP system using synchronized zoning picking. Consequently, it formulates the problem as an integer programming model. The formulation is tested by using data from the warehouse of a mail-order company in the Netherlands. It turns out that the problem can be solved efficiently.

1.5 Outline of the thesis

In this chapter, we have mentioned the background literature on order picking and consequently introduced the research problems that we are going to consider. From a broad view, the rest of the thesis can be divided in three parts. The first part (Chapters 2, 3 and 4) concerns several issues in designing efficient OP processes. This part is based on Le-Duc and De Koster (2005a,b,e, 2004a) and De Koster and Le-Duc (2005). The second part (Chapters 5 and 6) is related to control issues: order-batching and zoning. It is based on Le-Duc and De Koster (2005c, 2004b, 2003). The last part (Chapter 7) gives concluding remarks and suggests potential future research directions (see Figure 1.10).

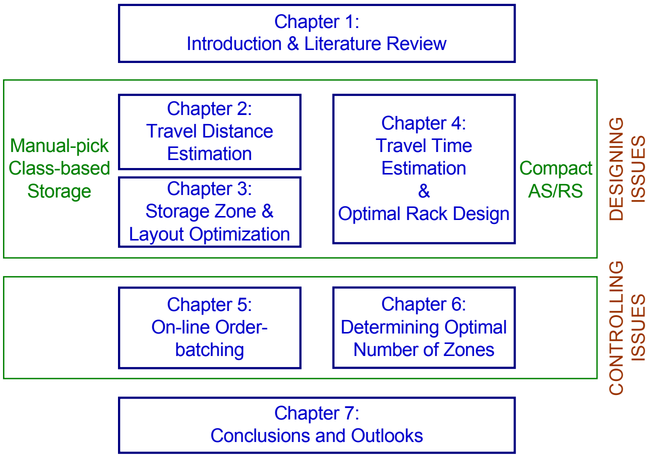

Figure 1.10 Outline of the thesis

2

Travel Distance Estimation in Manual-pick Class-based Storage Strategy Warehouses

2.1 Introduction and literature review

As mentioned in the introduction chapter, the travel time is proportional to the travel distance and minimizing the travel distance is therefore often considered as a primary objective function in order picking improvement studies. In the literature, the travel distance estimation problem in manual-pick OP has been explored thoroughly for the case of random and COI-based storage assignment (see Caron et al., 1998, De Koster et al., 1998). In this chapter, we consider the problem of estimating the travel distance in manual-pick class-based storage strategy warehouses. The developed probabilistic travel estimation models will then be used as the objective function for optimizing the storage zone and layout, which will be presented in the next chapter.

In the following, we review the most relevant publications on travel time estimation in manual-pick warehouses. For a general literature review, on design and control of OP processes, we refer to Section 1.3.

The average travel distance of a picking tour (or *average tour length*) depends mainly on the following factors: the layout, the aisle's width and length, the storage assignment and routing method used. Estimating the *travel distance within an aisle* is the basic start and is mentioned in all literature concerned. From the travel distance within an aisle and the statistical properties of the routing method used, we can estimate the *travel distance in multiple-aisle* warehouses. The travel distance in a one-block (i.e., no middle cross aisle) warehouse is the summation of the travel distance within (pick-) aisles for retrieving items and between (cross- or rear) aisles for changing aisles. When there are multiple blocks, the travel distance has an additional component: the distance that an order picker needs to traverse from one block to another. Caron et al. (1998, 2000) and Le-Duc and De Koster (2005a,b, 2004a) consider a warehouse with a cross aisle in the middle (2-block warehouse). Roodbergen (2001) studies the problem of estimating the travel distance for warehouses with more than two blocks, for the case of random storage strategy and the S-shape routing only (refer to Section 1.3.4 for a description of the routing methods).

Storage assignment certainly affects the pick locations that an order picker has to visit, and thus may affect the length of a picking tour. In random storage strategy warehouses, where items are randomly located, the probability of visiting any location along any aisle is the same. However, it is certainly not the case for other storage assignments (for example, class-based, COI…). As a consequence, we cannot apply the same travel distance estimation model for all storage assignments. Caron et al. (1998, 2000) propose travel distance models for a 2-block warehouse with a COI-based storage assignment (we refer to Section 1.3.2 for a definition of storage assignment methods). Jarvis and McDowell (1991), Tang and Chew (1997) and Chew and Tang (1999) estimate the average tour length in a single-block warehouse using a class-based storage assignment. They assume that the aisles are only one-way travel (i.e. only traversal or S-shape routing strategy is applicable) and each storage aisle can only be used for storing a certain class. In this chapter, we develop more general travel distance models and can represent both single and 2-block layouts, class-based storage assignments that allow storing different product classes per aisle, and two-way traffic aisles (thus both S-shape and return routing method are applicable).

The organization of this chapter is as follows. The problem is discussed in more detail in the next section. Section 2.3 presents probabilistic model for estimating the average travel distance in a single aisle warehouse. Section 2.4 shows how to estimate the tour length in a warehouse with multiple aisles. Concluding remarks are followed in Section 2.5.

2.2 Description of layouts, operating policies and assumptions

As noted above, the average tour length depends on the layout and the operating polices used (i.e. the routing and storage assignment method). Therefore, before introducing the probabilistic model for estimating the tour length, we clarify the layouts, operating policies and other assumptions employed by the model.

2.2.1 Layout

Figure 2.1 shows the types of layouts considered. Layout I was firstly studied by Caron et al. (1998). For this type of layout (2-block warehouse with *closed-end aisles*), as the pick aisles are bounded (*closed-end aisles*), the order picker cannot travel to the other aisles without going back to the cross aisle. It means that only the return routing method is applicable. Layout II (2-block warehouse with *open-end aisles*) is an extension of Layout I, where two rear aisles are added to make it possible for the order picker to travel to other aisles without making a turn in the aisle. This layout can be considered as a basic layout for a 2-block order picking area. Layouts I and II can be easily transformed into the corresponding single-block layouts: Layouts III and IV. If we call $T_1(q)$ and $T_2(q)$ the average tour length resulting from a single-block and the corresponding 2-block layout, there exists a one-to-one mapping between $T_1(q)$ and $T_2(q)$ irregardless of the routing method used and the number of picks per route q. It can be easily proved that $T_2(q) = T_1(q) + w_b x$, in which w_b is the centre-to-centre distance between 2 consecutive aisles and $1 \le x \le a/2$ is an integer related to the position of the last visited aisles (a is the total number of pick aisles and assumed to be even). Therefore, for the sake of simplicity, in this chapter and the next chapter we only focus on 2-block layout type II (it has to be noted that Layout I can be considered as a special case of Layout II, e.g. when the width of the rear aisles is zero).

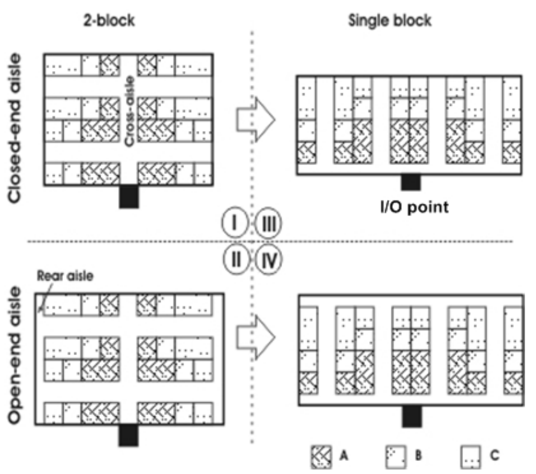

Figure 2.1 Possible layout configurations

2.2.2 Operating policy

The routing methods dealt with in this chapter are the traversal (or S-shape) and the return heuristic (see Section 1.3.4 for a description of routing heuristics). These methods are the simplest routing methods included in nearly every warehouse management software system, and widely used in practice (see, for example, Roodbergen and De Koster, 2001a). The applicability of the routing methods may depend on the type of aisle. The return heuristic can be applied in both open and closed-end aisle layouts. However, the S-shape can only be used in the open-end aisle layouts.

As mentioned in Section 1.3.2, the class-based storage strategy (i.e., the items are assigned to storage locations based on group basis), is widely used in practice because its advantages over other storage assignment methods. It is convenient to implement and maintain; it can easily handle assortment changes or changes in pick frequency. In addition, using a class-based storage strategy often leads to a substantial reduction in order pick travel distance as compared to random storage. Because of that, this chapter will focus on the class-based storage.

2.2.3 Assumptions

The following system and operational assumptions are made:

1. The warehouse consists of multiple identical rectangular racks (see Figure 2.2). Each rack can be used to store more than one product type.
2. The order picker can reach all items in the rack regardless of the rack's height and the vertical travel time within the aisle is negligible (this is typical for conventional shelf-storage warehouses, and pallet racks with manual picking from low levels).
3. The order pickers can pick items from both sides of the aisle by one pass; no additional time is needed for changing picking from one aisle side to the other (i.e. narrow aisle).
4. Aisle changes are possible in the cross and rear aisles. Picked orders have to be deposited at the I/O point, where the order picker also retrieves the instructions for the next tour.
5. The aisle’s storage space is defined as the aisle’s length. In reality, if the order picker could reach four levels without vertical transport (for example), the available storage space would be four times the aisle’s length.
6. Items in the same class have the same order frequency. The order frequency of each item-class is defined as the number of times that an item from that class is required

in a certain period (a planning period), it is known and constant throughout the planning period (see also Hausman et al., 1976).

7. A pick list may correspond to one customer order or several customer orders (the latter case might be the result of a batching policy).

8. It is also assumed that there is no demand dependent between products. It means that the probability of the occurrence of a product on an order is unaffected by the occurrence of any other product type on that order (Jarvis and McDowell, 1991).

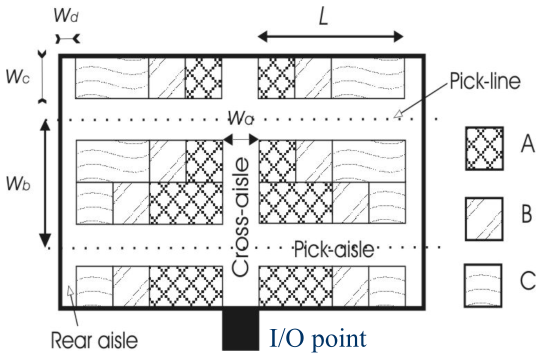

Figure 2.2 Warehouse layout and notations

2.2.4 *Main notations*

The following notations are generally used in this chapter; others are defined elsewhere or mentioned in the list of notations at the end of the thesis:

Data

a	number of pick aisles (also denoted as 'storage' aisles).
l_{ij}	partial length of pick aisle j used for storing of product class i.
q	number of picks (or order lines) in a picking tour (the pick-list size).
c	number of (product) classes.
L	length of a pick aisle.
w_a	width of the cross aisle.
w_b	centre-to-centre distance between two consecutive (pick) aisles.
w_c	width of the storage rack.
w_d	width of the rear aisle, $w_d = 0$ for closed-end aisle layouts.

f_i order frequency of product class i, $f_i = \sum_{j \in \ell_i} f_{ij} \Big/ \sum_{j \in \ell} f_{ij}$, where ℓ_i is the set of items belong to product class i and $\bigcup_{i=1}^{c} \ell_i = \ell$.

s_i percentage of the total storage space used for class i.

Intermediate (auxiliary) variables

P_{ij} the probability that the farthest pick in aisle j is in zone i

p_{ij} the probability that an item of class i located in aisle j is ordered (we assume this to be proportional to the pick frequency of class i)

$D_j(q,c)$ the expected travel distance (in a single direction starting from the cross aisle) within aisle j to pick up q items, given that there c classes

Decision variables

TD_z^{CA} travel distance within the cross aisle (called 'cross-aisle' travel distance), z denotes the name of the routing method used.

TD_z^{WA} travel distance within pick aisles (called 'within-aisle' travel distance).

TD_z (expected) average tour length.

2.3 Travel distance in a single aisle

In this section, a single aisle (aisle j) with a configuration given in Figure 2.3 is considered. Zone 1, zone 2, … and zone c are reserved for items of class 1, 2, …and c respectively. It is assumed that, within each zone, items are uniformly distributed. By conditioning on the farthest location of the requested items, the expected time from the starting point (see Figure 2.3) to the farthest pick location to pick up q_j picks can be computed as follows:

$$D_j(q_j,c) = \sum_{i=1}^{c} P_{ij} \mathrm{E}(\text{travel time} \mid \text{farthest pick in zone } i) = \sum_{i=1}^{c} P_{ij} d_{ij} \qquad \textbf{(2.1)}$$

From (2.1), it can be seen that in order to determine $D_j(q_j,c)$ (the expected travel distance from the starting point to the farthest pick location) we have to determine the probability that the farthest pick is in zone i (P_{ij}) and the corresponding expected one-way travel distance (d_{ij}). We consider the following situations:

- the farthest pick is in zone 1,
- the farthest pick is in zone 2 and
- the farthest pick is in zone i $(i = 3..c)$.

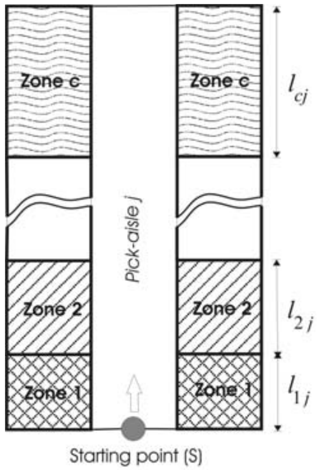

Figure 2.3 An example of class-based aisle

2.3.1 Farthest pick is in zone 1

If the farthest pick is in the zone 1, this means that all picks are in zone 1 and no pick is in the zones from 2 to c:

$$P_{1j} = p_{1j}^{q_j} \tag{2.2}$$

$$d_{1j} = \mathrm{E}(\mathit{travel}\ \text{distance} \mid \text{all picks in zone } 1) = \frac{l_{1j} q_j}{q_j + 1} \tag{2.3}$$

(2.3) is based on the well-known property that the expectation of the maximum of q_j continuous uniformly distributed [0,1] variables equals $\frac{q_j}{q_j+1}$. Recall that p_{ij} is the probability that an item of class i located in aisle j is ordered, given aisle j is visited: $p'_{ij} = p_{ij} \Big/ \sum_{i=1}^{c} p_{ij}$.

2.3.2 Farthest pick is in zone 2

The farthest pick in zone 2 is equivalent to at least one pick in zone 2 and no pick in the zones from 3 to c (or all picks in zone 1 and zone 2 but not all picks in zone 1):

$$P_{2j} = \left(p'_{1j} + p'_{2j}\right)^{q_j} - \left(p'_{1j}\right)^{q_j} \tag{2.4}$$

$$d_{2j} = l_{1j} + l_{2j}\mathrm{E}\left(\frac{N_2}{N_2+1}\right), \tag{2.5}$$

where N_2 is the number of picks in zone 2. It is rather difficult to compute d_{2j} based on (2.5). Therefore, we estimate d_{2j} as follows. First, we calculate $\mathrm{E}\left(N_2\right)$, the expected number of picks in zone 2:

$$\begin{aligned}
\mathrm{E}\left(N_2\right) &= \sum_{n=1}^{q_j} nP(\text{n picks in zone 2} \mid \text{all picks in zones 1\&2, and not all in zone 1}) \\
&= \sum_{n=1}^{q_j} n \frac{P(\text{n picks in zone 2, all picks in zones 1\&2, and not all in zone 1})}{P(\text{all picks in zones 1 and 2, and not all picks in zone 1})} \\
&= \sum_{n=1}^{q_j} n \frac{P(\text{n picks in zone 2 and } (q_j\text{-n}) \text{ picks in zone 1})}{P_{2j}} \\
&= \frac{\sum_{n=1}^{q_j} n \binom{q_j}{n} \left(\frac{p'_{2j}}{p'_{1j}+p'_{2j}}\right)^{n} \left(\frac{p'_{1j}}{p'_{1j}+p'_{2j}}\right)^{q_j-n}}{(p'_{1j}+p'_{2j})^{q_j} - p'^{\,q_j}_{1j}} \\
&= \frac{\dfrac{q_j p'_{2j}}{p'_{1j}+p'_{2j}}}{(p'_{1j}+p'_{2j})^{q_j} - p'^{\,q_j}_{1j}}
\end{aligned}$$

The last step is based on the property of Binomial distribution. Then, d_{2j} can be estimated as follows:

$$\begin{aligned}
d_{2j} &\approx l_{1j} + l_{2j}\frac{\mathrm{E}\left(N_2\right)}{\mathrm{E}\left(N_2\right)+1} \\
&= l_{1j} + \frac{l_{2j}\dfrac{\dfrac{q_j p'_{2j}}{p'_{1j}+p'_{2j}}}{(p'_{1j}+p'_{2j})^{q_j} - p'^{\,q_j}_{1j}}}{\dfrac{\dfrac{q_j p'_{2j}}{p'_{1j}+p'_{2j}}}{(p'_{1j}+p'_{2j})^{q_j} - p'^{\,q_j}_{1j}}+1}
\end{aligned}$$

$$= l_{1j} + \frac{\dfrac{l_{2j} q_j p'_{2j}}{p'_{1j} + p'_{2j}}}{\dfrac{q_j p'_{2j}}{p'_{1j} + p'_{2j}} + (p'_{1j} + p'_{2j})^{q_j} - p_{1j}^{\ q_j}}$$

$$= l_{1j} + \frac{l_{2j} q_j p'_{2j}}{q_j p'_{2j} + \left(p'_{1j} + p'_{2j}\right)\left[(p'_{1j} + p'_{2j})^{q_j} - p'^{\ q_j}_{1j}\right]} \quad \textbf{(2.6)}$$

2.3.3 Farthest pick is in zone i ($i = 3..c$)

If farthest pick is in zone i ($i = 3..c$) then we can apply the same procedure as in two previous situations, we have:

$$\underset{i\geq 3}{P_{ij}} = \left(p'_{1j} + p'_{2j} + ... + p'_{ij}\right)^{q_j} - \left(p'_{1j} + p'_{2j} + ... + p'_{i-1,j}\right)^{q_j} \quad \textbf{(2.7)}$$

$$\underset{i\geq 3}{d_{ij}} \approx \sum_{k=1}^{i-1} l_{ij} + \frac{l_{ij} q_j p'_{ij}}{q_j p'_{ij} + \psi\left(p'_{ij}, q_j\right) \sum_{k=1}^{i} p'_{kj}}, \quad \textbf{(2.8)}$$

where we define $\psi\left(\alpha, \beta\right) = \left(\sum_{k=1}^{i} \alpha_k\right)^{\beta} - \left(\sum_{k=1}^{i-1} \alpha_k\right)^{\beta}$, $i \geq 2$.

Finally, substituting (2.2), (2.3), (2.4), (2.6), (2.7) and (2.8) into (2.1), we obtain:

$$D_j(q_j, c) \approx p'^{\ q_j}_{1j} \frac{l_{1j} q_j}{q_j + 1} + \sum_{i=2}^{c} \left\{ \psi\left(p'_{ij}, q_j\right) \left[\sum_{k=1}^{i-1} l_{kj} + \frac{l_{ij} q_j p'_{ij}}{q_j p'_{ij} + \psi\left(p'_{ij}, q_j\right) \sum_{k=1}^{i} p'_{kj}} \right] \right\} \quad \textbf{(2.9)}$$

2.3.4 Simulation test

To test the quality of the probabilistic model (2.9), we built a simulation model in Microsoft Excel using VBA (Visual Basic for Applications). In the test, we consider the case that there are only three classes (namely A, B and C), which is the most popular one in practice.

In the case of three classes, it is easy to verify that:

$$D(q,3) \approx p_A{}^q \frac{l_A q}{q+1} + \left\{ \left(p_A + p_B\right)^q - \left(p_A\right)^q \right\} \cdot \left\{ l_A + \frac{l_B q p_B}{q p_B + \left(p_A + p_B\right)\left[(p_A + p_B)^q - p_A{}^q\right]} \right\}$$

$$+ \left\{ 1 - \left(p_A + p_B\right)^q \right\} \left\{ l_A + l_B + \frac{l_C q p_C}{q p_C + 1 - \left(p_A + p_B\right)^q} \right\} \qquad \textbf{(2.10)}$$

where q is the number of picks. p_A, p_B, p_C and l_A, l_B, l_C are the order frequencies and storage length of class A, B and C, respectively.

Table 2.1 Storage assignment schemes (storage space/ order frequency)

Assignment	A-class	B-class	C-class
Skewed	20/80	30/15	50/5
Medium	30/50	30/30	40/20
Random	33.33/33.33	33.33/33.33	33.33/33.33

We consider three different ABC-storage assignments, namely: skewed, random and medium. In the skewed assignment, frequently ordered items occupy only a very small portion of the total storage space. Typically, items occupying 20% of the storage space are responsible for 80% of the picks. In the random assignment case, no distinction between items classes in the term of order frequency and required space can be made; items are randomly located within the warehouse. Finally, the medium assignment is in between the two above-mentioned patterns. The percentages of assigned space and order frequencies of classes are listed in Table 2.1. The effective pick-list size varies from 4 to 60 picks per picking tour.

For each simulation experiment, the number of replications needs to be determined such that the mean (average tour length) has a relative error of less than γ, for $0 < \gamma < 1$, with a probability of $1-\alpha$. An approximation procedure for estimating the necessary number of replications is given in Law and Kelton (2000). A the replication size of 10000 is chosen for all simulation experiments in this thesis, as it is sufficient to guarantee a relative error of at most 2% with a probability of 98%.

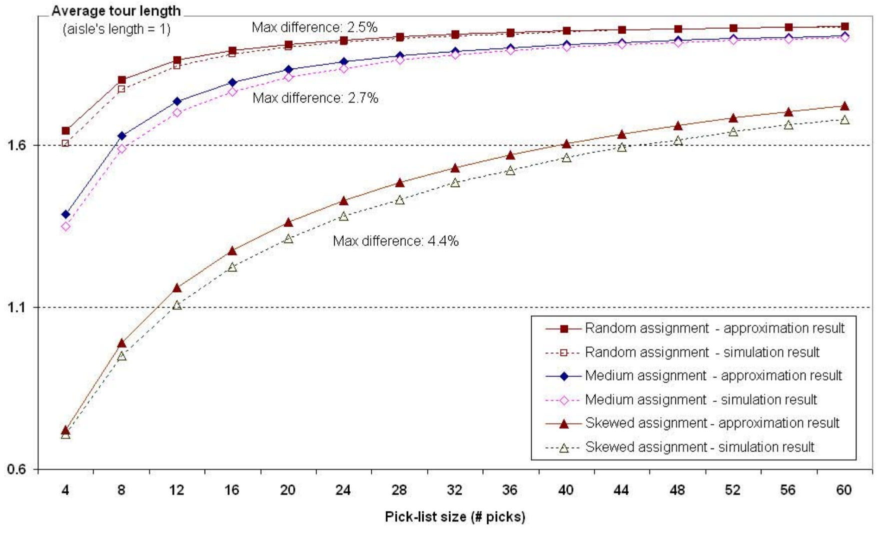

Figure 2.4 Difference between approximation and simulation results for travel distance in a single aisle

Figure 2.4 shows the results; the length of the pick aisle has been normalized to 1. The differences between approximation and simulations results are generally less than 5%. It confirms that the formula provides a good approximation for the travel distance in a single aisle. When the pick-list size is large, the difference is very small. It is because the order picker has to travel almost the entire pick aisle to pick up a large number of items; the average tour lengths found by the simulation and the estimation are close to the maximum travel distance (distance to travel the entire warehouse). Furthermore, the results from the approximation are always higher than the simulation results, since we overestimate the conditional expected travel distance d_{ij} .

The results show that ABC-storage can save travel distance up to 32% and 43% compared to the random strategy for the medium and skewed assignment, respectively. This finding is in line with the results in previous research on the ABC-storage assignment (see, for example, Hausman et al., 1976, Graves et al., 1977, Roodbergen, 2001). Also, the approximation performs better for the random assignment than for the medium and skewed assignment. In the worst case, the differences between the approximation and simulation results for the random, skewed and medium assignment are 2.5%, 3.7% and 4.4% respectively. The reason is that the variation in travel distance is higher in the cases of the skewed and medium assignment.

2.4 Travel distance in a warehouse with multiple aisles

In the previous section, the average travel distance within an aisle (i.e. the distance from the starting point to the farthest pick location in the aisle) has been estimated. In this section, the average tour length, including the travel distance within pick aisles (*within-aisle travel distance*) and the travel distance within the cross aisle (*cross-aisle travel distance*), is considered.

2.4.1 Within-aisle travel distance

The within-aisle travel time is the total travel time inside the pick aisles that an order picker has to traverse during a pick tour. Certainly, it depends on the pick-list size and the nature of the routing method. If we use the return heuristic, the within-aisle travel time (TD^{WA}_{Return}) can be estimated as the summation (over the set of all pick aisles) of the product of the probability that aisle j is visited (m_j) and the expected travel distance from the centreline of the cross aisle to the farthest pick in aisle j (k_j), given that the aisle j is visited.

For calculating the average tour length, we assume that l_{ij} and f_i are given. The probability that an item belonging to class i located in aisle j is ordered, $p_{ij}\ (i = 1..c,\ j = 1..a)$, can be then calculated by:

$$p_{ij} = f_i \frac{l_{ij}}{\sum_{j=1}^{a} l_{ij}} \quad \forall (i = 1..c, j = 1..a) \text{.}$$

Clearly, if one item is ordered then the probability that aisle j is visited is $\sum_{i=1}^{c} p_{ij}$. Thus, if q items are required then the probability that the aisle j is visited is:

$$m_j = 1 - \left(1 - \sum_{i=1}^{c} p_{ij}\right)^q \text{.}$$

The expected travel distance from the centreline of the cross aisle to the farthest pick in aisle j (given that the aisle j is visited) can be estimated as $k_j = w_a/2 + D_j(q_j, c)$, where q_j is the conditionally expected number of picks to be picked from aisle j :

$$q_j = \frac{\text{expected number of picks in aisle } j}{\text{probability that aisle } j \text{ is visited}} = \frac{q\sum_{i=1}^{c} p_{ij}}{m_j}.$$

$D_j(q_j, c)$ is the average travel distance from the starting point to the farthest pick in the aisle j to pick up q_j items, it is estimated by using formula (2.9) in Section 2.3.

The average 'within-aisle' travel distance of a tour can now be computed by:

$$TD_{\text{Return}}^{WA} \approx 2\sum_{j=1}^{a}\left[\left(\text{expected travel distance if aisle } j \text{ is visited}\right)*\left(\text{prob. that aisle } j \text{ is visted}\right)\right]$$

$$= 2\sum_{j=1}^{a} k_j m_j = 2\sum_{j=1}^{a}\left\{\left(w_a/2 + D_j(q_j, c)\right)\left[1-\left(1-\sum_{i=1}^{c} p_{ij}\right)^q\right]\right\}$$

$$= 2\sum_{j=1}^{a}\left\{\left\{w_a/2 + \left(p'_{1j}\right)^{q_j}\frac{l_{1j}q_j}{q_j+1} + \sum_{i=2}^{c}\left\{\psi\left(p'_{ij}, q_j\right)\left[\sum_{k=1}^{i-1} l_{kj} + \frac{l_{ij}q_j p'_{ij}}{q_j p'_{ij} + \psi\left(p'_{kj}, q_j\right)\sum_{k=1}^{i} p'_{kj}}\right]\right\}\right\}\right.$$

$$\left. \cdot\left[1-\left(1-\sum_{i=1}^{c} p_{ij}\right)^q\right]\right\}. \qquad \textbf{(2.11)}$$

In the case that the S-shape (traversal) routing heuristic is used, the within-aisle travel distance ($TD_{S-shape}^{WA}$) can be estimated as the summation (over the set of all aisles) of the product of the probability that aisle j is visited and the travel distance going through the aisle from the central line of the cross aisle to the central line of the rear aisle:

$$TD_{S-shape}^{WA} = \sum_{j=1}^{a}\left\{\left[1-\left(1-\sum_{i=1}^{c} p_{ij}\right)^q\right]\left[\left(w_a + w_d\right)/2 + L\right]\right\} + TD_{S-shape}^{correction} \qquad \textbf{(2.12)}$$

$TD_{S-shape}^{correction}$ is a correction term for the fact that the number of visited aisles in each block can be an odd number. If this is the case, then the order picker returns from the last pick position and leaves the aisle at the front-end (on the cross aisle's side). For single-block warehouse with random storage assignment, Hall (1993) assumes that the order picker has to return in the last aisle with probability 0.5 and the distance travel in this aisle is $2L$. Consequently, the correction term would equal $0.5L$. However, it can easily be seen that if the number of picks is high, $0.5L$ is either too low (if the number of aisles in one block is odd) or too high (even number of aisles).

