

## A Relative View on Tracking Error

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# A RELATIVE VIEW ON TRACKING ERROR

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*Keywords:* benchmarking, tracking error, information ratio, performance evaluation

JEL classification: C15, C43, G11

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## Abstract

When delegating an investment decisions to a professional manager, investors often anchor their mandate to a specific benchmark. The manager's exposure to risk is controlled by means of a *tracking error volatility* constraint. It depends on market conditions whether this constraint is easily met or violated. Moreover, the performance of the portfolio depends on market conditions. In this paper we argue that these mandated portfolios should not only be evaluated relative to their benchmarks in order to appraise their performance. They should also be evaluated relative to the opportunity set of all portfolios that can be formed under the same mandate – the *portfolio opportunity set*. The distribution of performance values over the portfolio opportunity set depends on contemporary market dynamics. To correct for this, we suggest a normalized version of the information ratio that is invariant to these market conditions.

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## 1. Introduction

Institutional investment decisions are generally centered around mandates, provided to professional investment managers. In a typical investment mandate, a benchmark portfolio is identified and the portfolio manager is assigned the task to beat that benchmark over a specified horizon. The benchmark portfolio as specified in the mandate should be attainable and investable<sup>1</sup>; some examples of benchmarks are the MSCI World Index (or MSCI region or sector indexes), the S&P500 Index, and the Dow-Jones Indexes. Depending on his expertise to identify mispriced securities, the portfolio manager can overweight underpriced securities and underweight overpriced securities, thus building a zero-investment active portfolio. The composition of this active portfolio reflects the selection bets made by the portfolio manager. When adding the active portfolio weight vector to the benchmark weight vector, the composition of the actual investment portfolio is obtained. The difference between the return on the managed portfolio and the return on the benchmark is termed the tracking error. Although the investor is concerned with the return and risk of the total portfolio, the portfolio manager focuses on the return and risk of the active portfolio only.<sup>2</sup>

In order to control the risk of the active portfolio, the investment mandate includes a constraint on the tracking error volatility, henceforth denoted by TEV. The TEV is defined as the standard deviation of the return differential between the managed portfolio and the benchmark.<sup>3</sup> Of course, the TEV should be monitored in order to determine whether the portfolio manager satisfies the risk limit on the TEV as specified in his mandate.<sup>4</sup> Over recent years, the financial markets experienced quite large swings

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<sup>1</sup> For the requirements a benchmark should satisfy, we refer to Reilly [1994, pp.975ff]. See also Rennie & Cowhey [1990] on this point.

<sup>2</sup> This induces the portfolio manager to optimize the active portfolio in excess return space only, thus neglecting the total risk of the resulting portfolio. This problem is identified and discussed by Roll [1992], Rohweder [1998] and Jorion [2003]. We do not pursue this particular problem here.

<sup>3</sup> The terminology can be confusing. Sometimes the standard deviation of the return differential is termed tracking error. Grinold & Kahn [2000] use the term “statistical tracking error” for TEV. In addition, the choice of standard deviation as a risk measure implies that the relative investment decision is molded in the familiar mean-variance framework. For an alternative formulation, see for example Rudolph, Wolter & Zimmermann [1999].

<sup>4</sup> In the broader context of asset allocation, the investment process is organized hierarchically. Strategic asset allocation decisions at the higher level are made on the basis of benchmark portfolios for asset classes such as equity, fixed income, real estate and cash. In the tactical asset allocation process, the weighting of these benchmarks is allowed to fluctuate between tactical asset allocation (TAA) bands. At the lower level, portfolio managers can in turn deviate from the composition of the asset class benchmarks under the restriction of TEV limits. As shown by Ammann & Zimmermann [2001], deviations from the compositions of the benchmark portfolios at the lower level have a

in volatility and this raised serious concerns about the ability of portfolio managers to obey the *ex ante* imposed TEV limit.<sup>5</sup>

The evaluation of the manager's performance will be of a relative nature, *vis à vis* the benchmark. Because the active portfolio is the source of not only return but also risk, the performance should be gauged on a risk-adjusted basis. For this goal, the Information Ratio (henceforth IR) is used.<sup>6</sup> The IR is defined as the average tracking error divided by the TEV. The IR hence measures the average excess return relative to the benchmark portfolio, expressed per unit of volatility in excess return.

The evaluation of the TEV limit and the risk-adjusted performance evaluation are key issues in the benchmark-based investment process. Within the *ex ante* imposed TEV limit, the portfolio manager can compose many portfolios. Given the constrained portfolio flexibility and given financial market dynamics, each of these portfolios can exhibit different excess risk – excess return characteristics. So each of these portfolios can reveal a different TEV *ex post facto*. In addition, each of these portfolios can show a different performance and hence generate a different IR.

In this paper we highlight the influence of portfolio flexibility and financial market conditions on TEV constraints and risk-adjusted performance. We argue that mandated portfolios should not only be evaluated relative to their benchmarks. They should also be evaluated relative to the set of all portfolios that can be formed under the same mandate – the *portfolio opportunity set*. All portfolios in this set obey the constraints as specified *ex ante facto* in the mandate. This portfolio opportunity set can be used to construct frequency distributions of (risk-adjusted) performance values such as realized tracking errors and IRs. The particular portfolio at hand is only one element of this portfolio opportunity set. The location of this portfolio can be plotted in this distribution and this in turn provides information on the relative position of the portfolio in the portfolio opportunity set. It may turn out, for example, that in some period 95% of the portfolios in the portfolio opportunity set performed better than the portfolio at hand. Irrespective of the absolute value of the chosen performance metric, this relative information would qualify (if not disqualify) the achievement of the portfolio manager over the period considered.

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greater impact on top level portfolio risk-return characteristics than deviations from the benchmarks' weights in the top level portfolio. This stresses the relative importance of TEV constraints.

<sup>5</sup> See for example Burmeister, Mausser & Mendoza [2005].

<sup>6</sup> See for example Goodwin [1998].

In addition to the *locus* of the investment portfolio in the frequency distribution of performance values, also the *width* of this distribution is relevant in evaluating investment performance. When the distribution of performance values over the portfolio opportunity set is narrow (broad), this implies that it was hard (easy) to obtain a specific level of outperformance. The width of this performance distribution is not constant over time but depends on contemporary market dynamics. When the range of returns on the individual securities comprised in the benchmark is narrow (we could term this a “homogeneous market”), differences in active portfolio composition translate to only a narrow range of active portfolio returns. When in contrast the market is heterogeneous and individual security returns have fluctuated over a wide range, small differences in portfolio composition can lead to large differences in portfolio returns. In the former case, the distribution of performance values over the portfolio opportunity set is narrow. In the latter case, this distribution will be broad. In practice, a “typical” distribution of IRs is used in order to assess how extraordinary the risk-adjusted performance of a portfolio was. Grinold & Kahn [2000], for example, stipulate that the IR exceeds 0.2 for the top 25% of portfolio managers. However, tracking errors – and hence the IR – depend on the dynamics in the security returns. Over some periods, then, attaining an IR of for example 0.1 may be quite extraordinary, whereas other periods allow achieving an IR of 0.3 or even higher. The information contained in the cross-sectional frequency distributions of performance values allows for normalizing the IR. In this way, we purge this performance metric for the influence of market dynamics on the width of the distribution of over time. In this paper, we discuss the use of portfolio opportunity sets for monitoring TEV constraints and for relative (risk-adjusted) performance evaluation. We illustrate our methodology for a fictitious investment portfolio where the Dow-Jones EURO STOXX 50 Index serves as a benchmark.

The structure of the paper is as follows. In section 2 we introduce notation and provide the definitions of tracking error and relevant performance metrics. Section 3 outlines the procedure to generate portfolio opportunity sets under TEV constraints. Section 4 discusses the data and explains how we constructed a managed investment portfolio. Section 5 contains our empirical results. We illustrate how the portfolio opportunity set approach can be used to reveal tracking error dynamics, check on potential TEV violation, and normalize risk-adjusted performance over time. Section 6 concludes and provides suggestions for further research.

