

## Online rules for container stacking

Bram Borgman · Eelco van Asperen ·  
Rommert Dekker

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**Abstract** Container stacking rules are an important factor in container terminal efficiency. In this paper, we investigate two concepts to increase efficiency and compare them to several benchmark algorithms, using a discrete-event simulation tool. The first concept is to use knowledge about container departure times, in order to limit the number of reshuffles. We stack containers leaving shortly before each other on top of each other. The second concept is the trade-off between stacking further away in the terminal versus stacking close to the exit points and accepting more reshuffles. It is concluded that even the use of imperfect or imprecise departure time information leads to significant improvements in efficiency. Minimizing the difference in departure times proved to be important. It was also found that the trade-off between stacking further away in the terminal versus stacking close by the exit points and accepting more reshuffles leads to improvements over the benchmark.

**Keywords** Container stacking · Marine terminals · Simulation · Container rehandling

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B. Borgman · R. Dekker (✉)  
Erasmus School of Economics, Erasmus University Rotterdam, Rotterdam, The Netherlands  
e-mail: rdekker@ese.eur.nl

B. Borgman  
e-mail: borgman.bram@gmail.com

E. van Asperen  
Center for Maritime Economics and Logistics, Erasmus University Rotterdam,  
Rotterdam, The Netherlands  
e-mail: vanasperen@ese.eur.nl

## 1 Introduction

One of the main problems in container terminals concerns the stacking of containers. Although it is also one of the main advantages of containers, namely, that they can be stacked on top of each other, additional work is required if the bottom container is needed. In that case the top containers have to be moved to another place, which is called a reshuffle or unproductive move.

Accordingly, every terminal needs a stacking strategy. The main objectives of such a strategy are: (1) the efficient use of storage space, (2) limiting transportation time from quay to stack and beyond, (and vice versa), and (3) the avoidance of reshuffles. Of course, the importance of each criterion depends from terminal to terminal. Ports like Singapore and Hong Kong have limited land space, so they need efficiently used storage spaces. Note also that these objectives are conflicting: you cannot maximize them all. For example, the third objective would be optimized by having stacks of only one container high; however, this would lead to very inefficient use of storage space.

In stacking containers, various decision horizons can be identified. An often used classification has four temporal categories: long term (years, decades), medium term (months), short term (days), and real time (minutes, seconds).

Long-term decisions are strategic decisions, e.g., decisions concerning the type of equipment (automated, manual), the stacking height, and the location and size of the stacking area.

Medium-term (or tactical) decisions concern capacity decisions, such as stack layout, number of vehicles used to move containers about and whether or not (and how) to do remarkshalling (i.e., performing reshuffles) in the yard when no ships are being served.

Short-term (or operational) decisions concern finding the storage location for a particular container and the allocation of the equipment to the various jobs scheduled in the coming hours.

Real-time decisions are decisions made when actually executing whatever part of the stacking process. It includes the speed and direction control of all vehicles, as well as the cranes, and is hence mostly of importance to automated equipment.

This paper focuses primarily on the short-term decision to allocate an incoming container to a stacking position. We consider an import container terminal with a high uncertainty regarding the departure time of the containers, such as terminals in Europe (Hamburg–Le Havre range) and the USA. We try to mimic the most common situation where imperfect or imprecise information about the departure time of a container is available. Moreover, we consider online stacking rules, which do not require extensive computations and can be used in many types of stacks and for large numbers of containers. We concentrate on the trade-off between traveling and finding a position which limits the likelihood of reshuffles. We use a quite realistic simulation program to test our ideas. As benchmark we take both random stacking policies as well as policies which use precise information on the container departure times. We do not consider different sizes of container in this study as that only complicates the research.

The paper is structured as follows. We start with giving an overview of existing literature on stacking in Sect. 2. Next we explain the set-up of our simulation model in Sect. 3. The basic stacking concepts are explained in Sect. 4. The experimental set-up

is presented in Sect. 5, while Sect. 6 presents our benchmark algorithms. The results from the experiments are given in Sect. 7 and we finish the paper with conclusions in Sect. 8.

## 2 Literature review

Academic literature on stacking problems is not very common yet, perhaps because the problem does not easily lend itself to analytical solutions (Dekker et al. 2006). However, in recent years, the subject seems to get more attention, because its importance is recognized (Steenken et al. 2004). In a recent overview paper on operations research at container terminals, Stahlbock and Voss (2008) looked at a number of aspects of container terminal operations. Among the topics surveyed were stowage planning, berth allocation, crane optimization, terminal transport optimization, and storage and stacking logistics. Their work is an extension of an earlier overview (Steenken et al. 2004), which also contains a paragraph on how stacking is done in practice.

Various methods are used to tackle the stacking problem, but two main approaches can be distinguished like in job scheduling (Dekker et al. 2006). Analytical calculations with full information on the moment a container will be retrieved from the stack. They are often based on integer programming and take relatively much computation time. Next there are detailed simulation studies which evaluate various stacking strategies. These strategies can be online, in which they determine for each container separately where to place it independent of other incoming containers and offline, where locations are found simultaneously for all containers to be offloaded from a ship. So far only online rule-based strategies have been studied in simulation studies. These rules can handle imperfect or imprecise information on departure times of containers. Their study takes a lot of time and the results may be dependent on the simulation set-up. Dekker et al. (2006) distinguish two types of stacking strategies: category stacking and residence time stacking. The former strategy assumes that containers of the same category (e.g., having the same size, destination, weight, etc.) are interchangeable, and can thus be stacked on top of each other without the risk a lower container in a stack is needed before the ones on top of it have been removed. The latter strategy does not use categories, but instead looks at the departure times of the containers: a container can only be stacked on top of containers that all have a (planned) departure time that is later than the departure time of the new container.

Recent examples of the analytical approach include Kim and Hong (2006), who use a branch-and-bound method to find an optimal solution to a stacking problem and then propose several heuristics to try to come close to the optimum, and Kang et al. (2006), who use simulated annealing to find good solutions reasonably fast. Caserta et al. (2010) combine metaheuristics and dynamic programming to improve upon the known results of Kim and Hong (2006). The problem most of these optimization approaches have, is that they assume perfect prior knowledge on the order in which the containers will be picked up. However, this information is usually not known in advance. Nevertheless, finding a theoretical optimum can be very useful as a benchmark (although the metaheuristic approaches used to make this approach computationally feasible are not guaranteed to find the global optimum). Other methods

that have been used for this include Q-Learning (Hirashima et al. 2006) and critical-shaking neighborhood search (Lim and Xu 2006). Han et al. (2008) use integer programming with Tabu search to generate an entire yard template, which should minimize reshuffling moves. Froyland et al. (2008) also use integer programming, optimizing the entire terminal in an effort to maximize quay crane performance.

Detailed simulations were performed by several authors. Dekker et al. (2006) simulated different stacking policies for containers in automated terminals. In particular, several variants of category stacking (with up to 90 different categories) were examined and compared with a base case in which containers are stacked randomly. The simulations demonstrated very high peak workloads during the handling of very large container ships. Category stacking was found to significantly outperform random stacking. Considering the workload of each automated stacking crane (when selecting the lane for an incoming container) and stacking close to the export transfer point were found to provide additional performance benefits. There was no significant benefit to using category stacking for containers with onward transport by short-sea/feeder, rail, truck, or barge. As the category definitions are based on information used in stowage planning, they advocate an integration of terminal operations and stowage planning.

Duinkerken et al. (2001) also used simulation and category stacking, albeit with only a limited number of categories. Several (reactive) reshuffling rules were tested. Reactive reshuffling means the reshuffling is done when a container on which other containers are stacked is demanded for retrieval, leading to a number of reshuffling operations. This in contrast to “proactive” reshuffling, which is done when stacking cranes are idle. They evaluated several reshuffling rules (random, leveling, closest position, and minimizing remaining stack capacity reduction). The use of categories was compared with a model that required specific containers and found that the categories lead to much better performance. Also, it was shown that the remaining stack capacity strategy lead to big improvements when compared to the other three. Duinkerken et al. (2001) also tested two “normal” stacking strategies (i.e. for when a container has just arrived), namely random and with dedicated lanes for a quay crane. However, using dedicated lanes is hard to do in practice, as load plans are not known in advance. Also, this strategy did not yield much improvement over random stacking.

Saanen and Dekker (2006a,b) went into great detail in simulating a (transshipment) container terminal with rubber-tired gantry cranes (RTG), carefully simulating all movements of the trucks and cranes. They made a “comparison between a refined, but still traditional, strategy for operating a transshipment RTG terminal with a simple random stacking strategy for this type of terminal”, and measured the differences in quay crane productivity (in lifts per hour), as this is considered the most important indicator for terminal efficiency. They found that the differences between the strategies was very small (about 0.7 lifts per hour). However, it was found that the number of gantry movements is a major factor in limiting the quay crane productivity.