For our case, in order to estimate $TD^{correction}_{S-shape}$, it is assumed that all aisles are identical (i.e. the storage space for each class in each aisle is the same). It then follows that all aisles have the number of picks $q^* = q \Big/ a\left[1-\left(1-\frac{1}{a}\right)^q\right]$, where $a\left[1-\left(1-\frac{1}{a}\right)^q\right]$ is the expected number of visited aisles. The expected travel distance inside the odd aisle(s), L', can be estimated by formula (2.9):

$$L' = D_j(q^*, c) \approx p_{1j}'^{\,q_j} \frac{l_{1j} q^*}{q^*+1} + \sum_{i=2}^{c} \left\{ \psi\left(p'_{ij}, q^*\right) \left[\sum_{k=1}^{i-1} l_{kj} + \frac{l_{ij} q^* p'_{ij}}{q^* p'_{ij} + \psi\left(p'_{ij}, q^*\right) \sum_{k=1}^{i} p'_{kj}} \right] \right\}$$

The probability that the number of visited aisles in one block is odd is 0.5, and in both blocks is 0.25, thus,

$$TD^{correction}_{S-shape} = 0.5\left(2L' - L\right) + 0.25\left(2\right)\left(2L' - L\right) = 2L' - L \tag{2.13}$$

In principle, it is possible to obtain a better approximation of $TD^{correction}_{S-shape}$ by finding the probability that the last visited aisle in each block is odd. For interested readers, it is advisable to read De Koster et al. (1998) and Roodbergen (2001).

2.4.2 Cross-aisle travel distance

To estimate the cross-aisle travel distance, we have to determine where the farthest (from the I/O point) visited pick-line is. It is similar to the situation of estimating the farthest pick location in a single aisle. Therefore, it can be estimated as follows:

$$\begin{aligned} TD^{CA}_z &\approx 2\sum_{j=1}^{a/2} \left\{ \left(2j-1\right)\left(w_b/2\right)(\text{prob. that } j \text{ is the farthest pick-line}) \right\} \\ &= \sum_{j=1}^{a/2} \left\{ \left(2j-1\right) w_b \left[\left(\sum_{i=1}^{j} n'_i \right)^q - \left(\sum_{i=1}^{j-1} n'_i \right)^q \right] \right\}, \end{aligned} \tag{2.14}$$

where $n_j = 1-\left(1-m_j\right)\left(1-m_{a-j+1}\right) = 1-\left(1-m_j\right)^2$ is the probability that aisle j and/or aisle (a-j+1) is visited (it has to be noted aisle j and aisle (a-j+1) are symmetrical), and $n'_j = n_j \Big/ \sum_{j=1}^{a/2} n_j$ $\left(\forall j = 1..a/2\right)$ is the probability that pick-line j is visited.

Adding TD_z^{CA} and TD_z^{WA} we obtain a formula for estimating the average tour length TD_z.

2.4.3 Simulation of multi-aisle layouts with the return routing method

We used simulation to examine the performance of the proposed formulas for the return heuristic: (2.11) and (2.14). In the experiment, we consider a warehouse with 6 aisles (see Figure 2.2 for an example of 4-aisle warehouse). We also assume that items are grouped into 3 classes and assigned to storage locations by either the skewed, medium or random assignment as mentioned in Table 2.1. The effective pick-list size varies from 4 to 60 items per picking tour. The aisle's length is normalized to 1. Other input parameters can be found in Table 2.2.

Table 2.2 Parameters for experimented layouts ($L=1$, $w_a = 0.11$, $w_b = 0.18$, $w_d = 0.06$)

	Medium assignment	Skewed assignment
Layout 1: 6 aisles	$l_{11}=0.53, l_{21}=0.4, l_{31}=0.07$	$l_{11}=0.39, l_{21}=0.4, l_{31}=0.21$
	$l_{12}=0.29, l_{22}=0.29, l_{32}=0.43$	$l_{12}=0.14, l_{22}=0.29, l_{32}=0.57$
	$l_{13}=0.09, l_{23}=0.21, l_{33}=0.7$	$l_{13}=0.07, l_{23}=0.22, l_{33}=0.71$
Layout 2: 10 aisles	$l_{11}=0.71, l_{21}=0.29, l_{31}=0$	$l_{11}=0.57, l_{21}=0.43, l_{31}=0$
	$l_{12}=0.57, l_{22}=0.43, l_{32}=0$	$l_{12}=0.29, l_{22}=0.42, l_{32}=0.29$
	$l_{13}=0.21, l_{23}=0.50, l_{33}=0.29$	$l_{13}=0.14, l_{23}=0.36, l_{33}=0.50$
	$l_{14}=0, l_{24}=0.29, l_{34}=0.71$	$l_{14}=0, l_{24}=0.29, l_{34}=0.71$
	$l_{15}=0, l_{25}=0, l_{35}=1$	$l_{15}=0, l_{25}=0, l_{35}=1$
Layout 3: 16 aisles	$l_{11}=0.71, l_{21}=0.29, l_{31}=0$	$l_{11}=0.64, l_{21}=0.36, l_{31}=0$
	$l_{12}=0.71, l_{22}=0.29, l_{32}=0$	$l_{12}=0.39, l_{22}=0.61, l_{32}=0$
	$l_{13}=0.49, l_{23}=0.51, l_{33}=0$	$l_{13}=0.29, l_{23}=0.20, l_{33}=0.51$
	$l_{14}=0.49, l_{24}=0.51, l_{34}=0$	$l_{14}=0.14, l_{24}=0.36, l_{34}=0.5$
	$l_{15}=0, l_{25}=0.8, l_{35}=0.2$	$l_{15}=0.14, l_{25}=0.36, l_{35}=0.50$
	$l_{16}=0, l_{26}=0, l_{36}=1$	$l_{16}=0, l_{26}=0.21, l_{36}=0.79$
	$l_{17}=0, l_{27}=0, l_{37}=1$	$l_{17}=0, l_{27}=0, l_{37}=1$
	$l_{18}=0, l_{28}=0, l_{38}=1$	$l_{18}=0, l_{28}=0, l_{38}=1$

The simulations were established in the same way as for the single aisle case. For each simulation run we drew q picks (q varies between 4 and 60 picks per route), which were first randomly assigned to pick classes based on the class order frequencies. Then, items of class i $(i = 1..c)$ are assigned to aisle j $(j = 1..a)$ proportional to l_{ij} (note that within each storage zone, items are randomly stored). In comparison, we used 10000 replications for each value of q (which is sufficient to obtain a relative error of at most 2% with a probability of 98%, see Section 2.3.4). Figure 2.5 shows the results obtained from the probabilistic model and simulations. In accordance with our expectation, the random assignment always provides the longest average tour length. The average tour length is an increasing concave function of the pick-list size.

We also considered two other warehouses: 10-aisle and 16-aisle. The results that we obtained from these two warehouses are very similar (see Figures 2.6 and 2.7). The maximum approximation error increases slightly when the number of aisles increases (7.3%, 7.4% and 8.0% for 6-aisle, 10-aisle and 16-aisle warehouse, respectively).

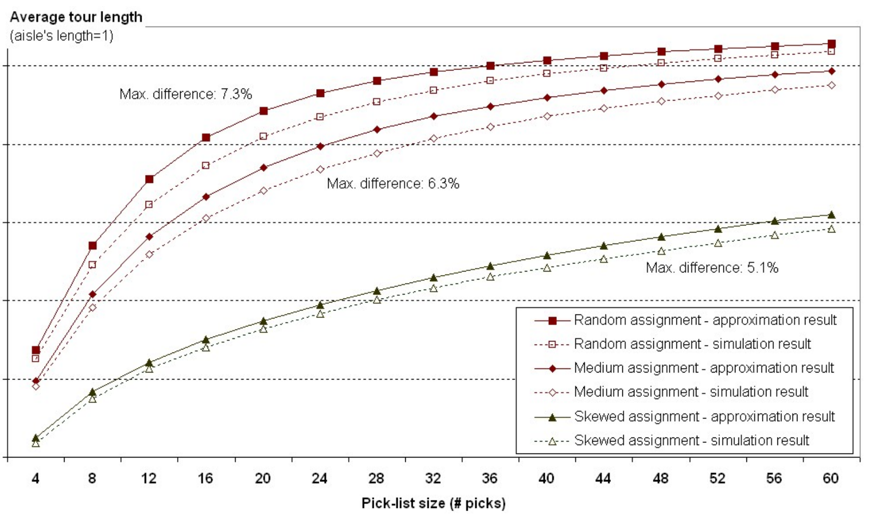

Figure 2.5 Differences between approximation and simulation results for the 6-aisle layout when the return routing method is used

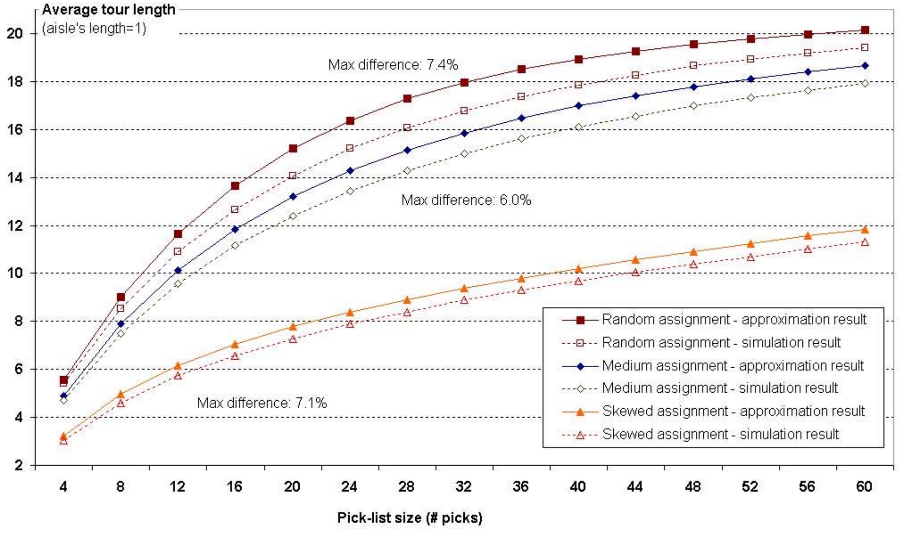

Figure 2.6 Differences between approximation and simulation results for the 10-aisle layout when the return routing method is used

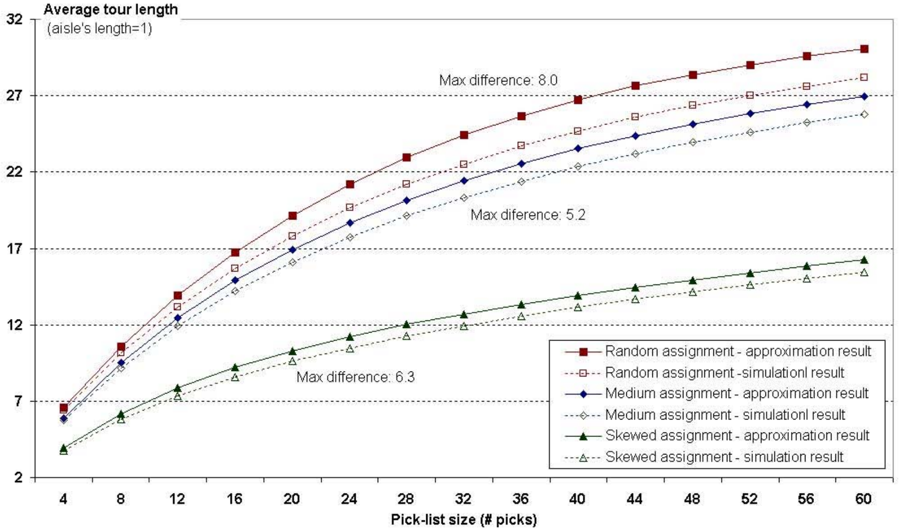

Figure 2.7 Differences between approximation and simulation results for the 16-aisle when the return routing method is used

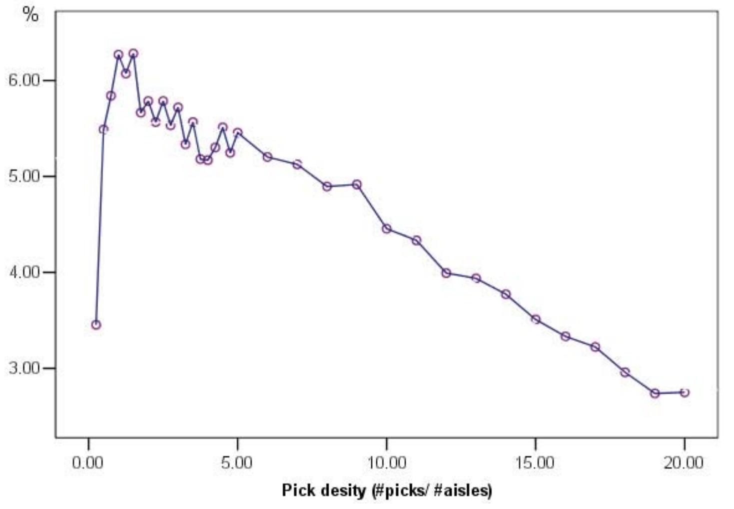

Figure 2.8 % difference between approximation and simulation results for 16-aisle warehouse with skewed assignment and the return routing method.

Figure 2.8 delineates the shape of the difference in the case of the layout with 16 aisles and a skewed assignment scheme. From the figure, we can see that the difference between approximation and simulation results first increases, when the pick density – defined as the average number of picks (or order lines) per aisle – increases. It reaches a maximum, and from there it decreases. When the pick-list size is very large, the difference is very small. We can explain this behavior as follows. The difference (or error ζ) consists of the following two components:

- the accumulated error resulting from estimating the average travel distance within all visited aisles (ζ_a). This amount is proportional to the expected number of visited aisles (η) and the error in estimating the travel distance in a single aisle (ε).
- the error in estimating the cross-aisle travel distance (ζ_b). This depends on η only.

When the pick-list size increases first, both η and ε increase. As a result, ζ_a, ζ_b and therefore, ζ increase. When the pick-list size is substantially large, η approaches the maximal possible number of visited aisles (a), and ζ_b stops increasing. For even larger pick-list sizes, ε will start to decrease since the order picker has to visit the entire warehouse. As a result, ζ will decrease as well. The estimate also approaches the maximum travel distance in this case.

2.4.4 Simulation of multi-aisle layout with the S-shape routing method

In this section, we use simulation to estimate the error of the approximation for the S-shape routing heuristic (by using Formulas 2.12, 2.13 and 2.14). We used the same warehouse instances as mentioned in Section 2.4.3 (see Table 2.2 for warehouses' parameters). The way the simulations are established is the same as mentioned previously. The only difference is that the routing method used is different (see Figure 1.8 for an example of an S-shape route).

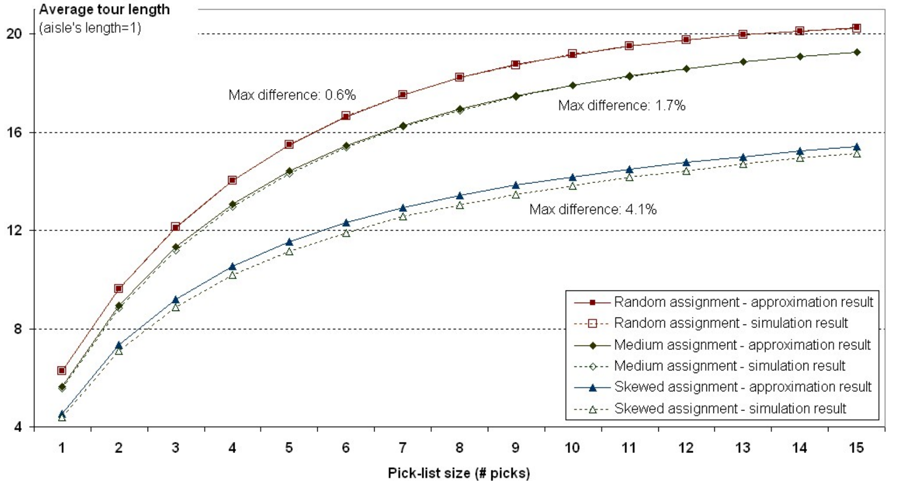

Figure 2.9 Differences between approximation and simulation results for the 16 aisle layout when the S-shape routing method is used

Figure 2.9 shows the results for the 16-aisle warehouse. As we can see the approximated error is less than that of the return heuristic. The reason is that, in order to estimate the within-aisle travel distance for the return heuristic, we need to keep track of the farthest pick location in every visited aisle. While in the case of the S-shape heuristic, we need to estimate only the number of visited aisles and the correction term. It has to be noticed that the cross-aisle travel distance is the same for both routing methods. We also tested warehouses with 6 and 10 aisles. These produced similar results. In the worst case, the error is 2.9%, 3.3% and 4.1% for 6, 10 and 16-aisle warehouses respectively. It appears that the error slightly increases when the number of aisles grows.

2.4.5 Comparison between S-shape and return routing method

Comparing the two routing methods, we find that the approximation error is smaller for the S-shape and larger for the return method. As mentioned before, the reason is that for the return method we need to keep track of the farthest pick location in every aisle. For the S-shape routing method, it is not necessary to do so.

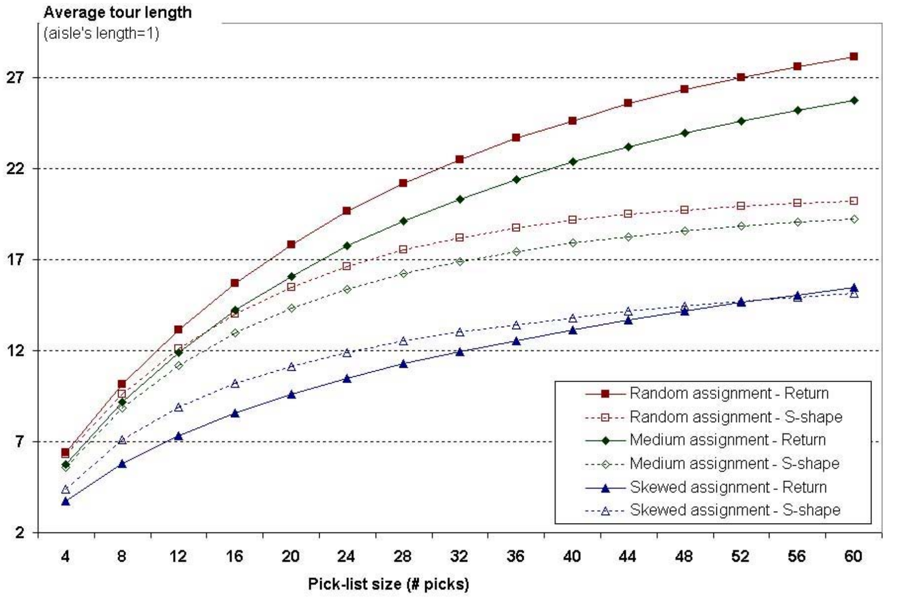

Figure 2.10 Comparison between the S-shape and return routing method (for 16-aisle warehouse)

Figure 2.10 shows the average tour lengths for two routing methods for the warehouse with 16 aisles. The S-shape clearly outperforms the return routing method in the case of the random and medium storage assignment. This result is in line with conclusions of Goetschalckx and Ratliff (1988), Hall (1993) and Caron et al. (1998). The gaps are small for small pick-list sizes and larger when the pick-list size grows. In the case of the skewed assignment, the return method results in a shorter tour compared to S-shape for small pick-list sizes $q \leq 54$, but in a longer one for large pick-list sizes $q > 54$. The reason is that with the skewed storage assignment and the return heuristic, when the pick-list size is small, we have to travel only a small part of most of the visited aisles, thus it will result in a shorter travel distance. For larger pick-list sizes, the order picker has to travel further into every visited aisle, and thus it will result in a longer travel distance than for the S-shape, since the order picker returns to the cross aisle before moving to the next aisle. In this situation, it is smarter to travel visited aisles entirely. We can conclude that in general S-shape outperforms the return routing method. The exception is the case of a skewed assignment with sufficiently small pick-list sizes, where the return method may provide a shorter average tour length. The results we obtained for other layouts (6-aisle and 10-aisle) also support this conclusion.

2.5 Concluding remarks

Travel distance estimation has been long considered as an essential problem in optimizing OP processes in particular and warehouse operations in general. The average tour length mainly depends on the following factors:

- the layout (number length and width of aisles and cross aisle, location of the I/O point)
- the routing method
- the storage assignment method
- the number of picks per route (q)

This chapter focuses on some typical layout types, these layouts can be considered as the most basic (and simple) form of major warehouses in practice. For routing order pickers, two common heuristics (the return and S-shape method) are used. The employed storage strategy is class-based assignment method. The effective pick-list size (q) varies between 4 and 60, which covers a wide range of pick-list sizes in order picking practices, we believe.

As pinpointed in the computational results, in the worst case, the difference between approximation and simulation result is about 8% for the return and less than 5% for the S-shape routing method. The error is small for small warehouses and appears to be larger for larger warehouses (i.e. large number of aisles, or equivalently smaller number of picks per aisle). Regarding the pick-list size, the gap between approximation and simulation result becomes smaller when the pick-list size grows; it is very tight for large pick-list sizes. The error is not very small, especially for the return and large warehouses. However, our intention is to use the travel distance estimation to support for rapid warehouse design in practice (i.e. storage zone and layout optimization), where data (e.g., customer demand, product availability) are stochastic variables. Therefore, it is not necessary to pursue high accurate estimation. In other words, in this situation we refer a quick and efficient layout to the optimal one.

So far, we have considered only some basic (small and simple) layouts. In practice, we may encounter much more complicated layouts (i.e. more than one cross aisle). It can be foreseen that the average within-aisle travel distance can be estimated in a same manner, but for estimating the cross-aisle travel distance, it is more difficult. The reason is that we have to keep track not of only the visited aisles (and their farthest pick locations if necessary) but also the visited blocks and where the transition from one block to another is made. For the case of random storage assignment, Roodbergen (2001) estimates the average tour length in a warehouse with multiple cross aisles. We can combine the models that we present in this chapter and Roodbergen's model to be able to estimate the average tour length for multi-block class-based warehouses. Besides considering more complicated layouts, we can also think of more sophisticated routing methods. As shown by Roodbergen and De Koster (2001), a good routing heuristic for order pickers is the combined heuristic, which is a combination of the return and S-shape heuristic and a decision between these two options is made per aisle by using dynamic programming. It may be possible to estimate the average tour length if the combined heuristic is used.

3

Storage Zone and Layout Optimization for Manual-pick Class-based Storage Strategy Warehouses

3.1 Introduction

In the previous chapter, we proposed a probabilistic model for estimating the average tour length. That is to say, for a given layout ($L, a, c, w_a, w_b \ldots$), item class pick-frequency (f_i), storage assignment scheme (l_{ij}), routing method (either S-shape or return) and pick-list size q, we can estimate the average tour length (TD). An ABC (or Pareto) analysis is carried out when the warehouse is first put into service or when there is a major change in product assortment and/or demand pattern. The purpose of this analysis is to make a distinction between products based on turnover speed (i.e. pick-frequency). In most cases, products are divided in pick-frequency classes A, B, C etc. where the product class A contains about 5 to 15% fastest moving items (see also Section 1.3.2). Next, each product class needs to be assigned 'optimally' to storage locations. In other words, if we divide storage space along each aisle into storage zones, each zone for storing a product class, then we have to define the optimal boundaries between zones in each aisle. The common objective to use here is minimizing the average tour length, which can be calculated by the formulas proposed in the previous chapter. We define this problem as the *storage zone optimization* problem.

The layout of the picking area is given (meaning that the number of aisles and the aisle lengths are given) in the case of storage zone optimization problem. However, if only the total storage space is given, then besides the storage zoning we have to determine also the optimal number of aisles and the aisle lengths. We will discuss this problem in depth in Section 3.5.

Several publications consider the problems of determining the optimal storage zone and/or layout of picking area for manual-pick OP systems are concerned. As far as we know the most recent ones are Roodbergen (2001), Caron et al. (2000), Petersen and Schmenner (1999) and Jarvis and McDowell (1991). (See Section 1.3.1 for an overview of the literature on the internal layout design.) Caron et al. (2000) use the travel time model proposed in Caron et al. (1998) to address the problem of optimal layout design for 2-block warehouses with COI-based storage assignment. In their study the optimal layout means the combination of the (pick-) aisle length and number of aisles that result in the shortest average tour length. They conclude that the picking area layout significantly influences the expected average tour length. Furthermore, they state that non-optimal layouts could be preferred in practice, since they are much less sensitive to changes in the operating conditions (routing method, pick-list size, and storage assignment scheme) compared to the optimal layout and cause only moderate increases in picking travel distance. Roodbergen (2001) proposes non-linear layout optimization models for single and multiple block warehouses. Based on his approach, we can find the best layout by varying a number of parameters (pick-list sizes, aisle length, and number of aisles). A missing point, in both Caron et al. (2000) and Roodbergen (2001), is that no specific conclusion about the shape of the optimal layout (in conjunction with the pick-list size, storage assignment or routing method) can be made. Jarvis and McDowell (1991) prove that when the S-shape routing method is applied for solving the routing problem, the optimal strategy is to begin by placing the most frequently selected products in the aisle nearest to the I/O point. By means of extensive simulation experiments, Petersen and Schmenner (1999) suggest that: "Organize the warehouse storage so that the high volume items are concentrated in a few aisles close to the I/O point". They also add that this type of product-to-location assignment provides average tour length savings of 10-20% compared to other types of storage.

The OP system considered in this chapter is described in the previous chapter (i.e. manual-pick, shelf-rack type of warehouse). In particular, it is assumed that the portion of total storage space (s_i) and the pick frequency (f_i) for each product class is known. In general, it is not too complicated to determine s_i after a pick-frequency Pareto (ABC) analysis has been carried out (setting class boundaries for the different pick-frequency classes). The storage space needed for each class mainly depends on the physical size of items in the class, the quantity stored per item and the storage assignment method that we use. Examples of making an ABC-storage classification in practice can be found in Hausman et al. (1976), Petersen and Schmenner (1999), and Dekker et al. (2004).

In this chapter, we use the same notations as in the previous chapter (all notations are listed in the list of abbreviation and notations at the end of the thesis). For the storage zone optimization problem the l_{ij} 's are decision variables. However for the layout optimization problem, not only the l_{ij} 's but also the a are decision variables.

3.2 Storage zone optimization problem formulation

The problem of optimizing storage zones for class-based storage assignment warehouses can be formulated as follows. For a given number of aisles (a), length (L) and width of aisles (w_c), a given width of the storage rack ($w_b - 2w_c$) and cross aisle (w_a), the pick-frequencies of classes (f_i), and fraction of total storage space needed for each class (s_i), our objective is to determine the optimal storage space (i.e. storage zone boundaries) for each class in each aisle (l_{ij}). We propose the following mathematical formulation for the problem:

$$Min \qquad TD_z\left(l_{ij}\right) \tag{3.1}$$

Subject to

$$\sum_{i=1}^{c} l_{ij} = L \qquad \forall j = 1..a \tag{3.2}$$

$$\sum_{j=1}^{a} l_{ij} = s_i La \qquad \forall i = 1..c \tag{3.3}$$

$$p_{ij} = l_{ij} f_i / \left(s_i La\right) \qquad \forall \left(i = 1..c,\ j = 1..a\right) \tag{3.4}$$

$$l_{ij} = l_{i,a-j+1} \qquad \forall (i = 1..c,\ j = 1..a/2) \tag{3.5}$$

$$l_{ij} \geq 0 \qquad \forall \left(i = 1..c,\ j = 1..a\right) \tag{3.6}$$

In the objective function (3.1), we minimize the average tour length to pick up q picks. It consists of two components: the within-aisle, $TD_z^{WA}\left(l_{ij}\right)$, and cross-aisle travel distance, $TD_z^{CA}\left(l_{ij}\right)$, where z denotes the routing method used (either the return or S-shape). The within-aisle travel distance is calculated by using either formula (2.11) or (2.12) depending on whether the return or S-shape routing method is used. The cross-aisle travel distance is determined by formula (2.13). In our formulation we consider these distances as functions of l_{ij}. In total, we have five sets of constraints. (3.2) concerns the aisle's length conservation. (3.3) concerns the conservation of the total storage space for each class. (3.4) shows the relationship between p_{ij} and l_{ij}. (3.5) ensures the layout symmetry. And (3.6) ensures the non-negativity of decision variables l_{ij}.

In the above formulation, we have five sets of linear constraints. The first two sets are similar to those in the classical transportation problem, which is known to be solvable in polynomial time. However, we have a non-linear objective function. The computation time for this objective function is significant when the pick-list size is very large. The total solution space of the problem can be very large as it rapidly increases with the number of storage aisles, product classes, and number of space slots per aisle (we divide each aisle into a number of identical pieces, called *space slots*). To get a rough idea about the running time, we considered a warehouse with 3 product classes and only 6 aisles, each divided into 100 space slots. The computation time to find the optimal layout achieved by total enumeration of the state space was about 20 minutes (on a Pentium IV 2.4 GHz CPU computer) when the batch size was 40 order lines per picking tour. From this computational experience we can conclude that for large warehouses, it is hard to solve the problem to optimality. Additionally, the objective function is an estimate of the real average travel distance. Therefore, we propose the following heuristic approach.

