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European Economic Review 48 (2004) 563–593
EUROPEAN ECONOMIC REVIEW
www.elsevier.com/locate/econbase

The political economy of regionalism

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Received 17 January 2000; accepted 8 March 2002

Abstract

We examine the incentives of regions to unite and separate. Separation allows for greater influence over the nature of political decision making while unification allows regions to exploit economies of scale in the provision of government. Our paper explores the influence of size, location and the diversity within regions in shaping this trade-off. We then examine the way in which alternative political institutions aggregate regional preferences and thereby define the number of countries.

JEL classification: D7; H2; H7

Keywords: Diversity; Referendum; Secession; Unequal union

1. Introduction

In the period between 1946 and 1997 the number of nations increased from 74 193. Many of these countries were born out of the decolonization process in Africa and in the rest of the world. During the same period, we witnessed a move towards greater integration in Europe, accompanied by lowering of boundaries between countries. In addition, during this period, more than 20 boundaries between nations were changed, without creating or eliminating a nation.[1] More recently, referenda have been held in many countries and these have resulted in substantial changes in boundaries (as in Northern Ireland, Scotland and Wales in Great Britain and East Timor in Indonesia).

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[1] See The Times Atlas (1993, Plate 8) for a survey map on border changes and changes in sovereignty since 1945 and Alesina et al. (2000) for more data on country formation since 1870.

doi:10.1016/S0014-2921(02)00329-X

In this paper we develop a framework to examine the incentives of regions to separate and unite. We then examine the social welfare implications of voting decisions made on the basis of regional preferences.

Our framework has the following features: there are two regions which can choose to be independent countries or to unite and form one country. Regional preferences are derived on the basis of majority voting in each region. If there is disagreement between the regions then the two regions separate. After the decision on unification and separation, individuals in each political territory choose the type/location of government they want to have. This determines, for example, where the capital, the national airport, the universities and other facilities are located. In this spirit, location choices may be interpreted geographically. The individuals living close to the capital then have the highest payoff. The model also permits an interpretation in terms of individual preferences more generally.[2] We suppose that a country requires a government and that there is a fixed cost of this government. This assumption generates a trade-off: separation allows individuals within a region to exercise greater influence in political decision-making, while union allows them to exploit economies of scale in the provision of government.[3]

We *first* examine the role of regional size in shaping this trade-off. Our finding is that *unification takes place between similar sized regions* (Proposition 3.2). This result arises out of the different ways in which the political costs of unification compare with the tax advantages. In particular, political costs of unification vary linearly with the size of the other region, while the tax advantages are increasing and convex in the size of the other region. This implies that relative to the costs the gains from unification increase for the small region as it gets smaller; on the other hand, the gains from unification decline for the larger region as the small region becomes smaller. Thus large regions are reluctant to form unions with small regions while small regions are keen to form unions. One implication of these preferences is that unification only occurs if costs of government are relatively high and if regions are roughly of the same size. Moreover, small regions are more in favor of unification as compared to large regions.

We next examine the social welfare implications of decisions made on the basis of these regional preferences. Our main finding is that *majority voting leads to excessive separation from a social point of view* (Proposition 3.4). The excessive incentives arise out of the way the costs and benefits of unification are distributed and the externalities this generates. The costs of separation in terms of higher per capita tax rates are borne equally by individuals in a region. On the other hand, the benefits of separation depend on an individual's location. Individuals located close to the

[2] In the latter interpretation, individuals who are close to each other are assumed to have the same preferred type of government. Governments located far from an individual differ more from the preferred type of government of this individual. The choice of the type of government, for example, can determine which social security system will prevail. The people who prefer the prevailing social security system have a higher payoff than other individuals.

[3] This trade-off as well as some other features are similar to the model presented in Alesina and Spolaore (1997). We discuss the relationship of our paper with their work in detail below.
Alesina and Wacziarg (1998) present empirical evidence that supports the existence of scale effects in the provision of government.

boundary between the regions loose relatively more from separation while individuals away from the boundaries gain more from separation. Thus an individual's vote on unification/separation generates externalities on other voters; in particular, our analysis shows that the voting rule tends to under-represent the interests of the former set of voters and that this leads to excessive separation. The rest of the paper explores the influence of different features of the regions and the political institutions on these findings.

We start by examining how diversity within the regions affects the basic trade-off between unification and separation identified above. Our *first* observation is that given a size configuration of the regions, clustering in the large region makes unification more attractive for the large region but less attractive for the small region. The intuition for this comes from noting that tax benefits are independent of distribution, while the political costs vary with change in location of the government under unification. If preferences are clustered around the median voter in the large region then government will not move much under unification. Hence the political cost is modest and the large region prefers unification. The *second* observation is that the distribution of preferences in the smaller region is essentially irrelevant for the trade-off between union and separation (Propositions 4.1–4.3). These findings suggest that individuals in a large region may be more willing to form a union with small regions if they are themselves clustered/concentrated, while small regions will be less eager to form a union with a large region in spite of the tax advantages, if this is the case.

We then examine the strategic role of regional location. We suppose that there are three regions located on the unit interval. We are able to completely solve the model for the case where the corner regions are of the same size. In this setting, we find that *irrespective of the sizes of the different regions, the central region's most preferred alternative is a union between all three regions*. This is because a union delivers tax advantages while there are no political costs since the regions on the corners are of equal size and the location of the government remains unchanged under unification. This preference in combination with the incentives of very small and relatively equal sized regions to unite (from the basic model) yields us the outcome that *the three regions unite and form one country if the size of the corner regions is very small as well as when it is relatively large*.

One of the main findings in the basic model is that majority voting generates excessive incentives for separation. We next examine the role of alternative political arrangements in mitigating this inefficiency. Recall that in the basic model, political outcomes are based on majority voting in each region followed by separation in case of disagreement between the regions. We *first* explore the effects of this default option. We now suppose that the outcome in case of disagreement is unification. When we apply this default outcome, we find that *the excessive incentives for separation persist* (Proposition 5.1).[4] It is possible that a nation-wide referendum could be better at internalizing the externalities discussed earlier. This motivates an exploration of a nation-wide referendum. We find, however, that *the outcome under a nation-wide*

[4] Inefficient outcomes under majority voting have also been pointed out in other contexts; see Besley and Coate (1998) for a general analysis of inefficient outcomes under majority voting in the context of repeated elections.

referendum is the same as in the case where separation only takes place when both regions agree on separation (Proposition 5.2). Excessive separation thus obtains under these alternative voting rules as well. If there are only a few individuals in the small region then the tax burden on these individuals is very heavy. One way out of the outcome of separation would be for the small region to accept unequal or unconditional union. Under this arrangement, voters in the large region decide the location or type of the government, but people in both regions pay for it. We note that under this rule, a large region is always in favor of union. However, the small region is willing to pay the political costs only if it is sufficiently small. This suggests that *unequal union is likely to take place if the regions are of very different sizes*.

We finally examine the stability of different political arrangements and their normative appeal. Our main finding is that within the class of majority voting rules investigated, *the voting rule with two referenda* (*one in each region*), *supplemented with union as the default outcome*, *is stable—in the sense that it is chosen in a vote among different voting rules—as well as normatively appealing*.

Our paper is a contribution to the study of country formation and secession. Inspired by the recent redrawing of boundaries in Eastern Europe and the former Soviet Union, there has been considerable interest in these issues in recent years, see e.g. Alesina and Spolaore (1997), Bookman (1993), Bolton and Roland (1993), Casella and Feinstein (1990), Dagan and Volij (2000), Ellingsen (1998) and Wei (1991a, b).[5] In particular, our paper is closely related to the paper by Alesina and Spolaore (1997) and Wei (1991a, b).

Alesina and Spolaore (1997) study the influence of different factors, such as the level of market integration and democratization, in determining the number and size of countries. They use the same trade-off as we use: the economic advantages of unification are compared with the political costs of a government which is located further away in a larger country. In this setting, they find that democratization leads to an inefficiently large number of countries. In their analysis, the boundaries between nations are endogenous but they restrict attention to outcomes with equal sized countries. By contrast, in the present paper the focus is on different features of the regions, such as size, location and their internal diversity in shaping the trade-off noted above. We are particularly interested in the role of initial asymmetries along these dimensions. To focus on these factors, and to keep the model tractable we assume that the boundaries of the regions are exogenously specified.[6] The assumption of exogenous boundaries

[5] This recent political economy work is related to the local public good literature and the literature on fiscal federalism (Bolton et al., 1996). For the local public good theory, see Austin (1993), Benabou (1993), Bewley (1981), Epple and Romer (1991), Jehiel and Scotchmer (2001), Rubinfeld (1987), Scotchmer (1996), Stahl and Varaiya (1983) and Tiebout (1956). For literature on fiscal ferderalism, see Oates (1972), Persson and Tabellini (2000) and Wildasin (1988).