## 2. Relative Investing and Tracking Error

Over some period  $t$ , the return on the benchmark portfolio  $B$ ,  $\tilde{r}_{Bt}$ , is defined as:

$$(1) \quad \tilde{r}_{Bt} = \sum_{i=1}^n b_{it} \tilde{r}_{it} \quad \text{with budget restriction} \quad \sum_{i=1}^n b_{it} = 1$$

where  $n$  is the number of securities in the benchmark,  $\tilde{r}_{it}$  is the return on security  $i$ , and  $b_{it}$  is the weight of security  $i$  in the benchmark at the beginning of period  $t$ . Tildes denote stochastic variables. The portfolio manager deviates from the benchmark by choosing a portfolio  $P$  with weights  $\{w_{it}\}_{i=1,\dots,n}$ . The return on this portfolio over period  $t$  is:

$$(2) \quad \tilde{r}_{Pt} = \sum_{i=1}^n w_{it} \tilde{r}_{it} \quad \text{with budget restriction} \quad \sum_{i=1}^n w_{it} = 1$$

Consequently, the return on the self-financing active portfolio is:

$$(3) \quad \tilde{r}_{Pt} - \tilde{r}_{Bt} = \sum_{i=1}^n (w_{it} - b_{it}) \tilde{r}_{it} \quad \text{with} \quad \sum_{i=1}^n (w_{it} - b_{it}) = 0$$

and the tracking error volatility TEV is:

$$(4) \quad TEV = \sqrt{\text{var}(\tilde{r}_{Pt} - \tilde{r}_{Bt})} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n (w_{it} - b_{it})(w_{jt} - b_{jt}) \text{cov}(\tilde{r}_{it}, \tilde{r}_{jt})}$$

where  $\text{var}(\cdot)$  and  $\text{cov}(\cdot, \cdot)$  denote the variance and covariance operators, respectively.

The information ratio IR is defined analogous to the Sharpe [1966, 1994] ratio with the excess returns now measured with respect to the benchmark return:

$$(5) \quad IR = \frac{E(\tilde{r}_{Pt} - \tilde{r}_{Bt})}{TEV}$$



### 3. Portfolio Opportunity Sets for TEV-restricted Investments

In Pouchkarev [2005], we developed a novel approach for evaluating investment performance. The central idea behind our approach is not to consider a single benchmark index to describe a financial market, or a specific peer group to measure investment performance, but instead to consider all possible portfolios.<sup>7</sup> We call the set of all portfolios that are feasible under a specific investment mandate the *portfolio opportunity set*. This set signifies a level playing field for any portfolio that aims for the same investment objectives and respects the same constraints as specified in the mandate. The portfolio opportunity set approach can be considered as a formalization of the *Wall Street Journal*'s dartboard portfolio approach, in which a random portfolio is constructed by "throwing darts" at the financial pages of the *Wall Street Journal*.<sup>8</sup> But instead of focusing on only one random portfolio, the portfolio opportunity set approach considers the whole set of feasible portfolios. Indeed, in general, the undisputable advantage of our approach is that the set of generated portfolios can be made commensurate with any specific investment environment such as an investment mandate, defined by goals and constraints. Hence, we evaluate the managed portfolio against that set of all available investment alternatives.

Although the portfolio opportunity set approach is completely general and can be made commensurate with any investment mandate, we limit ourselves here to a single constraint on the formation of portfolios: the TEV constraint. Given the portfolio opportunity set generated under the prevailing TEV constraint, we can measure the performance of all these alternative portfolios according to the selected performance criteria: realized return, TEV and IR. We can next evaluate the performance of the investment portfolio at hand against the performance of this complete opportunity set. In this way, the portfolio opportunity set can be considered as a synthetic peer group for the specific investment portfolio. This peer group exactly matches the restrictions under which a portfolio manager is allowed to operate. We next outline in detail how the portfolio opportunity set under a TEV constraint can be generated.

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<sup>7</sup> In Hallerbach *et al* (2005), we describe an application to market description.

<sup>8</sup> For a description of the *Wall Street Journal*'s dartboard portfolio approach, we refer to Moy [1994] and Adams & Cyree [2004]. An early attempt to such a formalization of random portfolios is provided by Cohen & Fitch [1966]. Fisher [1965] and Fisher & Lorie [1970] also used random portfolios to analyze investment performance.

The portfolio opportunity set, which consists of  $n$  assets and which is subject to a TEV constraint, can be specified through the following formal description (see eqs. (3) and (4)):

$$(6) \quad \sum_{i=1}^n \sum_{j=1}^n (w_{it} - b_{it}) \cdot (w_{jt} - b_{jt}) \cdot \text{cov}(\tilde{r}_{it}, \tilde{r}_{jt}) \leq \psi^2$$

$$\sum_{i=1}^n (w_{it} - b_{it}) = 0$$

where  $\psi$  denotes the maximum allowable level of TEV and where as before  $b_{it}$  is the weight of security  $i$  in the targeted benchmark;  $w_i$  is now the weight of the security  $i$  in a feasible portfolio. It is more convenient to work with such opportunity sets if we remove the last dependent weight from the system (6). Reorganizing the second (budget) equation from (6), we have:

$$(7) \quad w_{nt} - b_{nt} = -\sum_{i=1}^{n-1} (w_{it} - b_{it})$$

Substituting eq.(7) into the first equation of (6), we obtain:

$$(8) \quad \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \left[ \text{cov}(\tilde{r}_{it}, \tilde{r}_{jt}) - 2 \text{cov}(\tilde{r}_{it}, \tilde{r}_{nt}) + \text{cov}(\tilde{r}_{nt}, \tilde{r}_{nt}) \right] \cdot (w_{it} - b_{it}) \cdot (w_{jt} - b_{jt}) \leq \psi^2$$

Denoting the expression in the square brackets by  $\hat{\text{cov}}(\tilde{r}_{it}, \tilde{r}_{jt})$ , we obtain the modified expression for representing all active alternative portfolios, which satisfy a given TEV constraint:

$$(9) \quad \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \hat{\text{cov}}(\tilde{r}_{it}, \tilde{r}_{jt}) \cdot (w_{it} - b_{it}) \cdot (w_{jt} - b_{jt}) \leq \psi^2$$

The portfolio opportunity set thus forms a convex polytope in the security weight space  $\mathbb{R}^{n-1}$ .

Given a portfolio opportunity set of the form (9) and a performance measure (e.g. realized differential return, IR), we are interested to quantify how many of the feasible portfolios take on a particular performance value. In other words, we are interested in calculating the frequency distribution of feasible portfolios in terms of values of this performance measure. These frequency distributions (in our case for differential returns and IRs) can be estimated numerically. The procedure is based on statistical sampling: we estimate the distribution of performance values of the whole portfolio opportunity set through the distribution of performance values of a reasonably large sample of active portfolios, which are feasible under a given TEV-constraint.

There exist several universal algorithms that can generate a random variable  $X$  over any bounded region  $Q \subset \mathbb{R}^{n-1}$  (as a portfolio opportunity set of the type (9)) with a given density (in our case uniformly distributed over  $Q$ ). The general idea of all algorithms is to construct an ergodic Markov chain, recursively generating a sequence of points all in  $Q$  whose stationary distribution is uniform. If the initial point of the chain is uniformly distributed over  $Q$  and the transition kernel is symmetric, then the points of the generated sequence will be independent and uniformly distributed over  $Q$  asymptotically. The most known algorithms from the family are the *Hit-and-run* algorithm by Smith [1984] and Bélisle et al. [1993], and the *Sequential Direction* algorithm by Telgen.<sup>9</sup> We adopt the last algorithm to sample random feasible portfolios for a given TEV-constrained investment mandate.

Having  $n$  assets in the opportunity set (for example, 50 stocks constituting the EURO STOXX 50 index) and transforming the set into the form of eq.(9), the sampling procedure is formulated as follows:<sup>10</sup>

1. Choose uniformly a feasible random portfolio  $\mathbf{p}^{(0)}_t = \langle w_1^{(0)}, w_2^{(0)}, \dots, w_{n-1}^{(0)} \rangle$  in a given portfolio opportunity set  $\hat{Q}$  (the simplest solution is to sample uniformly in the neighborhood of the benchmark<sup>11</sup>);

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<sup>9</sup> See Berbee et al. [1987] and Ritov [1989]. In the last reference, the Sequential Direction algorithm is called the *Coordinate Direction* algorithm. Generally, the Sequential Direction algorithm is a slight modification of the Hit-and-Run algorithm proposed by Smith [1984].