Finally, Park et al. (2006) used simulation to determine the best combination of an AGV dispatching rule with a reactive reshuffling rule, for various amounts of containers and AGV's in an automated container terminal. Their goal is to minimize the number of reshuffling operations. Park et al. used the random, closest position (but with residence time taken into account) and minimal RSC reduction reshuffling rules from Duinkerken et al. (2001). In most cases, the minimal RSC reduction rule,

combined with the Container Crane Balancing (CCB) dispatching rule (the AGV is sent to the crane which has the most containers waiting for it), lead to the least reshuffling operations.

This paper elaborates on residence time stacking. In particular we consider several residence time classes and use that information to limit the number of reshuffles. We compare a number of stacking rules where we consider trade-offs between further traveling and the possibility of reshuffles. We also consider the case of full information, on one hand as a benchmark, but also to get some insight into the structure of good policies. The research approach and experimental setup of this paper build on prior work (Dekker et al. 2006) but here we have limited ourselves to relatively simple stacking rules that use less information in order to get more insight into the basic performance of these rules. The ideas in this paper can easily be combined with the category stacking from Dekker et al. (2006).

### 3 Simulation model

The simulation model that was developed for the experiments in this paper consists of two major components: a generator and a simulator. Although based on the same specifications as the simulator model described in Dekker et al. (2006), the code for both programs was rewritten from scratch. The existing code could not easily facilitate some of the new experiments. As the tools for developing discrete-event simulation models, especially the language and library that were used in the original implementation (Pascal and MUST) are less prevalent today, and programming languages in general have improved since the original implementation, we have chosen to rebuild the system in a modern programming language (Java) using a solid discrete-event simulation library (SSJ, L'Ecuyer and Buist 2005).

The generator program creates arrival and departure times of some 76,300 20ft. containers covering a period of 15 weeks of operation, including a 3-week warm-up period to initialize the stack. The generator is based on the same data as the generator in Dekker et al. (2006), including sailing schedules and a modal-split matrix. The output of the generator is a file that contains the ship arrivals, details of the containers to be unloaded and loaded, and the specification of the destination of each container. The departure time is specified as the planned (a.k.a. expected) departure time and the actual (a.k.a. real) departure time. In the implementation of the generator program the actual departure time is generated first on the basis of the sailing schedules; the expected departure time is created from this actual departure by applying a perturbation function (the parameters of this function depend on the departure mode: ship, short-sea vessel, train, or truck). The destination can be another deep-sea vessel or (for import containers) a short-sea vessel, barge, train or truck. For each container the location of the individual container within a ship is specified. The generator takes the detailed quay crane sequences for loading and unloading into account.

The average residence time of a container is 3.8 days; the 90%-percentile of the dwell time is 5.3 days, and the maximum dwell time is 8 days. The specifications of the input for the generator are detailed in Voogd et al. (1999); the current implementation is documented in Borgman (2009). Most experiments in this paper are done with a

stack that has a total capacity of  $6 \times 34 \times 6 \times 3 = 3,672$  TEU. The average utilization of the yard is therefore  $(76,300 \times 3.8)/(3,672 \times 12 \times 7) \approx 75\%$ . In some experiments we use a stack of  $8 \times 34 \times 6 \times 4 = 6,528$  TEU with an average utilization of 42%.

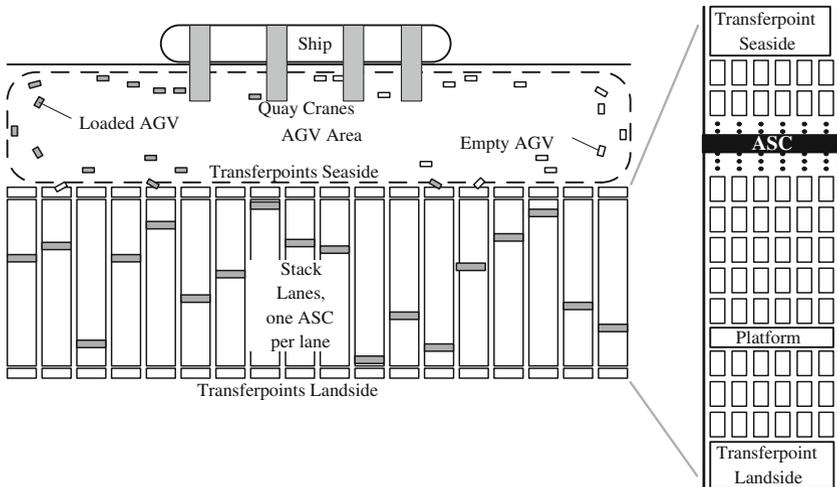
The simulator program reads the output of the generator and performs the stacking algorithms. The core of the simulator itself is deterministic: the stochastic components are in the generator and, optionally, in the stacking algorithm. This setup facilitates a comparison of stacking algorithms as any changes in the statistical output of the simulator must be caused by the stacking algorithm.

Within the simulation program, the containers are loaded and unloaded from ships and other transport modes (trucks and trains). The transport of containers from the quay to the stack is performed by Automated Guided Vehicles (AGV's); the simulation does not contain a detailed model of the AGV's (issues such as routing and traffic have not been modeled). Once an AGV with a container arrives at the stack, the Automated Stacking Crane (ASC's) for the lane is tasked with lifting the container from the AGV and storing it in the stacking lane. As previous research into this container terminal has shown the ASCs to be performance bottlenecks, they have been modeled in detail: the simulator calculates the time it takes for all the motion components (hoisting, lengthwise and widthwise movement) where hoisting and movement are sequential and the length- and widthwise movement are done simultaneously. There is a single ASC per lane (based on the ECT terminal, v.i.) and the simulation program maintains a job queue for each ASC. Containers that have to be reshuffled are always stored within the same lane. On the land side, the containers are moved to and from the stack using straddle carriers.

To verify that the generator works correctly, we have performed a number of tests on the output (such as testing the distribution of the container dwell times). The simulator was verified using a number of test scenarios for which the values of the statistical indicators could be determined analytically. Once the simulator had passed these tests, the simulator was benchmarked against the model described in [Dekker et al. \(2006\)](#); the performance of random stacking and category stacking (experiments A0 and A from [Dekker et al. \(2006\)](#)) was similar but not identical. The main difference in the two models is that the current simulation model has a very detailed simulation of the ASC's whereas the original model had a very simplistic model for the ASC's. As other authors (such as [Axelrod 1997](#)) have noted, achieving numerical identity for simulation models is hard. The detailed descriptions required to achieve this can rarely be published in papers and are too much work (with little reward) to describe in internal reports.

## 4 Basic concepts

In this section we present some generic concepts that form the basis of the stacking rules we will evaluate in this paper. The precise formulations will depend on the layout of the stacking area. While we only present these formulations for one particular layout (in order to clarify the presentation and analysis), the formulations can be adapted for other layouts.



**Fig. 1** Terminal layout with details of a single lane

We want to investigate the basic concepts of stacking containers in a yard. The core dilemma is that we would like to stack a container that arrives and departs at the quay side as close to the transfer point quay-side as possible because this will minimize the total travel time of the stacking crane when the container enters and exits the stack. As there are many of these sea-sea containers, this would require us to stack high. Unfortunately, when we start to stack containers on top of each other, we face the risk of stacking a container on top of a container that will depart before the incoming container. This will lead to a reshuffle, which takes time. We will thus have to balance the travel and hoisting time of the stacking crane and the time taken by reshuffle moves.

If we consider a single lane of the type of container terminal under investigation, we see that there is a single rail-mounted stacking crane that has to perform all the stacking moves for that lane. We can distinguish between containers that are moving into the lane (i.e., that are being stacked) and containers that are moving out of the lane (they are being “unstacked”). Containers can enter and leave the lane at two sides: at the quay side (for containers that are coming from or going to deep-sea ships) and at the land side (for all other modes of transport). Figure 1 provides a schematic overview of the terminal layout. The layout of this terminal has the stacking lanes perpendicular to the quay. Each lane has a length, a width, and a height. We will refer to a single line along the length of the lane as a lane segment. A layout within this terminal configuration will be denoted as ‘number of lanes × length × width × height’; the basic configuration for our experiments will be ‘6 × 34 × 6 × 3’.

The first trade-off that is worthy of investigation is the trade-off between the time it takes the ASC to travel to a certain location and the amount of time required to (un)stack a container. For a container that has arrived on a deep-sea vessel and that will also depart on another deep-sea vessel, it is attractive to stack it as close to the transfer point at the quay side as possible. If we can stack the container close to the transfer point we save travel time of the ASC both when the container is stacked and

when it is unstacked. Clearly the same applies for containers that arrive and depart at the land-side of the stack. There is no obvious best location for containers that arrive at the quay-side and will depart at the land side and for containers that arrive at the land side and will leave the stack at the quay side. In both cases the ASC will have to (in two stages) move the container along the entire length of the lane. At first glance stacking the container close to the planned exit transfer point seems beneficial; however, this would imply a longer travel time of the crane. Since sea-to-land moves will occur most when large ships are being unloaded, it would seem more beneficial to stack these containers as fast as possible in order to release the crane more quickly for other moves. This would however conflict with the desire to stack sea-sea containers as close to the quay-side transfer point as possible.