[6] While tractability is the primary motivation for our formulation, it is worth mentioning that in some cases pre-existing borders do play an important role when interstate borders are redrawn. For instance, when the Soviet Union broke up, Belarus and Moldavia became independent partly because they already existed as states within a federal union. Similarly, when Italy unified, decisions on unification were made separately in the Bologna region and in Tuscany because these regions existed as separate political units. We are grateful to an anonymous referee for suggesting these examples as a motivation for the assumption of exogenous boundaries.

also allows us to examine in detail the impact as well as the stability of alternative political institutions.

Wei (1991a, b) examines a model in which the size of the regions is exogenously specified. Moreover, he allows for the level of a public good in a nation to vary depending on the level of economic development and the size of the nation. The trade-off in his model is between the higher efficiency of the public good under union and the lower coordination costs under separation. Our analysis differs from Wei's in that we consider a fixed-costs public good and this leads to a very different trade-off: we compare the efficiency gains in terms of one as against two governments with the political costs of greater distance to the government. In addition, we also study the nature of socially desirable outcomes.

The paper is organized as follows. In Section 2 we present the basic model. Section 3 presents the outcomes under majority voting as well the socially optimal outcomes. Section 4 examines diversity within regions and the strategic role of regional location, while Section 5 explores the impact of alternative political arrangements. Section 6 concludes.

2. The basic model

We suppose that one public good identifies a nation (i.e. a country); we call this public good the 'government'. The range of all possible governments is normalized in the segment $[0,1]$. The location of a government is denoted by l. In addition, we assume that the total population has mass one and that individuals from this population are located at ideal points, which indicates their preferred government. The individuals are uniformly distributed on the segment $[0,1]$. The utility of each individual is decreasing with the distance from his government to his location (i.e. his ideal point). The distance between the ideal point of a consumer i and the government in his country is denoted by d_i.

We assume that there are two regions with a fixed (exogenous) boundary α. The region located on the left-hand side of α is called region A, while the region on the right-hand side of α is called region B. We suppose that $0 < \alpha < \frac{1}{2}$. We assume that there is a fixed cost F per country, regardless of its size.[7] This F includes for example the costs of building airports and hospitals and the costs of having a machinery of government. In the basic model every individual has the same, exogenous income y, and pays the lump-sum tax t_i.[8] Now, we can define the utility function for each

[7] When the costs of a government depends on the size of the country, we could model the costs as $F = f + \zeta s$ where s denotes the size of the country. We conjecture that, as long as f is positive, our main results will carry over.

[8] Proportional taxation with different tax levels across regions is not sustainable when the subject of taxation (e.g. capital or labor) is mobile in a union. In the model with exogenous income levels which are equal across the regions lump-sum taxation is equivalent to proportional taxation. We assume that individual wealth is equal in the two regions. We examine the case of unequal wealth across regions in Appendix B.

individual i as follows:

$$U(i) = g(1 - ad_i) + y - t_i, \tag{1}$$

where g and a are two positive parameters. The parameter g measures the utility of the public good when the preference distance d_i is zero and the parameter a measures the loss in utility if the government is farther away (i.e. when d_i increases). The utility function is thus linear in the preference distance. We assume that $a < 1$, which ensures that a higher g increases utility. The parameter a can then be interpreted as the marginal utility of a government located at a distance d_i. We look at the incentives for separation and unification under majority voting and we assume that separation occurs if a majority of voters is in favor of separation in at least one region.

3. Regional incentives and the social optimum

In this section we will first examine the outcomes when decision to form one or two countries is taken by majority voting and then we will derive the socially optimal number of countries.

Majority voting: We first observe that if α is very small then the per capita cost of supporting an independent government, F/α, becomes very large and the individuals in region A will benefit significantly from unification. Hence small regions will typically prefer to have a union. The individuals in region B also compare the benefit of a lower tax rate under unification with the disadvantage of a change in the location of the public good under unification. This comparison depends in turn on how political costs and the tax advantage varies as the size of regions varies. Our analysis of these issues is summarized in Proposition 3.1. We define $\alpha_A = 2F/ga$ and $\alpha_B = 1 - 2F/ga$.

Proposition 3.1. *Region A prefers unification if and only if $\alpha < \alpha_A$ while region B prefers unification if and only if $\alpha > \alpha_B$. Thus unification only takes place if $\alpha_B < \alpha_A$ and $\alpha \in [\alpha_B, \alpha_A]$.*

We first observe that the preferences of the person in the center of a region reflect the majority opinions in each region: *In a region, there is a majority in favor of separation if and only if the median voter in that region prefers separation*. This is a direct implication of the median voter theorem.[9] The incentives for unification and separation can therefore be derived by comparing the payoffs of the median voter in each of these cases. These computations are now presented. Let $U_{\mathrm{I}}(i)$ be the utility of voter i under unification, and let $U_{\mathrm{II}}(i)$ be his utility under separation.

[9] In our setting, preferences over the location of the public good are single-peaked and the policy space is single-dimensional. It follows that in case of separation the public good will be located at the mid-point of the region, while in case of unification, the public good will be located at the point $\frac{1}{2}$. It is now straightforward to check that if the median voter (say) in region A (who is located at $\alpha/2$) prefers separation then all voters in the interval $[0, \alpha/2]$ will prefer separation. Likewise, if the median voter prefers unification then all voters in the interval $[\alpha/2, \alpha]$ will prefer unification.

Proof of Proposition 3.1. There will be a majority in favor of unification in region A if the median voter $\alpha/2$ prefers unification:

$$U_{\mathrm{II}}\left(\frac{\alpha}{2}\right) = g + y - \frac{F}{\alpha} < g\left(1 - a\left|\frac{1}{2} - \frac{\alpha}{2}\right|\right) + y - F = U_{\mathrm{I}}\left(\frac{\alpha}{2}\right). \tag{2}$$

That is, if

$$\alpha < \frac{2F}{ga} = \alpha_A. \tag{3}$$

There is a majority in favor of unification in region B if the median voter $(1+\alpha)/2$ prefers unification:

$$U_{\mathrm{II}}\left(\frac{1+\alpha}{2}\right) = g + y - \frac{F}{1-\alpha} < g\left(1 - a\frac{\alpha}{2}\right) + y - F = U_{\mathrm{I}}\left(\frac{1+\alpha}{2}\right). \tag{4}$$

That is, if

$$\alpha > 1 - \frac{2F}{ga} = 1 - \alpha_A = \alpha_B. \tag{5}$$

This completes the proof. □

Our interest is in the relationship between the size of a region and the incentives for unification and separation. The incentives for unification depend on the magnitude of the tax advantage as against the costs of political distance. The above computations show that the two effects—the political costs and the tax advantages of unification—do not influence utility in the same way. In particular, the tax advantage from unification is $F/\alpha - F$ for Region A, and $F/(1-\alpha) - F$ for Region B. It follows that the tax advantage from unification for region B is increasing and convex in the size of region A. Fig. 1 illustrates this aspect of the trade-off.

This figure allows us to derive the outcomes under majority voting. We summarize them in the following result.

Proposition 3.2. *The outcomes under majority voting are given as follows*: (a) *If* $F < ga/4$ *then the regions remain independent for all* $\alpha \in [0, \frac{1}{2}]$, (b) *if* $ga/4 < F < ga/2$ *then there is unification if and only if* $\alpha \in [\alpha_B, \frac{1}{2}]$, *and* (c) *if* $ga/2 < F$ *then there is unification for all* $\alpha \in [0, \frac{1}{2}]$.