<sup>10</sup> For the sake of simplicity we drop in the algorithm the subscript  $t$  denoting the time period, as all portfolios from the sample belong to the same period of time.

<sup>11</sup> A more sophisticated technique for this preprocessing step with polynomial time is discussed in Lovász [1998].

2. Having the last generated portfolio  $\mathbf{p}^{(s)} = \langle w_1^{(s)}, w_2^{(s)}, \dots, w_{n-1}^{(s)} \rangle$  (the subscript  $s = 0, 1, 2, \dots$  denotes the actual step of our ergodic Markov chain), determine the “changing” weight  $w_j^{(s)}$  as the  $j$ -th element of  $\mathbf{p}^{(s)}$  where  $j$  is taken consecutively from the set  $\{1, 2, \dots, n-1\}$ ;
3. Substituting all  $w_i^{(s)}$  but  $w_j^{(s)}$  into the left-hand side of eq.(9) and replacing the inequality sign with equality, we obtain a quadratic equation of only one variable  $w_j^{(s)}$ . Solving the equation, we obtain two boundary values for  $w_j^{(s)}$ ,  $\dot{w}_j^{(s)}$  and  $\ddot{w}_j^{(s)}$ . Compute the next value of  $w_j^{(s+1)}$  by choosing a point on the segment  $[\dot{w}_j^{(s)}, \ddot{w}_j^{(s)}]$  uniformly, i.e.

$$w_j^{(s+1)} \sim \mathcal{U}(\dot{w}_j^{(s)}, \ddot{w}_j^{(s)});$$

4. Create a new random portfolio extending  $\mathbf{p}^{(s+1)} = \langle w_1^{(s+1)}, w_2^{(s+1)}, \dots, w_{n-1}^{(s+1)} \rangle$  with  $w_i^{(s+1)} = w_i^{(s)}$  for all elements but  $w_j^{(s+1)}$ , a new  $w_j^{(s+1)}$  from step 3, and the last depending weight as:

$$\left\langle w_1 = w_1^{(s+1)}, w_2 = w_2^{(s+1)}, \dots, w_{n-1} = w_{n-1}^{(s+1)}, w_n = 1 - \sum_{i=1}^{n-1} w_i^{(s+1)} \right\rangle.$$

Add this new random portfolio to the sample;

5. If the sample is big enough, then stop. Otherwise return to step 2.

Having  $n$  assets in the opportunity set, we need to generate at least  $\mathcal{O}(n^3)$  random portfolios in order to guarantee the uniformity of the sample over a given TEV-constrained opportunity set (cf. Lovász [1998]).

The last but not the least issue is the computational speed of the sampling algorithm. Because we are moving across orthogonal directions by forming a new random portfolio, the portfolios can be generated rather efficiently. Hence, we can use large samples (e.g. one million portfolios) when estimating frequency distributions for TEV-restricted sets. Furthermore, the algorithm can be easily extended to incorporate additional risk constraints as discussed by Jorion [2003] and Alexander [2004]. In that

case, we introduce an additional acceptance-rejection step, which filters out portfolios violating at least one of these supplementary constraints.

#### 4. Data and Investment Mandate

In the illustration of our methodology, we choose the Dow-Jones EURO STOXX 50 Index as the benchmark portfolio  $B$ . This index was introduced on February 28, 1998 and provides a blue-chip representation of industry sectors in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.<sup>12</sup> It contains 50 stocks and captures almost 60% of the free-float market capitalization of the represented countries. The weighting of individual stocks is according to their free-float market capitalization, subject to a 10% weighting cap. The composition is reviewed annually, in September.

We use end-of-month values of the price index to compute monthly discretely compounded returns over the period January 1995 through May 2005 (125 monthly returns). Over the same period we computed monthly price returns on all stocks comprised at any moment in the index.<sup>13,14</sup> This enables us to replicate the index at any time with individual stocks. In addition, we construct an equally-weighted price index,  $EW$ . In each month, the return on this index is computed as the unweighted arithmetic average of the price returns on all stocks comprised at that time in the EURO STOXX 50 Index.

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<sup>12</sup> Index history is available daily back to December 31, 1986. For details, see <http://www.stoxx.com>.

<sup>13</sup> For consistency, we use price returns on both the index and the individual stocks. In our illustrative setting, the exclusion of cash dividends is not important. For actual applications, the use of total returns is appropriate.

<sup>14</sup> When a stock is incorporated in the EURO STOXX 50 Index which does not have at least 36 months of return history, we use a recursive regression approach to fill in missing observations. We start with a window of 36 return observations on the stock  $i$  and the index  $B$ , which begins immediately after the missing return data point for the stock. Within this window, we use OLS regression to estimate the parameters  $\alpha_i$  and  $\beta_i$  of the market model:

$$r_{it} = \alpha_i + \beta_i r_{Bt} + \varepsilon_{it} \quad t = 1, \dots, 36$$

Given these parameters and the available index return for the first observation  $t=0$  before the start of the window, we next construct the stock return, conditional on the index return:

$$r_{it} | r_{Bt} = \hat{\alpha}_i + \hat{\beta}_i r_{Bt} \quad t=0$$

Finally, we randomly sample one residual term from the 36 residuals in the estimation window and add that to  $r_{i0} | r_{B0}$ . We substitute this return for the missing return  $r_{i0}$ . We repeat the procedure by moving the data window backwards in time until 36 months of history is generated. We implemented this procedure for 6 stocks out of 66.

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insert Figure 1 and Table 1 about here

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Figure 1 shows the course of the indexes over the complete sample period and Table 1, Panel A, provides some descriptive return statistics. From January 1995, the EURO STOXX 50 Index shows a gradual increase until it reaches a peak in April 2000, on the way up only fiercely interrupted by the August-September 1998 crash, following the Russian debt crisis. Then a gradual but steady descend sets in until March 2003, from whereon the index slowly recovers. The benchmark return has an arithmetic average of about 10% p.a., with a volatility of 20% p.a. Note that the equally-weighted index has both a higher average return and a higher standard deviation. However, the 100 bps higher standard deviation does not seem to outweigh the 300 bps higher average return, and the equally-weighted portfolio clearly outperforms the benchmark portfolio over the sample period.

One explanation for this outperformance could be the small-cap effect. The benchmark portfolio is value-weighted (under restrictions on the maximum weights) and is hence biased towards large cap stocks. The equally-weighted portfolio, in contrast, is biased towards small cap stocks. When small cap stocks outperform large cap stocks, the equally-weighted portfolio outperforms the value-weighted portfolio. An alternative (and perhaps additional) reason for outperformance is the “volatility pumping” effect.<sup>15</sup> The value-weighted portfolio is a buy-and-hold portfolio (at least between revision moments, which is once a year). The equally-weighted portfolio, however, is rebalanced every month. This implies that stocks that have performed well against the portfolio are sold and stocks that have underperformed the index are bought, in order to keep the portfolio equally-weighted. When the returns on the individual stocks fluctuate within a range, the “buy low, sell high” strategy generates outperformance with respect to a buy-and-hold portfolio.

Given the outperformance of the equally-weighted index over the benchmark portfolio, we synthetically construct an investment portfolio as a linear combination of the benchmark and the equally-weighted portfolio. The return on this managed portfolio  $P$  in month  $t$  is computed as:

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<sup>15</sup> See for example Luenberger [1998, pp.422ff].

$$(10) \quad \tilde{r}_{Pt} = (1 - \lambda) \tilde{r}_{Bt} + \lambda \tilde{r}_{EWt}$$

where  $\tilde{r}_{EWt}$  is the return on the equally-weighted portfolio and the weight  $\lambda$  is a constant. Hence, the tracking error of portfolio  $P$  is:

$$(11) \quad \tilde{r}_{Pt} - \tilde{r}_{Bt} = \lambda (\tilde{r}_{EWt} - \tilde{r}_{Bt})$$

and its tracking error volatility, TEV, is:

$$(12) \quad TEV = |\lambda| \sqrt{\text{var}(\tilde{r}_{EWt} - \tilde{r}_{Bt})} = \lambda \sigma_{EW-B}$$

where  $\sigma_{EW-B}$  is the standard deviation of the return differential between the equally-weighted portfolio and the benchmark portfolio, and where  $\lambda > 0$  by construction.