The second trade-off we want to research is between the time required to stack a container and the number of reshuffles. Although reshuffles as such should be avoided, it is interesting to test a strategy that favors a fast stacking time during peak times with a resulting reshuffle that may occur at an off-peak time.

The overall approach of the experiments in this paper is focused on the operational decisions that have to be made by terminal operators. Specifically, we take the arrivals and departures that are specified as part of the generator output and perform these operations. There is no global optimization or explicit planning; the operations are performed one at a time, i.e., in a greedy fashion, whenever a container arrives. We do not consider future events such as other incoming containers.

## 5 Experimental setup

The experiments in this paper all use the following configuration. Experiments are run for a 15-week period, of which 3 weeks are used for warm-up (to initialize the stack). As some stacking rules have a stochastic component (such as selecting a position at random), we use ten replications to get statistically robust results. These replications are used to compute the 95% confidence intervals of the mean.

We assume that there are sufficient AGVs and straddle carriers to ensure that these resources do not act as bottlenecks. The basic configuration for the stacking area is modeled on part of the automated ECT Delta Terminal at the Port of Rotterdam, The Netherlands. We have chosen to use only a part of the actual stack area in these experiments to clarify the discussion and to facilitate the analysis. Thus, our stacking area has far fewer lanes than the actual terminal; the length of the lanes, the maximum stacking height, and the number of ASC's per lane are based on the configuration at the ECT Delta terminal.

There are thus just six lanes, each equipped with a single ASC (Some container terminals have multiple ASC's per lane. This would make the problem more difficult to study as we would have to model the interaction between multiple cranes in detail. We therefore limit ourselves to a single ASC which is the configuration at the Delta terminal.) per lane to mirror the configuration at the Delta terminal and Each line is 34 TEU long, for a total of  $6 \times 34 = 204$  ground positions (measured in TEU). (The Delta terminal has some additional room for reefer containers in each lane but we have not taken these containers into account for our experiments so we present the layout

without this reefer area.) The maximum stacking height is three containers. Each lane has six lane segments. We denote the configuration as  $6 \times 34 \times 6 \times 3$  (six lanes, each lane being 34 TEU long, 6 segments per lane, and a maximum stacking height of 3 containers). The number of lanes and the maximum stacking height will be changed for some experiments to evaluate the performance of the stacking rules under investigation. For these experiments we use a single size of container, the standard 20 ft. container, as a mix of different sizes of container would make the comparison of the stacking rules more complicated. We thus skipped all other sizes of containers in the generator's output file. The number of 20 ft. containers is a good fit with the base layout.

We will use Random Stacking (RS) and an implementation of the Leveling algorithm (LEV) described in [Duinkerken et al. \(2001\)](#) as benchmarks for the experiments.

## 6 Benchmark algorithms

In this section we introduce the basic algorithms used for comparison in the experiments.

### 6.1 Random stacking

Random stacking is a straightforward way of determining a stacking position for a new container. Basically, the new container is placed at a randomly chosen allowed location, with every allowed location having an equal probability of being chosen. We have implemented this as follows:

1. Select a random lane.
2. Select a random position in the lane.
3. Check whether we could stack at this position.
4. If so: stack here.
5. If not: start again in the next lane.

Given enough tries this algorithm is guaranteed to find an available location (in our implementation, we have set the limit at 5,000 tries, which has proven sufficient). This algorithm is also applied for reshuffling, with the difference that we then only want to search the lane the container is in.

### 6.2 Leveling

The idea is to fill lanes in layers, so that all empty ground positions are filled with containers first, before containers are stacked upon others. The stacking lane is filled from the transfer point quayside on. This strategy is taken from the earlier work of [Duinkerken et al. \(2001\)](#). It is an intuitive strategy, but it does not use most of the available information. We thus get the following steps:

1. Choose a random lane with at least one available position.
2. Search for the first empty location, from the transfer point quayside towards the transfer point landside, row for row (i.e., widthwise).

3. If found: stack there.
4. If not found: search all existing piles (of the same size and type), from the transfer point landside towards the transfer point quayside, row for row, for the lowest (i.e., search lowest piles first) stack location and stack on the location found first.

This algorithm is also applied for reshuffling, with the difference that we only search the lane the container is in.

## 7 Experiments

In this section we will present our experiments with a number of stacking rules. We follow the same structure for each experiment. We first present the design of the stacking rule. Next, we formulate a number of hypotheses regarding the performance of the stacking rule. The results are presented in tabular form and we discuss the results in terms of our hypotheses. The hypotheses are tested using the 95% confidence intervals of the mean; we accept that there is a significant difference if these intervals do not overlap. In the interest of clarity and brevity we only present a subset of the total experimental results; a comprehensive list of results is listed in [Borgman \(2009\)](#).

The performance of a stacking algorithm is measured with the following statistics:

- |                           |  |
|---------------------------|--|
| Exit time (ETQ and ETL).  | The exit time is the time (in hours) it takes to remove a container from the stack and have it ready for onward transport (to the quay or to a truck/train/barge). This time is measured for each side (quay-side and land-side) of the stack and will be listed as ETQ and ETL, respectively. The exit time is the main performance indicator for a stacking algorithm. It is negatively influenced by stacking further away from the exit point and by any reshuffles that are needed to retrieve the container. When a container enters the stack, the time it takes to perform this operation is determined by the workload of the ASC (how many jobs are in the current job queue) and the time it takes the ASC to move the container to its position. There are no reshuffles when containers are stored in the stack; reshuffles only occur when a container has to leave the stack. |
| ASC workload (ASC).       | The automated stacking cranes are critical components for the overall performance so we measure the percentage of time that the ASC's are busy. (The ASC workload will be denoted as ASC in the results.)  |
| Reshuffles (RDC and ROC). | For the unproductive reshuffle moves, we measure the number of reshuffles (denoted as RDC) as a percentage of the total number of container movements. To get an indication of the number of reshuffles that happen per move, we also measure the reshuffle occasions  |

(as a percentage of the total number of container movements, denoted as ROC); a single reshuffle occasion implies one or more reshuffles. These numbers are not absolute indicators of performance as the time of the reshuffle is not taken into consideration. A reshuffle that occurs when the workload is low has less impact on the overall performance than a reshuffle during a peak workload, for example when (un)loading a very large vessel.

**Ground position usage (GPU).** We report only the average percentage of ground positions that are in use (denoted by GPU) as the various stacking strategies have differing preferences for stacking on the ground.

### 7.1 Experiment 1: leveling with departure times (LDT)

The first experiment is on the influence of knowing the exact departure times of all containers. This can be exploited by only stacking containers on top of other containers, if the new container departs at an earlier time than the one below it, i.e., residence time stacking.

In practice the actual departure time is not known. However, it is valuable as a reference case to determine the best possible performance of such a stacking rule. For a more realistic scenario, we use the expected departure time that is also part of the generator output file.

We differentiate between containers both arriving and leaving at the quay and other containers (i.e., containers arriving or departing at the truck loading point). The containers both arriving and leaving at the quay (the sea–sea containers) should be stacked close to the transfer point quayside, because every meter they move towards the transfer point landside is a waste.

The dwell time, or better the departure time of a container, can be used when determining where to stack it. Containers departing before the containers below them will never lead to a reshuffle. We would like to exploit this to the maximum and stack as high as possible, because this means other positions remain free for other containers which depart later. The first priority when searching for a place to stack is thus to find the piles which have such a container on top.

Second, and again in the interests of keeping options open, we want to find the position where the difference between departure times is as small as possible, since this means that there are more possibilities for stacking other containers on top of them, using as little space as possible. We thus select from the piles the one where the difference in departure times would be smallest.

If we can find no pile with a container on top that will depart after the new container, we have to stack it elsewhere. Preferably, we do this on the ground, so no reshuffles will occur. From the available positions on the ground, we want to stack it as close to the transfer point as possible, so that travel times are minimized.

Should we still not be able to find a position, we have to stack the container on an existing pile, rendering a reshuffle inevitable. To minimize the number of reshuffles, we place the container on the highest pile available, so that no or few containers can be stacked on top of them, each of which would lead to another reshuffle. In case several of these piles are available, we select the pile closest to the transfer point, in order to minimize travel time for the ASC.

In summary, we use the following algorithm, which we shall call “LDT” (Leveling with Departure Times) (in each step we look at all lane segments):

1. Stack the new container (departing at  $T = T_n$ ) at that pile, where the top container departs at time  $T = T_o$ ,  $T_o > T_n$ , and  $T_o - T_n$  is minimal and on which the container may technically be stacked (i.e., pile is not full).
2. If no position was found yet: stack the container on an empty ground location. Sea–sea containers are to be stacked as close to the transfer point quayside as possible.
3. Stack at a pile of the highest height available, as close to the transfer point as possible.