This Proposition says that if costs of having a government are very small or very large then the political outcomes are independent of the size of the two regions: in case they are very small the regions remain independent, while if costs of having a government are large they form a union. The role of size surfaces if the costs of having a government are at an intermediate level, where $ga/4 < F < ga/2$, since in this case union occurs for $\alpha \in [1 - 2F/ga, \frac{1}{2}]$. Note that the expression $1 - 2F/ga$ is decreasing with respect to F. A rise in the costs of having a government will therefore make unification more likely. If the maximum benefit from the government, g, increases then

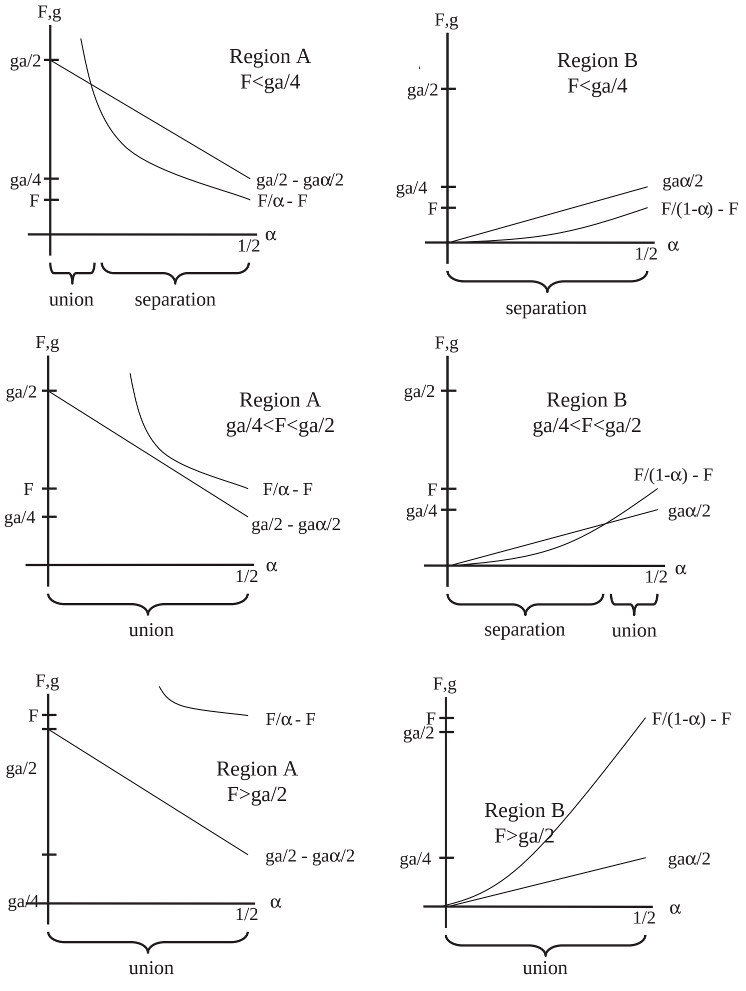

Fig. 1. Majority voting outcomes.

$1 - 2F/ga$ will also increase and unification becomes less likely. The same holds for an increase in the preference intensity a.

The social optimum: It is socially optimal to have two independent nations when the gain due to lower political costs outweigh the additional costs of having two governments. It is therefore only socially optimal to have two independent nations when the fixed costs of the public good are low. If the small region is very small, then there will be only a few individuals in the small region which will benefit from a separate government. It is therefore only socially optimal to have two independent nations if the two regions are both of reasonable size. These considerations are summarized in the next proposition. Let $F_{SP} = ga/8$ and $\alpha_{SP} = \frac{1}{2} - \sqrt{\frac{1}{4} - 2F/ga}$.

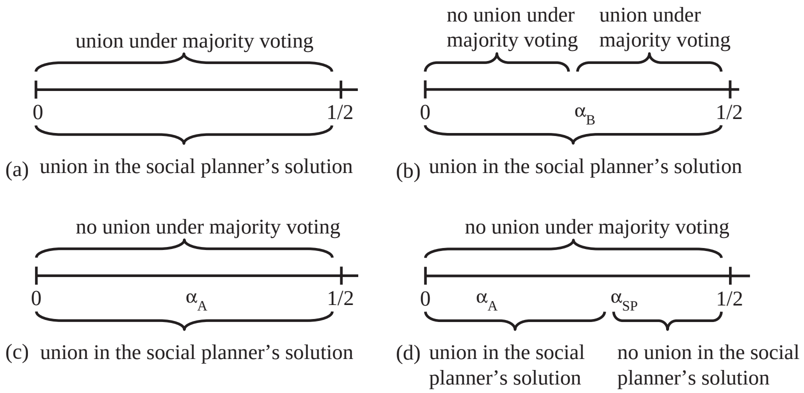

Fig. 2. Majority voting and social optima. (a) $F > ga/2$; (b) $ga/4 < F < ga/2$; (c) $ga/8 < F < ga/4$; (d) $F < ga/8$.

Proposition 3.3. *If $F < F_{SP}$ then union is optimal if and only if $\alpha < \alpha_{SP}$. If $F > F_{SP}$ then union is the unique optimal outcome.*

The computations are provided in Appendix A. We now turn to a comparison between the outcomes under majority voting and the socially optimal outcomes.

Majority voting and social optimum compared: A comparison of the outcomes under majority voting and the social optima reveals:

Proposition 3.4. (i) *If $F < ga/8$ then unification is socially optimal for all $\alpha \in [0, \frac{1}{2} - \sqrt{\frac{1}{4} - 2F/ga}]$ but separation obtains under majority voting for all $\alpha \in [0, \frac{1}{2}]$*, (ii) *if $ga/8 < F < ga/4$ then unification is socially optimal for all $\alpha \in [0, \frac{1}{2}]$ but separation obtains under majority voting for all $\alpha \in [0, \frac{1}{2}]$*, (iii) *if $ga/4 < F < ga/2$ then unification is socially optimal for all $\alpha \in [0, \frac{1}{2}]$ but separation obtains under majority voting for all $\alpha \in [0, 1 - 2F/ga]$, and* (iv) *if $F > ga/2$ then union is socially optimal as well as the majority voting outcome for all $\alpha \in [0, \frac{1}{2}]$.*

These results are illustrated in Fig. 2.[10] The main finding of the above result is that there exist excessive incentives for separation under majority voting: for certain parameter values majority voting obtains separation but the socially optimal solution is unification. We now elaborate on the sources of this inefficiency.

The excessive incentives arise out of the way the costs and benefits of unification are distributed and the externalities this generates. The costs of separation, $F/\alpha - F$ for the small region and $F/(1-\alpha) - F$ for the large region, are borne equally by the individuals

[10] Note that $F < ga/8$ implies $2F/ga < \frac{1}{2} - \sqrt{\frac{1}{4} - 2F/ga}$ and $1 - 2F/ga > \frac{1}{2} - \sqrt{\frac{1}{4} - 2F/ga}$ (and thus $2F/ga > 1 - 2F/ga$), that $F > ga/4$ implies $1 - 2F/ga < 2Fga$ and that $F > ga/2$ implies that $2F/ga > 1$ and $1 - 2F/ga < 0$ (and thus $1 - 2F/ga < 2F/ga$).

in each region because of the lump-sum taxation system. On the other hand, the benefits of separation to an individual depend on this location. The individuals located close to the boundary between the regions will loose the most from separation, and in both regions the individuals away from the boundary gain the most from separation. The aggregate increase in the payoff of these individuals (located towards the corners), however, is less than the aggregate decrease in the payoff of the individuals located close to the boundary between the two regions.

The analysis of the basic model yields us two principal insights. *Firstly*, that the large region typically is less keen on unification as compared to the small region. This result is due to the fact that the tax advantages from unification are increasing and convex in the size of the other region, while the political costs are linear. Thus a union with a very small region fails to generate adequate tax advantages for the large region (in the interesting class of parameters). *Secondly*, we find that outcomes under majority voting typically lead to too much separation as compared to what is socially optimal. This is due the fact that whereas the tax advantages of unification are shared evenly by voters in a region, the political costs are unevenly distributed and this generates externalities which lead to inefficient outcomes. We will now discuss the empirical and normative implications of our findings.

Relation with empirical patterns: The main prediction of the basic model is that unification will only take place between similar sized regions and that there are situations in which a small region prefers unification while a large region is averse to unification and hence no unification takes place. How does this prediction square with empirical observation? Our observations pertain to unification among unequal sized regions. The *first* remark is that in many instances large regions are relatively more interested in unification as compared to small regions. One manifestation of this is the desire of regions to secede or seek greater autonomy from large countries, who in turn resist these attempts (examples of this include Kashmir and Assam from India, and Chechnya from Russia). This seems to go against the prediction of the model and motivates a closer examination of the assumptions of our model. One way to proceed is to allow a region to choose between independence, union with one country or union with the other country. It is possible, for example, to think of Northern Ireland as a part of Ireland or as a part of the United Kingdom, as it (still) is at present. It is unlikely, however, that Ireland and the United Kingdom will form a union. Another example is South Tirol (between Italy and Austria; in Italy this region is now part of Trentino Alto Adige). In Section 4 below we will explore the role of the distribution of preferences and the strategic role of regional location in shaping incentives for unification and secession. Our *second* observation pertains to the phenomenon of *unequal union*—a situation in which a small region contributes to the public good but has virtually no say in the policy making. There are several instances of this arrangement; for example, the Dutch central bank used to base its monetary policy on the German Bundesbank; presumably the costs of running an independent policy were too high. Other examples include small countries like Luxembourg being essentially passive members of NATO, Liechtenstein participating in a customs union with Switzerland, using the Swiss Francs as its national currency, and Puerto Rico whose citizens do not vote in the US presidential elections. Our finding that there are situations in which small regions may want union while large regions are averse to union provides an explanation for why such unequal union arise. We explore the scope of unequal union formally in Section 5 below. A *last* remark concerns wealth differences. It is widely argued that wealth and income differences are one of the main factors behind regional movements which seek secession. Some examples of this are Belgium (between Flanders and Wallonia), Italy (between north and south Italy), Catalonia (vis-à-vis the rest of Spain) and Norway (vis-à-vis the European Union). Our analysis of wealth differences is presented in Appendix B.