But how do we set the appropriate value for the weight  $\lambda$ ? We assume that the investment mandate sets an upper bound of  $\psi$  on the TEV:

$$(13) \quad TEV \leq \psi$$

Given  $\sigma_{EW-B}$  and using eq.(12), we obtain the composition of portfolio  $P$  that exhibits the maximum allowed TEV by setting the weight  $\lambda$  in eq.(10) as:

$$(14) \quad \lambda = \frac{\psi}{\sigma_{EW-B}}$$

In practice, the mandate will specify the upper bound on TEV on an annual basis. In our illustration, we work with monthly returns. When returns are intertemporally uncorrelated and volatility is constant (at least over a year), a TEV constraint of  $x\%$  per annum translates into a monthly allowed TEV of  $x / \sqrt{12} \%$ . In our illustration, we choose

$\psi=1.15\%$  per month, which is derived from a maximum allowed TEV of 4% per annum.

To replicate a feasible investment strategy, we generate out-of-sample returns on the managed portfolio  $P$ . We use the first 36 months of data in our sample to estimate the standard deviation of the return differential between the equally-weighted portfolio and the benchmark,  $\sigma_{EW-B}$ . Using the TEV constraint and eq.(14), we infer the weight  $\lambda$  and compute the return on the investment portfolio for month 37 according to eq.(10). Next we shift the estimation window one month forward and repeat the procedure to obtain the return on the investment portfolio for month 38, and so on. At the end of our sample period, we finally have 89 out-of-sample returns on portfolio  $P$  over the period January 1998 through May 2005. These returns are generated by a portfolio that *ex ante* exactly satisfies the TEV constraint. From an *ex ante* perspective, i.e. on the basis of the trailing 36 months windows that are used to compose the managed portfolio, the annualized projected IR ranges from 0.02 to 1.76, with an average value of 0.80.<sup>16</sup>

Table 1, Panel B, shows the descriptive statistics of this investment portfolio and its tracking error. The managed portfolio  $P$  shows an average realized return of 7.73% p.a. (with standard deviation of almost 23% p.a., which is 1.2% higher than the benchmark volatility) and it outperforms the benchmark on average by 2.72% p.a. The realized TEV is 4.46% which is almost 50 bps higher than the mandated 4%. The *ex ante* estimated TEV thus seems to underestimate the *ex post* TEV. Over the total out-of-sample period January 1998 through May 2005, the realized mean and standard deviation of the tracking error yield an annualized IR of 0.61. Compared to the *ex ante* IR, this shows the effect of the underestimated risk on the portfolio's *ex post* performance.

In section 5, we analyze the performance of the constructed investment portfolio  $P$  in more detail. More specifically, we use the portfolio opportunity set approach to evaluate this investment portfolio *vis à vis* all feasible portfolios under the formulated investment mandate.

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<sup>16</sup> Plugging eqs.(11) and (14) in (5) gives  $IR = E(\tilde{r}_{EWt} - \tilde{r}_{Bt}) / \sigma_{EW-B}$ , which does not depend on the TEV constraint.



## 5. Empirical Results

We use the procedure as outlined in section 3 to generate portfolio opportunity sets. Each random investment portfolio satisfies *ex ante facto* the maximum allowed TEV of 4% p.a. (i.e.  $\psi=1.15\%$  per month), just as the constructed portfolio  $P$  under the mandate. Since we use a trailing window of 36 monthly observations to estimate the portfolios' TEVs, we generate a portfolio opportunity set for each of the 89 months from January 1998 through May 2005. Each portfolio opportunity set contains 50,000 random portfolios.

In this section, we first illustrate how the portfolio opportunity set approach can be used to analyze tracking error dynamics over time. This allows us to evaluate the tracking error relative to the tracking errors of all portfolios that could have been formed under the same mandate. We then show how one can use this information to identify cases where the portfolio manager, given some level of confidence, has exceeded the TEV constraint on an *ex ante* basis. Finally, we turn to risk-adjusted performance evaluation and first show how to put the IR of the managed portfolio in a relative perspective. We then suggest a normalization of the IR to purge risk-adjusted performance from market dynamics which determine the range of *ex ante* attainable IRs.

### tracking error dynamics

For each portfolio opportunity set, we compute the frequency distribution of the tracking errors with respect to the benchmark return over all 50,000 portfolios. For clarity of presentation, we summarize the frequency distributions by five quantiles: 2.5%, 25%, 50%, 75% and 97.5%.

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insert Figure 2 about here  
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Figure 2 shows the quantiles of the tracking error distributions over time. From the graph, we not only observe that the median out-of-sample tracking error fluctuates widely over time, also the range of potentially attainable tracking errors under the mandate varies from narrow to broad. In September 1998, for example, following the

Russian debt crisis, the difference in annualized tracking error between the 25% and 75% quantile portfolios in the portfolio opportunity set is a staggering 34% p.a. Also in July through November 2002, the range of annualized realized tracking errors of the middle 50% of feasible portfolios is about 25% p.a. The distributions narrow, notably in the last year of the sample period. In May and June 2004, for example, the difference in annualized tracking error between the top 25% and bottom 25% portfolios in the portfolio opportunity set is reduced to about 7% p.a. Figure 2 also shows that in November 1999 and January 2000, most feasible portfolios realized a negative tracking error under the mandate. In January 2001 and especially November 2002, in contrast, most of the attainable portfolios outperformed the benchmark. In these months, a positive tracking error is not due to the specific ability of the portfolio manager, but instead induced by favorable market conditions.

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 insert Figure 3 about here  
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It is interesting to see how our constructed investment portfolio  $P$  performs relative to the tracking error distributions over time. Not only the *locus* of our investment portfolio in the frequency distributions of tracking errors provides useful information, also the *width* of these distributions is relevant in evaluating the managed portfolio. When in some month the tracking error distribution is narrow, it was difficult for the portfolio manager to realize a large (positive) tracking error. Conversely, when this distribution is broad, a large tracking error was relatively easy to obtain. In Figure 3, we substituted the tracking error of our managed portfolio  $P$  for the median value. This allows us to evaluate the tracking errors of  $P$  relative to the top 25% and bottom 25% of generated portfolios. It clearly shows that in the months January 1998, September 1999, March 2000 and February 2001, our investment portfolio outperformed more than 75% of the feasible portfolios under the mandate. However, the true performance can only be gauged on a risk-adjusted basis. Before discussing this issue, we first show how the portfolio opportunity set approach can be used to infer (on a confidence level basis) whether the portfolio manager has obeyed the TEV constraint as specified in the mandate.

## TEV violation diagnostics

For the investor who has provided the mandate to a portfolio manager, an interesting question is whether the investment portfolio  $P$  satisfies the formulated TEV constraint. On an *ex post* basis, this can be checked by estimating the TEV of realized portfolio returns. The obvious drawback of this procedure is that in order to estimate the realized TEV with some reliability, one would have to wait to collect a reasonable number of return observations (for example 36 monthly observations). In addition, the realized volatility can be different from the projected volatility and the investor who provides the mandate can choose not to blame the portfolio manager for these volatility surprises which affect the whole market.

More interesting, then, is the question whether the investment portfolio  $P$  satisfies the formulated TEV constraint on an *ex ante* basis. In other words: when constructing his portfolio, did the portfolio manager take the TEV constraint into account?<sup>17</sup> Without explicit information on the composition of the managed portfolio, the portfolio opportunity set approach allows for checking whether the tracking error of the managed portfolio falls reasonably well within the frequency distribution of tracking errors. After all, when the tracking error of the managed portfolio is outside the range of tracking errors realized by  $q\%$  of the portfolios formed under the same mandate, it is likely – at a confidence level of  $q\%$  – that the TEV constraint was violated *ex ante facto*. For example, when in a specific month 99% of the random portfolios under the mandate show a smaller tracking error than the managed portfolio, it is very likely that the portfolio manager has violated the mandate. Note that since the portfolio opportunity set is generated under the same *ex ante* TEV constraint, but the *realized* tracking errors of all generated portfolios are subject to volatility surprises, this check on potential TEV violation is insensitive to changes in volatility.