For sea–land and land–sea containers we do not have a preference for a particular part of the lane, since they have to traverse it in full anyway. However, because sea–sea containers prefer the sea (quay) side of the lane, sea–land containers should be stacked away from them, as close to the transfer point landside as possible, when the two types are both included in an experiment. Since we select the stacking location based on departure times, the two types may become mixed. If only sea–sea containers are included, the algorithm automatically stacks the containers near the transfer point quayside, but with land containers included, it may not do this. Since this removes the advantage of stacking near the quayside, we can choose to separate the piles, so that sea–sea containers may not be stacked on land–sea containers and vice versa. However, as this means there are less options for optimizing residence times, it remains to be seen which is best.

We compare the LDT algorithm with random stacking, leveling, and a modified version of random stacking (RS-DT), in which the algorithm searches for a random pile with the top container’s departure time being after the new container’s departure time. If no such pile is found, the container is stacked randomly, according to the random stacking algorithm. We use this to see which part of the differences between ordinary random stacking and the LDT algorithm is caused by the “random” part and which part is caused by the lack of perfect information.

### 7.1.1 Hypotheses

Because of the great advantage of perfect information regarding the departure times, we expect to see a very big improvement for relevant statistics, compared to random stacking and leveling. In particular, we look at the reshuffle percentages (which we expect will be lower for this algorithm), time to exit (will also be lower), ASC workload (will also be lower, as it is related to the previous ones), stack usage (will be slightly lower due to containers exiting quicker), and ground position usage (will be much

lower than with leveling, which maximizes ground usage. We cannot say in advance how it compares with random stacking).

The effects of the RS-DT algorithm should be similar to those of LDT with regard to random stacking and leveling, because of the extra information. However, because the pile selection process is still very basic, it probably will not perform as good as LDT.

The experiments with expected, rather than actual, departure times should still be better than random stacking and leveling, i.e., the same effects should occur as with perfect information. We do expect these effects to be somewhat weaker, since the information is less reliable and hence some poor decisions are likely to be made.

On the basis of these considerations we formulate the following hypotheses:

- Hypothesis 1.1 The LDT stacking algorithm will have a lower number of reshuffles, a lower exit time and a lower ASC workload than the benchmarks RS and LEV.
- Hypothesis 1.2 The RS-DT stacking algorithm will have a better performance than RS, but worse than LDT.
- Hypothesis 1.3 Mixing piles in the LDT stacking algorithm will lead to less reshuffles when compared to not mixing piles.
- Hypothesis 1.4 Mixing piles in the LDT stacking algorithm will lead to higher exit times compared to not mixing piles.

### 7.1.2 Experimental setup

We have varied two parameters for this experiment; the first is the departure time (actual or real departure time vs. expected departure time) and the second parameter controls whether mixed piles are allowed (mixed vs. unmixed).

In this experiment we are particularly interested in the value of the perfect information regarding departure times. We therefore compare the results to random stacking. We would also like to know whether it is better to allow mixed piles or not. The difference is measured by looking at the times it takes for a container to enter and leave the stack.

### 7.1.3 Results

We have listed the results of all experiments in a single table to facilitate comparison (see Table 4). The first column of Table 4 lists the number of the experiment. The results for the benchmark algorithms are included as experiment “0”. As expected, the LDT algorithm in its various forms outperforms the random stacking and leveling benchmarks. Less reshuffles occur and exit times and ASC workloads are lower. Ground position usage is also lower.

The relative performance of RS-DT is as predicted: better than the benchmark but worse than LDT. It is also of note that the modified random stacking algorithm (using real departure times), outperforms the LDT when the latter is using expected departure times. Using expected data, rather than actual, leads to a big performance drop for LDT.

The effects of mixed piles are inconclusive. When using actual data, mixed piles lead to less reshuffles but an increased exit time. However, using expected data, they lead to more reshuffles (and also an increased exit time). These effects are also found in the other stack layouts, albeit somewhat weaker.

We have tested this strategy for some larger stacks as well. In larger stacks there is little improvement in the results for expected times, unlike those for actual times. Moreover, the results for expected times actually deteriorate when going from 6 lanes, 3 high to 6 lanes, 4 high.

#### 7.1.4 Discussion

From the results, it becomes clear that LDT's departure times have a big impact, even when they are not exactly known. Even partially random stacking, using departure times only to a limited extent, leads to big improvements across the board. Still, there remains a big gap between the results of expected and actual departure times, especially in bigger stacks, where little improvement, if any, is seen in comparison to smaller stacks. This is most dramatically the case when going from height 3 to 4 with 6 lanes, where performance actually drops, despite an increase of stacking options. This is probably due to any mistakes made being punished more heavily with extra reshuffles at higher stack levels.

Using mixed piles seems to be not such a good idea. In some cases, there is an improvement in the number of reshuffles (as was expected), but in others there is none. In all cases, mixed piles lead to longer exit times.

Our conclusion with respect to the hypotheses is:

Hypothesis 1.1: Confirmed.

Hypothesis 1.2: Confirmed.

Hypothesis 1.3: Rejected.

Hypothesis 1.4: Confirmed.

## 7.2 Experiment 2: LDT with departure time classification (DTC)

In the first experiment we have used the information from the generator program on the actual and expected departure times. We now take a different approach to model uncertain departure time information.

We use the data from the arrivals file to define a limited number of classes. The boundaries of these classes are calculated from the arrivals file by taking the quintiles or the 20th, 40th, 60th, 80th and 100th percentiles of the residence time. This gives us five classes of almost equal size, for the initial residence times at least. We use five classes because the maximum stacking height in these experiments is either three or four. It is likely that using more classes would yield better results but a larger number of classes would be a less accurate reflection of the uncertainty of the residence time. The classes are listed in Table 1.

We use the algorithm from experiment 1 for this experiment too, only when the time difference is calculated we do not use the actual time or the expected time from the file, but instead use the class value from Table 1 (based on either the actual or expected

**Table 1** The five classes of container departure times used in experiment 2

Class	Actual times		Expected times	
	From (h)	To (h)	From (h)	To (h)
1	0	68.2	0	68.0
2	68.2	79.4	68.0	79.4
3	79.4	92.4	79.4	92.3
4	92.4	111.6	92.3	112.5
5	111.6	$\infty$	112.5	$\infty$

The maximum time any container in the used arrivals file will stay is 192.2 h

departure time). This means that lower classes will be stacked on top of higher classes, thereby ensuring no reshuffles occur, unless no suitable pile or ground position could be found.

Note that a container's class will change over time as its departure time comes nearer. This means that the lower classes will be more prevalent in the stack, because every high class will at one time become a low class.

### 7.2.1 Experimental setup

To test this algorithm, we used the settings of Experiment 1 (see Sect. 5), because we want to compare the different method of estimating departure times with the original and with perfect knowledge. We also experiment with classes based upon expected departure times, to see whether the double uncertainty gives any different results. We compare this algorithm to the entire Experiment 1 (LDT), including random stacking (RS), leveling (LEV), and RS-DT. We also modified the RS-DT algorithm to work with departure time classes (this version being referred to as RS-DTC).

### 7.2.2 Hypotheses

We expect this algorithm to have a better performance than LDT-exp because it uses real departure times to define the classes. The algorithm may have a problem with small stacks, where space is in short supply and some suboptimal decisions may have to be made. In larger stacks, we expect that LDT-DTC will perform almost as good as normal LDT with real departure times. We expect similar effects with the use of expected departure times. Because the classes are then based on imperfect information, we do expect a drop in performance, when compared to LDT-DTC.

**Hypothesis 2.1** The LDT-DTC algorithm with real departure times will have a lower number of reshuffles, a lower exit time, and a lower ASC workload than LDT-exp, but higher than LDT.

**Hypothesis 2.2** The LDT-DTC algorithm with expected departure times will have a higher number of reshuffles, a higher exit time, and a higher ASC workload than LDT-exp and LDT.

Hypothesis 2.3 The LDT-DTC-exp will have a higher number of reshuffles, a higher exit time, and a higher ASC workload than LDT-DTC-real.

### 7.2.3 Results

The results for a  $6 \times 34 \times 6 \times 3$ -stack are in Table 4. For brevity, we have not included the results for mixed piles, since their performance is similar to the previous experiment and they only complicate the presentation.

For the  $6 \times 34 \times 6 \times 3$ -stack, the LDT-DTC performs much worse than normal LDT with expected departure times. The random stacking version of the departure time classes algorithm (RS-DTC) also performs significantly weaker, compared to RS-DT, and even compared to LEV; in fact, the performance of RS-DTC is similar to RS. Interestingly, the results for LDT-DTC-exp are slightly (yet significantly) better than LDT-DTC-real. For the RS-DTC algorithm, the same applies. This is probably due to the following effect: when no “nice” stacking position can be found (this happens in as much as 40% of the cases, we found), a container is put on another pile, certainly causing a reshuffle when real departure times are used. However, when expected departure times are used, there is a small probability that, because of the error in departure time estimation, no reshuffle is caused. This leads to a slight advantage for expected times, and hence to this counterintuitive insight.