3.3 A heuristic approach for storage zone optimization problem

The following terminologies are used. An *identical-aisle* layout is a layout in which all aisles are identical (thus the storage space for a certain class is the same in all aisles). Class i and j are called *proximity classes* if $|i-j|=1$. Our idea is that we first start with a zoning scheme and then, step by step, exchange storage spaces between classes to get closer to an optimal solution. Clearly, the starting solution plays a role here. We know that the idea behind the class-based storage strategy is to locate fast moving classes as close as possible to the I/O point, by doing so we may reduce the average tour length. In a same manner, starting from the identical-aisle layout, we exchange space slots of fast moving classes in far-from-depot aisles with those of slower moving classes in the closer-to-depot aisles. When exchanging space slots between aisles, the average tour length changes. At each step, we evaluate the average tour length by using the approximation formula that has been proposed. We limit ourselves to consider only exchanges of single slots between proximity classes. As the layout is symmetrical, we need to consider only a half of the warehouse. The exchange program in pseudo code reads as follows:

```
procedure ZoningOpt
initialise                                        /*start with identical-aisle layout*/
for i from a/2 downto 2 do
   for k from 1 to i-1 do                          /*exchanging space slots
      for j from 1 to c-1 do                       between class j in aisle i and class
         repeat EXCHANGE                           j-1 in aisle k (i > k) */
         until LegalExchange=false or ΔTD > 0;
         undo the last EXCHANGE;
      end do;
   end do;
end do;
end sub;
```

In the *EXCHANGE* procedure, we exchange one space slot of class j in aisle i for one space slot of class ($j-1$) in aisle $k\ (i>k)$. An exchange is called a *legal exchange* if after the exchange all l_{ij} 's are non-negative. It is noted that as we start from a feasible solution and just swap storage classes between aisles, the other conditions are automatically satisfied. ΔTD is the difference between the average tour length after the exchange and the current best average tour length. This difference can be calculated by using the formulas that we mentioned before with appropriate values of the l_{ij} 's.

To illustrate the method, suppose that we have a half layout with three aisles (numbered from the I/O point as aisle 1 to 3) and three classes (A, B and C). Starting from the farthest aisle (aisle 3), we first do the exchanges between aisle 3 and aisle 1. It means that we first swap, one by one, space slots of class A in aisle 3 for B slots in aisle 1. Then we swap B slots in aisle 3 for C slots in aisle 1. Next, we do the exchanges between aisle 3 and aisle 2. Finally, we consider aisles 2 and 1.

The running time of the algorithm depends on the number of aisles, the number of classes, the number of space slots per aisle, and the time needed to compute ΔTD (which can be negligible for small pick-list sizes). It is easy to verify that the complexity of the algorithm is $O\left(a^2cs\right)$, where a is the number of aisles, c is the number of classes and s is the number of space slots per aisle (assuming that all aisles are divided into an equal number of space slots). The heuristic we have proposed is a type of 2-opt exchange technique, which belongs to the neighborhood-search-heuristics family (see, for example, Aarts and

Lenstra, 1997). These types of heuristics are widely used for facility design problems (Tompkins et al., 2003).

3.4 Numerical results and discussions

To evaluate the performance of the heuristic and to determine the optimal zone sizes in different warehouses, we carried out various numerical experiments. In the experiments, we considered the skewed and medium assignment (see Table 2.2 in Chapter 2) for five layouts (1, 2, 3, 4 and 5 with 4, 6, 8, 12 and 24 aisles respectively) and a varying pick-list size. We keep the aisle lengths constant over all layouts. Details about the parameters of the layouts are shown in Table 3.1 and Figure 3.1. The shape ratio (ρ) is defined as the ratio between the warehouse's width, $aw_b/2$, and the aisle length, L. Based on the experiment results, we discuss the quality of the heuristic, the shape of the optimal storage zone and the robustness of the identical-aisle layout.

3.4.1 Quality of the heuristic

The optimal average tour lengths for layouts 1, 2 and 3 are found by total enumeration of the state space when the return heuristic is used. Because the number of aisles is large, we could not find optimal results for layouts 4 and 5 (within 240 minutes of CPU running time).

Table 3.1 shows the differences between average tour lengths obtained from the optimal method, the exchange heuristic, and the average tour lengths from the identical-aisle layout. The gaps between the exchange heuristic and the corresponding optimal results appear to be very small. In the worst case, the heuristic result is only about 1.2% off the optimal result. For large pick-list sizes the heuristic provides the optimal solution. It is because of the fact that for large pick-list sizes, the order-picker has to travel the entire warehouse, and consequently the zoning scheme does not influence the average travel distance much. In all cases, the running time of the heuristic was less than 5 seconds.

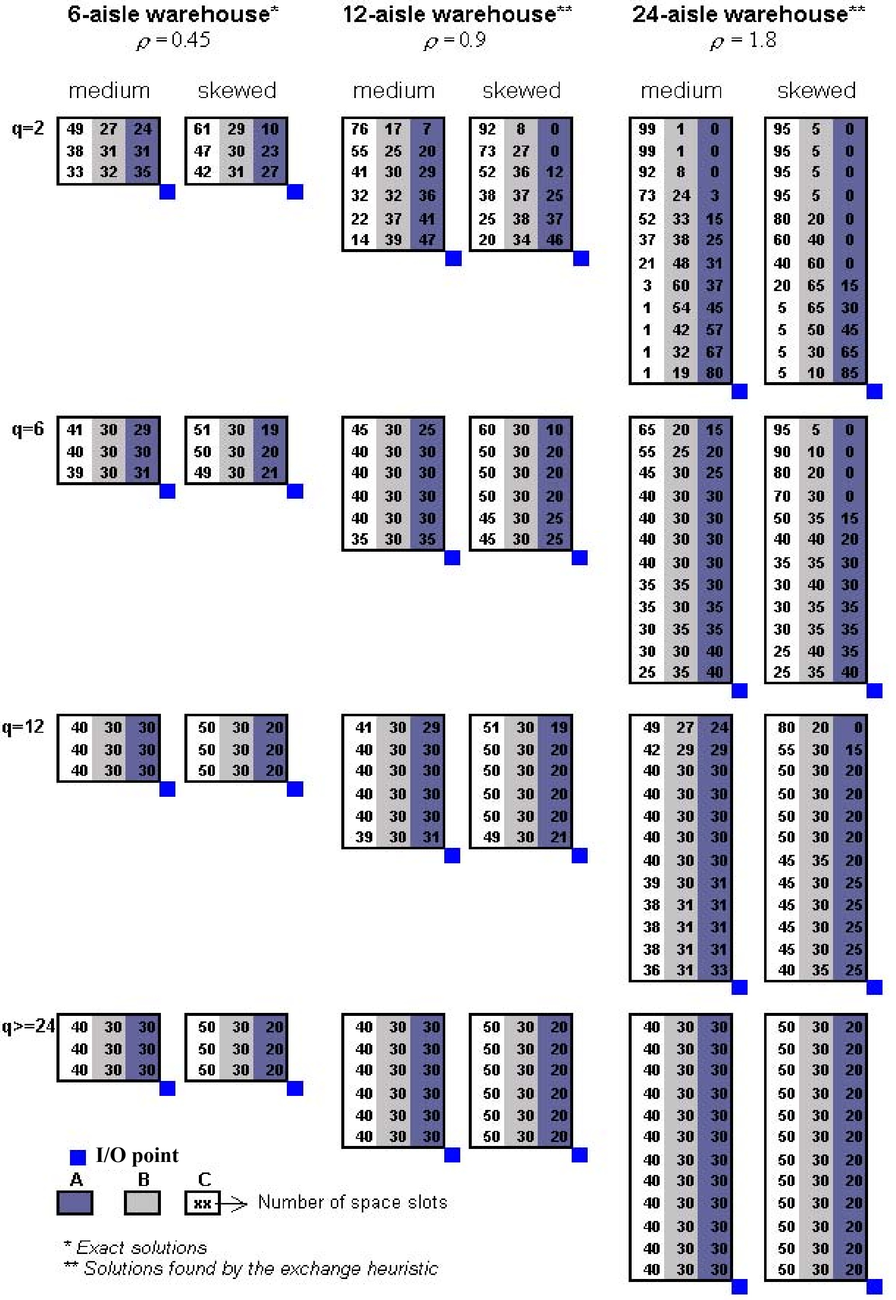

Figure 3.1 'Optimal' storage zones for layout 2, 4 and 5 when the return routing method is used (only left-parts of the warehouses are shown)

Table 3.1 Comparison between optimal average tour length, average tour length obtained by the exchange heuristic and the average tour length of the identical-aisle layout (using the return routing method)

	Pick-list size (q)	Optimum	Heuristic	Identical	% diff. Heu.- Opt.	% diff. Iden.- Opt.
Layout 1: 4 aisles, $L=100$, $w_a=10$, $w_b=15$, $w_d=5$, $s=100$ and $\rho=0.3$						
Medium assignment	1	114.52	114.52	115.00	0	0.41
	2	196.48	196.63	197.77	0.08	0.65
	12	578.23	578.23	578.23	0	0
Skewed assignment	1	72.49	72.50	74.12	0	2.19
	2	119.19	120.63	125.49	1.19	5.02
	12	330.66	330.66	330.67	0	0
Layout 2: 6 aisles, $L=100$, $w_a=10$, $w_b=15$, $w_d=5$, $s=100$ and $\rho=0.45$						
Medium assignment	1	128.57	128.58	130.00	0	1.10
	2	220.63	221.22	221.25	0.27	0.28
	12	729.06	729.06	729.06	0	0
Skewed assignment	1	84.14	84.15	89.12	0	5.58
	2	140.09	140.92	142.67	0.59	1.81
	12	416.55	416.56	416.56	0	0
Layout 3: 8 aisles, $L=100$, $w_a=10$, $w_b=15$, $w_d=5$, $s=20$ and $\rho=0.6$						
Medium assignment	1	142.19	142.24	149.84	0.03	5.10
	2	242.18	242.53	246.79	0.14	1.87
	12	833.34	833.34	833.35	0	0
Skewed assignment	1	94.39	94.50	104.75	0.12	9.90
	2	157.89	158.34	164.91	0.28	4.26
	12	482.02	482.02	482.03	0	0

Optimum = optimal average tour length found by total enumeration;

Heuristic = average tour lengths found by the exchange heuristic; Identical = average tour length for the identical- aisle layout.

% diff. Heu.- Opt. = % difference between solutions obtained by the exchange heuristic and the optimal method.

% diff. Iden.- Opt. = % difference between the solutions from the identical-aisle layout and the optimal results.

s = number of space slots per aisle (s = 100 means the aisle is divided into 100 space slots).

3.4.2 Shape of the optimal storage zones

When the return heuristic is used

Figure 3.1 shows the 'optimal' zone sizes for layout 2 ($\rho = 0.45$), layout 4 ($\rho = 0.9$) and layout 5 ($\rho = 1.8$) when the return heuristic is used. We can see that the optimal number of space slots in the aisles for each class depends on the pick-list size, pick-frequency of classes and shape ratio change. The optimal zoning scheme is summarized in Figure 3.2. The names of the layout types have been adapted from Petersen and Schmenner (1999). The identical-aisle layout is defined as before (Section 3.3). The across-aisle layout means that we allocate the A-class items to the aisles closest to the I/O point, after allocating all A-class items, the B-class items are considered and so on. The diagonal layout is in between the identical- and across-aisle layout. Layout type I (identical-aisle layout) appears to be the best for sufficiently large pick-list sizes, or small shape-ratio (thus short-aisle) warehouses, while layout type III (across-aisle layout) is only good for large shape-ratio (long-aisle) warehouses with rather small pick-list sizes per route. It is surprising that the identical-aisle layout is optimal for large pick-list sizes regardless of the shape ratio. When we consider the continuous-storage space case, the optimal storage layout depends on the I/O position (see Figure 3.3, partly based on Tompkins et al. 2003). However, with the presence of the cross aisle, the I/O position does not influence the optimal storage layout when the pick-list size is sufficiently large, leading to the optimality of the identical-aisle zoning scheme. The reason is that for large pick-list sizes, it is likely that we have to visit all storage aisles and, therefore, any position of the I/O point on the cross aisle results in the same average tour length.

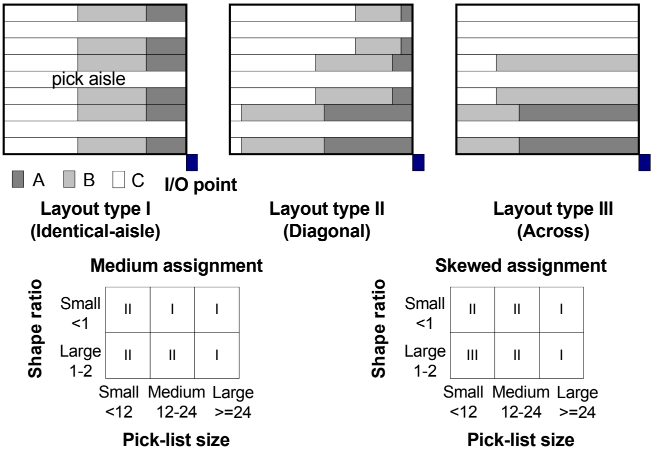

Figure 3.2 Shape of the optimal storage zone in relation with pick-list size, storage assignment and shape ratio (for the return routing method)

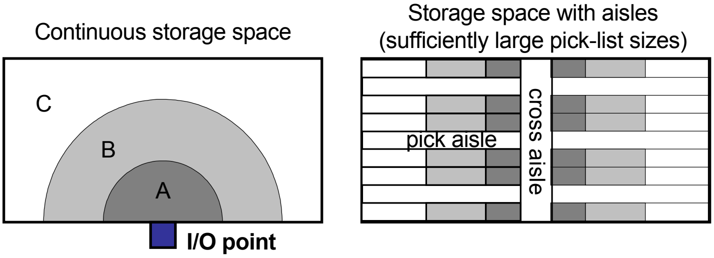

Figure 3.3 Optimal storage zoning in continuous storage space, and in space with aisles for sufficiently large pick-list sizes and the return routing method

When the S-shape heuristic is used

Figure 3.4 shows the optimal storage zone shapes for layouts 2, 4 and 5 when the S-shape is used. For all three layouts, the across-aisle layout (see Figure 3.2) appears to be the optimal, irrespective of the layout, the storage assignment and pick-list sizes (between 1 and 60).

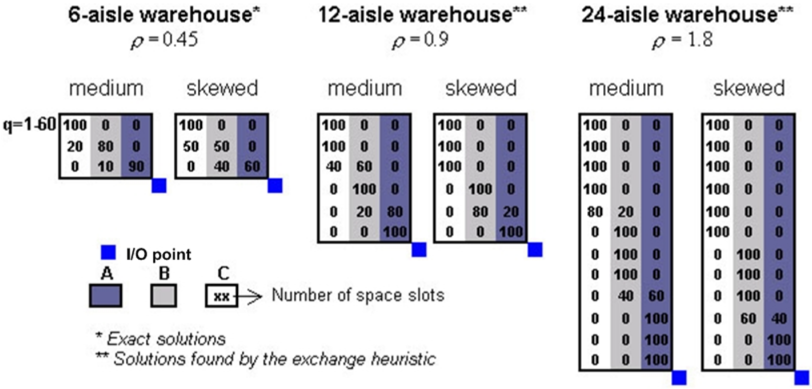

Figure 3.4 'Optimal' storage zones for layout 2, 4 and 5 when the S-shape routing method is used (only left-parts of the warehouses are shown)

The reason is that, among other layout types, the across-aisle layout is the one that minimizes the number of visited aisles, and hence the cross-aisle travel distance. Furthermore, for the S-shape method the average within-aisle average travel distance is mainly determined by the number of visited aisles. (It has to be noted that the correction term is only a minor portion of the average within-aisle travel distance, especially in the case when the number of visited aisles is large.) Therefore, the across-aisle layout is favorable for the S-shape method. This result is in line with Jarvis and McDowell (1991) and Petersen and Schmenner (1999) who suggest that when the S-shape routing method is applied, to reduce the average travel distance, the most frequently ordered items should be concentrated in a few aisles close to the I/O point.

3.4.3 Robustness[3] of the identical-aisle layout

When the return heuristic is used

From Table 3.2 we can also see that the differences between results from the identical-aisle layouts and the corresponding optimal layouts are small for small pick-list sizes (less than 10%) and the gaps decrease rapidly when the pick-list size grows (less than 0.01% when the pick-list size is larger than or equal to 12). To study this effect further, we considered layout 1 (4-aisle warehouse). For each possible value of the number of space slots used for the A-class in the aisle closest to the I/O point (l_{11}), we found the corresponding optimal layout.

Figure 3.5 and 3.6 show the experimental results for the skewed and medium assignment, for *q=2* and *q=12*, respectively. As we can see, when the pick-list size is small (q = 2), the average tour length is a convex function of l_{11}. However, this curve becomes very flat when the pick-list size is rather large (q = 12). It means that when q is large, multiple optima (or near optima) exist. As a result, the optimal average tour length and the best travel distance found by the exchange heuristic and from the identical-aisle layout are very close. This fact also explains why the heuristic performs very well for large pick-list sizes).

From a practical point of view, this result (robustness of the identical-aisle layout) is very interesting: instead of adjusting the optimal zone borders for a single pick-list size, we now can select the identical-aisle layout as a good approximation for all pick-list sizes. No doubt, this approximation is better for the medium assignment than for the skewed storage assignment.

[3] In this thesis, robustness is referred as ‘generally’ good or good in many cases.

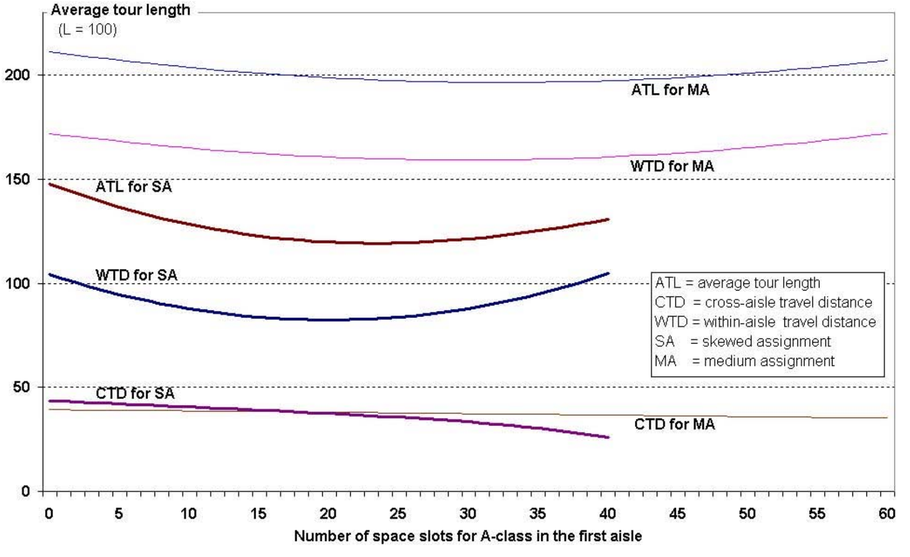

Figure 3.5 Optimal average tour length for layout 1 with 2 picks per route, and when the return routing method is applied

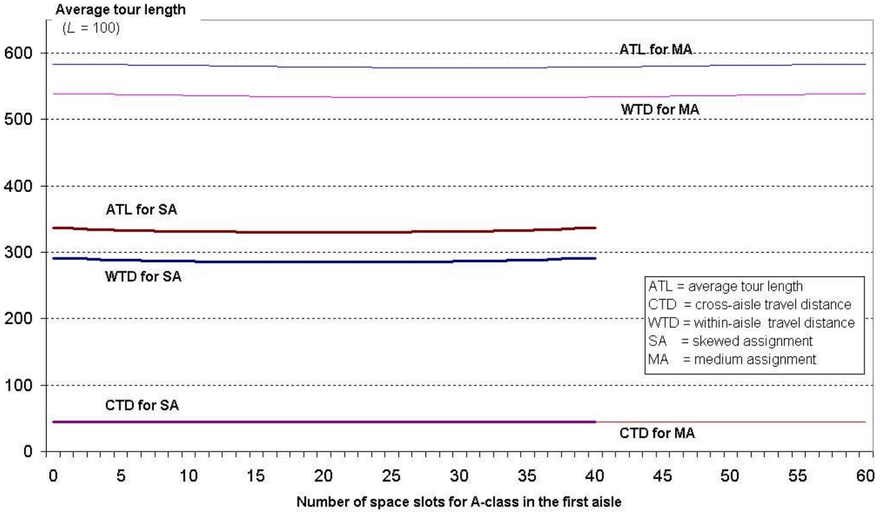

Figure 3.6 Optimal average tour length for layout 1 with 12 picks per route, and when the return routing method is applied

When the S-shape heuristic is used

The identical-aisle layout can be far from the optimum in the case of the S-shape heuristic. Table 3.2 shows the minimum and maximum differences between the average tour length resulting from the optimal layout and the identical-aisle layout when the pick-list size varies from 1 to 48. As we can see the difference between two layout types becomes smaller when either the pick-list size or the shape ratio decreases. And, it becomes larger when the skewed assignment is used.

Table 3.2 % differences between the optimal and identical-aisle layout (when the S-shape routing method is used and the pick-list size varies from 1 to 48)

Layout	Shape ratio (ρ)	Medium assignment		Skewed assignment	
		Max. diff.	Min. diff.	Max. diff.	Min. diff.
6-aisle	0.6	4.60	0.20	10.38	7.93
12-aisle	0.9	10.17	1.66	24.55	15.37
24-aisle	1.8	15.51	3.06	38.93	18.67

3.5 Layout optimization model

As mention in Section 3.1, for the storage zone optimization problem the layout dimensions (i.e. number of aisles, aisle's length and width) are given. However, at the layout design state, usually only the storage space (e.g. floor area) rather than the exact dimensions of the warehouse are fixed. In this situation, besides l_{ij}'s, the number of aisles a is also a decision variable. We call this problem (determining the optimal number of storage aisles and storage zone) *layout optimization problem*. More specifically, for a given total floor space (S), the width of a pick aisle, the width of the storage rack and the cross aisle, pick-frequencies of classes and portion of total space needed for each class, the layout optimization is the problem of determining the joint optimal number of aisles and storage zones.

3.5.1 Mathematical formulation

We propose the following mathematical formulation for the problem:

$$Min \quad TD_z\left(l_{ij}, a\right) \tag{3.7}$$

Subject to

$$\sum_{i=1}^{c} l_{ij} = S/\left(aw_b\right) - w_d - w_a/2 \qquad \forall j = 1..a \tag{3.8}$$

$$\sum_{j=1}^{a} l_{ij} = s_i a\left[S/(aw_b) - w_d - w_a/2\right] \qquad \forall i = 1..c \tag{3.9}$$

$$p_{ij} = l_{ij} f_i \Big/ \left\{ s_i a\left[S/(aw_b) - w_d - w_a/2\right]\right\} \qquad \forall (i = 1..c,\ j = 1..a) \tag{3.10}$$

$$l_{ij} = l_{i,a-j+1} \qquad \forall (i = 1..c,\ j = 1..a/2) \tag{3.11}$$

$$2w_c a\left[S/(aw_b) - w_d - w_a/2\right] \Big/ S \geq u_0 \tag{3.12}$$

$$2 \leq a \leq \left\lfloor \frac{2S}{(w_a + 2w_d + 2L_{\min})w_b} \right\rfloor \tag{3.13}$$

$$a \in N^+,\ l_{ij} \geq 0 \qquad \forall (i = 1..c,\ j = 1..a) \tag{3.14}$$

Compared to the formulation in Section 3.2, (3.7), (3.8), (3.9), (3.10) and (3.11) are about the same as (3.1), (3.2), (3.3), (3.4) and (3.5) respectively, except the number of aisles (a) now becomes a decision variable. Furthermore, there are two new constraints (3.13) and (3.14). (3.13) ensures that floor space utilization of the layout is always greater than or equal to a predetermined level (u_0). Floor space is rather expensive, thus using it efficiently is often desired. (3.14) shows how to calculate the upper bound of the number of aisles from the total floor space (S), the minimum aisle length ($L_{\min}$), the width of the cross aisle (w_a), the width of the marginal aisle (w_d), the width of the pick aisle ($w_b - 2w_c$) and the centre-to-centre distance between two consecutive aisles (w_b). Again, we are confronted with a difficult situation: a mixed-integer and non-linear program. We propose to solve the problem heuristically.

A variant of the problem is the situation in which not the total floor space but the total storage space (total capacity of racks) is fixed. However, given a storage system, we can easily convert the total storage capacity to the total floor space and vice versa.

3.5.2 A heuristic approach for layout optimization problem

If we fix the number of pick aisles and relax the space utilization constraint then the layout optimization problem reduces to the zoning optimization problem. Therefore we can chase for a best solution by considering all possible values of the number of pick aisles, for each we evoke the *ZoningOpt* procedure to find the best zoning which does not violate the space utilization constraint. We can do that because the running time of the *ZoningOpt* procedure is negligible and the maximum number of aisles is limited. It can be determined by using the sixth constraint in Formulation (3.2) mentioned above. The heuristic is described by the following pseudo code:

procedure *LayoutOpt*

for $\alpha = 1$ *to* $\left\lceil 2S \big/ \left(w_a + 2w_d + 2L_{\min}\right) w_b \right\rceil$ *do*

$a = 2\alpha$

while $2w_c a\left(S/aw_b - w_d - w_a/2\right)/S \geq u_0$ *do*

call ZoningOpt(a)

end do;

save L^{opt} *and* a^{opt}*;*

end do;

end sub;

L^{opt} is the current minimum average tour length and a^{opt} is the corresponding value of the number of aisles (a).

3.5.3 Numerical results and discussions

It can be seen that the *LayoutOpt* procedure simply bases on iteratively solving the zoning optimization problems and then selecting the best one among the solutions that give the shortest average tour length and satisfy the space utilization constraint. Therefore, the performance of the *ZoningOpt* procedure itself guarantees the quality of the heuristic for the layout optimization problem.

Figure 3.7 shows the optimal number of aisles obtained by using the *LayoutOpt* procedure for a picking area of 495 square units, w_a =3, $w_b = 5$, $w_c = 2$ and $w_d = 1.5$ units (refer to Figure 2.2 for the layout notations). The skewed assignment is defined as before (see Table 2.1) and the space utilization lower bound is 0.5 (i.e. $u_0 = 0.5$). Besides this layout, we also considered several others. They all show similar graphs as in Figure 3.7. We can draw the following conclusions from the experimental results.

- *Rule of thumb for selecting 'good' layout regardless of the routing method*. Layouts with many aisles (or short aisles) are better for small pick-list sizes, while layouts with long aisles are better for very large pick-list sizes, regardless of the routing method used. The reason can be explained as follows. When the pick-list size is small, the order picker only has to visit few storage aisles. So, the average travel distance will be shorter if the aisles are short. However, when the pick-list size is large, the order picker has to visit a large part of the warehouse (i.e. many aisles). Hence, it may be better if the number of aisles is reduced. Furthermore, by reducing number of aisles, the cross-aisle travel distance decreases as well.

- *Influence of demand skewness.* It appears that when the pick-list size increases, the optimal number of aisles reduces more gradually in the case of the skewed storage assignment than in the cases of random storage assignment. The reason is that, with a given pick-list size, the number of visited aisles in the case of the skewed storage assignment is less than in the case of random storage assignment. Consequently, the effect of increasing pick-list size is less for the skewed assignment.
- *Influence of routing method.* Given a pick-list size and an assignment scheme, it appears that using the return routing heuristic leads to a greater (or equal) optimal number of aisles (thus shorter aisles) than using the S-shape heuristic. This seems logical as with the return heuristic the order picker has to return in every visited aisle, thus short aisles are preferred.

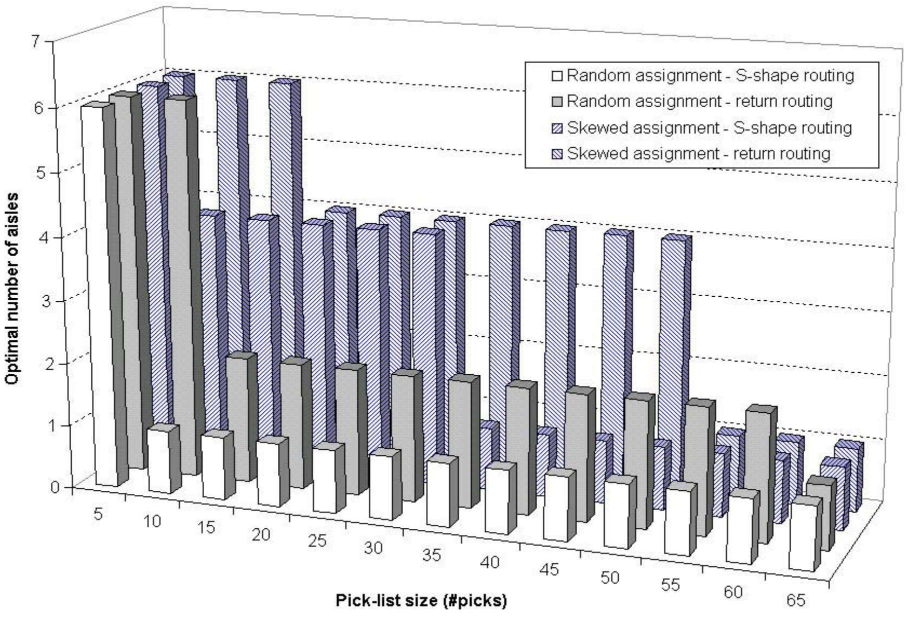

Figure 3.7 Optimal number of aisles for a picking area of 495 square units, $w_a = 3$, $w_b = 5$, $w_c = 2$ and $w_d = 1.5$ units

3.6 Concluding remarks

This chapter deals with the problem of finding the optimal storage zones and layout that minimize the average tour length for manual-pick class-based storage strategy warehouses. These problems are crucial in warehouse design and control; they occur whenever a warehouse is (re)designed, or the assortment or the order pattern changes. To solve the problems, we first consider a precise approach. However, the exact algorithm is time consuming; it cannot handle large warehouse instances (regarding the number of aisles, classes and space slots per aisle). Hence, we propose a heuristic approach to solve the problems. This heuristic exchanges proximity classes between aisles, from far-to-depot aisles to closer-to-depot aisles. The approach is rather simple, but fast and proves to be of very good quality. It can therefore be applied to many practical warehouse design or improvement situations.