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insert Figure 4 about here  
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In Figure 4, we highlight the frequency distributions of realized tracking errors for the months January 1998 and January 2000. Our managed portfolio is composed under a

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<sup>17</sup> Of course, the specific method employed to estimate the TEV should be contained in the mandate.

TEV constraint of 4% p.a. In addition, we consider two alternative managed portfolios  $P'$  and  $P''$  which are constructed according to the same recipe as our portfolio  $P$ , but for which the *ex ante* TEV constraint is relaxed to 5% and 6%, respectively. In January 1998, our portfolio  $P$  realized a positive tracking error of 3.37% (monthly basis). This implies an outstanding performance since 93.45% of the portfolios in the portfolio opportunity set realized a lower tracking error under the mandate. Portfolio  $P'$  has realized a tracking error of 4.21% and outperformed 98.60% of the portfolios in the portfolio opportunity set. Portfolio  $P''$  even outperformed 99.77% of the portfolios. These quantiles are extremely high and cast doubt on the presumption that these portfolios, just as portfolio  $P$ , are formed under a TEV constraint of 4%. Instead of attributing outstanding performance to the portfolios  $P'$  and  $P''$ , we would instead conclude with high confidence that these portfolios were composed under a relaxed mandate (which in this case is true by construction). The high performance of these portfolios is thus very likely due to a higher *ex ante* active risk level. Switching to January 2000, the case is less clear for portfolio  $P'$  since it underperformed about 89% of the portfolios. However, portfolio  $P''$  is outperformed by 98.25% of the portfolios and this again signals that the portfolio manager has very likely exceeded the TEV constraint.

### **a relative perspective on risk-adjusted performance**

The risk-adjusted performance of a managed portfolio will be evaluated by means of the IR. We here focus on expected risk-adjusted performance of the managed portfolio  $P$ .

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insert Figure 5 about here  
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For all portfolios in the portfolio opportunity sets, we calculated the *ex ante* IR, based on tracking errors from a preceding 36 months trailing data window. Figure 5 shows the quantiles of the distributions of IRs over time. Parallel to Figure 2, the median and range of IRs fluctuate over time. Depending on market dynamics, in some months it is clearly easier to achieve a high IR than in other months.

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insert Figure 6 about here  
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But how does our investment portfolio  $P$  fare? Figure 6 plots the *ex ante* monthly IR of our portfolio over time. In the first two years, the IR is quite high, about 0.4. Then the projected risk-adjusted performance drops sharply, where after the performance recovers from January 2003 on (monthly IR of about 0.25). On the basis of this information, and except for the years 2000 through 2002, the projected performance of the portfolio manager looks quite well.

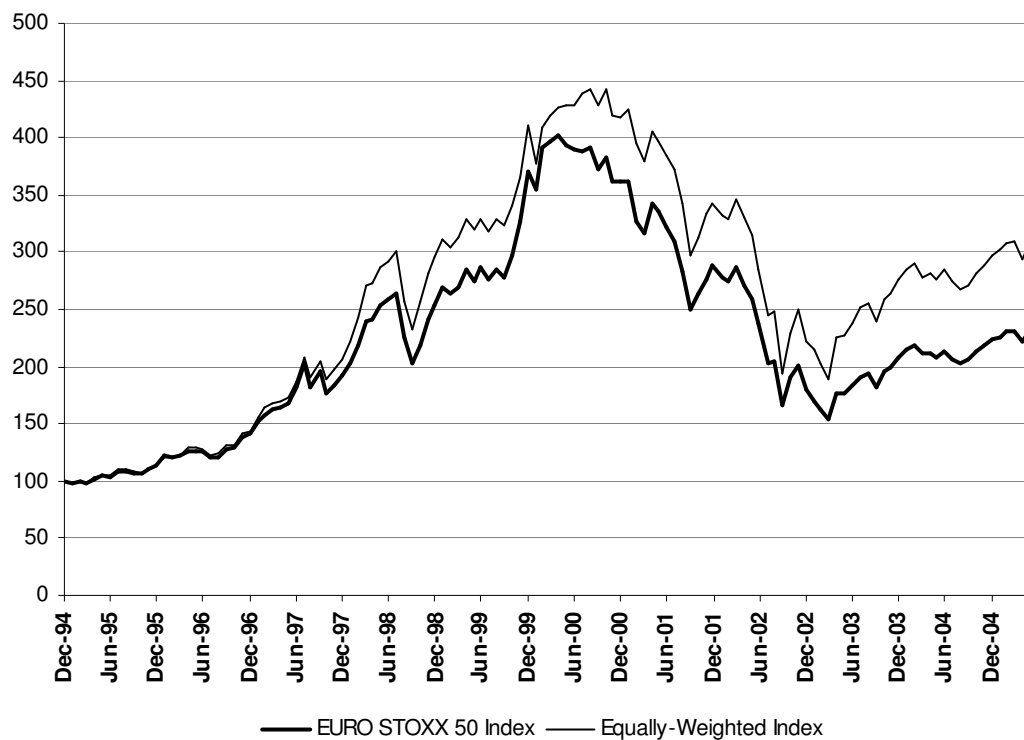
Next we evaluate the projected performance of our portfolio relative to all alternative feasible portfolios. One way to do this is to plot the IRs of our portfolio in the distributions of IRs over the portfolio opportunity sets in Figure 6. For space considerations, we do not show this graph. Instead we normalize the IR of our portfolio with respect to the distributions of IRs over the portfolio opportunity sets. We compute the normalized IR by first subtracting the median information ratio from the corresponding opportunity set and then dividing by the standard deviation of information ratios (calculated over the cross-section of all feasible portfolios). This relative perspective radically changes our perception of the portfolio manager's projected performance. Figure 6 reveals that during the years 1998 and 1999, our portfolio manager expected to outperform the median feasible portfolio. This is in line with the high level of the non-normalized IRs. However, during the next three years (2000-2002) we see that the projected IR is not only low in absolute value, but also low when compared to the IRs of alternative feasible portfolios: most portfolios in the portfolio opportunity set outperform the managed portfolio on an *ex ante* basis. And finally, during the last three and a half years, during which the performance seems to be restored when judged by the IRs, the normalized IRs deteriorate even further. The vast majority of alternative portfolios have a projected IR that is way above the IR of our constructed portfolio. The managed portfolio's *ex ante* IR ranges well outside the two-standard deviation interval of IRs. This is in flagrant contrast with the information provided by the non-normalized IR. We can only conclude that relative to the level playing field of portfolio opportunities, our hypothetical portfolio manager performs very poorly... We therefore advice to evaluate the performance of mandated portfolios from a relative perspective: not only relative to the targeted benchmark, but also relative the corresponding portfolio opportunity set.

## 6. Summary & Conclusions

In this paper, we argue that mandated portfolios should not only be evaluated relative to their benchmarks, but also relative to the portfolio opportunity set. The portfolio opportunity set comprises all portfolios that satisfy the constraints as specified in the investment mandate. This portfolio opportunity set can be used to construct frequency distributions of (risk-adjusted) performance values such as realized tracking errors and information ratios. This allows for evaluating the relative position of the investment portfolio in the portfolio opportunity set. Both the *locus* of the investment portfolio in the frequency distribution of performance values, as the *width* of this distribution provides valuable information for evaluating relative investment performance. To purge the information ratio for the influence of market dynamics over time, we suggest normalizing this performance metric. In addition, we show how the portfolio opportunity set approach can be used for monitoring *ex ante* tracking error volatility constraints. We illustrated our methodology for a fictitious investment portfolio where the EURO STOXX 50 Index served as a benchmark.