Normative aspects: Our results suggest that using majority voting in each region to make decisions on unification and separation may lead to socially suboptimal outcomes. This finding leads us to investigate alternative voting mechanisms. In the basic model the default outcome in case of disagreement between regions is separation; this outcome is realistic when in an initially unified nation, the central government is too weak or does not want to prevent secession through military means. An instance of this is the disintegration of the Soviet Union; it is arguable that parts of the country such as Russia did not favor secession, but it took place nonetheless because the central government could not prevent other regions from seceding. This voting rule is also relevant when two initially independent nations are considering political integration. In many cases, however, these conditions may not hold. There may, for example, exist an authoritarian regime in an initially unified nation which can stop secession supported by just one region but which cannot stop secession supported by both regions. In some institutional settings, democratic approvals of secessions require separate majorities in each region. For example, Aruba, an island in the Caribbean Sea is still a part of the Netherlands. The Dutch constitution allows changes in the status of Aruba if the governments of Aruba and the Netherlands both agree. Although there was at least a wish by Dutch politicians for an independent Aruba, this was blocked by the island. Alternatively a regime in an initially unified nation may want to prevent secession because secession may mean a loss of prestige or lower tax revenues. These considerations motivate the examination of a default rule that if the preferences of the two regions differ then the outcome is union. The final decision rule we consider is a nation-wide referendum. This rule is motivated by examples of countries where the parliament decides on the borders of individual states as well as on whether a region can be allowed to secede. If the parliament is elected in a nation-wide election, then we may interpret a national parliament deciding on the break up of a nation as analogous to a nation-wide referendum. An example of this is the vote on the break up of Czechoslovakia in the parliament.[11] The analysis of alternative rules and their stability is presented in Section 5.

[11] In fact, in the Czechoslovakia example there were three votes on the separation agreement: one in the Czech part, one in the Slovak part, and a third one in the Czechoslovakian parliament. In all votings there was a majority in favor of separation. Proposition 5.2 below may be interpreted as saying that either the vote in the Czechoslovakian parliament or the two votes in the Czech and the Slovak parliaments were superfluous.

4. The role of diversity and location

4.1. Diversity

One of the assumptions in the basic model is that the individuals are uniformly distributed over the unit interval. In general, populations are not uniformly distributed over regions and the preferences of individuals are typically clustered. This motivates the examination of the role of concentration of preferences. We note that an explicit model of clustering/concentration also allows us to distinguish between the influence of the size of a region and the diversity within the region.

We start with an examination of the incentives of the large region. The boundary between the two regions is at α and an α proportion of the individuals is in region A and the remaining $(1-\alpha)$ proportion of the individuals is in region B, as before. To keep matters simple, we shall suppose that individuals are uniformly distributed over the interval $[(1+\alpha)/2 - b/2, (1+\alpha)/2 + b/2]$, with $b < 1-\alpha$. When b is small, we have a high clustering/concentration of preferences, while for large b we approximate the case of uniform distribution.

If $b=0$, then all voters in region B are identical and located at the middle of region B, which is then also the location of the median voter. If $\alpha < \frac{1}{2}$, then it follows that the public good will be located at $(1+\alpha)/2$, both in case of separation as well as in case of unification. Thus there are no political costs and definite tax advantages from union for the large region and that therefore it will always prefer unification. We note that this preference is independent of the location of the median voter in the small region. The following result elaborates on the incentives of the large region more generally.

Proposition 4.1. *Suppose that $b < 1-\alpha$. Then there exists a cut-off cost of the public good $F_B(b)$ such that a majority of the individuals in the large region prefers unification if and only if $F > F_B(b)$. Moreover, $F_B(b) = 0$ at $b = 0$ and $F_B(b)$ is strictly increasing with respect to b. These preferences are independent of the distribution of preferences in the small region.*

Proof. For a fixed α, the median voter compares the tax benefit with the political costs of unification. The tax benefit $F/(1-\alpha) - F$ is independent of the distribution of individuals. The political cost of unification, however, depends on the distribution of individuals in the large region. Under separation, the public good is located at $(1+\alpha)/2$. Under unification, the public good will be located at

$$\frac{1+\alpha}{2} - \frac{b}{1-\alpha}\frac{\alpha}{2}. \tag{6}$$

Hence the political costs of unification are given by

$$\frac{b}{1-\alpha}\frac{\alpha}{2}ga. \tag{7}$$

The median voter in the large region therefore prefers unification if

$$F > \frac{gab}{2} = F_B(b). \tag{8}$$

Note that $F_B(b)$ is increasing in b and is equal to 0 at $b = 0$. The result follows upon noting that the computations are valid for any distribution of preferences in the small region, given a value of $\alpha < \frac{1}{2}$. □

We now study how the incentives for unification and separation in the smaller region depend on the distribution of preferences.

Proposition 4.2. *Fix a location of the median voter in the small region. Preferences over unification and separation in region A are independent of the distribution of preferences in that region.*

Proof. The tax benefit $F - F/\alpha$ of unification does not depend on the distribution of the individuals in the smaller region. Given the assumption on the size of the regions, that $\alpha < \frac{1}{2}$, the public good will always be located in the larger region. Moreover, the precise location of the public good will depend only on the distribution of individuals in the larger region and is independent of the distribution of individuals in the smaller region. For a fixed α and a given median voter, it follows that both the tax benefit as well as the political cost of union for an individual in the small region does not depend on the distribution of the individuals in the small region. The preferences over unification and separation in region A are therefore insensitive to the distribution of the individuals in region A. □

We next examine the impact of preference distribution in the large region on the incentives for unification and separation in the small region. Recall that b is a measure of diversity of preferences in the large region.

Proposition 4.3. *Let the median voter in the small region be located at $\alpha/2$. There exists a cut-off cost of the public good, $F_A(b)$, such that a majority of the individuals in the small region prefers unification if and only if $F > F_A(b)$. Moreover, $F_A(b)$ is strictly decreasing in b.*

Proof. We examine the preference of the median voter in the small region. The utility from separation is given by

$$U_{\mathrm{I}}\left(\frac{\alpha}{2}\right) = g + y - \frac{F}{\alpha}. \tag{9}$$

Under unification, the public good is located at $(1+\alpha)/2 - b\alpha/2(1-\alpha)$. The payoff in a union is therefore given by

$$U_{\mathrm{II}}\left(\frac{\alpha}{2}\right) = g(1 - a|D|) + y - F, \tag{10}$$

where $D = \frac{1}{2} - b\alpha/2(1-\alpha)$. Thus, the median voter in the smaller region prefers separation over unification if

$$F > \frac{\alpha}{2(1-\alpha)^2}[1 - \alpha - b\alpha]ga = F_A(b). \tag{11}$$

Note that $F_A(b)$ is decreasing in b. □

Remark. The qualitative properties of $F_A(b)$ hold for any location of the median voter; the assumption that the median voter is located at $\alpha/2$ is used to compute the specific function $F_A(b)$.

Propositions 4.1–4.3 clarify the influence of the distribution of preferences on the essential trade-off identified earlier. We find that the distribution in the large region is very important while the distribution in the small region is essentially irrelevant. Greater diversity in the large region makes unification more attractive for the small region, while greater concentration (lesser diversity) makes unification more attractive for the larger region. The intuition behind this is that the greater the diversity in the large region, the greater the influence of the small region on the location of the government after unification; correspondingly, the greater this influence the greater the political cost for the large region and the smaller the political cost for the small region. This illuminates an interesting point: small regions prefer to unite with diverse large regions, while large regions are more in favor of unification if they are less diverse themselves. Our findings are consistent with the stylized fact that democratization leads to greater secession and break-up in countries. This is because democratization takes diversity more into account and this can lower the incentives for unification in the large region.

4.2. The strategic role of regional location

In the basic model, we have assumed that there are only two regions and that the boundary between the two regions is fixed. This section will examine the implications of relaxing these assumptions. We suppose that there are three regions located on the unit interval: region A is located on the left hand corner, region B is located on the right hand corner, while region C is located in the center.