We define a *best layout* as a layout that provides the shortest average tour length. Based on the experimental results from this and the previous chapter, we can establish several *design guidelines* for manual-pick class-based storage strategy warehouses.

A. When the warehouse's dimensions (i.e. warehouse's length and width, the number of aisles) are given (see Sections 3.3&3.4):

1. When the return method is applied to route order pickers (note that in real warehouse environments, one may be forced to return in the aisle because of the nature of aisles, of the warehouse organization and the relation between storage areas, see Dekker et al., 2004 for an example).

   (a) For large pick-list sizes: the identical-aisle layout is the best layout, irrespective of demand skewness (or skewness level of the storage assignment).

   (b) For small pick-list sizes: the across-aisle layout is the best for long-aisle[4] warehouses with skewed storage assignments.

2. When the S-shape routing method is applied to route order pickers: *the across-aisle layout is always the best layout, irrespective of the pick-list size, shape ratio and storage assignment.*

3. From Chapter 2 we know that the S-shape method outperforms the return method, except the case of small pick-list sizes with the skewed assignment. Therefore, if no preference has been made for either the S-shape or return routing method, we suggest that:

   (c) For small pick-list size, skewed demand and long-aisle warehouses: choose the across- aisle layout and apply the return routing method.

   (d) For other cases: choose the identical-aisle layout and apply the S-shape routing method.

B. When only the total floor area of warehouse is given (thus the warehouse's dimension are not fixed, see Section 3.5):

[4] i.e. shape ratio is greater than 1 (see Section 3.4.2)

(e) *Long-aisle layouts appear to be the best for large pick-list sizes, while short-aisle layouts appear to be better than long-aisle layouts for small pick-list sizes, regardless of the routing method and demand skewness.*

The layouts for testing the quality of the proposed heuristics are rather small compared to real warehouse sizes (i.e. number of aisles and space slots or storage locations). The reason is that for large warehouse instances, it is not possible (for us) to find the (exact) optimal layout. Our conjecture is that the gap between heuristics and optimal result increases when the size of the warehouse becomes larger. However, in our situation, due to stochastic natures of the demand, it is not necessary to trace for the optimal layout; a robust layout (that is 'good' on average) may be better than the optimal layout (that is the best in a certain situation, but not necessarily good in others).

4

Travel Time Estimation and Optimal Rack Design for a 3-dimensional Compact AS/RS

4.1 Introduction

In the previous chapter, we considered the problem of storage zone and layout optimization in manual-pick OP systems. In this chapter, we consider a layout design problem for a 3-dimensional compact storage system. The system uses gravity flow racks with conveyors, working in pairs.

Although their application is still limited, compact storage systems become increasingly popular for storing products (Van den Berg and Gademann, 2000, Hu et al., 2005) , with relatively low unit-load demand, on standard product carriers. Their advantage is the full automation, making it possible to retrieve and store unit loads around the clock, on a relatively small floor area. In principle, every load can be accessed individually, although some shuffling may be required. They are also used to automatically presort unit loads within the system, so that these loads can rapidly be retrieved when they are needed.

Several compact storage system technologies exist with different handling systems taking care of the horizontal, vertical and depth movements. In this chapter, we calculate the travel time and investigate the optimal dimensions for minimizing the travel time under a random storage strategy, for a given storage capacity, of the compact storage system as sketched in Figure 4.1. This system has been designed for several application areas.

The compact storage system consists of an S/R machine taking care of movements in the horizontal and vertical direction (the S/R machine can drive and lift simultaneously). A gravity conveying mechanism takes care of the depth movement. Conveyors work in pairs: unit loads on one conveyor flow to the rear end of the rack, in the neighboring conveyor unit loads flow to the S/R machine. At the backside of the rack, an inexpensive simple elevating mechanism lifts unit loads from the down conveyor to the upper conveyor, one at a time.

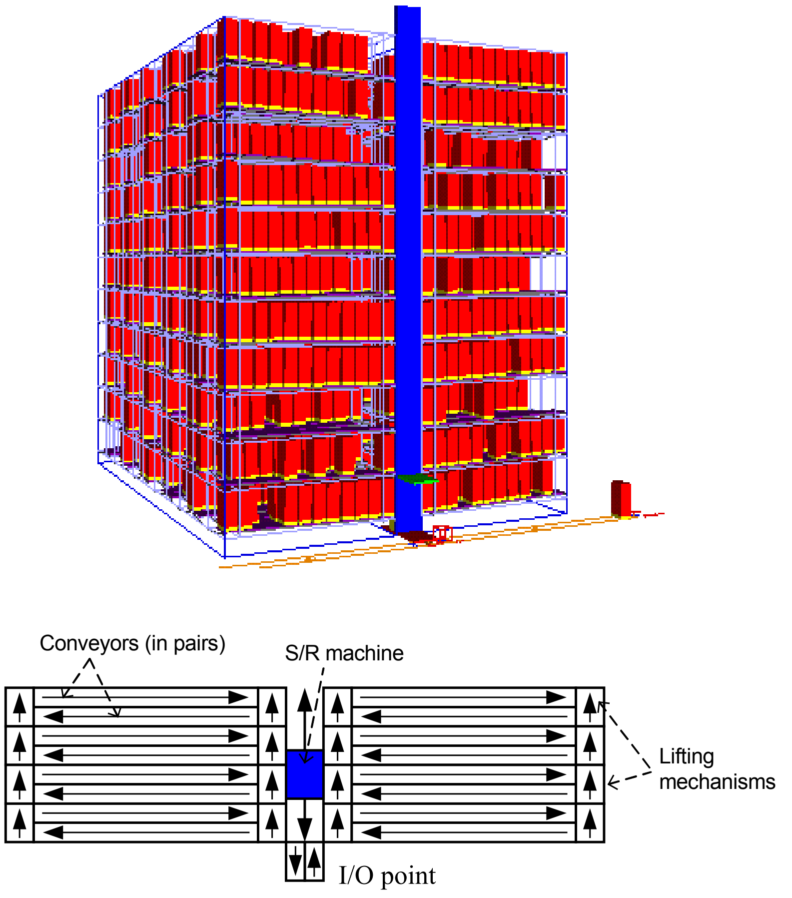

Figure 4.1 A compact S/RS with gravity conveyors for the depth movements

The innovation of the system is in its cheap construction: no motor-driven parts are used for the conveyors and the construction of the lifting mechanisms is simple as well. The unit loads move by (controlled) gravity. Potential application areas are also innovative. We have studied applications in dense container stacking at a container yard and the Distrivaart project in the Netherlands, where pallets are transported by barge shipping between several suppliers and several supermarket warehouses. This project has actually been implemented and has resulted in a fully automated storage system on a barge (see

Figure 4.2). Although this project was a technical success, it was stopped after two years for lack of transport pallet loads committed by suppliers and retailers.

Figure 4.2 Distrivaart: A conveyor-supported automated compact storage system on a barge (*source: De Koster and Waals, 2005*).

The throughput capacity of the system depends on not only the physical design, the speeds of handling systems used, but also on the dimensions of the system and the storage and retrieval strategy used. We assume that only single cycles are carried out (in fact, we investigate only retrievals, since storage and retrieval are likely to be decoupled in these systems) and that the storage strategy is random. This is more or less a worst-case scenario, since in reality pre-sorting is often possible. Although finding the S/R machine travel time is not too difficult for the general case, finding closed-form expressions for the three dimensions that minimize the total travel time is more complicated. Analytically, we have been able to find these dimensions for the case that the rack is SIT (i.e. length and height of the rack are equal in horizontal and vertical travel time of the S/R machine respectively). For the none-square-in-time (NSIT) case, we have to rely on different methods. For a given total storage space, we use the nonlinear solver of What'sBest®7.0 (LINDO optimization software for Excel users) to find the optimal dimensions and the corresponding travel time. After considering a wide range of total storage space values, we propose regression formulas for estimating the expected travel time (for single-command cycles) and the optimal dimensions.

This chapter is organized as follows. In the next section, we review literature concerning travel time models for AS/RS and mention assumptions and notations used in the chapter. In Section 4.3, we present the travel time models for estimating the expected single-command travel time. In Sections 4.4 and 4.5, we find the optimal rack's dimensions that minimize the travel time. We illustrate the results found in Section 4.6 by an example in Section 4.7. Finally, we conclude and propose some potential directions for future research in Section 4.8.

4.2 Literature review, assumptions and notations

A considerable number of papers exist that analyze AS/RS performance (e.g. estimating expected travel time, rack's dimensions, system throughput, etc.). Figure 4.3 lists major problem characteristics and solution methods used in AS/RS performance models in the literature. The following common assumptions are commonly used (see also Bozer and White, 1984, 1990, 1996, Ashayeri et al. 2002, and Foley et al., 2004):

- The S/R machine is capable of simultaneously moving in vertical and horizontal direction at constant speeds. Thus, the travel time required to reach any location in the rack is approximated by the Tchebyshev metric. In contrast, in manual-pick OP systems, which use humans to retrieve items from storage area, the travel distance (or, equivalently, travel time) is measured by the Euclidean metric.

- The rack is considered to have a continuous rectangular pick face, where the I/O point (also: depot) is located at the lower left-hand corner.

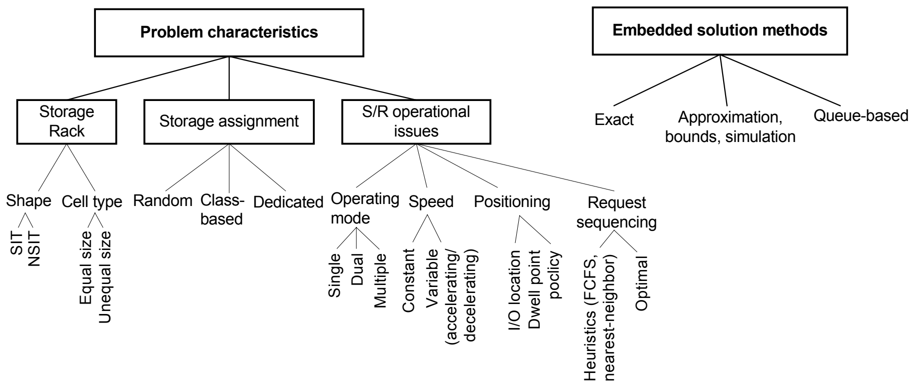

Figure 4.3 Problem characteristics and solution methods used in AS/RS performance models

In this section, we review most recent publications (i.e. most of the articles are published after 1995, except for some important radical ones) concerning AS/RS performance analysis. We discuss the publications mainly based on the system characteristics embedded in the model and solution methods applied. For a general review on the design and control of automated material handling systems, we refer to Johnson and Brandeau (1996). For an overview of travel time models for AS/RS published before 1995, it is advisable to see Sarker and Babu (1995).

- *Storage rack.* Storage shape may influence the performance of AS/RS. It is proved that under the random storage assignment and with a constant AS/RS speed, the SIT rack is the optimal configuration (Bozer and White, 1984). However, this is not necessarily true for other storage assignments. Pan and Wang (1996) propose a framework for the dual-command cycle continuous travel time model under the class-based assignment. The model is developed for SIT racks using a first-come-first-serve (FCFS) retrieval sequence rule. Foley and Frazelle (1991) derive the distribution of dual-command travel time for SIT rack with uniform distributed turnover. Recently, Park et al. (2005) propose the distribution of the expected dual-command travel time and throughput of SIT racks with two storage zones: high and low turnover. Ashayeri et al. (1997, 2002) compute the expected cycle time for an S/R machine where racks can be either SIT or NSIT. Park et al. (2003a) compute the mean and variance of single and dual-command travel times for NSIT racks with turnover-based storage assignment. They also show how to adjust the model if the class-based storage assignment is used. In general, AS/RSs have racks of equally-sized cells. However, in some cases, a higher utilization of warehouse storage can be archived by using unequal sized cells. Lee et al. (1999, 2005) develop travel time models for a rack with unequal cells under a random storage assignment, and both single and dual-command cycles. They also compare the proposed continuous-rack model with a discrete-rack model (through simulation) and conclude that the differences in expected travel times are small.

- *Storage assignment.* Using class-based and dedicated storage assignments may lead to a substantial saving on the travel time of the S/R machine (see Section 1.3.2). For a two-class-based storage assignment rack, Kouvelis and Papanicolaan (1995) develop expected command cycle time formulas for both single and dual-command cycles. They also present explicit formulas for the optimal boundary of the two storage areas in the case of single-command cycles. As exact expressions of the throughput are often lengthy and cumbersome, Foley et al. (2004) derive formulas bounding and approximating the throughput of a mini-load system with exponential distributed pick time and either uniform or turnover-based storage assignment. They report that for typical configurations, the worst-case relative error for the bounds is less than 4%.

- *S/R machine operational issues*. With one shuttle, the S/R machine can at most execute two commands (storage and retrieval) in one travel cycle. Single and dual-command cycles are studied in most of studies in the literature (for example, single-command cycles in Seidmann, 1988, Park et al., 2003a; dual-command cycles in Foley and Frazelle, 1991, Pang and Wang, 1996, and Wilhelm and Shaw, 1997). By using multiple shuttles, the S/R can perform more than two commands in one travel cycle, and thus the system performance can be enhanced. Meller and Mungwattana (1997) present analytical models for estimating the throughput in multi-shuttle AS/RS systems. Potrč et al. (2004) present heuristics travel time models for AS/RS with equal-sized cells in height and randomized storage under single- and multi-shuttle systems. Almost all studies concerning AS/SR assume that the S/R speed is constant. Certainly it is not true in practice (Hwang and Lee, 1990), although the impact of accelerating and decelerating is limited (especially for large racks). Chang et al. (1995) propose a travel time model of S/R machines by considering the speed profiles that exist in real-word applications. They consider the system under random storage assignment, single and dual-command cycles. Chang and Wen (1997) extent this travel time model to investigate the impact on the rack configuration. The results demonstrate that the optimal rack configuration of the single-command cycles is still SIT whereas the dual-command cycles may not be. Wen et al. (2001) also adjust the travel time model in Chang et al. (1995), but for the class-based and turnover-based storage assignment.

- *Solution approach*. Most of the travel time models were developed based on statistical analysis and simulation (for example, Hausman et al., 1976, Graves et al., 1977, Bozer and White, 1984, Foley et al., 2002, 2004). Lee (1997) uses a single-server model with two queues to estimate the throughput of a mini-load system, where the cycle times are assumed to be independent, identical, and exponentially distributed (iid) random variables, while requests arrive according to a Poisson process. Simulation results in this study show that the method performs well and can be easily adapted for other AS/RS. However, Hur et al. (2004) claim that the exponential distribution of travel times does not reflect the dynamic aspect of the system. They propose to use an M/G/1 queuing model (also with a single server and two queues). According to their computational results, the proposed approach gives satisfactory results with very high accuracy. Park et al. (1999) study an end-of-aisle OP system with inbound and outbound buffer positions (a mini-load system with a horse-shoe front-end configuration). They model the system as a two-stage cyclic queueing system consisting of one general and one exponential server queue with limited capacity. They assume that the S/R machine always executes dual-command cycles and that the dual-command cycle times are independent of each other. With known results for a two-stage cyclic queueing system, they obtain closed form expressions for the stationary probability and the throughput of the system. To compute the mini-load system throughput, the distribution of order arrivals is needed (usually the pick time distribution is assumed to be exponential or uniform, see for example Bozer and White, 1990, 1996, and Foley and Frazelle, 1991). However, this information is not completely available at the designing phase (only partial information is known). Foley et al. (2002) determine upper and lower throughput bounds for mini-load systems under several different types of the partial information: no information, mean only, and NBUE (i.e. New Better than Used in Expectation, roughly it means that the mean pick time of a partially processed bin is smaller than the mean pick time from a new bin).

In the above-mentioned publications, only two travel directions are considered (vertical and horizontal). However, situations exist where the S/R machine can travel in three orthogonal directions simultaneously, i.e. vertical, horizontal and cross-aisle direction. Park and Webster (1989b) propose a conceptual model that can help a warehouse planner in the design of 3-dimensional, pallet storage systems. Park and Webster (1989a) deal with the problem of finding a rule for assigning rack locations to product turnover classes to minimize the expected travel time. In these publications, however, the rack dimensions are given or, in other words, the problem of determining the optimal rack dimensions is neglected. For the AS/RS described in Section 4.1, the S/R machine can only travel vertically and horizontally. However, there is another travel time/direction associated with each travel cycle of the S/R machine; the time needed to convey the load to the pick position or to reveal an empty location to store the load. For this reason, we also use the terminology 3-dimensional compact storage for our system. We have not found any literature on travel time estimation and/or optimal system dimensioning for this or similar AS/RS types. In the following sections, we will step by step estimate the single-command travel time of the S/R machine for the system that we introduced in Section 4.1.

4.3 Travel time estimation

Besides the common assumptions mentioned in the previous section, we use the following explicit assumptions for our travel time model:

- The S/R machine operates on a single-command basis (multiple stops in the aisle are not allowed). This restriction is relaxed when we develop travel times models for dual-command cycles (see Appendix 4 at the end of this chapter).
- The total storage space, the speed of the conveyor (s_c), as well as the S/R machine's speed in the horizontal (s_h) and vertical direction (s_v), are known.
- The S/R machine travels simultaneously in the horizontal and vertical direction. In calculating the travel time, constant velocities are used for the horizontal and vertical travel: no accelerating or decelerating effects. These effects should be taken into account for the cases of short travel distances. However, in our model they are reflected (or included) in the pick-up/ deposit time.
- We use random storage. That is, any point within the pick face is equally likely to be selected for storage or retrieval.
- The pick-up and deposit (P/D) time for a given load is known and constant. The P/D time is identical for all loads.

The length (L), the height (H) of the rack and the perimeter (or length $2S$) of the conveyor form three orthogonal dimensions of the system. Without loss of generality, we suppose that the travel time to the end of the rack is always no less than the travel time to the highest location in the rack: $\frac{H}{s_v} \le \frac{L}{s_h}$. To standardize the system, we define the following quantities.

$t_c = \frac{2*S}{s_c}$: length (in time) of the conveyor.

$t_h = \frac{L}{s_h}$: length (in time) of the rack.

$t_v = \frac{H}{s_v}$: height (in time) of the rack.

$T = \max\{t_h, t_v, t_c\}$

$b = \min\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$. Note that $0 \le b \le 1$ and $b = 1$ if and only if $t_h = t_v = t_c$.

a is the remaining element (besides b and 1) of the set $\left\{\frac{t_h}{T}, \frac{t_v}{T}, \frac{t_c}{T}\right\}$, thus $0 < b \le a \le 1$.

For determining the optimal dimensions of the rack, we suppose that $2 * H * L * S$ is given. Consequently, $t_h t_v t_c = V$ is also given (*V* is so called the total handling capacity of the system)

Assume that the retrieval location is represented by (x, y, z), where X, Y and Z refer to the movement directions of the S/R machine and conveyor. We can see that the S/R machine's travel time for single-command cycles (*ESC*) consists of the following components:

- Time needed to go from the I/O point to the pick position and to wait for the pick to be available at the pick position (if the conveyor circulation time is larger than the travel time of the S/R machine), W. In other words, W is the maximum of the following quantities:
  - time needed to travel horizontally from the I/O point to the pick position,
  - time needed to travel vertically from the I/O point to the pick position,
  - time needed for the conveyor to circulate the load from the current position to the pick-up position, R.
- Time needed for the S/R machine to return to the I/O point, U.
- Time needed for picking up and dropping off the load, c (assumed to be constant).

Hence, the expected travel time can be expressed as follows:

$$ESC = E(W) + E(U) + c \qquad \textbf{(4.1)}$$

As c is a constant, it does not have any influence on the rack layout so from now on we will not consider this component.

As proven by Bozer and White (1984), in the case of a 2-dimensional rack, the travel time from a random pick location to the I/O point can be calculated as:

$$E(U)=\left(\frac{\beta^2}{6}+\frac{1}{2}\right)t_h, \tag{4.2}$$

where $\beta=\frac{t_v}{t_h}(\beta\le 1)$ is the shape factor of the rack (recall that we assume $t_h \ge t_v$).

Let $F(w)$ denote the mass probability function that W is less than or equal to w. We assume that the x, y, z coordinates are independently generated, where: $0 < x \le a$, $0 < y \le b$ and $0 < z \le 1$ (that is, we consider the 'normalized' rack). Similar to the case of 2-dimensional racks (see Bozer and White, 1984), we have:

$$F(w)=\mathrm{P}(W\le w)=P(X\le w).P(Y\le w).P(Z\le w)$$

Furthermore, as we use randomized storage; the location coordinations are uniformly distributed. Therefore,

$P(Z\le w)=w$, with $0\le w\le 1$

$$P(X\le w)=\begin{cases} w/a & \text{if } 0\le w\le a \\ 1 & \text{if } a<w\le 1\end{cases}$$

and

$$P(Y\le w)=\begin{cases} w/b & \text{if } 0\le w\le b \\ 1 & \text{if } b<w\le 1\end{cases},$$

Hence,

$$F(w)=\begin{cases} w^3/ab & \text{if } 0\le w\le b \\ w^2/a & \text{if } b<w\le a \\ \text{w} & \text{if } a<w\le 1\end{cases}$$

$$\Rightarrow f_W(w)=\begin{cases} 3w^2/ab & \text{if } 0\le w\le b \\ 2w/a & \text{if } b<w\le a \\ 1 & \text{if } a<w\le 1\end{cases}$$

Therefore,

$$E(W)=T\int_{w=0}^{1} g(w)wdw=T\left(\int_{w=0}^{b}\frac{3w^3}{ab}dw+\int_{w=b}^{a}\frac{2w^2}{a}dw+\int_{w=a}^{1}wdw\right)$$

$$\Rightarrow E(W) = \left(\frac{b^3}{12a} + \frac{a^2}{6} + \frac{1}{2}\right) \qquad \textbf{(4.3)}$$

From (4.1), (4.2) and (4.3), it is possible now to find the single-command travel time if we know the relative magnitude of each dimension compared to others (i.e. which one is the longest, shortest). And therefore the ratio between three dimensions which minimizes the expected travel time can be determined. To facilitate the analysis, we distinguish two situations: SIT racks (section 4.4) and NSIT racks (section 4.5).

4.4 Optimal dimensions for the square-in-time (SIT) rack

As shown in Bozer and White (1984): "*For 2-dimensional racks, the expected travel time will be minimized if the rack is SIT*". Suppose that this type of rack is used we further consider two situations:

- when the conveyor's length is the largest dimension (section 4.4.1),
- when the conveyor's length is the shortest dimension (section 4.4.2).

4.4.1 Conveyor's length is the largest dimension (SIT_CL)

In this case, we have $T = t_c$, $a = b$ (thus $\beta = 1$), $t_h = at_c$, $t_v = at_c$ and $a^2 t_c^3 = V$. From (4.1) and (4.2):

$$\begin{cases} E(U) = \dfrac{2}{3} a t_c \\ E(W) = \left(\dfrac{a^2}{4} + \dfrac{1}{2}\right) t_c \end{cases}$$

$$\Rightarrow ESC_{SIT\_CL} = \left(\frac{a^2}{4} + \frac{2}{3}a + \frac{1}{2}\right) t_c \qquad \textbf{(4.4)}$$

At this point, our problem turns out to be the following constrained-optimization problem:

$$\textit{Minimize} \qquad f_{SIT\_CL}(a, t_c) = \left(\frac{a^2}{4} + \frac{2}{3}a + \frac{1}{2}\right) t_c$$

$$\textit{subject to} \qquad D = \left\{ (a, t_c) \middle| a^2 t_c^3 = V,\ 0 < a \le 1, t_c \ge 0 \right\}$$

We use the Lagrangian multiplier method to include the constraint $a^2 t_c^3 = V$ in the objective function and obtain: $L(a, t_c, \lambda) = \left(\frac{a^2}{4} + \frac{2}{3}a + \frac{1}{2}\right)t_c + \lambda\left(a^2 t_c^3 - V\right)$, where λ is the Lagrangian multiplier. The critical points of $L(a, t_c, \lambda)$ must be the solutions of the following system:

$$\begin{cases} \dfrac{\partial L(a, t_c, \lambda)}{\partial a} = 0 \\ \dfrac{\partial L(a, t_c, \lambda)}{\partial t_c} = 0 \\ \dfrac{\partial L(a, t_c, \lambda)}{\partial \lambda} = 0 \end{cases} \Leftrightarrow \begin{cases} \left(\dfrac{a}{2} + \dfrac{2}{3}\right)t_c + 2a\lambda t_c^3 = 0 \\ \dfrac{a^2}{4} + \dfrac{2}{3}a + \dfrac{1}{2} + 3\lambda a^2 t_c^2 = 0 \\ a^2 t_c^3 - V = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda = -0.46 / \sqrt[3]{V^2} \\ a = 0.72 \\ t_a = t_b = 0.89\sqrt[3]{V} \\ t_c = 1.24\sqrt[3]{V} \end{cases}$$

It is easy to see that the sufficient condition for the critical point to be the minimum point is satisfied (meaning that Hessian matrix H is positive semi-definite at the critical point). Thus, this critical point is the minimum point and the optimal value is $ESC^{*}_{SIT\_CL} = 1.38\sqrt[3]{V}$.

We conclude:

> *"Given an SIT rack with a total storage capacity V and provided that the conveyor's length t_c is the longest dimension, the estimated travel time of the S/R machine will be minimized if $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$ and the optimal travel time is $1.38\sqrt[3]{V}$ ".*

4.4.2 Conveyor's length is the shortest dimension (SIT_CS)

In this case $a = b$ (so $\beta = 1$), $T = t_h = t_v$, $t_c = bt_h$ and $bt_h^3 = V$. From (4.2) and (4.3) we have:

$$\begin{cases} E(v) = \dfrac{2}{3}t_h \\ E(w) = \left(\dfrac{b^2}{4} + \dfrac{1}{2}\right)t_h \end{cases}$$

$$\Rightarrow ESC_{SIT\_CS} = \left(\frac{b^2}{4} + \frac{7}{6}\right)t_h \qquad \textbf{(4.5)}$$

At this point, our problem turns out to be the following constrained-optimization problem:

Minimize $\quad f_{SIT\_CS}(b,t_h) = \left(\frac{b^2}{4} + \frac{7}{6}\right) t_h$

subject to $\quad D = \left\{(b,t_h) \middle| bt_h^3 = V,\ 0 < b \le 1, t_h \ge 0\right\}$

In a fashion similar to SIT racks, we obtain:

$$\begin{cases} b = 0.97 \\ t_c = 0.98\sqrt[3]{V} \\ t_v = t_h = 1.01\sqrt[3]{V} \end{cases}$$

The optimal value is $ESC^*_{SIT\_CS} = 1.42\sqrt[3]{V}$. We can conclude:

> *"Given an SIT rack with a total storage capacity V and provided that the conveyor's length t_c is the shortest dimension, the estimated travel time of the S/R machine will be minimized if $t_v : t_h : t_c \equiv 1:1:0.97$ and the optimal travel time is $1.42\sqrt[3]{V}$ ".*

Comparing two situations, we can see the rack where the conveyor's length is the longest dimension provides a shorter expected (single-command) travel time. Therefore, we can draw the following general conclusion for the SIT rack:

> **Proposition 4.1** *Given an SIT rack with a total capacity V, the expected travel time of the S/R machine will be minimized if $t_v : t_h : t_c \equiv 0.72:0.72:1$ and the optimal travel time is $ESC^*_{SIT} = 1.38\sqrt[3]{V}$* **.**

4.5 Optimal dimensions for none-square-in-time (NSIT) rack

For this case, we make a distinction between the following situations:

- the conveyor's length is the longest dimension (NSIT_CL),
- the conveyor's length is the medium dimension (NSIT_CM),
- the conveyor's length is the shortest dimension (NSIT_CS).

If the conveyor's length is the longest dimension then we have: $T = t_c$, $t_h = at_c$, $t_v = bt_c$ $\left(\text{thus } \beta = \frac{b}{a}\right)$ and $abt_c^3 = V$. From (4.2) and (4.3) we have:

$$ESC_{NSIT\_CL} = \left(\frac{b^2}{6a^2}+\frac{1}{2}\right)at_c + \left(\frac{b^3}{12a}+\frac{a^2}{6}+\frac{1}{2}\right)t_c = \left(\frac{b^3+2b^2}{12a}+\frac{a^2}{6}+\frac{a}{2}+\frac{1}{2}\right)t_c$$

Similarly, if the conveyor's length is the medium dimension: $T = t_h, t_v = bt_h$, $(\text{thus } \beta = b)$, $t_c = at_h$ and $abt_h^3 = V$:

$$ESC_{NSIT\_CM} = \left(\frac{b^3}{12a}+\frac{a^2}{6}+\frac{b^2}{6}+1\right)t_h$$

And if the conveyor is the shortest dimension: $T = t_h, t_v = at_h$, $(\text{thus } \beta = a)$, $t_c = bt_h$ and $abt_h^3 = V$:

$$ESC_{NSIT\_CS} = \left(\frac{b^3}{12a}+\frac{a^2}{3}+1\right)t_h$$

It is easy to see that $ESC_{NSIT\_CL} \le ESC_{NSIT\_CM} \le ESC_{NSIT\_CS}\ \forall\left(0 < b \le a \le 1, V > 0\right)$. It means that the systems where the conveyor is the shortest or medium dimension cannot provide a better solution compared to the system where the conveyor is the longest dimension. For this reason, from now on, we can ignore ESC_{NSIT_CS} and ESC_{NSIT_CM}.