We showed only some applications of the portfolio opportunity set approach. In this paper, we focused on the relative performance evaluation of a mandated portfolio. Normalizing the information ratio with respect to the portfolio opportunity set revealed the clear inferiority of the strategy followed by our fictitious portfolio manager when compared to all strategies feasible under the mandate. This suggests an interesting route for further research: to use the portfolio opportunity set approach to *ex ante* evaluate and select alternative investment strategies.

**Figure 1: The EURO STOXX 50 Index (benchmark) and the equally-weighted index over time. January 1, 1995 (=100) through May 31, 2005.**



**Table 1 Descriptive statistics.**

Mean and standard deviation (st.dev.) of monthly discretely compounded price returns, expressed in percent per annum. Means and standard deviations are annualized by multiplying monthly figures with 12 and  $\sqrt{12}$ , respectively.

**Panel A: The benchmark index  $B$ , the equally-weighted index  $EW$ , and the return difference  $EW-B$ .**

The benchmark  $B$  is the EURO STOXX 50 Index,  $EW$  is the equally-weighted portfolio, and  $EW-B$  indicates the return differential between the benchmark and the equally-weighted portfolio. The sample period is January 1995 through May 2005 (125 monthly observations, annualized statistics).

% p.a.	$B$	$EW$	$EW - B$
mean	10.19	13.15	2.96
st.dev.	20.23	21.32	4.25

**Panel B: The benchmark index, the investment portfolio, and the tracking error.**

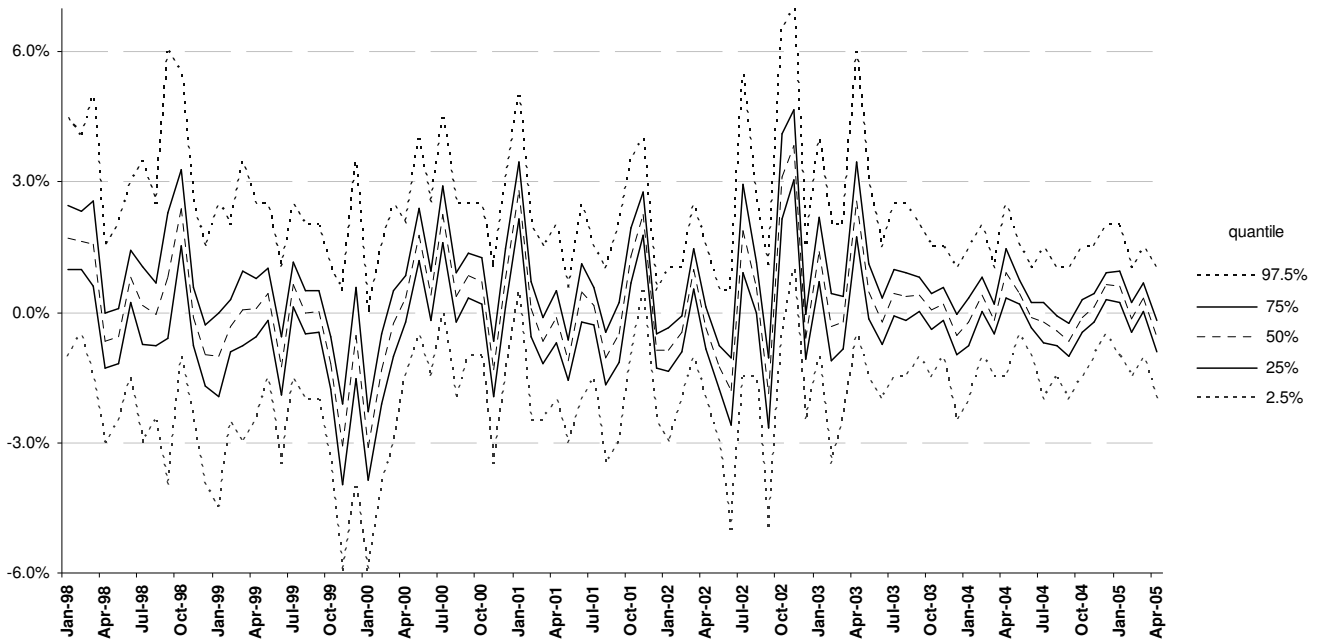
The investment portfolio  $P$  is formed with *ex ante* tracking error volatility of 4% p.a., based on a window of the preceding 36 months. The benchmark  $B$  is the EURO STOXX 50 Index and  $P-B$  indicates the tracking error. Out-of-sample returns over the period January 1998 through May 2005 (89 monthly observations, annualized statistics).

% p.a.	$B$	$P$	$P - B$
mean	5.01	7.73	2.72
st.dev.	21.80	22.99	4.46



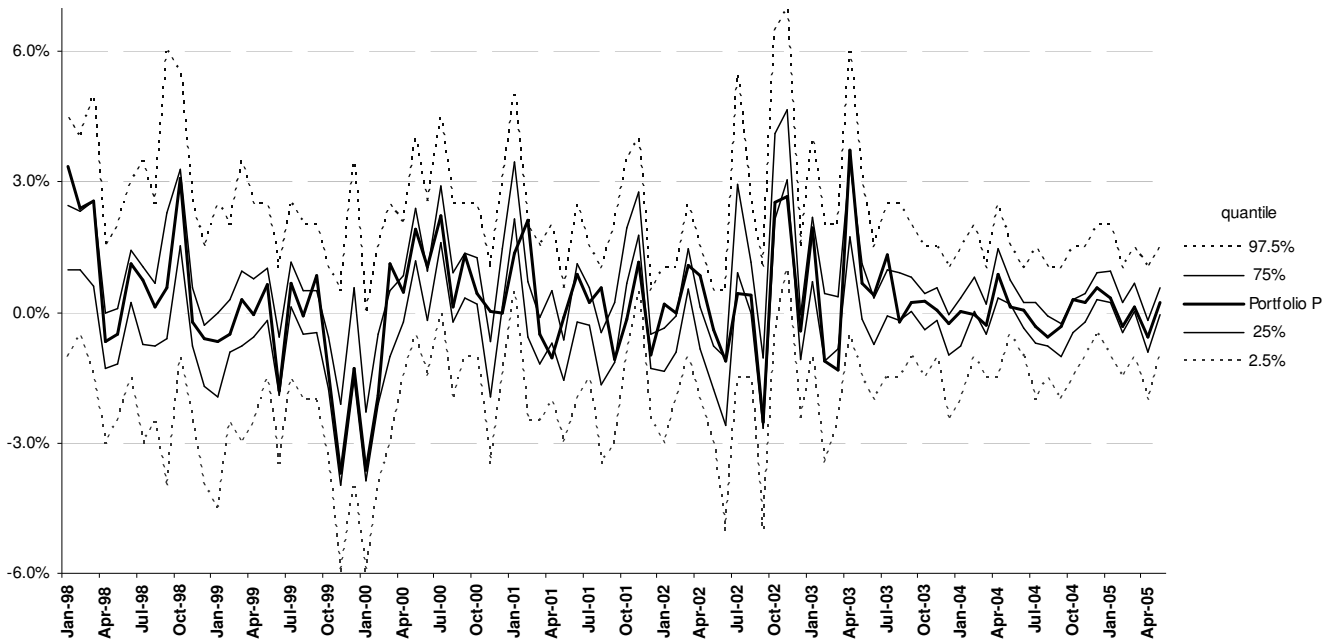
**Figure 2: The distributions of tracking errors over portfolio opportunity sets, summarized by quantiles.**

All portfolios satisfy a TEV constraint of 4% p.a. (estimated *ex ante* from a 36 months trailing data window). Out-of-sample tracking errors in percent per month with respect to the EURO STOXX 50 Index over the period January 1998 through May 2005.