However, the results differ significantly for larger stacks, as can be seen in Table 5 (a  $8 \times 34 \times 6 \times 4$ -stack). In this larger stack, there are no problems finding a suitable spot for the LDT-DTC-real algorithm, and thus its advantage of certainly knowing whether a reshuffle will occur is enough to yield better results. This leads to results almost as good as when using normal LDT. There is also a big improvement for the LDT-DTC-exp algorithm, which now actually performs better than LDT-exp. This is probably due to the used classes providing a bigger “margin of error”, leading to less mistakes in determining which container departs first.

Interestingly, RS-DTC-real algorithm does not seem to benefit much from a larger stack. Apparently, its method of trying to find any suitable pile, without regard for the smallest class difference, leads to very inefficient stacking.

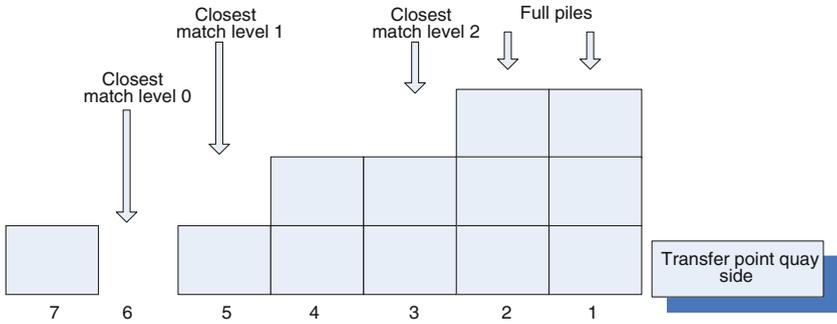
### 7.2.4 Discussion

Using the suggested five classes gives a very good result and a very good approximation of the results of actual departure times, provided there is enough space in the stack. For that case, we can confirm the hypotheses but as the results for the smaller stack differ, we cannot confirm them.

Hypothesis 2.1: Rejected.

Hypothesis 2.2: Rejected.

Hypothesis 2.3: Rejected.



**Fig. 2** Example case for experiment 3. Maximum stacking height is 3 containers. The two piles to the right are closest to the transfer point, but are full. Three other options are available though

### 7.3 Experiment 3: traveling distance versus reshuffling (TVR)

In this experiment, we do not use residence time knowledge, but rather try to optimize the selection of a location in some other way.

Suppose we only have sea-sea containers. As stated previously, these containers should be stacked as close to the transfer point quayside as possible, because they both arrive and leave the lane there. Leveling all the way down the lane would thus not be a good idea; we want to stack as high as possible at the beginning of the lane, and leave the end of the lane empty, if possible. Of course, we could just stack new containers as close to the transfer point as possible, by stacking them on top of each other, but this has the unwanted side effect of leading to reshuffles, because the order in which containers arrive and leave is mostly random.

However, since we also do not want to level too much, a compromise solution should be better. Whenever a container arrives at a lane (maximum stacking height  $n$ ), we can say there are  $n$  possibilities to stack it: at every possible height layer, as close to the transfer point quayside as possible. For example, if  $n = 3$ , we may get the scenario from Fig. 2. The ground positions closest to the transfer point are full, but the next one is only stacked 2 out of 3 high. This is the best position from the distance viewpoint, but it has a high risk of leading to reshuffles, with two containers under it. The next position has the same risk, but it is further away, so therefore not as good, and we henceforth discard it for this decision.

We then encounter a pile with only one container. This means that the risk of reshuffling is lower, but unfortunately the pile is also further away from the transfer point. Still, it may be a good candidate to investigate. The sixth ground position is empty, and while it is still further away from the transfer point, there is no risk of causing reshuffles. This is the final candidate for stacking, because although there are more ground positions ahead, none of them is better in terms of reshuffling risk than the ones we already selected, and they are all worse in terms of distance to travel.

Now that we found the “best” (note that we do not use residence times or categories here, these factors seriously complicate matters) three piles, we need to choose which one is best, i.e., which one has the lowest associated cost in time. Time costs consist

of two parts: extra ASC driving time and time due to reshuffling. The latter will often be greater, but it is not certain it will occur.

The ASC driving time can easily be calculated using the distance from the transfer point to the selected ground position and the ASC's lengthwise and widthwise speeds. Lifting times can also be calculated, since we know exactly at which level the container is and will be stacked. The cost of reshuffling is more difficult to determine, because it is not known in advance where to a container would be reshuffled. We assume this to be a (configurable) time period of driving away. The lifting times are also not known, so we estimate the lifting time to be that of a container in the middle of a pile (i.e., at the level of half the maximum height). The cost of a reshuffle is then multiplied by the expected number of reshuffles that putting the container at a particular place will generate. This gives the following cost function to minimize:

$$\min[2 \cdot TT_{qp} + L_{qp} + (TT_r + L_r) \cdot ER(n)] \quad (1)$$

where  $TT_{qp}$  is the travel time from the transfer point quayside to the target pile,  $TT_r$  is the (fixed) travel time for reshuffles,  $L_{qp}$  and  $L_r$  are the lifting times per container (including pickup, lift up, lift down and set down) for the container itself and reshuffles, respectively, and  $ER(n)$  is the expected number of reshuffles. The travel time to the pile has to be doubled because containers need to go back to the transfer point at one time; reshuffling can lead to a position closer to the transfer point, so we do not double that time. We will use  $TT_r$  as a penalty factor to vary the relative cost of reshuffles in our experiments.

A new container can, by itself, only generate one extra reshuffle, at most. Any other reshuffles were already in the pile when the container was stacked, or are added later. This means that we have to calculate the probability  $P(nr)$  of the new container leading to an extra reshuffle. Since we assume every container in a pile has an equal chance of being chosen, this gives the following formula:

$$P(nr) = \frac{n-1}{n}, \quad (2)$$

where  $n$  is the height of the pile. We use this probability as  $ER(n)$  in Eq. 1.

Summarizing, we use the following algorithm:

1. Select for every possible stacking level the (available) position closest to the transfer point quayside, if any position is available.
2. Calculate the costs of every position, according to Eq. 1 (using Eq. 2).
3. Select the position with the lowest cost and stack there.

While this algorithm can stack sea–sea containers of a single type, it is easy to extend the algorithm for other containers. As discussed in the previous experiment, sea–land and land–sea containers should be stacked as close to the transfer point landside as possible. We can determine costs in the same way.

### 7.3.1 Experimental setup

In this experiment we are particularly interested in the value of putting sea–sea containers close to the transfer point quayside. We therefore compare the results to random stacking. We would also like to know what the effects of different penalties (i.e.,  $TT_r$  in Eq. 1) are. (Here, we report a subset of the experiments described in [Borgman \(2009\)](#) with values of the reshuffling movement penalty ranging from  $-0.03$  to  $0.04$ .) The difference is measured by looking at the times it takes for a container to enter and leave the stack.

We compare this algorithm with random stacking, leveling and a modified version of random stacking, which we shall refer to as TPRL (Transfer Point Random Level), in which the algorithm chooses one of the possibilities offered (i.e. it randomly selects the level where the container is to be stacked). This means containers will be near the quay, and since we know one of the positions is the “best” one, we can see the influence of the complicated calculations, when we compare it with chance.

### 7.3.2 Hypotheses

Again, we predict this algorithm will always outperform the basic random stacking and leveling algorithms. We again look at the reshuffle percentage (which we expect will be lower for this algorithm), time to exit (will also be lower), ASC workload (will also be lower, as it is related to the previous ones), stack usage (will be slightly lower due to containers exiting quicker) and ground position usage (will be somewhat lower than with leveling, which maximizes ground usage).

The ground usage depends on the reshuffling movement penalty applied. A higher penalty will lead to more containers being stacked on the ground and this leads to a higher ground position usage. A high penalty would lead to the algorithm behaving as normal leveling. This would also mean that the effect of allowing to stack 4 containers on top of each other, rather than 3, would be almost completely gone (there is still a minor effect on crane lifting times). Low penalties, on the other hand, would lead to a very low ground position usage, with high piles near the transfer points.