The problem of finding the optimal ESC_{NSIT_CL} turns out to be the following constrained-optimization problem:

$$\textit{Minimize} \quad f_3(a,b,t_c) = \left(\frac{b^3+2b^2}{12a}+\frac{1}{6}a^2+\frac{a}{2}+\frac{1}{2}\right)t_c$$

$$\textit{subject to} \quad D = \left\{(a,b,t_c)\middle|\, abt_c^{\,3} = V,\ 0 < b < a \le 1, t_c \ge 0, V > 0\right\}$$

It is hard to analytically solve this problem. For this reason, we opt for the numerical optimization. For a given total storage capacity, *V*, we used the nonlinear optimization module built in What'sBest to find the optimal dimensions as well as the optimal estimated single cycle time of the S/R machine. We carried out an extensive number of experiments (on a very wide range of *V*: from 10 to 2000). From the experimental results found:

- The optimal ratio between three dimensions does not depend on the system capacity *V*: ESC_{NSIT_CL} reaches the optimum if $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$.
- In order to estimate the relation between the system capacity *V* and the optimal estimated travel time $ESC^*_{NSIT\_CL}$, we carried out a regression analysis (implemented in SPSS). In the analysis, the total storage capacity varied from 10 to 2000. We used

different curve fitting models and found that the optimal estimated travel time is best estimated by the following relation: $ESC^*_{NSIT\_CL} \approx 1.38\sqrt[3]{V}$. The standard errors of the estimate is less than 10^{-5}.

- When the system is cubic-in-time (all dimensions are equal in time), it is easy to find that $ESC^*_{cubic\_in\_time} = 1.42\sqrt[3]{V}$. Interestingly, $ESC^*_{cubic\_in\_time} = ESC^*_{SIT\_CS}$.
- As shown in Figure 4.4, there is a difference between the overall optimal value and the other optimums with some restrictions on the dimensions. However, the gap is very small; the difference between the cubic-in-time configuration and the optimal one is:

$$\left[\left(1.42\sqrt[3]{V} - 1.38\sqrt[3]{V}\right)\Big/ 1.38\sqrt[3]{V}\right] * 100\% \approx 2.90\% .$$

- The reason that the cubic-in-time rack is not optimal is that the travel time consists of two components (see Section 4.3). The travel time from the I/O point to the pick location depends on the movement times on all three directions, but the time needed to go back to the I/O point depends only on the vertical and horizontal travel time.

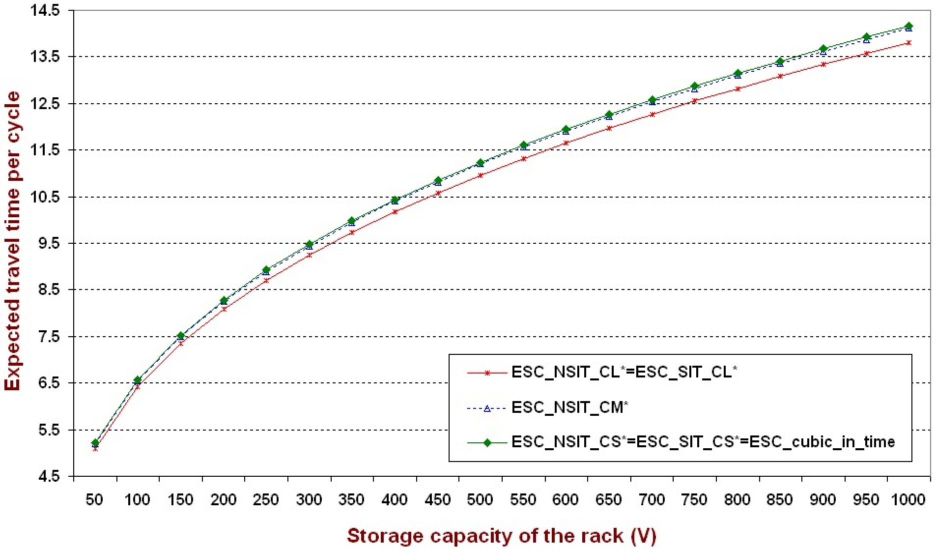

Figure 4.4 Comparison between optimal expected travel time of SIT and NSIT racks for different values of total storage capacity *V* (in cubic units)

We can make the following conclusion for the NSIT rack:

> ***Proposition 4.2*** *Given a NSIT rack with a total storage capacity V, the expected travel time of the S/R machine will be minimized if* $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$ *and the optimal estimated travel time is* $1.38\sqrt[3]{V}$.

Figure 4.4 shows all eligible possibilities (in both section 3 and 4). We can see that the SIT rack system (i.e. length and height of the rack are equal) results in the overall optimal configuration: it gives the overall shortest estimated single-command cycle time. Now, we are able to state the following proposition:

> **Proposition 4.3** *Given the 3-dimensional compact AS/RS (as described in Section 4.1) with a total storage capacity V, the expected single-command travel time of the S/R machine will be minimized if the system dimensions satisfy* $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$ *and the optimal travel time is* $1.38\sqrt[3]{V}$.

4.6 Effect of fixing one dimension

As shown above, if all three dimensions are 'open', we can find the optimal ratio (with regards to minimizing the estimated travel time) between these dimensions. However, in the Distrivaart project (see Section 4.1), we could not freely adjust all these dimensions, due to space limitations and equipment standardizations. The previous analysis can also be used to solve the problem with space restrictions. If two dimensions are fixed, then the problem is trivial as all dimensions are defined (given that we know the total system's storage capacity). If only one dimension is fixed, we can still adjust the others to reduce the estimated travel time. Clearly, the resulting optimal travel time can not be shorter than the 'overall' optimum (when we have three 'open' dimensions).

It is straightforward in this case to determine the expected travel time of the S/R machine (e.g. based on formulas (4.2) and (4.3)). Figure 4.5 shows the optimal estimated travel time for different values of the conveyor's length (t_c). From this figure, we can easily see the effect of fixing the conveyor's length. For example, if $t_c = 2\sqrt[3]{V}$ (200% of $\sqrt[3]{V}$), at best we can design a system with an expected travel time of 1.53 $\sqrt[3]{V}$ (time units), while the 'overall global' optimum, 1.38 $\sqrt[3]{V}$, is achieved for $t_c = 1.24\sqrt[3]{V}$. Similarly, Figures 4.5 and 4.6 show the cases when the rack's length and height (in time) are fixed.

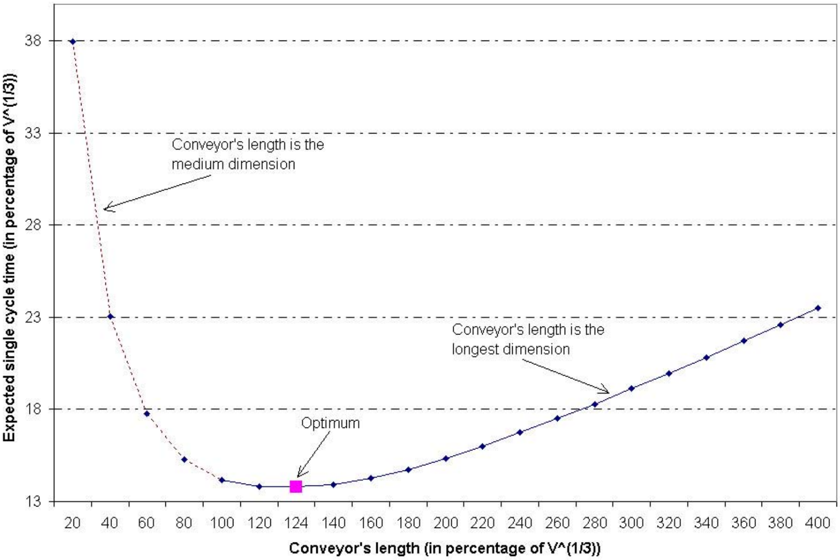

Figure 4.5 Optimal expected travel time when the conveyor's length is fixed

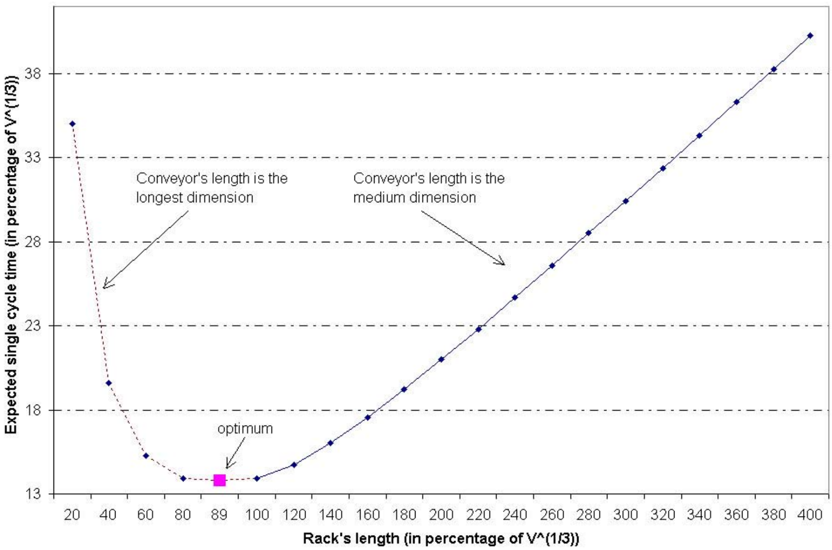

Figure 4.6 Optimal expected travel time when the rack's length (the longer dimension of the rack) is fixed

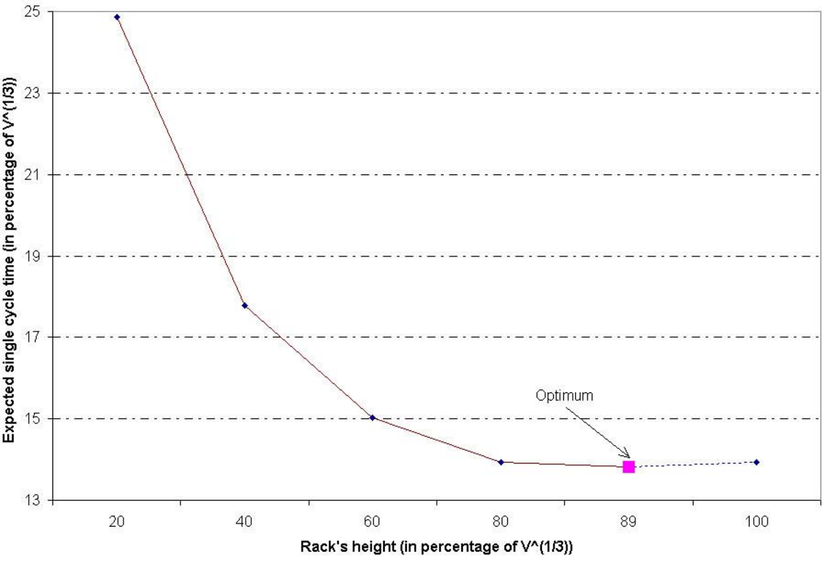

Figure 4.7 Optimal estimated travel time when the rack's height (the shorter dimension of the rack) is fixed

4.7 An example

As an illustrating example, assume that we have to design a 3-dimensional compact system that can store 1000 pallets (other data are given in Table 4.1). The decision problems are: (1) finding the optimal dimensions of the system and (2) the best position of the S/R machine so that the expected travel time is minimized. The S/R machine either dwells at one end of the rack (A) or between two rack parts (B) (referring to Figure 4.8).

For situation A: the expected pallet circulation time is S/s_c . Suppose that the length of the conveyors in the left part of the warehouse is $X(0 < X < S)$ (see Figure 4.8). As pallets are located randomly on the conveyors, in the situation (B) the expected time for a random pallet to be circulated from the current position to the position that the S/R machine can pick it up (the main rack) is:

$$\tau = \left(\frac{X}{S}\right)\left(\frac{X}{s_c}\right) + \left(\frac{S-X}{S}\right)\left(\frac{S-X}{s_c}\right) = \frac{X^2 + (S-X)^2}{Ss_c},$$

where s_c are the conveyor' speed and S is the diameter of the conveyors in situation (A). $\left(\dfrac{X}{S}\right)$ and $\left(\dfrac{S-X}{S}\right)$ are the probabilities that the pallet is located in the left-side and the right-side of the warehouse respectively. Applying the Cauchy-Schwarz inequality gives

$$\tau = \frac{X^2 + (S-X)^2}{Ss_c} \geq \frac{\left[X + (S-X)\right]^2}{2Ss_c} = \frac{S}{2s_c}.$$

Table 4.1 System parameters

*Total system capacity (*V)		*1000 pallets*
Storage policy		*Random storage*
Pallet size in seconds* (width x length x height)	Net	*0.4* x *0.4* x *2*
	Gross	*0.5* x *0.5* x *2.17*
S/R machine	Operating policy	*Single-command cycle*
	Vertical speed (s_v)	*0.6 (meter per second)*
	Horizontal speed (s_h)	*2 (meter per second)*
Conveyors' speed (s_c)		*2 (meter per second)*

* *For a given speeds of the S/R machine and conveyer, we can convert pallet sizes in distance unit to time unit*

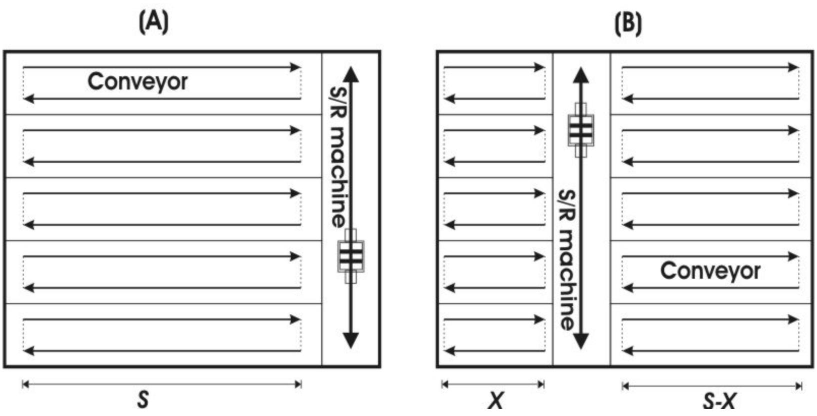

Figure 4.8 Possible positions of the S/R machine

This lower bound is tight with equality for $X = S/2$. Therefore, the optimal position, which minimizes the expected single-command travel time, is the middle of the storage rack.

We apply the theorem of Section 4.5 to calculate the optimal dimensions. We have: $t_c^* = 1.24\sqrt[3]{V} = 10.11\,(seconds)$ and $t_h^* = t_v^* = 0.72 t_c^* = 7.26\,(seconds)$. The rack dimensions must be multiples of the pallet's dimensions. Therefore, we choose the 'practical optimal' dimensions such that they are as closed as possible to the corresponding optimal dimensions found and result in a system with a storage capacity of not less than 1000 pallets (the required capacity). We obtain the practical optimal dimensions: *10* x *8* x *6.5 (seconds)* with an optimal expected travel time of 11.17 (*seconds*).

It turns out that it is not possible to dimension such a storage system to optimality at the Distrivaart project. The reason is that there are many restrictions to the ship's dimensions. The length and width of the ship are limited because of berth length and depth, and also of river locks and the dock. The height of the ship is mainly determined based on the ship's length and width (i.e. they must satisfy a certain ratio for the ship stability), and further restricted by the bridge heights.

4.8 Concluding remarks

This chapter considers a 3-dimensional compact system originating from the Distrivaart project that consists of rotating conveyors and an S/R machine. Bozer and White's travel time model (for 2-dimensional rack systems) is extended for estimating the expected single-command travel time of the S/R machine. Followings are the main findings:

- For a given 3-dimensional compact AS/RS (as above-mentioned) with a total storage capacity V, the optimal rack dimensions are $t_v = t_h = 0.89\sqrt[3]{V}$, $t_c = 1.24\sqrt[3]{V}$, and the optimal travel time is $1.38\sqrt[3]{V}$. Equivalently, the optimal ratio between three dimensions is $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$.
- The cubic-in-time system (all dimensions are equal in time) is not the optimal configuration (as intuitively we may think). However, it is a good alternative configuration for the optimal one as the resulting expected travel time is only about 3% away from the optimum. This is in line with the findings by Rosenblatt and Eynan (1989) and Chang and Wen (1997) for 2-dimensional SIT racks with single and dual-command cycle respectively. They conclude that "*The expected travel times are fairly insensitive to slight deviations in the optimal rack configuration*".

A disadvantage of the method is that the rack is assumed to be continuous. This simplification of reality is only justified if the number of storage positions is sufficiently large (see, for example, Graves et al., 1977 and Lee et al., 1999). The quality of the approximation of the real travel time depends on this.

Storage strategy used in this study is randomized storage. Clearly, other storage policies (like class-based or dedicated storage) could be considered as well. This is an interesting direction for further research. Another straightforward extension of the research is to analyze the system when the S/R operates in a dual-command basis. In the Appendix 4, we show how to estimate the expected dual-command cycle time for the S/R machine.

Appendix 4 Expected cycle time for dual-command cycles

As mentioned in Section 1.2.1, in many cases, the S/R machine can work more efficiently by a dual-command basis: it can both pick up and deliver loads in one cycle. Starting from the I/O station, it carries a load to the storage position. After putting away the load, it moves to the retrieval position and retrieves and brings another load back to the I/O point. In this appendix, we extend the travel time models developed for single-command cycles to dual-command cycles. All assumptions made before are kept unchanged. The cycle time of the S/R machine (*EDC*) consists of the following components:

- Time needed to go the storage position and waiting time for the conveyor to convey an empty location for the storage load, if any. We assume the rotation time to reveal an empty location has the same probability distribution function as the rotation time for a retrieval load to be at the pick position. Consequently, this time component is the same as in case of the single-command cycles: W (see section 4.3).
- Time needed for picking up and dropping off the two loads, c (assumed to be constant).
- Travel time from the storage point to the retrieval point: V. This is travel time between two random selected points. As shown in Bozer and White (1984):

$$f_V(v) = \begin{cases} f_V^{(1)}(v) = \dfrac{2-2v}{2v/\beta - v^2/\beta^2} + \dfrac{2v-v^2}{2/\beta - 2v/\beta^2} & \text{if } \ 0 \le v \le \beta \\ f_V^{(1)}(v) = 2-2\beta & \text{if } \ \beta < v \le 1 \end{cases}$$

$$E(V) = 1/3 + \beta^2/6 - \beta^3/30, \qquad \textbf{(4.6)}$$

where $0 < \beta \le 1$ is the shape factor of the rack.

- The waiting time, T, that may occur if the rotation time of the conveyor carrying the retrieval load, R, is longer than the time the S/R machine needed to be available at the retrieval position: $T = \max\left\{0, R - \left(W + V\frac{1}{2}c\right)\right\}$.
- Travel time needed for returning to the I/O point, U. This time component is as in the case of single-command cycles, and $E(U)$ can be calculated by (4.2).

As the conveyor with the retrieval load can be activated at the moment the S/R machine picks up a load to leave the I/O point, it is reasonable to assume that $\left(W+V+\frac{1}{2}c\right)\geq R$. Consequently, $T=0$. The expected dual-command travel time can now be expressed as follows:

$$EDC=E\left(W\right)+E\left(V\right)+E\left(U\right) \tag{4.7}$$

As in the case of single-command cycles, we make a distinction between the following situations:

- the conveyor's length is the longest dimension (EDC_{CL}),
- the conveyor's length is the medium dimension (EDC_{CM}),
- the conveyor's length is the shortest dimension (EDC_{CS}).

If the conveyor's the longest dimension, we have $T=t_c$, $t_h=at_c$, $t_v=bt_c$, $\beta=\frac{b}{a}$ and $abt_c^3=V$. From (4.2), (4.3), (4.6) and (4.7) and after some algebraic operations, we obtain:

$$EDC_{CL}=\frac{\left(b^3+2b^2+2a^3+6a^2+6a\right)\left(10a^3+5b^2a-b^3\right)\sqrt[3]{V}}{360a^4\sqrt[3]{ab}}.$$

If the conveyor's length is the medium dimension, we have $T=t_h, t_v=bt_h$, $t_c=at_h$, $\left(\text{thus } \beta=b\right)$, and $abt_h^3=V$. It then follows:

$$EDC_{CM}=\frac{\left(b^3+2b^2a+2a^3+12a\right)\left(10+5b^2-b^3\right)\sqrt[3]{V}}{360a\sqrt[3]{ab}}.$$

If the conveyor is the shortest dimension: $T=t_h, t_v=at_h$, $\left(\text{thus } \beta=a\right)$, $t_c=bt_h$ and $abt_h^3=V$. It then follows:

$$EDC_{CS}=\frac{\left(b^3+4a^3+12a\right)\left(10+5a^2-a^3\right)\sqrt[3]{V}}{360a\sqrt[3]{ab}}.$$

It is easy to see that $EDC_{CM}\leq EDC_{CS}\ \forall\left(0<b\leq a\leq 1\right)$. Numerically, we found $EDC_{CL} \leq EDC_{CM}\ \forall\left(0<b\leq a;\, 0.1\leq a\leq 0.4\right)$.

5

Online Order Batching Problem

5.1 Introduction

In chapters 3 and 4, we discussed the layout optimization problem. A good layout reduces the order picking travel distance and thus improves the efficiency of the OP system. In this chapter, we elaborate on the dynamic order batching problem, which is the problem of grouping on-line orders into batches such that each can be picked in one picking tour, and the total travel time (or distance) is minimized (see also Section 1.3.3). It has to be noticed that while the layout design is often considered at the tactical level (i.e. medium planning period) and reviewed when there is a major change in product assortment or order pattern, the order batching decision is made much more frequently, for example before every working shift.

Due to inventory minimization and just-in-time policies, many companies have changed their ordering behavior over the last decades. Few-but-large quantity orders are being replaced by many-but-small orders (i.e. few items per order), which have to be processed in very tight time windows. When orders are small, the order batching problem becomes more crucial. For example, online retailing companies that focus on specialized product types (such as books, computers or CD's) often receive orders with only one or few items. If the order picker starts a tour for every order, the capacity may even be insufficient to serve all orders. If the order picker waits to have a sufficiently large number of orders, the average time in the system of the orders may be longer than desired, so that orders miss shipping due time. Clearly, we can increase the efficiency of the OP process in such environments by serving a group of orders instead of individual orders.

As pinpointed out in Section 1.3.3, the static order batching problem has received considerable research attention: both optimal and heuristic algorithms have been developed to solve the problem. In these algorithms, the order profile (the number of orders, number of items per order, item-quantity per item) is often considered as known or given. This is true for many traditional order picking situations, where customer orders are received some time before the picking process starts (orders that arrive the night before are picked today). However, this may not be true in quick picking environments, where orders arrive online and need to be shipped in a short time (for example online retailing companies). In these situations, arrived orders should be processed as soon as possible. Furthermore, the service time (including traveling, picking, setup time) is often considered as a constant (i.e. average value) for a given number of picks per picking tour. Certainly, it is not true; the travel time depends also on the exact pick locations, besides other factors like traffic, aisle nature, order pickers…

In this chapter, we consider order batching in a quick picking environment (or *online order batching problem*), where order arrival times and picking times of a batch of orders are assumed to be stochastic random variables. We again consider 2-block manual-pick shelf-rack type warehouses (depicted in Figure 5.1). As shown in Roodbergen and De Koster (2001), a layout with a middle aisle (2-block) often results in a lower average travel time than the layout without a middle aisle (single-block). As far as we now, the only study which concerns the stochastic nature of the order arrivals and service time is Chew and Tang (1999). They model the order batching problem for a single-block warehouse as a queueing model and apply a series of approximations to calculate the lower bound, upper bound and an approximation value for the average throughput time. The limitation of this research is that they consider the average throughput time of the first order in a batch as estimation for the average throughput time of individual orders in the batch. Our research is mainly based on the approach given in this article, but distinguishes from it in two respects. First, we consider a different layout (2-block warehouse), which can be found commonly in practice. Second, we perform a direct analysis on the average throughput time of a random order.

This chapter is organized as follows. In the next section we describe the system, notations and assumptions to be used. Then, we elaborate on the first and second moment of the order picker's travel time in Section 5.3. We use these moments to estimate the average throughput time of a random order in Section 5.4. This enables us to estimate the optimal picking batch size (i.e. the number of orders to be served in one pick route). In Section 5.5, we illustrate the method by several numerical examples. Finally, we discuss some possible extensions of the model and give conclusions in Section 5.6 and Section 5.7 respectively.

5.2 Notations and assumptions

We initially use the following assumptions, some of which will be relaxed later.

- *Order pattern* (or orders' characteristics)*:* arrivals of orders follow a Poisson process with an order arrival rate[5] λ and every order contains one order-line (note that the quantity per order-line can be greater than 1). Although this assumption may seem restrictive, many practical situations exist in which single-line order batching occurs. This is due to the fact that order picking is often split up over multiple storage systems (piece pick, box pick, pallet pick systems) leading to small orders per subsystem. Single-line orders are often picked separately, since no additional consideration is necessary. In the warehouse of a Dutch mail order company, the inventory is even mirrored in two systems: one for multi-item orders and one for single-line orders. We will come back to multi-item OP situations in Section 5.6, where we also explain how to deal with the multiple order-lines per order situation. We presume that the storage capacity of a storage location is sufficiently large: to pick up one order line the order picker has to visit only one storage location.
- *Service:* we consider only one order picker and the service is carried out per batch of exactly k orders and order splitting is not allowed. The order picker's capacity is sufficiently large to handle multiple (k) orders per route.
- *Routing method*: the used routing method is the S-shape (or traversal) heuristic. We refer to Section 1.3.4 for a description of different routing methods. As mentioned in Section 1.3.4 and Section 2.4.5, the S-shape method outperforms the return routing method in almost all the cases, irregardless of the storage assignment methods. Also, the S-shape routing method is one of the simplest routing methods, included in nearly every warehouse management software system, and widely used in practice.
- *Storage policy*: we use a random storage strategy, which means that incoming products are randomly located to empty storage spaces.
- *Batching rule*: batching is carried out on a first-come first-serve basis; we assume that the system is empty at the beginning.
- *Picker's speed*: we assume that the order picker travels with a constant speed, which is normalized to 1 (length/time): if the distance is d, the corresponding travel time is also d.

As it is also the aim of the research to deal with real-life applications, some of these assumptions are relaxed later in Section 5.6, where we discuss the possibility of including

[5] λ can be changed over time, however we assume λ is a constant and given for each pick shift.

compound-Poisson arrivals, multiple order pickers and a class-based storage assignment method into the model.

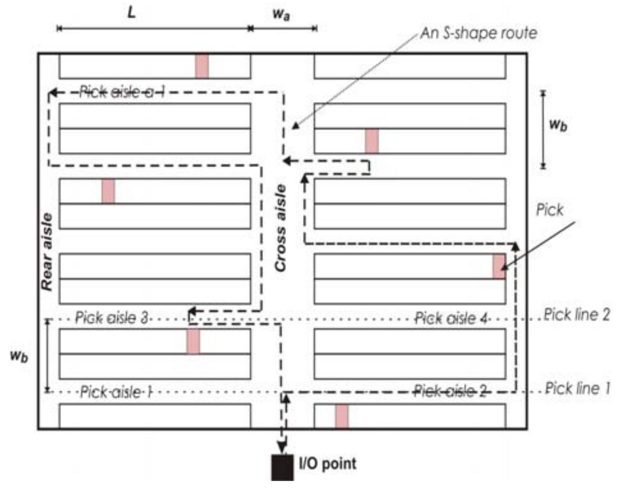

Figure 5.1 A 2-block warehouse layout with an S-shape pick route

We use the following notations throughout the chapter (some are recalled from Section 2.2.4):

Data

L	length (in travel time units) of a pick aisle
a	number of pick aisles (an even integer)
w_a	width (in travel time units) of the cross aisle
w_b	center-to-center distance (in travel time units) between two adjacent pick aisles
p_i	probability that a random item is picked from aisle $i\,(i=1..a)$; $p_i = 1/a$ $(i=1..a)$ for the random storage assignment
τ_s	setup time per batch (constant)
τ_p	picking time per item (constant)
λ	order arrival rate

Intermediate variables

TR_B^{WA}	travel time caused by traversing the pick aisles (within aisle (WA) traveling), B can be $+$, $-$ or $\approx$ (indicating upper, lower bound or approximation values respectively)

TR_B^{CA}	travel time caused by traversing the cross aisle (CA)
AT	adjustment time
$E(S)$	first moment of the service time (including setup, picking and traveling time)
$E(S^2)$	second moment of the service time
$\sigma(S)$	standard deviation of the service time

Decision variables

k	number of orders to be picked in a tour
q	number of items in a batch of k orders.

In the next section, we estimate the first and second moment of travel time to pick up q items.