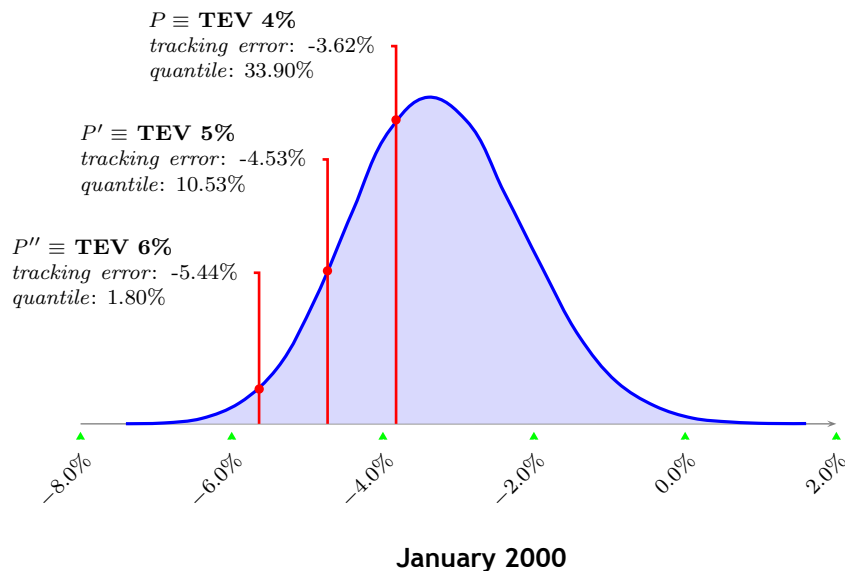
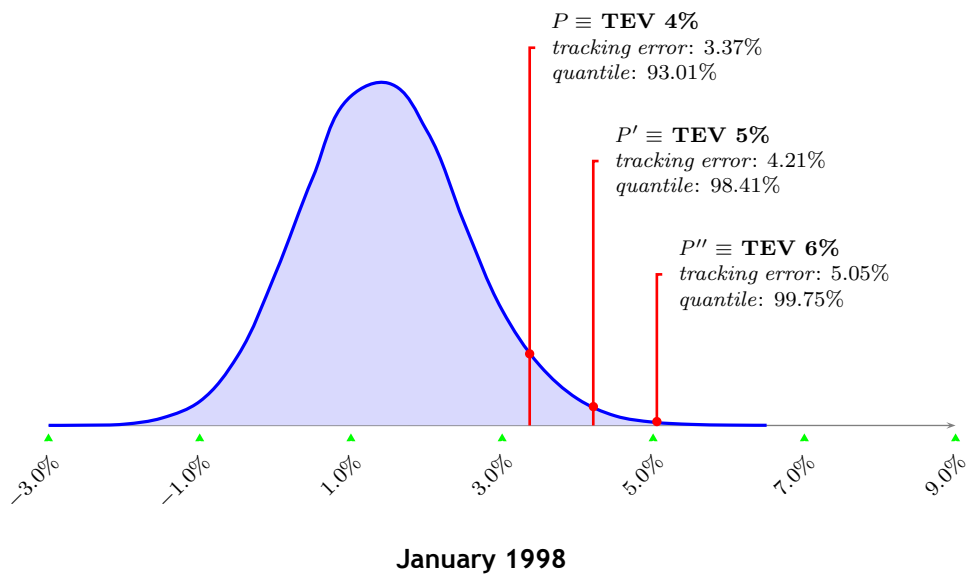
**Figure 3: The locations of portfolio *P* in the distributions of tracking errors over portfolio opportunity sets, summarized by quantiles.**

All portfolios satisfy a TEV constraint of 4% p.a. (estimated *ex ante* from a 36 months trailing data window). Out-of-sample tracking errors in percent per month with respect to the EURO STOXX 50 Index over the period January 1998 through May 2005.



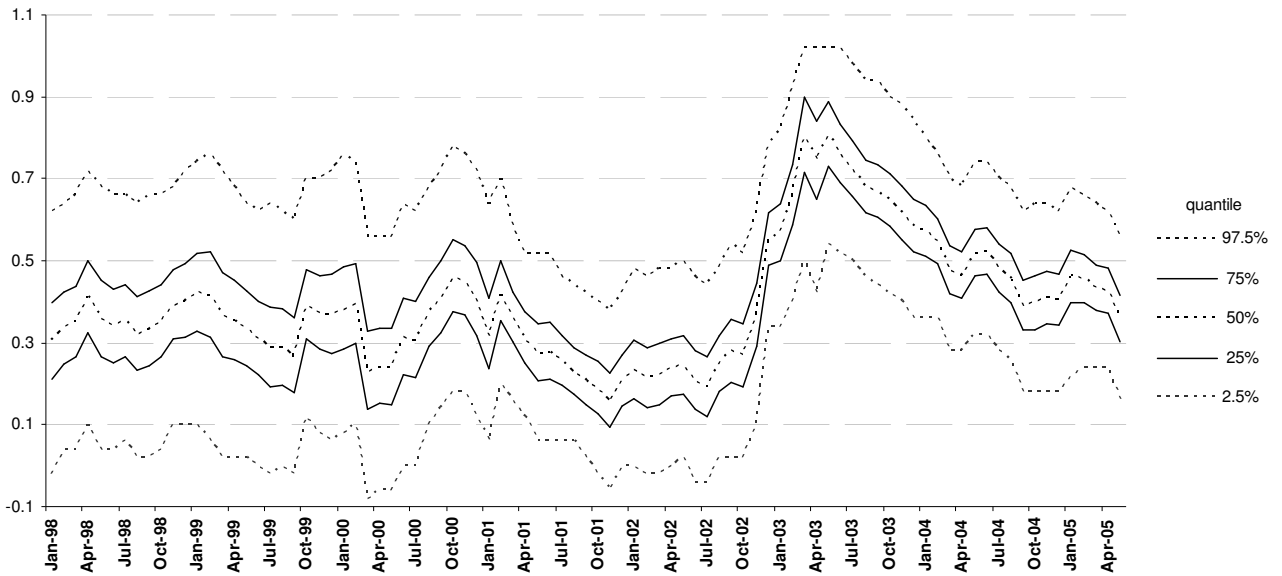
**Figure 4: Frequency distributions of realized tracking error, calculated over the portfolio opportunity sets for the months January 1998 and January 2000.**

The frequency distribution is constructed from monthly tracking errors from a portfolio opportunity set under a TEV constraint of 4% p.a. (estimated over the preceding 36 months). Our managed portfolio P is also composed under the same mandate (indicated by TEV 4%). In addition, we list the realized tracking error of our investment portfolio where the TEV constraint is relaxed to 5% ( $P'$ , TEV 5%) and 6% ( $P''$ , TEV 6%). In each case, we list the actual tracking error and the quantile (i.e. the percentage of portfolios in the opportunity set that realized a smaller tracking error in that month).