4.2.1. Symmetric case

We will first examine the nature of the majority voting outcome in the setting where regions A and B are of equal size. Our first observation concerns the preferences of the central region: this region always prefers unification of the three regions due to the fact that in a union with the other two regions the location of the public good will be the same as when region C remains independent. The tax rate, however, will be lower in a union, making region C strictly better off. The two regions at the ends of the interval, A and B, by contrast prefer to be independent if the fixed costs of a public good are low. We therefore have to specify what happens when the preferences of the majority in the different regions are different.

We will look at majority voting outcomes which have the core property: there does not exist a union of regions which is preferred over the current majority voting outcome by a majority of the individuals in each of the regions of the union. The boundary between region A and region C is fixed at α. In the symmetric case the boundary between region B and region C is therefore fixed at the value $1 - \alpha$. The formal analysis of core outcomes is given in Appendix A. We now discuss the findings and clarify the connections with the findings in the basic model.

First, we find that given a value of F, there is a cut-off level $\hat{\alpha}$ such that for all $\alpha < \hat{\alpha}$, the unique stable outcome is the union of all the three regions. This outcome appears to differ from the outcome we observed in the basic model. Recall that in that model, unification occurs only if $F > ga/4$ and even then it occurs only if the regions are of relatively equal size, i.e., for large α. The principal reason for this change in outcome is the change in the preferences of the large region. In the three regions model, the central region's most preferred outcome, independently of its size and the value of F, is the union between three regions. In the two regions case, by contrast, the large region prefers union only with a relatively large region. This contrast illustrates the importance of location in shaping regional preferences between union and separation.

Second, we find that if $F > ga/4$ then there exists some $\hat{\alpha}$ such that $\alpha \in (\bar{\alpha}, \frac{1}{2})$, the unique outcome is the union of all the three regions. The reasoning behind this outcome is as follows: the central region C always prefers such a union, while regions A and B prefer such a union due to cost-sharing reasons. This is analogous to the intuition behind the result obtained in the basic model that relatively equal regions prefer unification.

Our *third* finding pertains to the relation between α and the pattern of union and separation. In the three regions case, we find that union occurs for very small as well as for very large α, but generally there is either independence or a union of only two of the regions for intermediate values of α. The precise outcome depends on the value of F, with the three independent regions outcome being stable for low F and the union between two regions being stable for intermediate F.

Finally, we find that for fixed α, raising F typically increases the level of integration in the majority outcome. This is intuitive and also in line with our findings for the basic model.

4.2.2. Central region incentives

In the previous part of this section we assumed that the regions A and B were of equal size. This assumption allowed a complete characterization of the outcomes under majority voting. We would also like to explore the case where the two regions are of unequal size. This is the aim of the present section. There are several sub-cases involved here and we have been unable to solve the model completely. We shall therefore focus on the incentives of the central region, which is denoted throughout as region C. This region has the choice of joining region A or region B, or of remaining independent. This setup is realistic in cases where we have three regions and where two of the three differ too much to form a union but where the third region is in a position where it can join either of the two regions. Moreover the two regions are in principle willing to have a union with the central region.

We consider a special case to obtain insights into the issues involved. We fix the boundary between regions A and region C at $\frac{1}{10}$, and vary the boundary between region C and region B. Region C, in the centre, will prefer to be independent if the costs F are low. For large F, region C will prefer a union with the largest of the two other regions. In this case it is important to share the fixed costs of the public good with as many individuals as possible. For intermediate values of F, matters are more complicated. If region C is small then it prefers to join the larger region, while if it is itself large then it prefers union with the smaller of the two other regions. When a region is small it is important to share the fixed costs of the public good with more individuals but when a region is not too small the political costs of unification become more important and the region might prefer to join with a smaller region or it might prefer independence. A formal statement of these findings (along with the proof) is provided in Appendix A.

5. Alternative political institutions

In the basic model we assumed that unification occurs if a majority in each of the regions prefers unification. In this section we examine the robustness of the main findings under alternative political arrangements. We *first* consider the role of the default outcome, i.e. the rule that specifies what happens when the majority voting outcomes in the two regions are different. *Second*, we explore the nature of majority voting outcomes under one nation-wide referendum. *Third*, we examine the effects of unequal union: this is an outcome in which one region gives up the influence on the location of the government. *Finally*, we examine the endogenous determination of different voting rules.

5.1. The default outcome

In the basic model, we apply the following majority voting rule. In each region a referendum is organized over unification and separation. If there exists a majority in favor of unification in both regions then union takes place; otherwise, the regions remain separate and form two countries. In this section we consider the following alternative majority voting rule: *In each region a referendum is organized over unification and separation. If there is a majority in favor of separation in both regions then separation takes place*; *otherwise the two regions remain united.* Under majority voting a default outcome specifies what happens if the regions do not agree. For the majority voting rule used in the basic model the default outcome is separation. For the majority voting rule we will now use, the default outcome is unification.

We start by observing that socially optimal outcomes do not change with a change in default outcome rules: *The socially optimal solution is independent of the default outcome. Hence*, *the socially optimal solution in the model with union as default option is the same as the socially optimal solution in the model with separation as the default outcome*. The intuition behind this is straightforward. In the social optimal solution the decision on unification and separation is taken by maximizing total utility and not by considering a possible difference in preference of majorities in each region.

We next note that the conditions on α for a region to prefer unification are the same as in the basic model and therefore identical to those identified in Proposition 3.1. In fact a useful reformulation of Proposition 3.1 is: *There exists an α_A such that a majority in region A prefers separation if and only if $\alpha > \alpha_A$ and there exists an α_B such that region B prefers separation if and only of $\alpha < \alpha_B$*. In the proof of Proposition 3.1 we determine the conditions on α by comparing the payoffs of the median voter

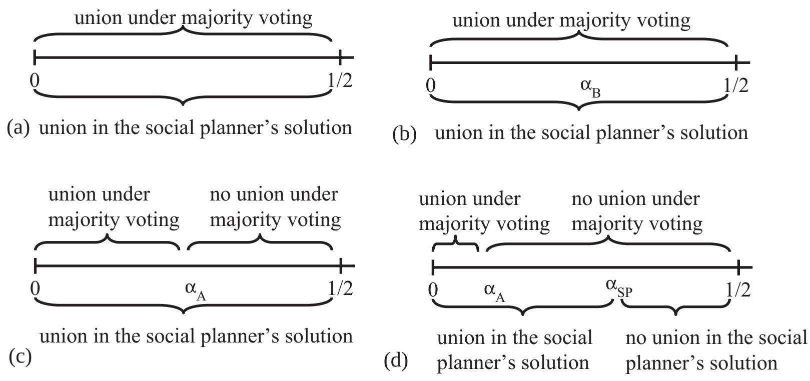

Fig. 3. Union as default option. (a) $F > ga/2$; (b) $ga/4 < F < ga/2$; (c) $ga/8 < F < ga/4$; (d) $F < ga/8$.

under unification and under separation. Using the new majority voting rule neither the median voter nor his utilities are changed, so the conditions on α will also be the same.

Since we have changed the default outcome from separation to unification, we require a majority in favor of separation in both regions for separation to occur. Using the majority voting rule with unification as default outcome and using Propositions 3.1–3.3, it is easy to derive Fig. 3. In the next result we provide the exact conditions for unification and separation under majority voting as well as the nature of socially optimal outcomes.

Proposition 5.1. (a) *If $F < ga/8$ then unification is socially optimal for all $\alpha \in [0, \frac{1}{2} - \sqrt{\frac{1}{4} - 2F/ga}]$ but separation obtains under majority voting for all $\alpha \in [2F/ga, \frac{1}{2}]$,* (b) *if $ga/8 < F < ga/4$ then unification is socially optimal for all $\alpha \in [0, \frac{1}{2}]$ but separation obtains under majority voting for all $\alpha \in [2F/ga, \frac{1}{2}]$ and* (c) *if $F > ga/4$ then unification is socially optimal as well as the majority voting outcome for all $\alpha \in [0, \frac{1}{2}]$.*

When the default outcome is changed from separation into unification we expect that union becomes more likely. Indeed, unification is now the majority voting outcome in more cases. However, even in this setting, there are parameter ranges where the majority voting outcome is separation, while the socially optimal outcome is unification. This highlights the robustness of the finding concerning the inefficiencies of the majority rule, obtained in the basic model.

5.2. One nation-wide referendum

In the majority voting rules we have studied so far separate referenda are organized in each region. We observed that these rules generate excessive incentives for separation. This motivates an examination of a more inclusive political rule: a nation-wide referendum. We note that if separate referenda are held in each of the regions, then it is possible that the results of these referenda are not the same. It is therefore necessary to specify a default outcome which defines what happens in that case. When there is a nation-wide referendum, there is no need to specify a default outcome.