5.3 Travel time estimation

The order travel route is sketched in Figure 5.1. Starting from the I/O point, the order picker travels to the nearest pick aisle containing picks, either in the left or right block. Aisle by aisle, he travels to the farthest pick aisle in the same block in such a way that all visited aisles are completely traversed. After accomplishing all pick requests in the first block, he moves to the farthest requested pick aisle in the second block. In a same manner but in the downward direction, he picks while going from the farthest to the nearest aisle containing picks. From there, he goes back to the I/O point to complete the tour. It should be noticed that it does not matter which block is served first, as in both cases the picker travels the same distance. Furthermore, it is obvious that picking block by block provides a shorter (or at most equal) travel distance than picking in two blocks simultaneously.

In order to estimate the throughput time, it is necessary to find the first and second moment of travel time.

5.1.1 First moment of travel time

The average travel time consists of three components: within-aisle travel time (TR^{WA}), cross-aisle travel time (TR^{CA}), and adjustment time (AT). (It should be noted that as we assume unity travel speed, it does not make any difference, in magnitude, between travel distance and travel time.) We define $E(TR_B) = E(TR) + E(AT_B)$, where $E(TR) =$

$E\left(TR^{WA}\right)+E\left(TR^{CA}\right)$ and B can be – (lower bound), + (upper bound) or $\approx$ (approximation).

The adjustment time AT consists of two components: AT_1 and AT_2 . AT_1 is the travel time from the central line of the cross aisle to the beginning of the first pick aisle and the travel time from the end of the last pick aisle to the central line of the cross aisle. AT_2 is the correction of travel time if the last visited aisle in each block is odd (pick aisles are numbered alternately from left to right from 1 to a as shown in Figure 5.1). In the following, we will determine the expected value: TR^{WA}, TR^{CA} and AT given that the pick list contains q items (in our case, each order consists of only one item thus $q = k$).

With the S-shape routing method, the expected within-aisle travel time depends only on the length of pick aisle L and the expected number of aisles visited: $E\left(J \mid q\right)$. Chew and Tang (1999) show that given q and the number of pick aisles a :

$$E\left(TR^{WA} \mid q\right) = LE\left(J \mid q\right) = L\sum_{j=1}^{a} jP\left\{J = j \mid q\right\} = L\left[a - \sum_{i=1}^{a}\left(1-p_i\right)^q\right]$$

where the term in brackets is the expected number of visited aisles.

On the other hand $E\left(TR^{CA} \mid q\right)$ is the doubled travel time from the I/O point to the farthest visited aisle. It is determined by the travel time between two neighboring pick aisles, w_b, and the position of the farthest visited aisle M . If we consider two pick aisles opposite the cross aisle as one pick line (see Figure 5.1) then we can make use of the formula for estimating in the $E\left(TR^{CA} \mid q\right)$ in single-block warehouses given in Chew and Tang (1999)[6]:

$$E\left(TR^{CA} \mid q\right) = 2w_b\sum_{m=1}^{a/2} mP\left\{M = m \mid q\right\} = 2w_b\left[a/2 - \sum_{m=1}^{a/2-1}\left(\sum_{r=1}^{m} p'_r\right)^q\right],$$

where $p'_r = 1-(1-p_{2r-1})(1-p_{2r})$ is the probability that the pick line $r\ (r = 1..a/2)$ is visited.

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[6] We can modified formulas (2.12) and (2.14), in Chapter 2 for class-based storage strategy warehouses, for estimating the first moment of travel time in this case (i.e. random storage). However, we choose Chew and Tang's approach to make it consistence with the approach of estimating the second moment of travel time later.

For the first adjustment term, we can see that if only a half of the warehouse (one block) is visited then $AT_1 = 2\left(w_a/2\right) = w_a$ (it is doubled because whenever the order picker enters an aisle he has to leave the aisle). If both halves of the warehouse are traversed then $AT_1 = 2w_a$. Hence, we can determine the conditional expected value of the first correction term:

$$E\left(AT_1 \mid q\right) = w_a\left(2*0.5^q\right) + 2w_a\left(1-2*0.5^q\right) = 2w_a\left(1-0.5^q\right).$$

The second adjustment term takes into account the fact that from the last pick position (in the last visited aisle) in each block, the order picker has to return to the center line of the cross aisle. It is easy to verify that: $0 \le AT_2 \le 2L$. The expected value: AT_2, $E\left(AT_2 \mid q\right)$, can be estimated by formula (5.8) given in Appendix 5B

From all estimates above, we now can come up with the following expressions of travel time:

$$E\left(TR \mid q\right) = L\left[a - \sum_{i=1}^{a}\left(1-p_i\right)^q\right] + 2w_b\left[a/2 - \sum_{m=1}^{a/2-1}\left(\sum_{r=1}^{m} p'_r\right)^q\right]$$

$$E\left(TR_- \mid q\right) = E\left(TR \mid q\right) + w_a$$

$$E\left(TR_+ \mid q\right) = E\left(TR \mid q\right) + 2\left(L + w_a\right)$$

$$E\left(TR_\approx \mid q\right) = E\left(TR \mid q\right) + 2w_a\left(1-0.5^q\right) + E\left(AT_2 \mid q\right)$$

We used Visual Basic for Application (VBA) on Microsoft Excel to simulate the system. In the simulations, we considered 3 layouts: 6, 10 and 16 aisles (see Table 1 for other layout parameters). Batch size varied from 1 to 40 orders (i.e. number of locations that an order picker has to visit in one tour is from 1 to 40). The average travel-time value of 10000 runs was taken as the simulation result, this number of runs is sufficient to obtain a 98% confidence interval with a half-width of less than 1% of the sample mean. We found that, in the worst case, the difference between the approximated travel time and simulation outcome is less than 3 percent. For all layouts, the difference decreases when the batch size increases. When the batch size is greater than 20, the difference between approximation and simulation value is less than 1 percent. It is because of the fact that for larger batch sizes the probability that the order picker has to travel the entire warehouse is close to 1. Consequently, both approximation and simulation results reach the maximum travel time.

When we know the first moment of travel time, it is rather straightforward to compute the first moment of service time. We call $E(S_B \mid q)$ the first moment of service time given the batch size q, where B can be the approximation, lower bound or upper bound notation. We assume that the setup time of a batch, τ_s, is independent from the batch size. The picking time per item, τ_p, is identical for all items. It follows that:

$$E\left(S_B \mid q\right) = \tau_s + \tau_p q + E\left(TR_B \mid q\right).$$

5.1.2 Second moment of travel time

Without considering the adjustment time AT, the second moment of travel time can be formulated as

$$E\left(TR^2 \mid q\right) = L^2 E\left(J^2 \mid q\right) + \left(2w_b\right)^2 E\left(M^2 \mid q\right) + 2\left(2w_b\right) LE\left(JM \mid q\right) \tag{5.1}$$

Chew and Tang (1999) calculated $E\left(J^2 \mid q\right)$ and $E\left(M^2 \mid q\right)$ for the single-block layout. However, their result for $E\left(J^2 \mid q\right)$ still holds for the case of two blocks. For $E\left(M^2 \mid q\right)$, if we consider pick lines instead of pick aisles (see Figure 5.1) then their formula can be easily adapted. Hence, we have:

$$E\left(J^2 \mid q\right) = a^2 - \sum_{i=1}^{a}\left(2a-1\right)\left(1-p_i\right)^q + 2\sum_{i=1}^{a}\sum_{r=i+1}^{a-1}\left(1-p_i-p_r\right)^q \tag{5.2}$$

$$E\left(M^2 \mid q\right) = \left(a/2\right)^2 - \sum_{i=1}^{a/2-1}\left(2i+1\right)\left(\sum_{r=1}^{i} p'_r\right)^q \tag{5.3}$$

where $p'_r = 1-(1-p_{2r-1})(1-p_{2r}),\ r = 1..a/2$.

$E\left[JM \mid q\right]$ is the term that describes the interaction between the number of aisles visited and the farthest pick line. It can be calculated by

$$E\left[JM \mid q\right] = \sum_{m=1}^{a/2} m\left(\sum_{j=1}^{2m} jP\left\{J=j, X_{2m}>0, X_{2m+1}=\ldots=X_a=0 \mid q\right\} + \right.$$

$$\left. + \sum_{j=1}^{2m-1} jP\left\{J=j, X_{2m-1}>0, X_{2m}=\ldots=X_a=0 \mid q\right\}\right.$$

$$= \sum_{m=1}^{a/2} m\left(\left(\sum_{r=1}^{2m} p_r\right)^q\left[2m-\sum_{i=1}^{2m}\left(1-p_i^*\right)^q\right] - \left(\sum_{r=1}^{2(m-1)} p_r\right)^q\left[2(m-1)-\sum_{i=1}^{2(m-1)}\left(1-p_i^{**}\right)^q\right]\right) \tag{5.4}$$

where $X_i = 1$ if pick aisle i is visited and $X_i = 0$ otherwise. $p_i^* = p_i \Big/ \sum_{j=1}^{2m} p_j$ and $p_i^{**} = p_i \Big/ \sum_{j=1}^{2(m-1)} p_j$ (the proof can be found in the Appendix 5A). Subsequently, $E\left(TR^2 \mid q\right)$ can be computed by substituting (5.2)-(5.4) into (5.1). We can see that TR differs from TR_+ and TR_- only by constants, thus their variances are identical:

$$\sigma^2\left(TR \mid q\right) = \sigma^2\left(TR_+ \mid q\right) = \sigma^2\left(TR_- \mid q\right) = E\left(TR^2 \mid q\right) - \left[E\left(TR \mid q\right)\right]^2.$$

However, $TR_\approx$ does not differ from TR by a constant. To make things easier, we assume that $\sigma^2\left(TR_\approx \mid q\right) = \sigma^2\left(TR \mid q\right)$.

For a given number of items per batch q, the variance of service time $\sigma^2\left(S \mid q\right)$ is just the summation of the variance of travel time and the variance of picking time, since the setup time is constant and the picking time is independent of the travel time. The variance of the picking time simply equals $q\tau_p^2$. Hence,

$$\sigma^2\left(S_B \mid q\right) = \left\{E\left(TR_B^2 \mid q\right) - \left[E\left(TR_B \mid q\right)\right]^2\right\} + q\tau_p^2,$$

where B can be the approximation, lower bound or upper bound notation.

5.4 Throughput time analysis for $M/G^k/1$ queueing model

Due to stochastic natures of both order arrivals and service, a natural way to deal with the order batching problem is to model the OP process as a queueing system. With the assumptions made earlier, our problem can be modeled as an $M/G^k/1$ queue, where G^k denotes that the service is performed per batch of exactly k orders and the distribution function of the service time has a general form, while inter-arrival times are exponentially distributed. In other words, the order batching problem in this case can be considered as the problem of determining the optimal service batch size (k) for the $M/G^k/1$ queue such that the average throughput time of a random customer is minimized.

In the literature, there are only few publications in which this type of queue is thoroughly studied. Foster and Perera (1964) show that the probability generating function of the system size at random epochs $P(z)$ can be expressed by the following formula:

$$P(z)=\frac{(1-z^k)(1-\frac{u}{k})\prod_{j=1}^{k-1}\frac{(z-\delta_j)}{(1-\delta_j)}}{1-\frac{z^k}{K(z)}} \tag{5.5}$$

where $K(z)=\psi\{\lambda(1-z)\}$ is the Laplace-Stieltjes transform of the cumulative service time distribution function. λ is the arrival rate, $u=\frac{\lambda}{\mu}$ is the utilization rate (or traffic density). $\mu=1/E(S\,|\,k)$ is the service rate of a batch consisting of k orders. δ_j, with $j=1,..,(k-1)$, are $(k-1)$ roots inside the unit circle of the characteristic equation $z^k=K(z)$. It follows from Rouche's theorem that this equation has exactly $(k-1)$ roots inside the unit circle (detailed explanations can be found in Gross and Harris, 1998, p. 282). If we know the form of the service time then the steady-state probabilities $\{p_n\}$ can be theoretically obtained by successive differentiation of $P(z)$. Nevertheless this work is cumbersome when k becomes large.

Chaudhry (1991) is also interested in this queue and he provides a closed-form expression in term of the roots of certain characteristic equations for computing the average queueing time of orders. However, he only considers the queueing time of the last customer in the service batch, which, certainly, differs from the waiting time of a random customer. Another type of queues that is also considered in the same article is $M/G^{[a,b]}/1$. In this queue, services can be performed as soon the number of orders waiting in the queue reaches the lower threshold a (b is the capacity of the server, $a \le b$). Chaudhry et al. (1987) discuss a numerical computation approach to compute the steady-state probability of this system. However, from a practical point of view, this approach is rather complicated to use. In order to obtain steady-state probabilities, we first have to find the roots of the characteristic equations and than successively take the derivative of the probability generating function. This requires tremendous computational efforts, especially when the batch size is large.

Apparently, it is too difficult from a practical point of view, to compute exact results for the $M/G^k/1$ queue. Furthermore, for the sake of the order batching problem, it is not necessary to find an extremely accurate throughput time. Therefore, in this research we are interested in finding a good and easy-to-compute approximation for the average waiting time of a random order. We use the well-known 2-moment approximation formulation (see, for example, Tijms, 1994, p. 335):

$$W_{M/G^k/1} = \left(1-c_S^2\right) W_{M/D^k/1} + c_S^2 W_{M/M^k/1},$$

where $c_S^2 = \sigma^2\left(S \mid k\right) / E^2\left(S \mid k\right)$ is the squared coefficient of variation of the service time; $W_{M/M^k/1}$ and $W_{M/D^k/1}$ denote the average throughput time of orders (or waiting time in the system of a random order) when the service time distribution is exponential and deterministic, respectively. As recommended in Tijms (1994), this method performs very well in the case that c_S^2 is not very high and the traffic density u is not very low. Our approach for determining the optimal batch size is sketched in Figure 5.2.

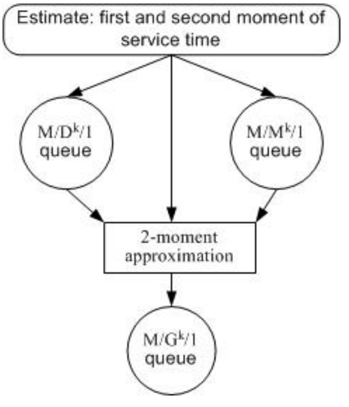

Figure 5.2 Two moment approximation approach for general service time distribution

When the service time is exponential, we have (Gross and Harris, 1998, p.125):

$$W_{M/M^k/1} = \frac{1}{\lambda}\left[\frac{k-1}{2} + \frac{\lambda}{\mu k}\frac{(r_0^{1-k} - r_0^{2-k})}{\left(1-r_0\right)^2} + \left(\frac{\lambda(r_0^{-k} - r_0^{1-k}) - \mu r_0}{k\mu}\right)\sum_{k=1}^{k-1} k r_0^k\right],$$

where r_0 is the unique real root of the characteristic equation $\mu r_0^{k+1} - \left(\lambda + \mu\right) r_0 + \lambda = 0$.

When the service time is deterministic it can be shown that $K(z) = e^{-u(1-z)}$. Substituting this into (5.5) we have:

$$P(z) = \frac{(1-z^k)(k-u)\prod_{j=1}^{k-1}\frac{(z-\delta_j)}{(1-\delta_j)}}{k\left(1-\frac{z^k}{e^{-u(1-z)}}\right)} \tag{5.6}$$

where δ_j, $j=1,..,(k-1)$, now become $(k-1)$ roots inside the unit circle of the equation $z^k = e^{-u(1-z)}$. In the literature, several solution methods have been proposed for finding roots of this equation. The common technique used is transforming the equation into $\lceil (k-1)/2 \rceil$ independent equations, each of which has only one root inside the unit circle. These roots and their conjugates form the $(k-1)$ roots we need (see Chaudhry et al., 1987 and 1990). When $(k-1)$ roots of the equation are known, we can find $W_{M/D^k/1}$ by taking the limitation of $P(z)$ when z reaches 1: $W_{M/D^k/1} = \frac{1}{\lambda}\frac{d}{dz}P(z)\big|_{z=1}$. We note that, for $z=1$, $P(z)$ is indeterminate of the $0/0$ form. Therefore, we proceed as follows. Let $N(z)$ and $D(z)$ denote the numerator and denominator of the right-hand side of equation (5.6) respectively. Then we use the following well-known result in queueing theory (see Madan (2000)):

$$W_{M/D^k/1} = \frac{1}{\lambda}\frac{d}{dz}P(z)\big|_{z=1} = \frac{1}{\lambda}P'(1) = \frac{1}{\lambda}\frac{N'(1)D''(1) - D'(1)N''(1)}{2\left(N'(1)\right)^2}.$$

As mentioned earlier, successive differentiations are cumbersome when the batch size is large; but in this case, we only need to take the first order derivation of the generating function. The derivative operator is available in many common mathematical software packages (such as Maple or Matlab). These make it possible to carry out a numerical analysis for the value of $W_{M/D^k/1}$, even for very high batch-size values.

5.5 Numerical examples

In order to illustrate the procedure, we consider three warehouse instances with parameters given in Table 1. These parameters are based on the OP instance mentioned in Chew and Tang (1999). Figure 5.3 shows the calculated throughput times of the deterministic, exponential and general form service time model for the three warehouses, where the average throughput time of the general form service time model is interpolated from the average throughput time of the corresponding constant and exponential service time model (by using the two-moment approximation method described in Section 5.4). It appears that the average throughput time is a convex function of the batch size. We can explain this behavior as follows. There are mainly three elements that affect the average throughput time of an order. They are the waiting time to form a batch, the waiting time for service (i.e. picking) and the service time. When the batch size is small the batch-forming waiting time and service time are small, but the service waiting time can be large (i.e. limited number of pickers). In contrast, when the batch size is large, the batch-forming waiting time and service time are large, but the service waiting time can be small. This trade-off indicates that the optimal batch size exists. This is in line with the finding of Chew and Tang (1999) when they considered single-block warehouses. In the figure, it can also be seen that the approximation curve is extremely close to the deterministic curve when the batch size is large. This is due to the fact that the squared coefficient of variation of service time is almost zero for large batch sizes. It suggests to us that the deterministic model is a good approximation for the general service time queue. This result is in line with the finding, for the case of single-aisle warehouses, mentioned in Le-Duc and De Koster (2003).

It should be noted that, to satisfy the equilibrium condition, the batch size can only be defined on a semi-bounded interval $\left[k^{-},\infty\right)$, where k^{-} is a minimum batch size value such that the traffic density, $\lambda k/\mu$, is less than 1.

Table 5.1 Parameters for the simulation experiment

Attributes	Quantities
a	6, 10, 16 aisles
λ	4 orders/ 10 minutes
L	30 seconds
w_a	6 seconds
w_b	10 seconds
τ_s	180 seconds
τ_p	12 seconds

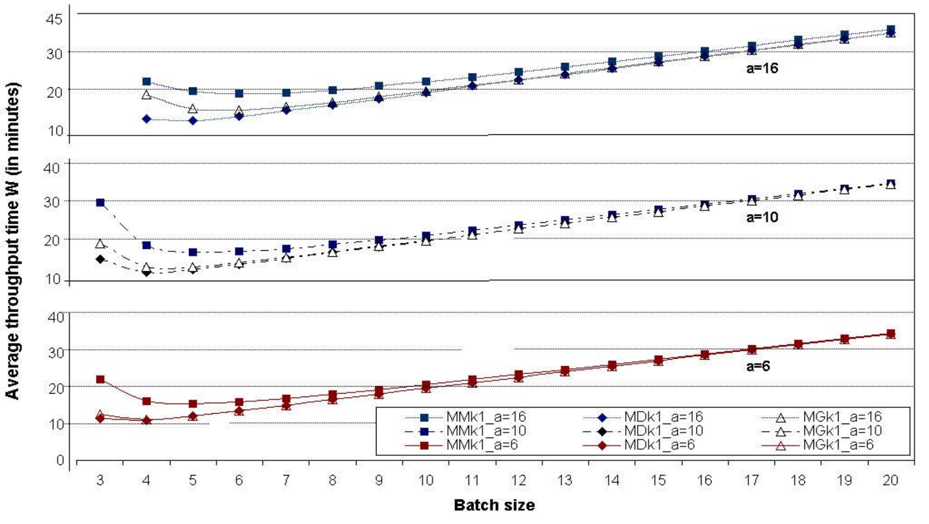

Figure 5.3 Average throughput time for different service time distributions (with the approximation value of the service time)

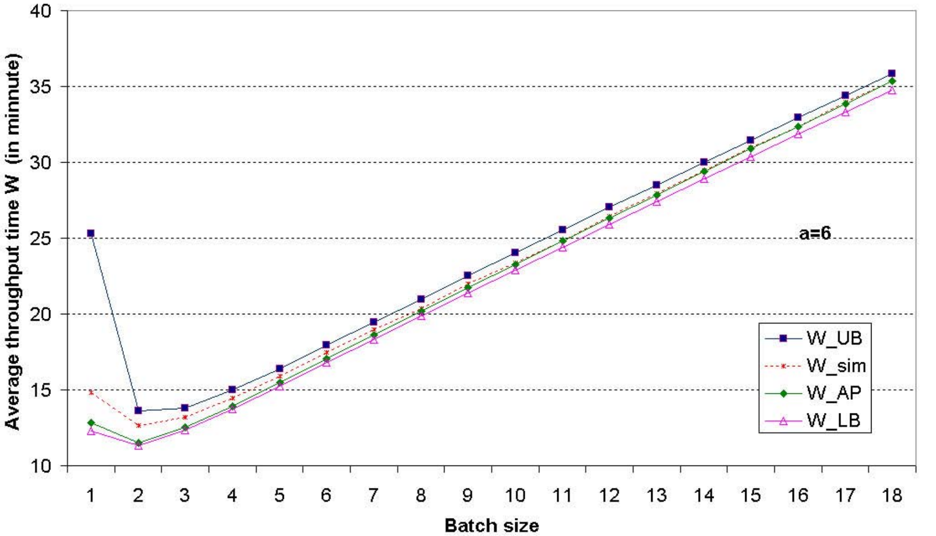

Figure 5.4 Average throughput time of a random order for the 6-aisle layout (W_LB is the approximated value of the average throughput time, by the 2-moment approximation approach, based on the lower bound value of the first and second moment of service time).

To find how good the estimation (the order throughput time) is, we used the AutoMod simulation package to simulate the OP system. In the experiment, the average order throughput times were taken after a run length of 48 hours (warming up time was 4 hours, determined by using AutoStat – a tool accompanying AutoMod for batch running and statistical analyses). Figure 5.4 shows the simulation results together with the expected lower bound, upper bound and approximation value of the throughput time for the layout with 6 aisles (we mention only one case as other cases - 10 and 16 aisles- bring similar pictures). The lower bound, upper bound and approximation of throughput time are correspondingly determined by the lower bound, upper bound and approximation of service time. For example, in order to find the lower bound of the throughput time, we first estimate the lower bound of the first and second moment of service time. Then using these moments we calculate the throughput time of the deterministic and exponential service time queue. Finally, we use the 2-moment approximation formula to obtain the lower bound of the throughput time. As we can see from Figure 5.4, the bounds are tight, especially when the batch size is large. It means that the approximation provides sufficient accuracy in estimating the average throughput time of a random order. This result is in accordance with the finding of Chew and Tang (1999) for single-block warehouses. Furthermore, the optimal batch size is relatively small; close to its lower bound. It means that we need not to search the optimum batch size on a large interval. This is an important point as it can help to reduce the search time significantly. Perceiving this, we propose a greedy procedure for determining the optimal batch size as follows. We first determine the lower bound of the batch size. Starting from the batch size lower bound, we iteratively increase the batch size by one, until the average throughput time (determined by two-moment approximation approach) increases. The optimal batch size is the value that minimizes the average throughput times found.

5.6 Some possible extensions of the model

We have considered the OP process with single-line orders, a single order picker and the random storage strategy. This can be considered as a basic model and it can be extended in several directions.

As the first extension, we can consider multiple order pickers instead of a single one. Under this situation, the OP process can be modeled as a batch processing and multi-server queue: $W_{M/G^k/c}$, where c is the number of servers (or order pickers). It is too difficult, if not impossible, to find the exact results for this type of queues. However, in the literature, a simple method exists for finding the bounds of the average waiting time of a multi-server queue from its corresponding single server queue (see, for example, Gross and Harris

(1998), p. 340). According to their method, the lower bound of $W_{M/G^k/c}$ can be found by assuming that $W_{M/G^k/c}$ is equivalent to $W_{M/G^k/1}$ where the service rate is c times faster. The upper bound can be obtained by assigning batches in cyclic order to the c servers with no jockeying allowed (See Figure 5. 5 - first batch to server 1, second batch to server 2, ..., $(c+1)st$ to server 1, ...). Then each server faces a single queue, in which the inter-arrival time is the c-fold convolution of the original inter-arrival distribution, with no change in the service time process. The waiting time of a random batch taken from one of these queues provides an upper bound for the multi-server queue. These bounds are very useful: we can use them to interpolate the expected value of the throughput time. One reasonable value of the throughput time could be the average value of the lower and upper bound. In this approximation we neglect possible aisle congestion which may occur when multiple order pickers work in the same picking area at the same time. However, we may expect that this effect is small when the S-shape routing method is used, as aisles are mostly (particularly for not too small batch sizes) traveled in a single direction.

The second extension could be that we consider the class-based storage assignment method. As mentioned earlier, when the random storage strategy is used, $p_i = 1/a \; (i = 1..a)$, where p_i is the probability that aisle i is visited. When the class-based storage assignment method is used, there are two possibilities, depending on whether partial-aisle assignment is allowed or not. A partial-aisle assignment means that we can store different product classes in the same aisle, while in the other cases product class is stored in one or more entire aisles. Our model already captures the latter case, because in the calculations we use the general expression of p_i (p_i can differ from $1/a$). It is also possible to consider the partial-assignment case. However, the expression for the second moment of the travel time may become very complicated (see Chapter 2).

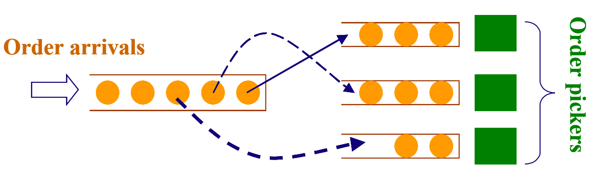

Figure 5. 5 Cyclic order assignment of orders to servers/order pickers

A single order-line order picking can be observed in warehouses where single and multi-line orders are picked separately or single-line orders form the majority. However, in other cases, orders may consist of more than one order line. Thus, another interesting extension could be that we consider compound-Poisson arrivals instead of Poisson arrivals. The OP process can then be modeled as the compound-Poisson arrivals with batch service queue. For this type of queues, it is still possible to trace the expected waiting time if both moments of the service time are known. Unfortunately, again it is very tough to come up with a closed-form formulation for the second moment of service time. We suggest that we approximate this system by $M/G^X/c$, where $X = E(k)E(i)$ with $E(k)$ and $E(i)$ are the expected number of orders in a batch and items per order, respectively. This means that we can still apply $M/G^X/c$ queue to estimate the optimal number of items per batch and than based on this value and $E(i)$ to determine the 'optimal' number of orders to be included in a batch.

5.7 Concluding remarks

This chapter focuses on finding a simple but efficient approach for determining the optimal picking batch size for order pickers in a typical 2-block warehouse. In order to do so, we first extend the results given in Chew and Tang (1999) for single-block warehouses to estimate the first and second moment of the service time. Then, we use these moments to estimate the waiting time of a random order based on the corresponding batch service queueing model. The optimal picking batch size is then determined in a straightforward manner. Results from the simulation experiments show that our approach provides a high accuracy level. Furthermore, the method is very simple; it can be easily applied in practice.

The average waiting time appears to be a convex function of the batch size. As a result, a unique optimum picking batch size exists. As the optimum batch size is close to its lower bound (obtained from the traffic density condition), we propose a simple greedy procedure, which can be used to search for the optimum in a short computational time.

The OP system that we considered is a simple one; we can extend it in several directions. It is easy to include multiple order pickers. However, in general it is rather difficult to capture the effect of aisle congestion, compound-Poisson arrivals or other storage strategies and different layouts. These topics issue a challenge for future research.