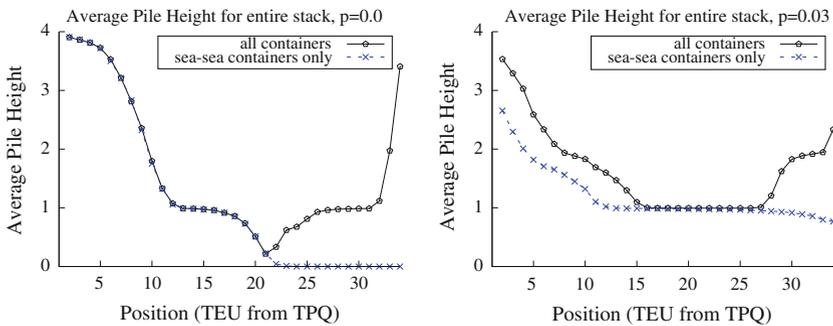
In any case, when only sea–sea containers are included, the algorithm will stack containers mostly next to the transfer point quayside and will stack less and less containers further away. This would mean that the average pile height would be decreasing in a monotone way, when going away from the transfer point.

We expect the modified random stacking algorithm TPRL to perform worse than the original random stacking (and, for that matter, all other algorithms tested in this section), especially when there is a lot of space in the stack. This is because modified random stacking will too often build high piles, while the other algorithms place more containers on the ground, which leads to less reshuffles.

On the basis of these considerations we define these hypotheses:

**Hypothesis 3.1** The best TVR stacking algorithm will have a lower number of reshuffles, a lower exit time and a lower ASC workload than the benchmarks RS and LEV.

**Hypothesis 3.2** The TPRL stacking algorithm will have a worse performance than TVR.



**Fig. 3** Time-average pile height for the entire stack for penalty value 0 (*left*) and 0.03 (*right*). Stack layout:  $8 \times 34 \times 6 \times 4$

Hypothesis 3.3 The TPRL stacking algorithm will have a worse performance than RS and LEV.

Hypothesis 3.4 The TVR stacking algorithm is equal to LEV with a high penalty and only sea–sea containers.

Hypothesis 3.5 The best TVR stacking algorithm will lead to a monotone decreasing average pile height away from the transfer point quayside, when using only sea–sea containers. and TPRL.

Hypothesis 3.6 The TVR stacking algorithm, with very low penalties, will have a worse performance than RS, LEV, and TPRL

### 7.3.3 Results

See Table 4 for the results of Experiment 3. We can see that the TVR algorithm performed better than random stacking on all statistics, but compared to basic leveling the differences are minute. Even the best TVR version, in this experiment with a moving penalty for reshuffles of 0.0 h, scores only marginally less reshuffles and slightly lower exit times. It should be noted, however, that these results are still statistically significant and outside the 95% confidence interval.

The very low penalty value of  $-0.03$  h results in very low ground position usage, meaning high piles. This in turn leads to more reshuffles and higher exit times.

What further becomes clear is that penalties from  $-0.01$  and higher yield almost the same results. Lower penalties give different (worse) scores (with the low ground position usage as predicted). With higher penalties, the results approach the benchmark result of leveling, but a penalty of 0.03 does not mean that the algorithm is the same yet. For larger stacks, the same behavior is observed, although the value of  $-0.01$ , from whereon the results are very similar, appears to be somewhat higher for larger stacks. (The full set of results are in Borgman (2009)).

The TPRL performs slightly worse than leveling, especially on larger stacks, in terms of exit times and reshuffle occasions, but not by the amount we expected. Moreover, it performs better than normal random stacking (RS). The number of actual reshuffles is higher though, because the ground position usage is lower and piles are higher.

To illustrate the effect of the penalty value on the pile heights along the length of the lane, we have graphed the time-averaged pile height in Fig. 3. We have selected a

slightly larger and higher stack configuration ( $8 \times 34 \times 6 \times 4$  layout) as this configuration provided the clearest graphical illustration. If the penalty is zero (left), then the average pile height is the same for both types of containers up to position 20. Beyond that, there are no sea–sea containers which means that the penalty value causes the two types of containers (sea–sea versus land–sea/sea–land) to be spatially separated. If we increase the penalty value to, e.g., 0.03, we see more sea–sea containers being moved along the entire length of the lane. The line for all containers shows that the sea–sea containers can no longer be stacked at the landside: more are now stacked towards the quayside and the average pile height increases.

### 7.3.4 Discussion

The TPRL algorithm performs not as bad as expected. This is likely partially due to the forced stacking near the transfer points, which leads to lower exit times, even compared to leveling. However, reshuffles are also consistently lower than with random stacking. A possible explanation lies in the fact that, on average, there is about a 33% probability (25% for stacking of height 4) of TPRL stacking in any of the possible stack layers. With random stacking, however, the probability for stacking on top of a high pile increases with an increased stack usage. In the biggest stack tested (8 lanes, 4 high, which naturally had the lowest stack usage), random stacking had a ground position usage of 76.1% (1,632 piles on average) and a stack usage of 42.1% (average 2,748.3 TEU in the stack at any time). This means piles were, on average, 1.68 (out of 4) containers high. Random stacking thus had a  $1 - 0.761 = 23.9\%$  probability of selecting an empty ground position and 76.1% probability of selecting one of another height (which was, on average, 1.68). This gives the expected pile height of random stacking as  $0.239 \times 0 + 0.761 \times 1.68 \approx 1.28$ . TPRL, on the other hand, had 25% probability of choosing an empty ground position, and 75% of choosing an existing pile, of which the average height was  $1 \times 0.25 + 2 \times 0.25 + 3 \times 0.25 = 1.5$ . This gives the expected pile height stacked upon as  $0.25 \times 0 + 0.75 \times 1.5 = 1.125$ . Since this is a lower number, the expected number of reshuffles caused is also lower for TPRL than for random stacking. For smaller stacks, the probability for random stacking to stack on an existing pile only becomes greater, so this explanation applies there as well.

A reshuffle movement penalty of  $-0.01$  hours appears to not lead to very bad results. There are only slight drops in performance compared to the higher penalties. We expect this is due to the feature of the algorithm which estimates the lifting time for reshuffles ( $L_r$  in Eq. 1). This variable is 0.019 h for a 4-high stack. When the penalty of  $-0.01$  is added, this still leaves a sizable reshuffling penalty of 0.009 h, which is usually more than the cost of driving a little further, which the algorithm chooses to do. Hence, the value of  $-0.01$  h, which is not possible to have as a travel time in reality, still gives acceptable results.

The results show that different reshuffle movement penalties lead to different outcomes. Also, the best value in terms of the primary performance measures (ETQ and ETL) is not the same for every stack configuration. See Table 2 for an overview of the best penalties per setup we found in the experiments. More specifically, a maximum height of 3 requires a lower penalty than one of height 4.

**Table 2** Best penalties found in Experiment 3

	Max height	Lanes	Best penalty all	Best penalty sea–sea
	3	6	0.01	0.01
	3	8	0.01	0.01
	4	6	0.03	0.03
	4	8	0.03	0.03
Penalties are reshuffle movement penalties in hours	5	5	0.04	0.04

TVR performs better than the benchmark tests if we focus on our primary performance indicator, the exit time; when no residence time information is available, this is a good strategy to use. TPRL is also better than normal random stacking, but TVR yields much greater benefits and, since it requires no extra information, is the preferred option.

Thus, we accept the hypotheses H3.1, H3.2, H3.4, H3.5, and H3.6; we reject hypothesis H3.3.

Hypothesis 3.1: Confirmed.

Hypothesis 3.2: Confirmed.

Hypothesis 3.3: Rejected.

Hypothesis 3.4: Confirmed.

Hypothesis 3.5: Confirmed.

Hypothesis 3.6: Confirmed.

#### 7.4 Experiment 4: peak-adjusted TVR

In Experiment 3, we argued that sea–sea containers should be stacked as close to the transfer point quayside as possible, because every move further on is a waste of time. Likewise, we stated that sea–land and land–sea containers should be stacked near the transfer point landside, because for them distance does not matter (as they need to traverse the entire lane anyway), and they are out of the way for sea–sea containers there.

There is a slight problem with this reasoning, though. The above is true only if the time spent driving across the lane by ASCs for sea–land containers is valued the same at every point in time. However, since sea containers often arrive and depart many at a time (i.e., in a jumbo or deep sea ship), there are big peaks in crane workload. At these times, it would be not such a great idea to move sea–land containers all the way across the lane.

In this experiment, we try to counter this problem and extend the algorithm of Experiment 3, by not putting the sea–land containers next to the transfer point landside, but somewhat further away. This is achieved by dividing every lane segment into two parts; one for sea–sea containers (near the transfer point quayside) and one for other containers (near the transfer point landside). We then stack all containers as close to the transfer point quayside, but in their own part of the segment. The size of the two parts is a configurable parameter. In the experiments it was set to 74% for sea–sea

containers (which is roughly the fraction of that type in 20 ft. containers). There is also an option to allow stacking of sea–sea containers in the land part.

#### 7.4.1 Experimental setup

In this experiment we are particularly interested in the value of putting sea–sea containers close to the transfer point quayside. We therefore compare the results to random stacking. We would also like to know what the effects of different penalties are. The difference is measured by looking at the times it takes for a container to enter and leave the stack.