The following result derives the conditions under which separation is supported in a nation-wide referendum.

Proposition 5.2. *There exists a majority in favor of separation in the whole nation if and only if there exists a majority in each region in favor of separation.*

The proof is given in Appendix A. This result is somewhat surprising and so we elaborate on the arguments underlying the proof. First note that the individual located at $\frac{1}{2}$ always prefers unification over separation. This follows from the fact that there are tax-advantages of unification as well as lower political costs, since the government is located at $\frac{1}{2}$ in a union. The second observation pertains to the linearity of the political costs. If one of the regions prefers unification, then it follows that the median voter in that region prefers unification over separation. The linearity of the political costs under both unification and separation implies that all the voters between the median voter (say) $\alpha/2$ and the end-point 0, also prefer unification. Moreover, it is obvious that if the median voter prefers unification then so do all voters located between $\alpha/2$ and α, since they have a similar tax burden but lower political costs of unification. Finally, the linearity of the political cost for voters between α and $\frac{1}{2}$ along with the fact that the voter at $\frac{1}{2}$ prefers unification always implies that they too prefer unification over separation. We have therefore shown that all voters in the interval $[0, \frac{1}{2}]$ prefer unification. The argument now follows from the continuity of the utility function, with respect to location.

Proposition 5.2 says that if majorities in both regions prefer separation there will be two separate countries and in all other cases, the outcome will be union with a single country. This is exactly the outcome we observed under the majority voting rule with two referenda and unification as the default outcome. Proposition 5.1 therefore also holds when we apply the voting rule with one nation-wide referendum and the outcomes are as illustrated in Fig. 3. The result on excessive incentives for separation thus also holds when a nation-wide referendum is held to decide on unification and separation.

5.3. *Unequal political union*

From the analysis of the basic model it follows that the smaller region (region A) prefers unification if α is small but the large region (region B) prefers separation in this case. In case α is small, the gain in terms of a lower tax rate is outweighed by the loss in terms of political relocation of the government. However, for region A the per capita cost of government becomes too big for small α. Therefore, it may be attractive for region A to ask region B for unification even if the location of the public good is determined solely by region B. We call this *unequal* or *unconditional* union. In this political arrangement, region A gives up its political influence in the hope of a significant tax reduction.

We begin by noting that the larger region, Region B, will always accept unequal union: the individuals in region B will then have higher utility since there is reduction in the tax rates while there is no loss of political influence. In cases where region B prefers equal union over separation and region A is willing to accept an unequal union there arises a bargaining problem. To keep matters simple we will assume that in such cases equal union will take place. Given this assumption, we find that the outcomes are as follows. Define $\alpha_{\mathrm{uu}} = 2F/(ga+2F)$; α_A and α_B are defined as before.

Proposition 5.3. *The outcomes under majority voting supplemented with the option of unequal union are as follows*: (a) *If* $F < ga/4$ *then there is unequal union for* $\alpha \in [0, \alpha_{\mathrm{uu}}]$ *and separation otherwise*, (b) *if* $ga/4 < F < ga/2$ *then there is unequal union if* $[0, \min\{\alpha_{\mathrm{uu}}, \alpha_B\}]$, *separation if* $\alpha \in [\min\{\alpha_{\mathrm{uu}}, \alpha_B\}, \alpha_B]$ *and equal union if* $\alpha \in [\alpha_B, \frac{1}{2}]$, *and* (c) *if* $ga/2 < F$ *then there is equal union for all* $\alpha \in [0, \frac{1}{2}]$.

Proof. We first note an implication of the median voter theorem: In region A there is a majority in favor of unequal union against the alternative of separation if the individual in the centre of region A is in favor of unequal union. Clearly, everyone in region A prefers equal union over unequal union. A majority in region A favours unequal union over separation if individual $\alpha/2$ prefers uneqaul union:

$$U_{\mathrm{uu}}\left(\frac{\alpha}{2}\right) = g - \frac{ga}{2} + y - F > g + y - \frac{F}{\alpha} = U_{\mathrm{II}}\left(\frac{\alpha}{2}\right). \tag{12}$$

That is, if

$$\alpha < \frac{2F}{ga+2F} = \alpha_{\mathrm{uu}}. \tag{13}$$

Thus *region A prefers unequal union over separation if and only if* $\alpha < \alpha_{\mathrm{uu}}$, where $\alpha_{\mathrm{uu}} = 2F/(ga+2F)$. We can now use Propositions 3.1–3.2 to complete the proof. □

From the above result it is clear that for some cases, such as when $\alpha \in [0, \alpha_{\mathrm{uu}}]$, unequal union is preferred over separation by the smaller region. Unequal union thus softens the negative consequences of excessive separation under majority voting. As one might expect, there will be unequal union if and only if there is a large difference in size between the regions.

We note that the nature of efficient outcomes remains the same: the socially optimal solution in the model extended with unequal union is the same as the socially optimal solution in the basic model. This is because in our computations of the socially optimal solution we allowed for arbitrary locations of the public good. A comparison of outcomes under majority voting and the socially optimal outcome is presented in Fig. 4.[12]

5.4. Endogenous political institutions

So far we have considered a number of political institutions. We have found that the rules under which voting is carried out have a significant influence on the nature of

[12] Note that $F > (\sqrt{5}-1)ga/4$ implies that $\alpha_{\mathrm{uu}} > \alpha_B$.

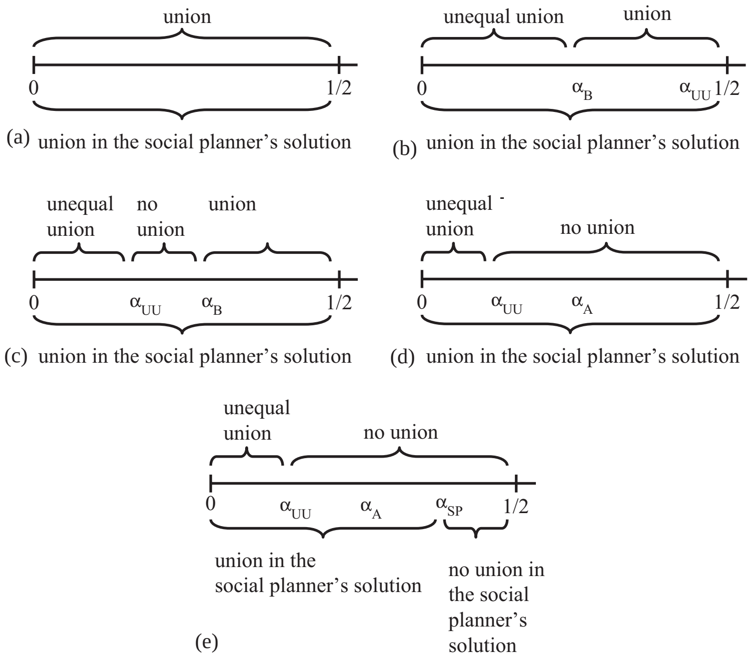

Fig. 4. Unequal union. (a) $F > ga/2$; (b) $(\sqrt{5}-1)ga/4 < F < ga/2$; (c) $ga/4 < F < (\sqrt{5}-1)ga/4$; (d) $ga/8 < F < ga/4$; (e) $F < ga/8$.

the outcomes as well as on the level of social welfare. This motivates an examination of their relative merits. In this section, we will first examine the normative appeal of the different rules and then we will examine their stability. Our principal finding is that the outcomes of the majority voting rule in each region supplemented with union as the default outcome is normatively appealing as well as enforceable.

In what follows we shall refer to the two region-wide referenda with separation as a default outcome as Rule I. We recall that outcomes under the two region-wide referenda with union as default outcome, and the outcomes under a single nation-wide referendum are identical. In what follows we will therefore only consider the latter rule and we will refer to it as Rule II. In addition we will refer to the unequal union option as Rule III.

From a normative point of view, we would like to implement the rule with outcomes closest to the socially optimal outcome. It follows from the analysis in Section 3 and 5 that Rule II induces outcomes closest to the socially optimal outcomes. Rule II is thus the most attractive from a normative point of view.

We now examine the stability of different rules. We shall say that a voting rule is *stable* if in a nation-wide vote a majority of voters prefers this rule to any other rule. We start with a comparison between Rules I and II. First note that in all cases where unification is the outcome under Rule I it is also the outcome under Rule II. Next note that in cases where separation is the outcome under Rule II it is also the outcome under Rule I. Hence, in all these cases individuals will be indifferent between the two rules. The interesting cases to check are therefore those in which the majority voting outcome is union under Rule II while it is separation under Rule I. Using Propositions 3.1–3.2 and Propositions 5.1–5.2 we can deduce that this difference in outcomes is only possible if a majority in the small region prefers unification, while a majority in the large region prefers separation. We next observe that all individuals located between α and $\frac{1}{2}$ will prefer unification over separation since in a union they will pay less taxes and the public good will be located closer to them. Finally, we note that if the median voter in the small region prefers unification then all other voters in the small region also prefer unification.[13] Thus the voters in the small region along with voters located between α and $\frac{1}{2}$ form a majority in favor of union and hence in favor of Rule II. Rule II therefore dominates Rule I for these cases.