Appendix 5A

We use the following definitions:

$M = m_+$: the farthest pick line is pick line m and pick aisle $2m$ is visited,

$M = m_-$: the farthest pick line is pick line m and pick aisle $2m$ is not visited.

$$X_i = \begin{cases} 1 & \text{if pick aisle } i \text{ is visited} \\ 0 & \text{otherwise} \end{cases}$$

$$E(JM \mid q) = \sum_{m=1}^{a/2} m \left(\sum_{j=1}^{2m} jP\{J = j, M = m_+ \mid q\} + \sum_{j=1}^{2m-1} jP\{J = j, M = m_- \mid q\} \right)$$

$$= \sum_{m=1}^{a/2} m \left(\sum_{j=1}^{2m} jP\{J = j, X_{2m} > 0, X_{2m+1} = 0, ..., X_a = 0 \mid q\} \right.$$

$$\left. + \sum_{j=1}^{2m-1} jP\{J = j, X_{2m-1} > 0, X_{2m} = 0, ..., X_a = 0 \mid q\} \right) \tag{5.7}$$

Applying the inclusion-exclusion rule, we have:

$$P\{J = j, X_{2m} > 0, X_{2m+1} = 0, ..., X_a = 0 \mid q\}$$

$$= P\{J = j, X_{2m+1} = 0, ..., X_a = 0 \mid q\} - P\{J = j, X_{2m} = 0, ..., X_a = 0 \mid q\} \infty$$

$$= P\{J = j \mid X_{2m+1} = 0, ..., X_a = 0, q\} * P\{X_{2m+1} = 0, ..., X_a = 0 \mid q\}$$

$$- P\{J = j \mid X_{2m} = 0, ..., X_a = 0, q\} * P\{X_{2m} = 0, ..., X_a = 0 \mid q\}$$

Thus,

$$\sum_{j=1}^{2m} jP\{J = j, X_{2m} > 0, X_{2m+1} = 0, ..., X_a = 0 \mid q\}$$

$$= \sum_{j=1}^{2m} jP\{J = j \mid X_{2m+1} = 0, ..., X_a = 0, q\} * P\{X_{2m+1} = 0, ..., X_a = 0 \mid q\}$$

$$- \sum_{j=1}^{2m-1} jP\{J = j \mid X_{2m} = 0, ..., X_a = 0, q\} * P\{X_{2m} = 0, ..., X_a = 0 \mid q\}$$

$$= \left(\sum_{r=1}^{2m} p_r \right)^q \sum_{j=1}^{2m} jP\{J = j \mid X_{2m+1} = 0, ..., X_a = 0, q\}$$

$$-\left(\sum_{r=1}^{2m-1} p_r\right)^q \sum_{j=1}^{2m-1} jP\left\{J=j \mid X_{2m}=0,\dots,X_a=0,q\right\}$$

Similarly,

$$\sum_{j=1}^{2m-1} jP\left\{J=j, X_{2m-1}>0, X_{2m}=0,\dots,X_a=0 \mid q\right\}$$

$$=\left(\sum_{r=1}^{2m-1} p_r\right)^q \sum_{j=1}^{2m-1} jP\left\{J=j \mid X_{2m}=0,\dots,X_a=0,q\right\}$$

$$-\left(\sum_{r=1}^{2(m-1)} p_r\right)^q \sum_{j=1}^{2(m-1)} jP\left\{J=j \mid X_{2m-1}=0,\dots,X_a=0,q\right\}$$

The conditional expectation $\sum_{j=1}^{2m} jP\left\{J=j \mid X_{2m+1}=0,\dots,X_a=0,q\right\}$ is just the expected number of aisles visited given q and aisles from $2m$ to a are not visited. From Chew and Tang (1999), this amount is $2m-\sum_{i=1}^{2m}\left(1-p_i^*\right)^q$, where $p_i^* = p_i \Big/ \sum_{j=1}^{2m} p_j$ is normalized probability. A similar argument holds for $\sum_{j=1}^{2(m-1)} jP\left\{J=j \mid X_{2m-1}=0,\dots, X_a=0,q\right\}$. At this step, (5.7) can be simplified as follows:

$$E\left(JM \mid q\right)=\sum_{m=1}^{a/2} m\left\{\left(\sum_{r=1}^{2m} p_r\right)^q\left[2m-\sum_{i=1}^{2m}\left(1-p_i^*\right)^q\right]-\left(\sum_{r=1}^{2(m-1)} p_r\right)^q\left[2(m-1)-\sum_{i=1}^{2(m-1)}\left(1-p_i^{**}\right)^q\right]\right\}$$

where $p_i^{**} = p_i \Big/ \sum_{j=1}^{2(m-1)} p_j$.

Appendix 5B Adjustment time estimation

The second adjustment term (AT_2) takes into account the fact that from the last pick position in the last visited aisle (in each block) the order picker has to return to the center line of the cross aisle. For each block, such a turn has to be made if and only if the number of visited aisles (in the visited block) is odd. The probability that the turn occurs in one of the blocks and all i picks fall into exactly g aisles ($g \in \{G \mid 1 \le g \le a/2, g \text{ is odd}\}$) is:

$$\binom{a/2}{g}\left(\frac{g}{a/2}\right)^i X(g,q)$$

where $X(g)$ is 1 minus the probability that all i picks fall into less than g aisles, conditional on the fact that all i items fall into at most g specific aisles (see Roodbergen, 2001):

$$X(g,q) = 1 - \sum_{i=1}^{g-1} (-1)^{i+1} \binom{g}{g-i}\left(\frac{g-i}{g}\right)^q$$

We call CR_1 and CR_2 are the correction time if the turn happens in only one and two blocks respectively. As items are randomly located within the warehouse, we assume that if g aisles are visited then the expected items in each visited aisle will be n/g. It then follows:

$$CR_1 = 2\left(0.5^q\right)\sum_{g\in G:odd}\left[\binom{a/2}{g}\left(\frac{g}{a/2}\right)^q X(g,q)\left(2L\frac{\frac{q}{g}}{\frac{q}{g}+1} - L\right)\right]$$

$$CR_2 = \left[1-2(0.5^n)\right]\sum_{k=1}^{q-1}\left[\frac{0.5^n q!}{k!(q-k)!}\right]\left\{\sum_{g\in G:odd}\left[\binom{a/2}{g}\left(\frac{g}{a/2}\right)^k X(g,k)\left(2L\frac{\frac{k}{g}}{\frac{k}{g}+1} - L\right)\right]+\right.$$

$$\sum_{g\in G:odd}\left[\binom{a/2}{g}\left(\frac{g}{a/2}\right)^{q-k} X\left(g,n-k\right)\left(2L\frac{\frac{q-k}{g}}{\frac{q-k}{g}+1}-L\right)\right]\Bigg]\Bigg\}$$

where $0.5^{q}\dfrac{q!}{k!\left(q-k\right)!}$ is the probability that $k\left(1\le k\le q-1\right)$ items fall into one block and $\left(q-k\right)$ items into the other.

Finally, the adjustment time due to making a turn if the number of visited aisles in a block is odd would equal:

$$E\left(AT_2 \mid q\right)=CR_1+CR_2 \qquad \textbf{(5.8)}$$

6

On Determining the Optimal Number of Work Zones in a Pick-and-Pack Order Picking System

6.1 Introduction

In the previous chapters, it was explicitly assumed that order pickers can be assigned to pick items from any location in the pick area. This chapter considers the situation where the pick area is organized into distinct sub-areas (or work zones), with one order picker or a group of order pickers assigned to each zone to pick requested items stored in that zone (zone picking, see Section 1.3.3). Orders are picked simultaneously from the zones (synchronized picking). After picking, the picked items are brought to an order consolidation area (by a transportation conveyor) where they are combined into complete orders before shipment. This type of order picking can be observed in many distribution centers (see Section 1.3.5 for a description of similar systems).

Compared to other picking methods, zoning has the following advantages.

- Zoning reduces the congestion in the aisles, since zoning reduces the number of order pickers working simultaneously in an aisle (in many cases, only one worker per zone).
- It reduces the picking time. By using zoning, the pick area per order picker is smaller, thus with a same pick-list size per pick tour, the average travel distance (or time) of a pick tour is likely to be shorter. Added to this, the familiarity of the order pickers with item locations leads to a reduction in searching time.

- It is easier to administer and control. Zoning allows items with similarities in physical characteristics or product carriers to be stored in the same zone. It also facilitates the use of relevant storage, handling equipments, and special labor skills in each zone.

Due to these advantages, zoning is widely used in practice. One example is warehouse of Wehkamp, a mail-order company. The case worked out in Section 6.4 is based on Wehkamp's data.

At the tactical decision level, a critical problem associated with zoning is to define the work zone storage capacity (or work zone borders). More specifically, for a given layout, operational policies (routing, batching method) and a storage assignment policy, it is the problem of how to divide the picking area into work zones such that a certain objective is maximized or minimized. Example objectives include the system throughput (Petersen, 2002) and the work load balance between zones (Jane and Laih, 2005). If we assume that all aisles are identical and all zones are of the same size (an equal number of identical aisles), then the zone partitioning problem becomes the problem of determining the optimal number of aisles constituting a work zone. It should be mentioned here that this problem has not been studied in the literature (see Section 1.3.3). The most related publication is Petersen (2002), where the effects on the travel distance in a zone of the number of aisles in the zone, of storage assignment methods, and of the number of items in the pick list are investigated. However, the zone storage capacity is fixed (i.e. aisle length is a decision variable). Therefore, the problem essentially differs with the problem of determining work zone storage capacity. In this chapter we shall investigate this problem for an OP system where picked orders are consolidated for packaging.

The rest of the chapter is organized as follows. We describe the OP system with zoning in Section 6.2. Then, we introduce an integer programming formulation for the problem of assigning items to pick routes in each zone (*item-to-route assignment* problem) and discuss its computational time in Section 6.3. In Section 6.4, we consider a case study to determine the optimal number of zones, by solving the item-to-route problem. Finally, we conclude the chapter in Section 6.5.

6.2 Order picking system

The schematic layout of the OP system under investigation is sketched in Figure 6.1. Basically, we have two functional areas: one area for picking and one for packing. Items are stored in rectangular bin-shelving storage racks. Batched orders are picked simultaneously (synchronized zone picking) from different zones in the picking area by a group of order pickers. After an order picker has completed a pick tour, the picked items are deposited on a conveyor and transported to the buffer area. When all items of an order have been picked, they are sorted and picked.

- *Batch generation*: orders (requests from a customer consist of one or several *items*[7]) arriving within a predetermined interval are grouped together in one *batch* for joint release to the order pickers. Within a batch, orders are spread over the zones based on the storage locations of their items. They are consolidated later at the packing area.
- *Picking operation*: all batched items from the same work zone are picked by one order picker or a group of order pickers designated to the work zone. Each order picker can only be assigned to at most one work zone (zone picking). As each order picker can only pick a limited number of items (e.g. due to the capacity limitation of the picking cart) in one pick route, the batched items from a work zone may require *t pick shifts* to be completed, where $1 \le t \le \tau$ with $\tau = \max_{zones}\{t\}$. (In the case of a single order picker per zone, the number of pick shifts required is the number of pick routes.) The order picker starts a batch by obtaining a picking cart and pick lists (each is a list of items to be picked in one pick route) from a central location. The order picker then goes to the left-most aisle in the work zone to start a pick route. After picking all requested items, the order pickers place them on the transportation conveyor, and go back to the left-most aisle to start a new pick route. The transportation conveyor runs continuously to move all picked items to the buffer area. For each batch of orders, it is assumed that the order picker receives all pick instructions at the beginning of the batch. For the ease of discussions later on, we divide the throughput time of a batch into periods from *1* to τ *+1*, where periods are defined as follows. Period *1* is the time lapse between the starting time (to pick the batch) and the moment when all the order pickers (from all zones) have finished the first pick route. Period *2* starts from the end of the period *1* and ends when all the second pick routes in all zones have been completed, and so on. The last period starts from the moment when all last pick routes have been completed, and ends when all items are sorted (no picking operation is carried out, only the packing).
- *Packing operation*: a conveyor runs continuously in the buffer area for buffering incomplete orders (an order is called incomplete if not all of its items are picked). Orders only enter the sorter when they are complete. It means that newly-picked items enter the sorter if and only if all the items in the orders they belong to either have been picked or were previously picked (waiting in the buffer area). The complete orders are sorted to sorter exits (see Figure 6.1) according to destinations (e.g. each shipping lane

[7] 'Item' here means stock keeping unit (SKU), in the literature it is also called 'order line'

is assigned to a group of proximity destination postcodes). A group of packers manually pack the orders. After packing, orders are transported to the shipping docks for delivery to the customers.

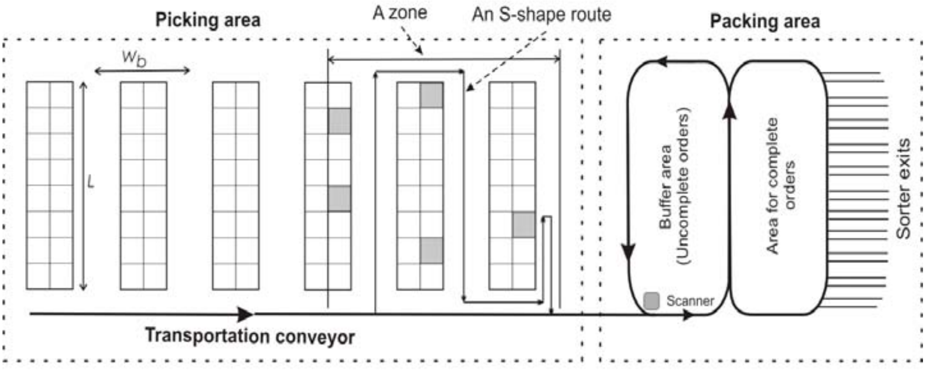

Figure 6.1 Schematic layout of the pick-and-pack OP system

With a given work force level (the number of workers at both picking and packing stages), the objective of our study is to minimize the total time to complete a batch of orders (*throughput time*). There are two decision problems that may impact the overall time to complete an order batch.

- At the operational level, the problem is how to assign items to different routes in each zone (recall that completion of a batch in one zone may require more than one pick route to be completed). The item assignment and sequence in which we pick routes in each zone has an important impact on the latter stage when the items are consolidated. Let us consider a simple example. We have two picking zones A and B, each with one order picker, with a pick capacity of one unit per pick route. In a batch, we have to complete two orders: order1=A1+B1, order2=A2+B2. For this situation, we have four possible pick sequences: (A1→A2, B1→B2), (A1→A2, B2→B1), (A2→A1, B1→B2) and (A2→A1, B2→B1). It is clear that the second and third sequence result in the longest throughput time, as there is no order to pack after the first pick shift. In the general case, when we have a set of orders, a given layout (number of zones, the size of zone), and a work force level at both the picking and packing area, we can formulate this problem as a mixed integer-linear program. We will discuss this in the next section.

- At the tactical level, we have to decide the number of zones into which the overall picking area should be divided (or in other words, how large the zone size should be). When the zone size increases, the route time (to pick a given number of items) also increases. And consequently, the throughput time may also increase. However, on the other hand, large zones reduce the consolidation problem, as orders are spread over fewer zones. This makes it simpler to arrange the pick sequence (item-to-route assignment in each zone) in such a way that the number of complete orders arriving at the packing area (per time unit) increases. And thus, the throughput time may be shorter. The best zoning scheme is the one that brings the best compromise between these two opposite effects.

In practice, the number of aisles in a warehouse is limited. Therefore, when we assume that zones are identical, we can choose from only a limited number of possible zone sizes (number of aisles per zone). For example, if we have 20 aisles then we have the following zone-size possibilities: 1, 2, 4, 5, 10 and 20 aisles (with 20, 10, 5, 4, 2 and 1 zones respectively). Because of that, our solution strategy is as follows. For each zoning scheme, we first solve the item-to-route assignment problem. In a next step, we vary the zone sizes and choose the zone size that provides the shortest overall throughput time. In the next section we will step by step formulate a mathematical model for the item-to-route assignment problem and discuss a solution approach.

6.3 Mathematical model for item-to-route assignment problem

In the model, the following assumptions are made:

- (Storage) aisles are identical.
- A work zone is a set of adjacent entire aisles (e.g. one aisle can not belong to more than one zone). All zones have the same number of aisles; this assumption is made to keep the workload balanced among zones.
- The picking capacity per pick route is determined by the number of items to be picked in one pick route.
- Order pickers always start from the left-most aisle (of the assigned zones). Within a zone, the average route length depends only on the number of items per route, the zone size, the storage assignment and the routing method used.
- The travel time between from one side of the aisle to the other is negligible. It means that an order picker can pick items from both sizes of the aisles in a single pass. No additional time is needed to reach the higher-level storage locations in an aisle.
- Multiple order pickers can work in one work zone at the same time (i.e. traffic congestion is negligible).

- The item transportation time (μ) between the picking and packing area is a constant.
- Routes between order pickers in different zones are synchronized.
- Packing can only start from period 2 onward.
- Only complete orders can enter the sorter, incomplete orders are buffered. The buffer capacity is sufficiently large to buffer all order needed.

Data

q	the maximum number of items that an order picker can pick in a pick route. We assume that this is identical for all order pickers as the pick capacity of an order picker depends on the picking vehicle or cart.
a	number of aisles per zone
L	length (in travel time unit) of a storage aisle
w_b	centre-to-centre distance (in travel time unit) between two consecutive storage aisles
t_s	set-up time of a pick route
μ	transportation (conveyor) time
r_{pi}	picking rate (number of units per time unit that an order picker can pick). It is assumed to be identical for all order pickers.
r_{pa}	overall packing rate (number of orders per time unit). This rate depends on the average order size (number of items per order) and the average packing time per unit.
N_k	number of order pickers in zone k
t,i,o,k	indices of period, item, orders and zones
K	set of zones
O	set of all orders
I_o	set of all items in order o
I_k	set of all items in zone k
I	set of all items, $I = \bigcup_{o\in O} I_o = \bigcup_{k\in K} I_k$
τ	the maximum number of required pick shifts in the zones, $\tau = \max_{k\in K}\left\{\left\lceil \frac{\lvert I_k\rvert}{qN_k}\right\rceil\right\}$.
$\Re(q,a)$	time needed to finish a pick route of q items (or picks) in a zone containing a aisles and return to the left-most aisle of the assigned zone. It consists of four components: travel time, setup time, picking time and correction time. (It has

to note that the number of items in the last pick route (in each zone) can be less than the route's capacity.) If the random storage assignment and the S-shape routing method are used, then it can be calculated by (see details in Appendix 6A):

$$\Re(q,a) = La\left[1-\left(1-\frac{1}{a}\right)^q\right] + 2w_b\sum_{i=1}^{a}(i-1)\left[\left(\frac{i}{a}\right)^q - \left(\frac{i-1}{a}\right)^q\right] + CR(q,a) + t_s + \frac{q}{r_{pi}} \quad \textbf{(6.1)}$$

Decision variables

$$x_{ti} = \begin{cases} 1 & \text{if item } i \text{ is picked in period } t (t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{to} = \begin{cases} 1 & \text{if order } o \text{ has been completely picked in period } t (t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

TL_{to} total number of items of order o completely picked by the end of period t

NCO_t number of newly complete orders in period $t\,(t = 1..\tau)$

UCO_t number of complete (but unpacked) orders transferred from period $t\,(t = 1..\tau)$ to period $t+1$. This is because in a period of length $\Re(q,a)+\mu$, we can only pack a limited number of complete orders: $P = \lfloor[\Re(q,a)+\mu].r_{pa}\rfloor$.

PAC_t number of complete order packed in period t $(t = 1..\tau)$

The whole batch is completed only when all orders have been packed. Therefore, the throughput time, the overall time (ψ) to complete a batch, is the summation of time required to pick all items (the total picking time), the transportation (for all pick shifts) and the time needed to pack all remaining unpacked orders after the last pick shift. The throughput time can be calculated by:

$$\psi = (\tau-1)\{\Re(q,a)+\mu\} + \{\Re(q_M,a)+\mu\} + UCO_\tau / r_{pa} \quad \textbf{(6.2)}$$

where $\Re(q_M,a)$ is the longest pick-route time in period τ; q_M is the maximum number of items which need to be picked from some zone in period τ (q_M is known if order profile is given). Having mentioned all assumptions and variables, we now can formulate the item-to-route assignment problem as follows.

MODEL

Objective $\quad Min\ \ UCO_\tau$

Such that

$$\sum_{t=1}^{\tau} x_{ti} = 1 \qquad \forall\left(k \in K, i \in I_k \right) \qquad \textbf{(6.3)}$$

$$\sum_{i \in I_k} x_{ti} \le qN_k \qquad \forall\left(k \in K,\, t = 1..\tau\right) \qquad \textbf{(6.4)}$$

$$TL_{to} = \sum_{j=1}^{t} \sum_{i \in I_o} x_{ji} \qquad \forall\left(o \in O, t = 1..\tau\right) \qquad \textbf{(6.5)}$$

$$\left|I_o\right| - TL_{to} \le M_1\left(1 - y_{to}\right) \qquad \forall\left(o \in O, t = 1..\tau\right) \qquad \textbf{(6.6)}$$

$$-\left|I_o\right| + TL_{to} \le y_{to} - 1 \qquad \forall\left(o \in O, t = 1..\tau\right) \qquad \textbf{(6.7)}$$

$$M_1 = \max_{o \in O}\left\{\left|I_o\right|\right\} \qquad \textbf{(6.8)}$$

$$NCO_t = \sum_{o \in O} \sum_{j=1}^{t} y_{jo} - \sum_{o \in O} \sum_{j=1}^{t-1} y_{jo} \qquad \forall\left(t = 1..\tau\right) \qquad \textbf{(6.9)}$$

$$PAC_1 = 0 \qquad \textbf{(6.10)}$$

$$PAC_t \le P \qquad \forall\left(t = 2..\tau\right) \qquad \textbf{(6.11)}$$

$$PAC_t \le UCO_{t-1} \qquad \forall\left(t = 2..\tau\right) \qquad \textbf{(6.12)}$$

$$UCO_t = NCO_t + UCO_{t-1} - PAC_t \qquad \forall\left(t = 1..\tau\right) \qquad \textbf{(6.13)}$$

$$UCO_0 = 0 \qquad \textbf{(6.14)}$$

$$UCO_t \ge 0 \qquad \forall\left(t = 1..\tau\right) \qquad \textbf{(6.15)}$$

$$x_{ti}, y_{to} \in \left\{0,1\right\} \qquad \forall\left(o \in O,\, t = 1..\tau,\, i \in I_k\right) \qquad \textbf{(6.16)}$$

In the objective function, we minimize the throughput time to finish a batch of q orders (note that in (6.1) two first components and r_{pa} are constant, thus minimizing ψ also means minimizing UCO_τ). Constraint (6.3) ensures that each item is assigned to exactly one pick route. Constraint (6.4) is the capacity constraint. It indicates that the maximum number of items that can be picked from zone k by N_k order pickers in one period cannot exceed the total capacity of the N_k order pickers. Constraints (6.5)-(6.8) imply that

$y_{to} = 1$ if order o is completed by the end of period t (meaning that all items belong to order o are picked in pick shift t), and $y_{to} = 0$ otherwise. Constraints (6.9)-(6.15) indicate that the number of complete orders left over period $t+1$ equals the number of newly complete order during period t plus the number of complete orders left over from period $t-1$ minus the number of orders that have been packed in period t. The last constraint defines the nonnegative and binary property of variables x_{ti} and y_{to}. It can be seen that we have a mixed linear-integer formulation. The hardest constraints are (6.3) and (6.4). Constraints (6.5)-(6.15) are used just for keeping track of the number of unpacked orders in the last period (UCO_τ).

6.4 Case study and numerical experiments

In this section, the investigated case is introduced and the results (obtained by using formulation presented in Section 3) are successively discussed.

6.4.1 Introduction

The case we consider is based on the distribution center of Wehkamp, a large online retailer in the Netherlands. Its mission is "being an innovative home-shopping organization with a wide assortment of consumer products against competitive prices and recognizable better service". The company uses a pick and pack system (which was simplified and sketched in Figure 6.1). About 15000 orders have to be picked per day, each containing 1.6 items (in total 2.3 units per order) on average. Since the picking and packing department have a limited capacity, orders received from customers are processed several times (in batches) a day; each batch contains about 1000 items in total. The picking process is described in Section 6.2. The order picker starts a batch by picking up a picking cart and obtaining pick lists from the central location. Pick routes always start from the left-most aisle in the zone. The picked items are dropped on the transportation conveyor, which conveys them to the packaging area. At the packaging area, complete orders are sorted by packing destination station (automatically) and then per order (manually), while incomplete orders (i.e. items) are buffered until they are complete (see Figure 6.1). In this case, all the buffering takes place at the packing station. When an order at packing station is complete, a light indicator turns on to signal the packers that packaging can start.

As previously discussed, the zone size may strongly influence the system throughput time. Therefore, it is a crucial decision for the manager to decide how large zones should be, or, equivalently, the number of zones the pick area should be divided into, such that the system throughput of the system is minimized. In the next section, we will use the model of Section 6.3 to answer this question for the case.

6.4.2 Numerical experiments and results

Table 6.1 shows the current operational data as well as the size of the picking area. The company has 36 storage aisles and uses 18 order pickers. Therefore, there are 6 possible zoning schemes (see Table 6.2). The packing rates depend on the average order size (average number of items per order); they are 8, 3, 1 and 0.5 order(s) per minute for order sizes of 1.6, 5, 10, and 20 items respectively.

Table 6.1 Operational data and system parameters

Operational data		System parameters	
Average number of items per batch	1000	Number of storage aisles	36
Average number of items per order	1.6		
Max. number of items per route (capacity or pick-list size)	40	Aisle length (L) in seconds	60
Number of order pickers	18	Distance between two consecutive aisles (w_b) seconds	5
Set-up time (t_s) in seconds	180		
Picking time per item ($1/r_{pi}$)	5		
Packing rate (r_{pa}): 8, 3, 1 and 0.5 order(s) per 60 seconds for 1.6, 5, 10, and 20 items order size respectively			

Table 6.2 Possible zoning schemes

Number of zones	Number of storage aisles per zone	Number of order pickers per zone
1	36	18
2	18	9
3	12	6
6	6	3
9	4	2
18	2	1

In order to determine the optimal number of zones, we carried out a number of experiments. We considered four pick-list sizes (10, 20, 30 and 40 items per pick route), and three order sizes (1.6, 5, 10, and 20 items per order on average). Combining this with 6 zoning schemes, we have 96 scenarios in total, including the current situations (1.6 items per order, maximum 40 items per pick route). An order batch was generated as follows. We fixed the number of items per batch. For each item, a storage location (in one of the 36 aisles) and an order (to which the item belongs, from 1 to κ) were randomly drawn from a uniform distribution (implying that random storage assignment is used). The average order size was controlled by adjusting κ: $\#\text{orders} = \kappa\left(1 - 1/\kappa\right)^{\#\text{items}}$. For each scenario we generated 5 order batches, and after solving the item-to-route assignment problem of Section 6.3, we calculated the average throughput time value. The average travel time per pick route can be calculated, based on the zone size, the number of items per route, and the routing method used. In our case, the S-shape method is used and the route time is calculated by using formulation (6.1). The route times for the different pick-list and zone sizes are tabulated in Appendix 6B.

Table 6.3 Average throughput time (in minutes)

Order size (items)	Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones	Mean
Small (1.6)	10	149.49	130.58	121.49	124.01	112.48	97.42	122.58
	20	105.75	105.61	104.13	103.71	99.61	97.42	102.71
	30	111.34	110.37	108.66	106.89	103.25	101.13	106.94
	40	116.41	114.92	112.15	110.66	106.88	104.83	110.98
	Mean	120.75	115.37	111.61	111.32	105.56	100.20	110.80
Medium (5)	10	149.64	130.73	121.64	94.15	96.95	111.33	117.41
	20	99.85	92.57	90.34	85.58	86.66	84.67	89.95
	30	97.88	96.95	95.07	88.26	89.46	87.58	92.53
	40	102.91	101.42	98.70	92.12	93.29	87.96	96.07
	Mean	450.28	421.67	405.75	360.11	366.36	371.54	395.95
Large (10)	10	150.47	131.56	122.47	116.98	116.02	119.41	126.15
	20	127.32	127.24	125.01	117.58	122.23	119.67	123.18
	30	132.21	131.62	129.41	120.26	124.50	122.25	126.71
	40	136.91	135.42	133.03	124.79	127.25	120.83	129.71
	Mean	136.73	131.46	127.48	119.90	122.50	120.54	126.44
Very large (20)	10	153.47	134.56	125.47	120.25	116.91	125.41	129.35
	20	127.38	127.24	127.01	121.80	125.61	122.67	125.29
	30	133.21	131.62	129.41	122.84	126.87	124.25	128.03
	40	137.91	135.42	133.03	124.79	128.63	125.58	130.89
	Mean	137.99	132.21	128.73	122.42	124.51	124.48	128.39

We used LINGO (version 8.0) to solve the item-to-route assignment problem (discussed in Section 6.3). It turns out that we can find the optimal solution for all 96 scenarios (within 25 seconds, 2.4 MHz Pentium CPU). The results of the experiments are presented in Table 6.3.

Table 6.3 shows that for the current demand situation (1.6 items per order on average) the 18-zone configuration gives the shortest throughput time for the system[8]. It means that for not very large order sizes, the configuration that minimizes the picking time (i.e. the 18-zone configuration) also minimizes the system throughput time. The reason is that when the zone size increases, the reduction in picking time is dominant the increase in packing time in the case of small orders. For very large orders (i.e. 20 items per order on average), it appears that the 6-zone configuration outperforms the other zoning schemes. It shows the effect of spreading orders over works zones: large zone may reduce the picking time, but may increase the consolidation time. This effect seems to be clear for large order sizes.

For the current situation, a pick-list of 40 items per route is not optimal. A pick-list size of 20 appears to be optimal in most of the cases. When the pick-list size changes from 10 to 40, the throughput time decreases and then goes up. We can explain this behavior as follows. If we increase the pick-list size, the overall travel time to complete a batch will decrease. Therefore, the overall picking time of a batch will be reduced. However, the accumulative number of complete orders, which have to be packed in the last period when the picking is completed, will grow (potentially). That increases the overall packing time, thus the throughput time. Clearly, there exists a trade-off between picking time and packing time when increasing the pick-list size.

6.5 Concluding remarks

In this chapter, the problem of choosing the right number of work zones at a manual pick-and-pack OP system has elaborated. At the first phase, the problem of assigning items to pick routes in each zone was formulated, such that the throughput time is minimized, as a mixed integer-linear program. At the second phase, this problem was used as a tool for evaluating different zone-size options to find the optimal one. The method was illustrated with data obtained from a distribution center of a large online retailer in the Netherlands.

Only random storage assignment and the S-shape routing method are used. However, our model can be applied for other operational policies (like the return routing, class-based or COI-based assignments), as long as we can estimate the travel time of a pick route.