It would not make much sense to test this algorithm with sea–sea containers only, because it is aimed only at improving the combination of both types.

#### 7.4.2 Hypotheses

In this experiment, we expect roughly the same results as in Experiment 3. The question is which algorithm will perform better.

We also expect a slightly lower exit time in Experiment 4 than in Experiment 3, when using a low penalty in both. This is because there should be slightly less pressure on the crane at peak times, while there is no other change (sea–sea containers should not be interfering with other containers). With a high penalty and a small stack, the leveling process is less efficient because of the two parts, which probably increases the exit time and the number of reshuffles, when compared to Experiment 3.

Regarding the mixed or unmixed version of the algorithm, we expect there to be very little difference (if any at all) between both results when using a big stack. This is because there will likely not be a need for any sea–sea containers to be put in the “land” part, especially with low penalties, and even if there was a need, there is plenty of space. In small stacks, on the other hand, there will probably be larger differences, since a lack of space is far more an issue. We cannot predict in advance which version is best, because they both have their advantages and drawbacks. Mixed segments leave more room for the sea–sea containers, but this goes at the expense of land containers.

Thus, allowing sea–sea containers in the land part offers some extra possibilities, which may be needed in a small stack. However, it could also limit the options to stack sea–land containers, which could undo this. Generally speaking, a high reshuffling penalty will level out the stack, and thus also increase the number of sea–sea containers in the land part. Conversely, there should be very little difference in the results of allowing and not allowing the mix, when a low penalty (which encourages high piles) is used.

Our hypotheses for TVR-PA are:

Hypothesis 4.1 The best TVR-PA stacking algorithm will have a lower exit time than the best TVR.

Hypothesis 4.2 Allowing sea–sea containers in the land part in the TVR-PA stacking algorithm will lead to more reshuffles for smaller stacks (compared to not mixing).

**Table 3** Results of experiment 3 versus 4

Exp.	Description	ROC %	RDC %	GPU %	ASC %	ETQ (h)	ETL (h)	90% ETQ	90% ETL
0	LEV	62.20	80.60	99.35	63.17	0.84	0.53	2.52	1.44
0	RS	69.32	134.11	84.68	69.41	1.83	1.15	5.48	3.64
3	TPRL	58.91	116.67	90.21	61.89	0.77	0.49	2.26	1.33
3	TVR (penalty 0.0)	45.64	92.03	89.30	45.66	0.21	0.16	0.38	0.28
4	TVR-PA (mixed, 0.0)	46.07	85.99	86.98	46.89	0.21	0.18	0.40	0.29
4	TVR-PA (unmixed, 0.0)	46.07	85.99	86.98	46.89	0.21	0.18	0.40	0.29
3	TVR (penalty 0.03)	45.12	64.38	99.52	47.20	0.19	0.17	0.34	0.26
4	TVR-PA (mixed, 0.03)	44.42	64.36	98.72	47.43	0.20	0.17	0.37	0.28
4	TVR-PA (unmixed, 0.03)	43.89	64.91	98.74	47.14	0.20	0.17	0.35	0.27

Stack layout:  $6 \times 34 \times 6 \times 4$ . The “90%” values are the average 90% percentile values

**Hypothesis 4.3** Allowing sea–sea containers in the land part in the TVR-PA stacking algorithm will lead to longer exit times (compared to not mixing).

### 7.4.3 Results

The results for Experiment 4 with the stack configuration that is used throughout this paper are in Table 4. Furthermore, we have also done some experiments with a stack that has a higher capacity and thus a lower utilization; see Table 3 for the results of this experiment for a relatively big 6 lane, 4 high stack.

In the case of a small stack, the TVR-PA algorithm seems to perform slightly worse compared to normal TVR. Both exit times and reshuffles are up, as well as ASC workload. In the table two penalty values are shown, but these results also appear with other penalties.

With a larger stack, however, the results are less clear. With a low penalty, reshuffles are up, but with a high penalty they are down, all compared to the TVR equivalent. Normal TVR’s exit times appear to be slightly lower than those of TVR-PA, but the differences are minute and well inside the 95% confidence intervals. ASC workloads also differ slightly, with a slightly lower value for normal TVR.

In both setups, there are differences between the results for mixed (i.e. allowing sea–sea containers to be stacked in the “land” part) and unmixed TVR-PA. Unmixed exit times are generally lower than mixed. These differences are small, but (for the small stack) outside the 95% confidence interval limits, so they are significant.

### 7.4.4 Discussion

When using TVR-PA, it appears that not mixing the two parts of the stack is best. The sea–sea containers in the land section take up much valuable space and also have to move further to get there. Whether to mix or not to mix only matters when space is tight. With the larger stack the algorithm almost never puts sea–sea containers in the land section.

TVR-PA is, compared to TVR, almost the same in terms of results. As predicted, TVR performs relatively best in a small stack, because the leveling part of the algorithm has to go up another stacking level a bit sooner. For larger stacks, it appears normal TVR still holds a tiny advantage. We can therefore not say that TVR-PA is better than TVR.

TVR-PA does not lead to improvements, compared to TVR. Mixing TVR-PA is not a good idea, because it increases exit times.

We infer the following from these experiments:

Hypothesis 4.1: Rejected.

Hypothesis 4.2: Rejected.

Hypothesis 4.3: Confirmed.

### 7.5 Experiment 5: TVR with departure time classes (TVR-DTC)

In the past two experiments, any knowledge about departure times was ignored. In this experiment, we put it back into the equation, to combine the two ideas of using residence time knowledge and calculating the costs and reshuffle probabilities of possible locations. We use the departure time classes idea from section 7.2.

Departure time classes are pretty easy to use with the existing TVR algorithm. With the classes, we can more accurately estimate the number of expected reshuffles, which leads to a better calculation of costs for each pile, which, in turn, leads to better stacking decisions.

One major difference with the original TVR algorithm that we will have to make, is that it is now no longer necessarily optimal to stack on the closest positions available near the transfer points. This is because piles further away may have more favorable departure times and lead to less reshuffles. We thus have to check every position from the transfer point on further down the lane. We can only stop once we have found a pile where stacking would lead to no extra reshuffles, for every possible stacking level. This will make the algorithm much slower.

In addition, Eq. 2 is not valid for this algorithm, because we no longer assume that every pile has the same probability of being chosen. Rather, the probability of an extra reshuffle for every pile is determined using the following algorithm:

1. Set  $c_{\min}$  as the earliest departure class in the current pile and  $c_{\text{new}}$  as the departure class of the container to be stacked.
2. if  $c_{\min} > c_{\text{new}}$ , there is no reshuffle ( $P(\text{nr}) = 0$ ).
3. else if  $c_{\min} < c_{\text{new}}$ , there is definitely a new reshuffle ( $P(\text{nr}) = 1$ ).
4. If the classes are equal, count the number of times the class occurs in the current pile as  $n$ . Then  $P(\text{nr}) = \frac{n}{n+1}$ .

We have added a variation of this TVR-DTC algorithm (TVR-DTC-MD) in which we also incorporate the minimization of the difference in departure time class between levels of the pile.

#### 7.5.1 Experimental setup

We use the same setup as Experiment 3 with the classes of departure times from Experiment 2 (see Table 1).

### 7.5.2 Hypotheses

Hypothesis 5.1 The best TVR-DTC stacking algorithm will have lower exit times, reshuffles and ASC workloads than the best TVR algorithm.

Hypothesis 5.2 The best TVR-DTC stacking algorithm will have lower exit times, reshuffles and ASC workloads than the best LDT-DTC algorithm.

Hypothesis 5.3 The TVR-DTC-MD stacking algorithm will have lower exit times, reshuffles and ASC workloads than the TVR-DTC algorithm.

Since this algorithm combines “the best of both worlds”, in this case of the two ideas we use to improve stacking efficiency (LDT and TVR), we expect it to perform better than the two ideas individually, when comparing the “best” penalties for both algorithms. For other penalties, this may not be the case, especially for negative penalties. This is because the residence time knowledge allows us to make better estimates of reshuffle probabilities, which will lead to a very high number of reshuffles, since these penalties favor reshuffles, rather than penalize them.

### 7.5.3 Results

The results in Table 4 show that TVR-DTC outperforms normal TVR and LDT-DTC. The TVR-DTC-MD variation further improves the exit times; this combination of features provides the best performance for this stack configuration. However, we have also performed this experiment for the larger  $8 \times 34 \times 6 \times 4$  stack (Table 5) and in that case neither TVR-DTC nor TVR-DTC-MD improve upon LDT-DTC.

### 7.5.4 Discussion

Apparently, the combination of TVR and LDT-DTC is a good idea for a relatively full stack. The TVR-DTC-MD algorithm displays better performance still, which indicates that minimizing the difference in classes between levels of the piles is worthwhile. For a larger stack, the performance of LDT-DTC and TVR-DTC is very similar. Here the TVR-DTC-MD algorithm does not show an advantage over the TVR-DTC algorithm. From this, we conjecture that there is little room for further improvement of these algorithms and that the larger stack that was evaluated in these experiments does not highlight the differences. From Experiment 5 we conclude:

Hypothesis 5.1: Confirmed.