We now compare Rules II and III. From Propositions 5.1 and 5.3 we deduce that if the outcome under Rule III is equal union then the same outcome obtains under Rule II. Similarly, if the outcome under Rule II is separation then the same outcome obtains under Rule III. So the only cases where the outcomes differ are those where there is unification under Rule II while the outcome under Rule III is unequal union or separation. There are two ways in which this can happen. In the first case voters in the small region prefer equal union over separation and separation over unequal union, while voters in the large region prefer separation over equal union. In the second case voters in the small region prefer equal union over unequal union and unequal union over separation. In the first case the outcome under Rule III will be separation. We can use the same argument we have used above (in comparing Rules I and II) to show that a majority of the individuals prefers Rule II over Rule III. In the latter case the outcome will be unequal union under Rule III. The individuals in the small region prefer equal union over unequal union. The individuals located between α and $\frac{1}{2}$ also prefer union over unequal union. In an equal union they will pay the same taxes as in unequal union but they will be located closer to the government in an equal union. Thus a majority of voters prefers Rule II to Rule III in all the cases where the outcome differs.

The above arguments lead us to conclude that Rule II is enforceable for all parameters[14] which is not the case for the other rules under consideration.

6. Conclusion

This paper has examined incentives of regions to unite/separate in terms of a basic trade-off: separation allows for greater influence over the nature of political decision

[13] If the median voter in a region prefers union over separation then all individuals in the region will have this preference, since utility is decreasing with the same rate under union and separation with respect to distance.

[14] Our arguments above also establish that Rule II is self-stable in the sense of Barbera and Jackson (2000).

making while unification allows regions to exploit economies of scale in the provision of government. We find that if regions have dispersed preferences then a small region is relatively more eager to form a union as compared to a large region. However, the incentives of a large region to form a union are increasing with regard to clustering/concentration of preferences in the large region, while the reverse holds true for the small region. Our analysis also identifies the important role of regional location in shaping preferences over unification and separation.

We then explore the social welfare of decisions made on the basis of majority voting in regions. Our main finding is that majority voting rules typically lead to excessive separation. A discussion of alternative voting systems reveals that a majority voting system supplemented with unification as a default option is normatively appealing as well as enforceable.

There are several directions in which our model can be developed further. First, we have examined a variety of political rules and also discussed their stability. There is another dimension to the stability question. There may be groups of persons who are unhappy with the outcome under majority voting and they may use violent means to have their way. This raises the question: to what extent are the outcomes robust to attempts by the disaffected group to secede or forcibly unite? We feel that these issues, while clearly important, require a model in which costs of violence as well as the technology of suppression of violence are explicitly formulated and therefore lie outside the scope of the present paper. Second, we have assumed that the public good/government is indivisible and has a fixed level and cost. Typically, public goods permit some decentralization and voters decide on the level of the public good to some extent. Third, we have assumed that public goods have no spillovers across regional/national boundaries. In many cases of interest, public goods have spillovers and, moreover, these spillovers are related to the location of different individuals. We hope to explore these issues in future work.

Acknowledgements

We are grateful to the editor, Jordi Gali, and two anonymous referees for useful comments and several suggestions that have improved the paper substantially. We would also like to thank Robert Dur, Matt Jackson and participants at presentations in Erasmus, ISI (Delhi), 8th World Congress ES (Seattle), 56th European Meeting ES (Lausanne) and the 16th Annual Congress EEA (Lausanne) for helpful suggestions. We also thank Vereniging Trustfonds Erasmus Universiteit Rotterdam for financial assistance.

Appendix A

Proof of Proposition 3.3. In the socially optimal solution the sum of all individual utilities is maximized. If it is optimal to have just one country, then it will be socially optimal to choose the location of the public good and the tax level to maximize the aggregate payoff $U_{\rm I}$ to all individuals in the union:

$$U_{\rm I} = \int_0^1 U_{\rm I}(i)\,{\rm d}i = g(1 - a{\rm E}(d_i|l)) + y - {\rm E}(t_i) \tag{A.1}$$

and if it is optimal to form two governments then it will be socially optimal to choose the location of the public good and the tax level to maximize the aggregate payoff $U_{\rm II}$ to all individuals under separation:

$$U_{\rm II} = \int_0^1 U_{\rm II}(i)\,{\rm d}i = \sum_{x=A,B} s_x[g(1 - a{\rm E}_x(d_i|l_x)) + y - {\rm E}_x(t_i)], \tag{A.2}$$

where ${\rm E}_x(d_i|l_x)$, s_x and ${\rm E}_x(t_i)$ are, respectively, the average distance in country x given the location of the government, the size of country x and the lump sum tax level in country x. Since the value of α is exogenously specified, the values of s_A and s_B are α and $1-\alpha$, respectively. In order to minimize ${\rm E}_x(d_i)$ it is socially optimal to locate the government in the middle of each country. Hence, ${\rm E}_A(d_i)$, ${\rm E}_B(d_i)$ and ${\rm E}(d_i)$ are, respectively, $\alpha/4$, $(1-\alpha)/4$ and $\frac{1}{4}$. Each country has to finance its own government, therefore ${\rm E}_A(t_i)$, ${\rm E}_B(t_i)$ and ${\rm E}(t_i)$ are, respectively, F/α, $F/(1-\alpha)$ and F.

Hence, the social utility expressions (A.1) and (A.2) can be rewritten as follows:

$$U_{\rm I} = g\left(1 - \frac{a}{4}\right) + y - F, \tag{A.3}$$

$$U_{\rm II} = \alpha\left[g\left(1 - a\frac{\alpha}{4}\right)\right] + (1-\alpha)\left[g\left(1 - a\frac{1-\alpha}{4}\right)\right] + y - 2F. \tag{A.4}$$

Comparing the total utilities of unification and separation determines the choice for either unification or separation. It is better to have one government (one nation) if and only if $U_{\rm I} > U_{\rm II}$:

$$g\left(1 - \frac{a}{4}\right) + y - F > \alpha\left[g\left(1 - a\frac{\alpha}{4}\right)\right] + (1-\alpha)\left[g\left(1 - a\frac{1-\alpha}{4}\right)\right] + y - 2F. \tag{A.5}$$

After rearranging terms, this inequality can be written as

$$\frac{ga}{2}\alpha^2 - \frac{ga}{2}\alpha + F > 0 \tag{A.6}$$

and this is equivalent with

$$\alpha^2 - \alpha + \frac{2F}{ga} > 0. \tag{A.7}$$

Note that this inequality will only have solutions if $F < ga/8 = F_{\rm SP}$.

Inequality (A.5) is satisfied for values of α when

$$\alpha < \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{2F}{ga}} \tag{A.8}$$

or when

$$\alpha > \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2F}{ga}}. \tag{A.9}$$

Note that the right hand side of inequality (A.9) is greater than $\frac{1}{2}$. Because α is, by assumption, smaller than $\frac{1}{2}$, we can omit inequality (A.9). This proves Proposition 3.3. □

A.1. Formal analysis for symmetric case

A coalition $\mathscr{C}$ is a subset of the set of regions $N = \{A, B, C\}$. An outcome specifies which, if any, subset of regions unites and which of the regions remain independent. Let $\mathscr{S}$ be the set of possible outcomes and let $s \in \mathscr{S}$ be a typical outcome in this set. Also denote by u_i^s the utility of the median voter in region i, under outcome s. Likewise, let $u_i^{\mathscr{C}}$ be the utility that accrues to the median voter of region i, in a coalition $\mathscr{C}$. A coalition is said to block an outcome s if $u_i^{\mathscr{C}} \geqslant u_i^s$, for all $i \in \mathscr{C}$ and there is some $i \in \mathscr{C}$ such that $u_i^{\mathscr{C}} > u_i^s$. An outcome is said to be stable (or to lie in the core) if there exists no blocking coalition.

We shall use the & sign to denote a union between two regions. We summarize the results of our analysis in the following proposition. Let $\alpha_1 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 8F/ga}$, $\alpha_2 = 2F/ga$, $\alpha_3 = 1 - 2F/ga$ and $\alpha_C = \frac{3}{4} - \frac{1}{4}\sqrt{1 + 16F/ga}$.

Proposition A.1.