There are several issues which have not been addressed in this study. First, the congestion in the aisles (resulting from having more than one order picker per zone) is not taken into account. Second, though the optimal solutions for all investigated instances are found, it does not guarantee that a 'good' solution for the item-to-route problem can be obtained for

[8] the 18-zone option would be more favorable if we take aisle congestion into account (in an 18-zone configuration, each zone has only one order picker, thus it is free from the travel congestion)

large-size instances (i.e. more aisles, periods, items). More experiments/analyses should be done in order to get more insights into the problem complexity.

Appendix 6A Picking time estimation

- *Travel time* consists of three components: travel time in the cross-aisles aisles, travel time within the storage aisles, and travel time back to the left-most aisle of the zone (e.g. to start a new pick route). As we assume that the order picker always starts a pick route from the left-most aisle of the zone, the last component equals to the cross-aisle travel time. With the S-shape routing method and random storage, the average travel time within storage aisles can be estimated by $L\left[a-a\left(1-\frac{1}{a}\right)^q\right]+\ CR(q,a)$, where the term in brackets is the expected number of visited aisles (see Chapter 2 for detailed explanations). $CR(q,a)$ is called the correction time, its takes into account the fact that from the last pick position (in the last visited aisle) the order picker has to return to the drop-off point (the transportation conveyor). Such a turn has to be made if and only if the number of visited aisles is odd. Applying a similar approach as mentioned in Appendix 5B:

$$CR(q,a)=\sum\nolimits_{g\in G:odd}\left[\binom{a}{g}\left(\frac{g}{a}\right)^q X(g)\left(2L\frac{\frac{q}{g}}{\frac{q}{g}+1}-L\right)\right],$$

where $X(g)$ is 1 minus the probability that all q items are in less than g aisles ($g\in\{G\,|\,1\le g\le a,\ g\text{ is odd}\}$), conditional on the fact that all q items in at most g specific aisles:

$$X(g)=1-\sum\nolimits_{j=1}^{g-1}(-1)^{j+1}\binom{g}{g-j}\left(\frac{g-j}{g}\right)^q.$$

If we number the aisles of a zone from 1 to a (from the left to the right), the cross-aisle travel time can be estimated by $w_b\sum_{i=1}^{a}(i-1)\left[\left(\frac{i}{a}\right)^q-\left(\frac{i-1}{a}\right)^q\right]$, where $\left(\frac{i}{a}\right)^q-\left(\frac{i-1}{a}\right)^q$ is the probability that q picks fall in aisles $1,\ldots,i$ minus the probability that q picks fall in aisles $1,...,i-1$, and w_b is the travel time between two consecutive storage aisles (see Figure 6.1).

Finally, $\Re(n,m)$ is estimated as follows (the first component is the within-aisle travel time, the second one is the cross-aisle travel time, the third one is the correction time, the fourth one is the set-up time, and finally the last one is the picking time):

$$\Re(q,a)=La\left[1-\left(1-\frac{1}{a}\right)^q\right]+2w_b\sum_{i=1}^{a}(i-1)\left[\left(\frac{i}{a}\right)^q-\left(\frac{i-1}{a}\right)^q\right]+CR(q,a)+t_s+\frac{q}{r_{pi}} \quad \blacklozenge$$

Appendix 6B Average route time (in minute) with different zone and pick-list sizes

Pick-list size	1 zone	2 zones	3 zones	6 zones	9 zones	18 zones
1	7.29	5.79	5.29	4.79	4.63	4.46
2	9.65	7.63	6.94	6.22	5.94	5.54
3	11.46	9.13	8.30	7.33	6.88	6.15
4	13.07	10.53	9.59	8.40	7.76	6.61
5	14.52	11.78	10.72	9.30	8.53	7.02
6	15.93	13.00	11.81	10.10	9.18	7.41
7	17.25	14.14	12.82	10.84	9.74	7.79
8	18.54	15.23	13.78	11.53	10.24	8.16
9	19.78	16.28	14.68	12.17	10.70	8.54
10	20.99	17.29	15.54	12.76	11.13	8.92
11	22.16	18.25	16.36	13.32	11.54	9.29
12	23.32	19.19	17.13	13.83	11.94	9.67
13	24.44	20.09	17.88	14.32	12.33	10.04
14	25.54	20.96	18.59	14.78	12.72	10.42
15	26.62	21.80	19.28	15.22	13.10	10.79
16	27.67	22.61	19.94	15.65	13.49	11.17
17	28.71	23.40	20.58	16.06	13.86	11.54
18	29.72	24.17	21.19	16.47	14.24	11.92
19	30.72	24.92	21.79	16.87	14.62	12.29
20	31.69	25.64	22.36	17.26	15.00	12.67
21	32.65	26.35	22.92	17.65	15.37	13.04
22	33.60	27.03	23.47	18.04	15.75	13.42
23	34.52	27.70	24.00	18.42	16.12	13.79
24	35.43	28.35	24.52	18.80	16.50	14.17
25	36.33	28.99	25.03	19.18	16.87	14.54
26	37.21	29.62	25.52	19.56	17.25	14.92
27	38.08	30.22	26.01	19.94	17.62	15.29
28	38.93	30.82	26.48	20.32	18.00	15.67
29	39.77	31.41	26.95	20.70	18.37	16.04
30	40.59	31.98	27.41	21.08	18.75	16.42
31	41.41	32.54	27.85	21.45	19.12	16.79
32	42.21	33.09	28.29	21.83	19.50	17.17
33	43.00	33.63	28.73	22.20	19.87	17.54
34	43.77	34.17	29.16	22.58	20.25	17.92
35	44.54	34.69	29.58	22.96	20.62	18.29
36	45.29	35.21	30.00	23.33	21.00	18.67
37	46.04	35.71	30.41	23.71	21.37	19.04
38	46.77	36.22	30.82	24.08	21.75	19.42
39	47.50	36.71	31.22	24.46	22.12	19.79
40	48.21	37.20	31.62	24.83	22.50	20.17

7

Conclusions and Future Research

Warehouses are an important part of almost every supply chain. As elaborated in Chapter 1, warehouses have multiple functions; they are used to maintain a source of supply, to buffer against demand uncertainties, to achieve transportation/production economies, etc. New trends in distribution, logistics, and manufacturing have brought several new functions to warehouses. A warehouse nowadays can be used as a cross-dock facility, a distribution or a return center, or light manufacturing (final assembly, product customization …). Among all the warehouse functions, order picking is the most critical one. Any inefficiency in order picking can lead to unsatisfactory service and high operational cost for its warehouse, and consequently for the whole supply chain. In order to operate efficiently, the order process needs to be robustly designed and optimally controlled. This thesis aimed at providing models to support the design and control of efficient OP processes. In detail, the following models are investigated.

7.1 Travel distance estimation for manual-pick class-based storage strategy warehouses

Travel time (or travel distance) is often used as an objective function for optimizing OP processes, since travel time immediately impacts operational cost and customer service (more elaborate reasons can be found in Section 1.2.3). Estimating the travel time is not trivial problem. The length of a pick route depends on order batching policy, the number of picks (or visit locations), layout of the picking area, the storage assignment method, and the routing method used.

In the literature, several travel time (or distance) models exist. Most of these models were developed for AS/RS (see Section 4.1). Only few models can be applied for random storage manual-pick warehouses. No analytical travel time model for class-based storage assignment manual-pick warehouses can be found in the literature, despite of their popularity in practice. Chapter 2 of this thesis presents an approximate travel-time model for this type of warehouse. The model focuses on some typical layout types, which can be considered as the most basic (and simple) forms of major warehouses in practice. To route order pickers, two common routing heuristics (the return and S-shape method) are used. The effective pick-list size (q) varies between 4 and 60, which covers a wide range of pick-list sizes in OP practices.

Numerical experiments shows that in the worst case (among the experimented instances), the difference between approximation and simulation result is about 8% for the return and less than 5% for the S-shape routing method. The error is small for small warehouses and appears to be larger for larger warehouses (i.e. large number of aisles, or equivalently smaller number of picks per aisle). Regarding the pick-list size, the gap between approximation and simulation result becomes smaller when the pick-list size grows; it is very tight for large pick-list sizes.

The presented travel time model can also be applied to more complicated (larger) warehouses. A feasible way of doing that is to partition the layout into sub-layouts so that the model is applicable. Travel time between sub-layouts can be estimated by taking into account the travel time and frequency between the sub-layouts.

7.2 Storage zone and layout optimization for manual-pick class-based storage strategy warehouses

In Chapter 3, the travel time model developed in Chapter 2 is used as the objective function for two problems: storage zone and layout optimization for manual-pick class-based storage strategy warehouses. In the first problem, it is assumed that the layout (i.e. number of aisles) is given. The decision variables are the storage zone divisions of product classes in each aisle. The second problem considers situations where only the storage area (or floor) is fixed. So, besides the storage zone divisions, the number of aisles is also a decision variable. These problems are crucial in warehouse design and control; they often occur when a warehouse is (re)designed, or the assortment or the order pattern changes. To solve the problems, we first consider a precise approach. However, the exact algorithm is time consuming; it cannot handle large warehouse instances (regarding the number of aisles, classes and space slots per aisle). Hence, a heuristic approach is proposed to solve the problems. This heuristic exchanges proximity classes between aisles, from far-to-depot aisles to close-to-depot aisles. The approach is rather simple, but fast and proves to be of very good quality. It can therefore be applied to many practical warehouse design or improvement situations.

Results from numerical results support the following warehouse design guidelines with respect to minimizing the average route length.

1. For a given layout, the across-aisle zoning layout type and the return routing is the best combination for the cases of small pick-list sizes, skewed demand and long-aisle warehouses, while the combination of the identical-aisle zoning layout and the S-shape routing method is the best for other cases.
2. For a given layout, when the S-shape routing method is used, the across-aisle zoning is the best zoning type.
3. For a given layout, when the return routing method is used: (a) for large pick-list sizes, the identical-aisle zoning type is the best; (b) for small pick-list sizes, the across-aisle zoning type is the best for long-aisle warehouses with skewed storage assignments.
4. For a given warehouse floor area, the long-aisle layout type is better for large pick-list sizes and short-aisle layout type is better for small pick-list sizes.

7.3 Travel times and rack design for a compact AS/RS

Chapter 4 discusses a compact system originating from the Distrivaart project that consists of rotating conveyors and an S/R machine. Because of the special structure and engineering design, the cycle time of the S/R machine depends on three rack dimensions (rack length, height and depth – or conveyor's perimeter). Every pallet position can be accessed individually. For a given storage capacity and an S/R operating mode (i.e. dual or single), Chapter 4 determines the rack dimensions (or, equivalently, the ratio between the dimensions) that minimize the cycle time of the S/R machine.

By extending Bozer and White's model for 2-dimensional AS/RSs, it is possible to estimate the single-command cycle times. Based on this travel model, we found

1. For a given 3-dimensional compact AS/RS (as above-mentioned) with a total storage capacity V, the optimal rack dimensions are $t_v = t_h = 0.89\sqrt[3]{V}$, $t_c = 1.24\sqrt[3]{V}$, and the optimal travel time is $1.38\sqrt[3]{V}$. Equivalently, the optimal ratio between three dimensions is $t_v : t_h : t_c \equiv 0.72 : 0.72 : 1$.

2. The cubic-in-time system (all dimensions are equal in time) is not the optimal configuration (as intuitively we may think). However, it is a good alternative configuration for the optimal one as the resulting expected travel time is only about 3% off the optimum.

The method assumes that the rack is continuous. This simplification of reality is only justified if the number of storage positions is sufficiently large.

7.4 Online order-batching problem

Order batching is the method of grouping a set of orders into a number of sub-sets, each of which can then be retrieved by a single picking tour, such that a specific objective is achieved. Several batching methods are mentioned in Section 1.3.4. Traditionally, information about orders and service time (including traveling, set-up, picking,…) are considered as deterministic variables. In reality they are stochastic variables. When orders arrive online and need to be picked and shipped in a tight time window, a trade-off has to be made between timeliness and picking efficiency. It would be inefficient, and also the capacity may be insufficient, if we start to pick too early or for each order. The decision is, therefore, how many orders should be picked together in one pick route.

Chapter 5 focuses on finding a simple but efficient approach for determining the optimal picking batch size for order pickers in a typical manual-pick shelf-rack type warehouse. In order to do so, the OP system is modeled as a queueing model with batch service. The first and second moments of the service time are estimated based on the batch size, the routing method and storage assignment used. The waiting time of a random order is estimated by using these moments and the corresponding batch service queueing model. The optimal picking batch size is then determined in a straightforward manner. Results from the simulation experiments show that our approach provides a high accuracy level. Furthermore, the method is simple; it can be easily applied in practice.

The average waiting time appears to be a convex function of the batch size. As a result, a unique optimum picking batch size exists. As the optimum batch size is close to its lower bound (obtained from the traffic density condition), we propose a simple greedy procedure, which can be used to search for the optimum in a short computation time.

In the numerical experiments, only random storage assignment and the S-shape routing method are considered. However, the proposed order batching method can be applied for other warehouse layouts, routing and storage assignment methods, as long as the first and second moment of the service time are tractable. For single or 2-block warehouses, it would not be difficult to estimate the first moment of the service time when other storage assignments are used (it is given in Chapter 2, for class-based storage). Nevertheless, obtaining the second moment may be cumbersome.

7.5 Determining number of work zones in a pick-and-pack OP system

Using zone picking reduces the overall picking time because of the smaller traversed area, of the familiarity of the order picker with the item locations in the zone, and less traffic congestion. One of the crucial decisions associated with zoning is to determine the optimal number of work zones (when the picking area is given, it is equivalent to determining the zone size). A larger number of zones may help to reduce the picking time, but may increase the time needed to consolidate the orders. Chapter 6 of this thesis addresses the problem of choosing the appropriate number of work zones at a synchronized pick-and-pack OP system. At the first phase, the problem of assigning items to pick routes in each zone, such that the throughput time is minimized, is formulated as a mixed integer-linear program. At the second phase, this problem is used as a tool for evaluating different zone-size options to find the optimal one. This approach is illustrated by data taken from an OP system used in a distribution center of a large online retailer in the Netherlands. The numerical experiments show that it is possible to determine the optimal number of zones for a given order pattern (number of orders and order size) and a range of pick-list sizes. A small-zone scheme (i.e., few aisles per work zone) may outperform a large-zone scheme when the order size is small (i.e. few lines). However, a large-zone scheme may outperform small-zone schemes for large order sizes.

Although in the experiments only random storage assignment and the S-shape routing method are used, the method can be applied for other operational policies (like the return routing method, class-based or COI-based storage assignment), as long as the tour length is tractable.

7.6 Future research

This thesis has revealed several future research directions. They can be divided into two streams. The first one focuses on extending the developed models to cope with more complicated OP situations. The second pinpoints several issues that have not been (thoroughly) treated in the thesis.

The presented models reflect several typical OP situations. When applying the models to different OP situations, some adjustments may be needed.

- The maximum number of (storage) blocks considered in Chapters 2 and 3 is 2. In practice we may encounter warehouses which contain more than 2 blocks. The presented models therefore need to be adapted in order to cope with larger warehouse instances.
- In Chapter 4, it is possible to determine the optimal rack dimensions for single-command cycles. Further effort is required in order to find the optimal rack dimensions for dual-command cycles.
- The queueing model in Chapter 5 can be extended in several directions. First, it would be more realistic if we consider Compound Poisson arrivals (instead of single Poisson), as in practice each order may contain several items. Second, the order batching problem is considered for only one isolated (work) zone. How can the model be adapted for the situation when there are several zones? What are the interactions between the zones? For example, when the batch size in one zone changes, what are its implications to other zones' performance and consequently to the system throughput? A network of queues can be a good approximate model for the system. And finally, aisle congestion may occur when there are multiple order pickers working in the same zone at the same time. Incorporating this effect into the model would be a challenging work.
- Although we found optimal solutions for the item-to-route assignment problem for all investigated instances in Chapter 6, it does not mean that we can find the optimal solution when larger instances are encountered. Further effort is need to testify the complexity of the problem.

Several issues have not been treated thorough in this thesis as well as in the literature.

- First is the interaction between picking and replenishment. Items need to be stored in storage locations (replenishment) before they are retrieved to fill customer orders (picking). Certainly, there are links between these two processes. For example, by using a dedicated storage assignment, the average tour length can be reduced, compared to when using the random storage assignment. However, the total travel time for replenishing may be longer (note that often the I/O point and reserve area are located on two opposite sides of the picking area). Most of the research in the literature focuses on the picking process only; the replenishment is largely ignored. Further research is required to identify the interactions between these two processes.
- Second is the problem of determining the optimal work force level for a picking area. The work force level (i.e. number of order pickers) certainly influences the system throughput time, especially for manual-pick OP systems. Although the total number of hired order pickers is often considered at the tactical level, the number of order picker

per a certain work area in a warehouse can be easily adjusted on short term basis. For example, we can appoint some order pickers from an exceed-pick-capacity area to an area when a high pick rate is desired. As far as we know, determining optimal number of order pickers for a pick area has not been investigated in the literature. This is an interesting future research.

- Third issue concerns the impact of storage assignment on traffic congestion in the aisles. For example, with class-based storage, one may expect that the aisle congestion will happen more often in a fast moving product area than in lower moving product areas. Integrating the aisle congestion effect into travel distance models (like ones presented in Chapter 2) will lead to a better estimation of the real tour length.
- Finally, the order picking, order sorting (and packaging) and truck dispatching problem are strongly related; 1 minute saving in order picking or order sorting time does not guarantee that customers will receive products 1 minute sooner. Integrating these problems into a unique model makes it possible to see the effect of order picking time on customer lead time. Certainly, it is a challenge research problem.

List of Abbreviations and Common Terminologies and Notations

COI	Cube-per-order index
EDC	Expected dual-command cycle time
ESC	Expected single-command cycle time
I/O	Input/output (point) (also: depot)
NBUE	New better than used in expectation
NSIT	Non SIT (rack)
OB	Order batching
OP	Order picking
P/D	Pick up/ deposit (point)
S/R	Storage/retrieval (machine or crane)
S/RS	Storage and retrieval systems
SIT	Square-in-time (rack)
SKU	Stock keeping unit

Customer order	A list of SKUs (each with a certain quantity) ordered by a customer
Cycle time	Time needed for the S/R machine to complete a travel cycle (leaving and returning to the I/O point)
Order-line	SKU appeared on customer orders
Pick list	A list of items (with quantity) to be picked in one pick route (or tour)
Pick tour (or route)	A travel tour by which the order picker has to make in order to pick up all requested items on a pick list
Pick-list size	Number of items in a pick tour
Tour length	The length (distance or time) of a pick tour

Main notations used in Chapters 2&3

a	number of pick aisles (also denoted as 'storage' aisles)
l_{ij}	partial length of pick aisle j used for storing of product class i
q	number of picks (or order lines) in a picking tour (the pick-list size)
q_j	expected number of picks to be picked from aisle j, given that the pick-list size is q
c	number of (product) classes
L	length of a pick aisle
w_a	width of the cross aisle
w_b	centre-to-centre distance between two consecutive (pick) aisles
w_c	width of the storage rack
w_d	width of the rear aisle, $w_d = 0$ for closed-end aisle layouts
f_i	order frequency of product class i, $f_i = \sum_{j\in\ell_i} f_{ij} \Big/ \sum_{j\in\ell} f_{ij}$, where ℓ_i is the set of items belong to product class i and $\bigcup_{i=1}^{c}\ell_i = \ell$
s_i	percentage of the total storage space used for class i
P_{ij}	the probability that the farthest pick in aisle j is in zone i
p_{ij}	the probability that an item of class i located in aisle j is ordered (we assume this to be proportional to the pick frequency of class i)
$D_j(q,c)$	the expected travel distance (in a single direction starting from the cross aisle) within aisle j to pick up q items, given that there c classes
TD_z^{CA}	travel distance within the cross aisle (called 'cross-aisle' travel distance), z denotes the name of the routing method used.
TD_z^{WA}	travel distance within pick aisles (called 'within-aisle' travel distance).
TD_z	(expected) average tour length.
ζ	accumulative error in estimating the average tour's length
ζ_a	accumulative error in estimating the within-aisle travel distance
ξ_b	error in estimating the cross aisle travel distance
ε	error in estimating the travel distance in an aisle (for the return routing method)
η	expected number of visited aisles

k_j	expected travel distance from the central line of the cross-aisle to the farthest pick location in aisle *j*, given that aisle *j* is visited: $k_j = w_a/2 + D_j(q_j, c)$
m_j	probability that the aisle j is visited
n_j	probability that aisle *j* and/or aisle (*a-j+1*) is visited
n'_j	probability that a pick-line *j* is visited
ρ	shape ratio: $\rho = {aw_b}/{2L}$
s	number of space slots per (pick-) aisle

Main notations used in Chapter 4

ESC	expected single-command travel time of the S/R machine
EDC	expected dual-command travel time of the S/R machine
$E(w)$	expected time needed to go from the I/O point to the pick position and to wait for the pick to be available at the pick position
$E(u)$	expected time needed for the S/R machine to return to the I/O point
$E(v)$	expected time to travel from a storage location to a pick location (in dual-command cycles)

Main notations used in Chapter 5

p_i	probability that a random item is picked from aisle $i\,(i = 1..a)$; $p_i = 1/a$ $(i = 1..a)$ for the random storage assignment
τ_s	setup time per batch (constant)
τ_p	picking time per item (constant)
TR_B^{WA}	travel time caused by traversing the pick aisles, B can be $+$, $-$ or $\approx$ (indicating upper, lower bound or approximation values respectively)
TR_B^{CA}	travel time caused by traversing the cross aisle
AT	adjustment time
$E(S)$	first moment of the service time (including setup, picking and traveling time)
$E(S^2)$	second moment of the service time
$\sigma(S)$	standard deviation of the service time
λ	order arrival rate
k	number of orders to be picked in a tour
q	number of items in a batch of k orders

Main notations used in Chapter 6

q the maximum number of items that an order picker can pick in a pick route (or tour)

a number of aisles per zone

L length (in travel time unit) of a storage aisle

w_b centre-to-centre distance (in travel time unit) between two consecutive storage aisles

t_s set-up time of a route

μ transportation (conveyor) time

r_{pi} picking rate (number of units per time unit that an order picker can pick). It is assume to be identical for all order pickers

r_{pa} overall packing rate (number of orders per time unit).

N_k number of order pickers in zone k

t, i, o, k indices of period, item, orders and zones

K set of zones

O set of all orders

I_o set of all items in order o

I_k set of all items in zone k

I set of all items, $I = \bigcup_{o \in O} I_o = \bigcup_{k \in K} I_k$

τ the maximum number of required pick shifts in the zones

$\Re(q,a)$ time needed to finish a pick route of q items (or picks) in a zone containing a aisles and return to the left-most aisle of the assigned zone

$$x_{ti} = \begin{cases} 1 & \text{if item } i \text{ is picked in pick shift } t (t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{to} = \begin{cases} 1 & \text{if order } o \text{ has been completely picked in or before period } t (t = 1..\tau) \\ 0 & \text{otherwise} \end{cases}$$

TL_{to} total number of items of order o completely picked at the end of period t

NCO_t number of newly completed picked orders in period $t\,(t = 1..\tau)$

UCO_t number of complete (but unpacked) orders transferred from period t $(t = 1..\tau)$ to period $t+1$.

PAC_t number of complete order packed in period t $(t = 1..\tau)$

ψ throughput time (time to pick and pack a batch of orders)

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Tóm Tắt Nội Dung Của Đề Tài (Summary in Vietnamese)

Quản lý hậu cần hay còn gọi là quản lý tiếp nhận vận chuyển (*logistics management*) là một khái niệm bao hàm việc lập kế hoạch, thực thi, điều hành một cách hữu hiệu nhất công việc vận tải, lưu kho, quản lý thông tin và các dich vụ liên quan đến hàng hóa, từ điểm nguồn đến điểm tiêu thụ cuối. Đây là một lĩnh vực hết sức quan trọng vì nó ảnh hưởng đến mọi mặt của đời sống con người, một cách trực tiếp hay gián tiếp. Một khâu đặc biệt quan trọng của quản lý hậu cần đó là quản lý kho bãi (*warehouse management*). Có rất nhiều lý do cho việc cần thiết phải sử dụng kho bãi. Một vài nguyên nhân thường gặp là để duy trì một nguồn hàng liên tục tránh trường hợp thiếu hàng khi cần đến; để giảm chi phí vận tải; để làm địa điểm trung chuyển hàng gom hàng từ nhiều khách hàng hoặc tách hàng cho nhiều khách hàng khác nhau … Trong nghiên cứu này, chúng tôi đề cập đến một khâu cốt lõi nhất của hầu hết các kho bãi đó là chu trình lấy hàng từ nơi lưu giữ ra theo yêu cầu của khách hàng (*order picking*). Nhiều nghiên cứu đã chỉ ra rằng với một kho bãi thông thường thì chu trình này chiếm hơn 55% tổng chi phí khai thác. Muốn nâng cao hiệu quả và giảm chi phí khai thác, chu trình cần phải được thiết kế và điều hành một cách tối ưu nhất. Mục đích của nghiên cứu này là đưa ra một số mô hình tối ưu hóa nhằm hỗ trợ cho quá trinh thiết kế và điều hành chu trinh này.

Quãng đường đi trong kho (để lấy hàng) có một ý nghĩa quan trọng, nó có thể được coi là tỉ lệ thuận với thời gian và với chi phí khai thác. Do đó nhà kho cần được thiết kế, bố trí, điều hành sao cho quãng đường đi để lấy hàng là ngắn nhất. Mô hình thứ nhất mà chúng tôi đề xuất ở nghiên cứu này là mô hình xác định quãng đường đi trong nhà kho (*warehouse travel distance estimation*) cho hai loại nhà kho thông dụng nhất, nhà kho sử dụng nhân công (chương 2) và nhà kho bốc lấy hàng tự động (chương 4). Ở chương 3 và 4, sử dụng mô hình này chúng tôi đã có thể đề ra các mô hình tối ưu hỗ trợ cho việc lựa chọn kích thước, số lượng dẫy đặt hàng (*storage racks*) và vị trí của từng loại hàng cụ thể trên mỗi dẫy (*layout and storage optimization*). Tiếp theo, ở chương 5, chúng tôi đã thiết lập mô hình xác định tối ưu số lượng hàng mỗi nhân công cần lấy mỗi lần để quãng đường trung bình là ngắn nhất (*order batching for minimizing the average travel distance*). Tập

trung vào tình huống cụ thể của các công ty bán lẻ qua mạng (*online retail companies*), chúng tôi đã áp dụng lý thuyết hàng đợi (*queueing theory*) để giải quyết vấn đề này một cách triệt để. Ở những nhà kho lớn, khu vực lấy hàng thường được chia ra thành các khu vực nhỏ hơn để dễ quản lý, giảm quãng đường đi, và cũng để giảm thời gian tìm kiếm hàng vì không quen thuộc với vị trí đặt hàng… Ở chương 6, chúng tôi đề ra một mô hình trợ giúp cho việc lựa chọn số lượng vùng lấy hàng (*number of picking zones*). Mô hình được áp dụng với số liệu thực tế lấy từ nhà kho của công ty Wehkamp, Hà Lan. Kết quả từ các mô hình đề xuất ở nghiên cứu này đã được so sách với kết quả lấy được từ các mô hình mô phỏng tương ứng (*simulation*). Các so sánh đã chỉ ra rằng các mô hình chúng tôi đề xuất đều đảm bảo một độ chính xác cao. Hơn thế nữa, chúng không yêu cầu thời gian tính toán lớn, có thể áp dụng vào các thực tế một cách khá dễ dàng.

Curriculum Vitae

Tho Le-Duc was born in 1974 in Quang Ninh, Vietnam. He received a Bachelor degree in Navigation Science from the Vietnam Maritime University in 1996 and a Postgraduate Diploma in Industrial Engineering from the Asian Institute of Technology Bangkok Thailand (AIT) in 1998. Thanks to the financial support from the Belgian Development and Co-operations, he obtained his master degree in Industrial Management from the Catholic University of Leuven in 2000. Since May 2001, he started as a Ph.D. candidate (AIO) at the RSM Erasmus University (formerly Rotterdam School of Management/ Faculteit Bedrijfskunde), the Erasmus University Rotterdam. For about more than four years, he performed research on order picking in warehouses. As the results, Tho Le-Duc has been presented his research at several conferences in the fields of operations research, material handling, logistics and supply chain management in both Europe and North America. His research papers have been published or accepted for publication in several refereed conference proceedings, scientific books and international journals.

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Design and Control of Efficient Order Picking Processes

Within a logistics chain, products need to be physically moved from one location to another, from manufacturers to end users. During this process, products are buffered or stored at certain places (warehouses) for a certain period of time. Order picking – *the process of retrieving products from storage (or buffer area) in response to a specific customer request* – is the most critical warehouse process. It is a labour intensive operation in manual systems and a capital intensive operation in automated systems. Order picking underperformance may lead to unsatisfactory service and high operational cost for the warehouse, and consequently for the whole chain. In order to operate efficiently, the order picking process needs to be designed and controlled optimally.

This thesis aims at providing analytical models to support the design and control of efficient order picking processes. Various methods for estimating picking tour length, determining the optimal storage zone boundaries, layout, picking batch size and number of pick zones are presented. The methods are tested by simulation experiments and illustrated by numerical examples.

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