Hypothesis 5.2: Rejected.

Hypothesis 5.3: Rejected.

## 8 Conclusion

In this paper we have evaluated the performance of a number of online stacking strategies. We have used data from practice to generate scenarios of container movements for an automated container terminal. These scenarios were then processed by a simulation model in which the various stacking strategies were implemented. The main results are listed in Tables 4 and 5.

**Table 4** Results of experiments 0–5

Exp.	Description	ROC %	RDC %	GPU %	ASC %	ETQ (h)	ETL (h)	90% ETQ	90% ETL
0	LEV	62.92	81.10	99.60	59.44	0.48	0.33	1.30	0.76
0	RS	69.44	107.69	89.56	62.70	0.73	0.47	2.09	1.23
1	RS-DT (real)	35.27	52.10	82.94	54.44	0.37	0.26	0.94	0.56
1	RS-DT (exp)	38.11	55.53	82.81	54.80	0.40	0.28	1.04	0.61
1	LDT (unmixed, real)	10.20	15.21	87.39	49.93	0.21	0.17	0.42	0.30
1	LDT (unmixed, exp)	17.49	23.19	87.12	50.61	0.24	0.19	0.51	0.35
1	LDT (mixed, real)	9.09	13.62	86.64	49.62	0.22	0.18	0.44	0.32
1	LDT (mixed, exp)	17.12	22.52	86.37	50.52	0.25	0.20	0.54	0.38
2	RS-DTC (real)	68.28	101.06	87.54	61.26	0.73	0.47	2.20	1.21
2	RS-DTC (exp)	67.59	99.87	87.44	61.17	0.72	0.46	2.18	1.18
2	LDT-DTC (unmixed, real)	39.83	62.13	96.72	56.53	0.53	0.34	1.59	0.73
2	LDT-DTC (unmixed, exp)	38.15	58.97	96.45	56.24	0.50	0.32	1.46	0.69
3	TPRL	62.82	95.35	96.41	57.78	0.43	0.29	1.13	0.66
3	TVR ( $p = -0.03$ )	80.01	131.68	75.85	57.90	0.66	0.42	1.87	1.09
3	TVR ( $p = -0.01$ )	64.85	102.93	96.68	55.87	0.43	0.29	1.11	0.66
3	TVR ( $p = 0$ )	62.24	94.95	99.35	57.01	0.40	0.28	1.02	0.62
3	TVR ( $p = 0.03$ )	63.71	86.50	99.65	57.57	0.40	0.28	1.00	0.61
3	TVR ( $p = 0.04$ )	63.52	84.76	99.65	57.61	0.40	0.27	1.00	0.60
4	TVR-PA (unmixed, $p = -0.03$ )	77.73	126.89	76.00	56.86	0.56	0.38	1.54	0.92
4	TVR-PA (unmixed, $p = 0$ )	63.41	95.58	98.70	57.11	0.41	0.28	1.07	0.62
4	TVR-PA (unmixed, $p = 0.03$ )	64.80	90.13	99.48	57.43	0.40	0.28	1.03	0.62
4	TVR-PA (mixed, $p = -0.03$ )	77.34	126.12	75.94	56.97	0.57	0.38	1.57	0.94
4	TVR-PA (mixed, $p = 0.0$ )	63.23	95.49	98.66	57.13	0.41	0.28	1.07	0.63
4	TVR-PA (mixed, $p = 0.03$ )	63.52	90.60	99.50	57.69	0.42	0.28	1.08	0.63
5	TVR-DTC ( $p = 0$ )	32.62	47.44	95.20	52.28	0.28	0.21	0.65	0.39
5	TVR-DTC ( $p = 0.01$ )	30.19	43.26	97.43	52.37	0.28	0.21	0.67	0.39
5	TVR-DTC ( $p = 0.03$ )	30.82	43.68	97.91	52.63	0.30	0.22	0.74	0.41
5	TVR-DTC-MD ( $p = 0.01$ )	22.78	29.88	91.60	50.65	0.19	0.17	0.38	0.30

Stack layout:  $6 \times 34 \times 6 \times 3$ . The “90%” values are the average 90% percentile values.  $p$  is used to indicate the values for the reshuffle penalty  $TT_r$

The relatively simple, greedy strategies that were evaluated in this paper provide more insight into the basic trade-offs for stacking at an automated container terminal. The strategies operate in an online mode: the stacking location is selected on the basis of the current state of the stack and the parameters of the incoming container. We do not consider the stream of containers that will follow it.

Using detailed simulation experiments we have evaluated stacking rules from two perspectives. On the one hand we have investigated stacking rules that are based on departure time information. As a reference case, precise information on the actual time of departure was used. For more realistic rules, we have considered expected

**Table 5** Results of experiments 0–5 for stack layout:  $8 \times 34 \times 6 \times 4$ 

Exp	Description	ROC %	RDC %	GPU %	ASC %	ETQ (h)	ETL (h)	90% ETQ	90% ETL
0	RS	65.77	118.35	76.14	54.38	0.41	0.30	0.99	0.61
0	LEV	52.63	52.64	99.54	48.53	0.21	0.19	0.38	0.31
1	RS-DT (real)	14.16	27.44	53.92	43.15	0.17	0.16	0.31	0.26
1	RS-DT (exp)	19.43	33.69	53.79	43.67	0.19	0.17	0.34	0.28
1	LDT (unmixed, real)	0.04	0.07	54.88	39.68	0.13	0.12	0.20	0.19
1	LDT (unmixed, exp)	10.55	11.97	54.62	40.42	0.14	0.13	0.22	0.20
2	RS-DTC (real)	51.82	97.88	68.32	51.29	0.35	0.26	0.82	0.50
2	RS-DTC (exp)	51.02	96.70	68.03	51.19	0.34	0.25	0.77	0.48
2	LDT-DTC (real)	1.94	3.97	75.64	42.06	0.13	0.13	0.21	0.20
2	LDT-DTC (exp)	3.64	5.49	75.54	42.16	0.13	0.13	0.21	0.20
3	TVR ( $p = 0$ )	47.68	96.34	85.22	45.85	0.21	0.18	0.40	0.30
5	TVR-DTC ( $p = 0$ )	11.74	20.35	74.96	41.51	0.14	0.13	0.22	0.20
5	TVR-DTC ( $p = 0.01$ )	1.16	1.25	81.61	41.22	0.13	0.13	0.20	0.19
5	TVR-DTC ( $p = 0.03$ )	0.02	0.02	82.22	41.24	0.13	0.13	0.20	0.19
5	TVR-DTC-MD ( $p = 0.01$ )	8.21	10.49	67.59	41.09	0.13	0.13	0.20	0.20

The “90%” values are the average 90% percentile values

departure times and departure time classes. For the latter, we have used the departure times to formulate a number of classes and then used this class information to evaluate potential stacking locations. The aim is to create the piles in such a way that the higher containers have a lower class (i.e., will depart sooner) than the containers below them. The best performance is achieved if every container has a class that directly precedes the class of the container below it.

On the other hand we have looked into the trade-off between the travel time of the stacking crane versus the probability of reshuffles. We aim to stack an incoming container as fast as possible but we are willing to accept a longer use of the stacking crane if we can reduce the probability of future reshuffles.

We have found that rules with a limited number of classes for the remaining residence time work very well. The experiments in this paper show that, for the larger stack, the algorithms that use these classes perform similar to algorithms that use the exact departure times. From this we conclude that even imprecise information on this departure time is very valuable.

The performance of the travel time versus reshuffling stacking rules shows a clear advantage with respect to the reference stacking rules. We have formulated an extension of this stacking rule to attempt to reduce the exit time during future peak workload periods but we found no significant improvement. A combination of the travel times versus reshuffling stacking rule with the departure time classes was also evaluated. Experiments with this rule confirm that it is beneficial to create piles of high quality, i.e., piles where the difference in departure time class between levels is exactly one.

We have tested the strategies using a small number of variations of the basic layout (in terms of the number of lanes, the lane width and length, and the maximum stacking height). The overall layout of the yard has been the same for all experiments and it would be interesting to evaluate these simple rules for other basic layouts such as lanes that are parallel rather than perpendicular to the quay or with multiple ASC's per lane. The experiments in this paper have been limited to the standard twenty foot container; further research is needed to investigate the performance of the online rules for heterogeneous container types (40-foot reefers). Another interesting direction for future research would be to differentiate the uncertainty regarding residence times for the various modes of onwards transport as the uncertainty regarding sea–land transports is typically higher than the uncertainty regarding sea–sea transports. Finally, a comparison of these greedy, online stacking rules with optimization approaches that do look ahead, such as strategies that process the containers of an entire ship at a time, would be interesting.

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