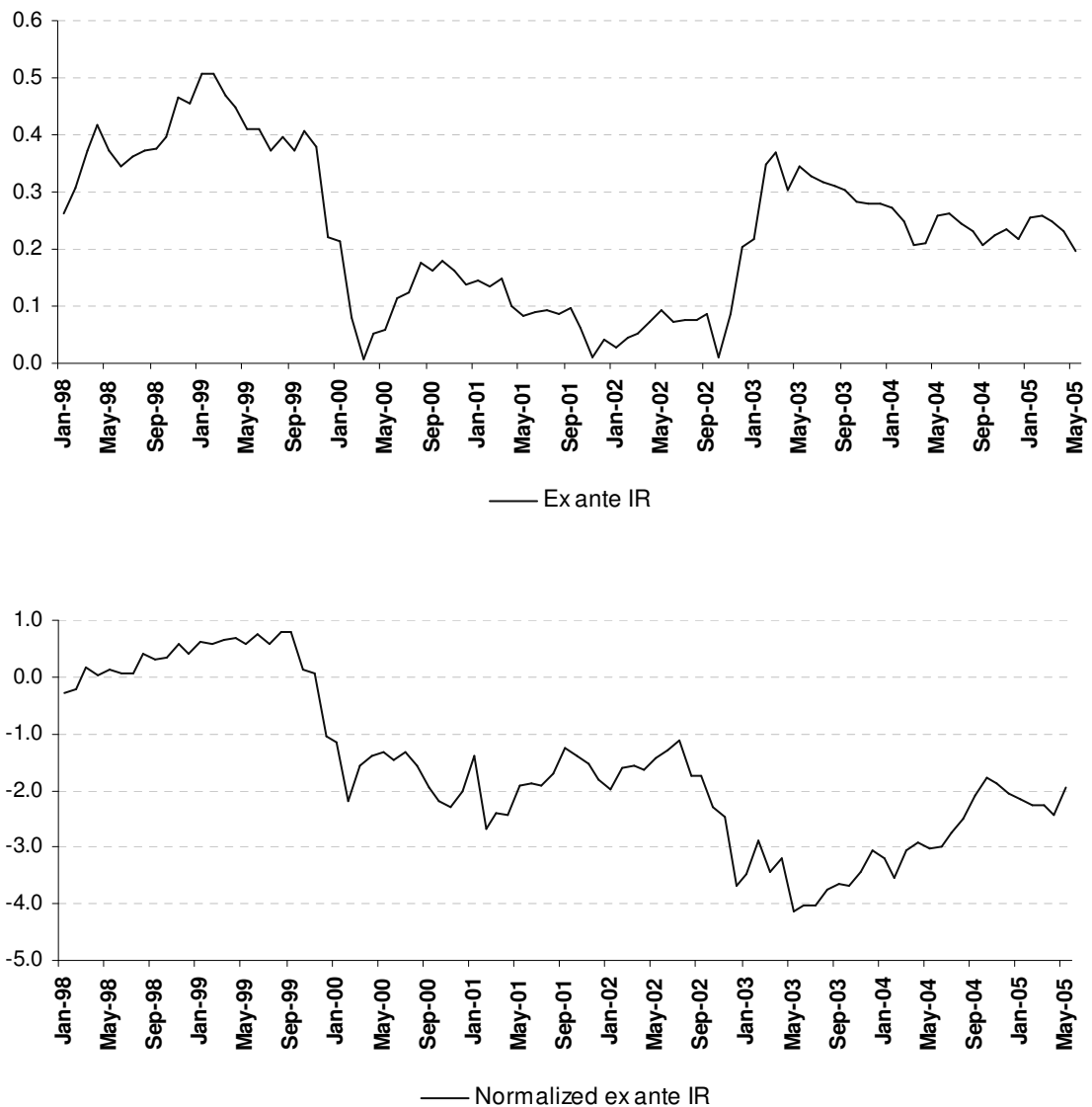
**Figure 5: The distributions of information ratios over portfolio opportunity sets, summarized by quartiles.**

All portfolios satisfy an *ex ante* TEV constraint of 4% p.a. Shown are *ex ante* information ratios with respect to the EURO STOXX 50 Index over the period January 1998 through May 2005. All information ratios are estimated *ex ante* from a 36 months trailing data window.



**Figure 6: The *ex ante* information ratio and normalized *ex ante* information ratio of the managed portfolio *P*.**

The investment portfolio *P* satisfies an *ex ante* TEV constraint of 4% p.a. Shown are *ex ante* monthly information ratios with respect to the EURO STOXX 50 Index over the period January 1998 through May 2005. All information ratios are on a monthly basis, estimated *ex ante* from a 36 months trailing data window. The normalized information ratio is computed by first subtracting the median information ratio from the corresponding opportunity set and then dividing by the standard deviation of information ratios over the portfolio opportunity set.



## References

- ADAMS, J.C. & K.B. CYREE (2004) "Market Efficiency and Diversification: An Experiential Approach Using the *Wall Street Journal*'s Dartboard Portfolio". *Journal of Applied Finance*, Fall/Winter, pp.40-51.
- ALEXANDER, G.J. & A. BAPTISTA (2004) "Active Portfolio Management with Benchmarking: Adding a Value-at-Risk Constraint". Working paper, University of Minnesota, available for download at <http://www.gloriamundi.org/picsresources/gaab3.pdf>.
- AMMANN, M. & H. ZIMMERMANN (2001) "The Relation Between Tracking Error and Tactical Asset Management". *Financial Analysts Journal*, Vol.57, Iss.2 (March/April), pp. 32-43.
- BÉLISLE, C.J.P., H.E. ROMEIJN & R.L. SMITH (1993) "Hit-and-Run Algorithms for Generating Multivariate Distributions". *Mathematics of Operation Research*, Vol.18, Iss.1, pp. 255-266.
- BERBEE, H.C.P., C.G.E. BOENDER, A.H.G. RINNOY KAN, C.L. SCHEFFER, R.L. SMITH & J. TELGEN (1987) "Hit-and-Run Algorithms for the Identification of Nonredundant Linear Inequalities". *Mathematical Programming*, Vol.37, Iss.1, pp. 184-207.
- BURMEISTER, C., H. MAUSSER & R. MENDOZA (2005) "Actively Managing Tracking Error". *Journal of Asset Management*, Vol.5, Iss.6, pp. 410-422.
- COHEN, K.J. & B.P. FITCH (1966) "The Average Investment Performance Index". *Management Science*, Vol.12, Iss.6, Series B, pp. B195-B215.
- FISHER, L. (1965) "Outcomes for 'Random' Investments in Common Stocks listed on the New York Stock Exchange". *Journal of Business*, Vol.38, Iss.2, pp. 149-161.
- FISHER, L. & J.H. LORIE (1970) "Some Studies of Variability of Returns on Investments in Common Stocks". *Journal of Business*, Vol.43, Iss.2, pp. 99-134.
- GOODWIN, TH.H. (1998) "The Information Ratio". *Financial Analysts Journal*, Vol.54, Iss.4 (July-August), pp. 34-43.
- GRINOLD, R.C. & R. KAHN (2000) *Active Portfolio Management*, 2<sup>nd</sup> edition. McGraw-Hill Trade, New York NY.
- HALLERBACH, W., C. HUNDACK, I. POUCHKAREV, & J. SPRONK (2005) "Market Dynamics from the Portfolio Opportunity Perspective: the DAX<sup>®</sup> Case". *Zeitschrift für Betriebswirtschaft*, Vol.75, Iss.7/8, July-August, pp.739-764.
- JORION, PH. (2003) "Portfolio Optimization with Tracking-Error Constraints". *Financial Analysts Journal*, Vol.59, Iss.5 (Sep/Oct), pp. 70-82.
- LOVÁSZ, L. (1998) "Hit-and-Run Mixes Fast". *Mathematical Programming*, Vol.86, Iss.6, pp. 443-461.

- LUENBERGER, D.G. (1998) *Investment Science*, 2<sup>nd</sup> edition. Oxford University Press, New York NY.
- MOY, R.L. (1994) "Pros, Darts, and the Dow: Portfolio Lessons From *The Wall Street Journal*". *Financial Practice & Education*, Fall/Winter, pp. 144-147.
- POUCHKAREV, I. (2005) *A General Framework for the Evaluation of Constrained Portfolio Performance*, PhD Thesis, Erasmus University Rotterdam
- REILLY, F.K. (1994) *Investment Analysis and Portfolio Management*. The Dryden Press, Fort Worth TX.
- RENNIE, E.P. & TH.J. COWHEY (1990) "The Successful Use of Benchmark Portfolios: A Case Study". *Financial Analysts Journal*, Vol.46, Iss.5 (Sep/Oct), pp. 18-26.
- RITOV, Y. (1989) "Monte Carlo Computation of the Mean of a Function with Convex Support". *Computational Statistics & Data Analysis*, Vol.7, Iss.3, pp. 269-277.
- ROHWEDER, H.C. (1998) "Implementing Stock Selection Ideas: Does Tracking Error Optimization Any Good?". *The Journal of Portfolio Management*, Spring, pp. 49-59.
- ROLL, R. (1992) "A Mean/Variance Analysis of Tracking Error". *The Journal of Portfolio Management*, Summer, pp. 13-22.
- RUDOLPH, M, H.-J. WOLTER & H. ZIMMERMANN (1999) "A Linear Model for Tracking Error Minimization". *Journal of Banking & Finance*, Vol.23, Iss.1, pp. 85-103.
- SHARPE, W.F. (1966) "Mutual Fund Performance". *Journal of Business*, January, pp. 119-138.
- SHARPE, W.F. (1994) "The Sharpe Ratio". *The Journal of Portfolio Management*, Fall, pp. 49-58.
- SMITH, R.L. (1984) "Efficient Monte Carlo Procedures for Generating Points Uniformly Distributed over Bounded Regions". *Operations Research*, Vol.36, Iss.2, pp. 1296-1302.

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