1. *If* $0 < F < \frac{1}{9}ga$ *then* (a) *for all* $\alpha \in [0, \alpha_2]$ *the stable outcome is* $A\&B\&C$; (b) *for all* $\alpha \in [\alpha_2, \frac{1}{2}]$ *the stable outcome is* A, B, C;
2. *If* $\frac{1}{9}ga < F < \frac{1}{8}ga$ *then* (a) *for all* $\alpha \in [0, \alpha_2]$ *the stable outcome is* $A\&B\&C$; (b) *for all* $\alpha \in [\alpha_2, \alpha_C]$ *the stable outcome is* A, B, C; (c) *for all* $\alpha \in [\alpha_C, \alpha_1]$ *the stable outcome is either* $A\&C, B$ *or* $A, B\&C$; (d) *for all* $\alpha \in [\alpha_1, \frac{1}{2}]$ *the stable outcome is* A, B, C.
3. *If* $\frac{1}{8}ga < F < (\frac{1}{2} - \frac{1}{4}\sqrt{2})ga$ *then* (a) *for all* $\alpha \in [0, \alpha_2]$ *the stable outcome is* $A\&B\&C$; (b) *for all* $\alpha \in [\alpha_2, \alpha_C]$ *the stable outcome is* A, B, C; (c) *for all* $\alpha \in [\alpha_C, \frac{1}{2}]$ *the stable outcome is either* $A\&C, B$ *or* $A, B\&C$.
4. *If* $(\frac{1}{2} - \frac{1}{4}\sqrt{2})ga < F < \frac{1}{4}ga$ *then* (a) *for all* $\alpha \in [0, \alpha_C]$ *the stable outcome is* $A\&B\&C$; (b) *for all* $\alpha \in [\alpha_C, \alpha_2]$ *the stable outcome is either* $A\&C, B$, $A, B\&C$ *or* $A\&B\&C$; (c) *for all* $\alpha \in [\alpha_2, \frac{1}{2}]$ *the stable outcome is either* $A\&B, C$ *or* $A, B\&C$.
5. *If* $\frac{1}{4}ga < F < \frac{1}{2}ga$ *then* (a) *for all* $\alpha \in [0, \alpha_C]$ *the stable outcome is* $A\&B\&C$; (b) *for all* $\alpha \in [\alpha_C, \alpha_3]$ *the stable outcome is either* $A\&C, B$, $A, B\&C$ *or* $A\&B\&C$; (c) *for all* $\alpha \in [\alpha_3, \frac{1}{2}]$ *the stable outcome is* $A\&B\&C$.
6. *If* $F > \frac{1}{2}ga$ *then for all* $\alpha \in [0, \frac{1}{2}]$ *the stable outcome is* $A\&B\&C$.

Proof. First, note that for all parameter values, the central region, region C, will prefer a union with the two other regions over all other possibilities. Secondly, note that the preferences over unification and separation for the two regions at the endpoints of the interval are essentially the same. Both regions have the same size as well as a 'similar' location.

A majority of the individuals in the central region prefer a union with one of the regions over independence if

$$U_{A\&C}\left(\frac{1}{2}\right) = U_{B\&C}\left(\frac{1}{2}\right) = g\left(1-\frac{a\alpha}{2}\right) + y - \frac{F}{1-\alpha}$$

$$> g + y - \frac{F}{1-2\alpha} = U_C\left(\frac{1}{2}\right). \tag{A.10}$$

That is, if $\alpha < \frac{3}{4}+\frac{1}{4}\sqrt{1+16F/ga}$ or if $\alpha > \frac{3}{4}-\frac{1}{4}\sqrt{1+16F/ga}$. Since α has to be less than $\frac{1}{2}$ and $\frac{3}{4}+\frac{1}{4}\sqrt{1+16F/ga} > \frac{1}{2}$ the relevant inequality is

$$\alpha > \frac{3}{4} - \frac{1}{4}\sqrt{1+\frac{16F}{ga}} = \alpha_C. \tag{A.11}$$

Similar comparisons of utility levels yield that a majority of the individuals in region A (and therefore also a majority of the individuals in region B) prefer a union with region C if

$$\alpha > \frac{1}{2} - \frac{1}{2}\sqrt{1-\frac{8F}{ga}} = \alpha_1. \tag{A.12}$$

Note that this implies that $F < ga/8$. A majority of the individuals in region A (region B) prefers independence over a union with the two other regions if

$$\alpha > \frac{2F}{ga} = \alpha_2 \tag{A.13}$$

and a union with three regions over a union with just the central region if

$$\alpha > 1 - \frac{2F}{ga} = \alpha_3. \tag{A.14}$$

To determine the ordering of α_1, α_2, α_3, α_C and $\frac{1}{2}$ we compare pairwise the six values and we get the following. For $0 < F < ga/9$, $0 < \alpha_2 < \alpha_1 < \alpha_C < \frac{1}{2} < \alpha_3$. For $ga/9 < F < ga/8$, $0 < \alpha_2 < \alpha_C < \alpha_1 < \frac{1}{2} < \alpha_3$. For $ga/8 < F < (\frac{1}{2} - \frac{1}{4}\sqrt{2})ga$, $0 < \alpha_2 < \alpha_C < \frac{1}{2} < \alpha_3$. For $(\frac{1}{2} - \frac{1}{4}\sqrt{2})ga < F < ga/4$, $0 < \alpha_C < \alpha_2 < \frac{1}{2} < \alpha_3$. For $ga/4 < F < ga/2$, $0 < \alpha_C < \alpha_3 < \frac{1}{2} < \alpha_2$. For $F > ga/2$, $\alpha_3 < \alpha_C < 0 < \frac{1}{2} < \alpha_2$. The combination of these relations, of inequalities (A.11)–(A.14), and the stability requirements mentioned above gives us the required result. □

A.2. Formal analysis of regional incentives in asymmetric case

We shall use & to denote a union between two regions and the notation $\succ$ to denote 'preferred under majority voting'. We summarize the results of our analysis of majority voting in the central region C in the following proposition. We define $\alpha_1 = \frac{1}{20} + \frac{1}{2}\sqrt{\frac{1}{100} + 8F/ga}$, $\alpha_2 = \frac{20}{9}F/ga + \frac{1}{10}$ and $\alpha_3 = \frac{20}{9}F/ga$.

Proposition A.2.

1. *If* $0 < F < \frac{9}{200}ga$ *then* (a) *for all* $\alpha \in [\frac{1}{10}, \alpha_2]$, $C\&A \succ C\&B \succ C$; (b) *for all* $\alpha \in [\alpha_2, \alpha_1]$, $C\&A \succ C \succ C\&B$; (c) *for all* $\alpha \in [\alpha_1, \frac{9}{10}]$, $C \succ C\&A \succ C\&B$; (d) *for all* $\alpha \in [\frac{9}{10}, 1]$, $C \succ C\&B \succ C\&A$.
2. *If* $\frac{9}{200}ga < F < \frac{72}{200}ga$ *then* (a) *for all* $\alpha \in [\frac{1}{10}, \alpha_3]$, $C\&B \succ C\&A \succ C$; (b) *for all* $\alpha \in [\alpha_3, \alpha_2]$, $C\&A \succ C\&B \succ C$; (c) *for all* $\alpha \in [\alpha_2, \alpha_1]$, $C\&A \succ C \succ C\&B$; (d) *for all* $\alpha \in [\frac{9}{10}, 1]$, $C \succ C\&B \succ C\&A$.
3. *If* $\frac{72}{200}ga < F < \frac{81}{200}ga$ *then* (a) *for all* $\alpha \in [\frac{1}{10}, \alpha_3]$, $C\&B \succ C\&A \succ C$; (b) *for all* $\alpha \in [\alpha_3, \frac{9}{10}]$, $C\&A \succ C\&B \succ C$; (c) *for all* $\alpha \in [\frac{9}{10}, \alpha_1]$, $C\&B \succ C\&A \succ C$; (d) *for all* $\alpha \in [\alpha_1, \alpha_2]$, $C\&B \succ C \succ C\&A$; (e) *for all* $\alpha \in [\alpha_2, 1]$, $C \succ C\&B \succ C\&A$.
4. *If* $\frac{81}{200}ga < F < \frac{90}{200}ga$ *then* (a) *for all* $\alpha \in [\frac{1}{10}, \frac{9}{10}]$, $C\&B \succ C\&A \succ C$; (b) *for all* $\alpha \in [\frac{9}{10}, \alpha_3]$, $C\&A \succ C\&B \succ C$; (c) *for all* $\alpha \in [\alpha_3, \alpha_1]$, $C\&B \succ C\&A \succ C$; (d) *for all* $\alpha \in [\alpha_1, 1]$, $C\&B \succ C \succ C\&A$.
5. *If* $F > \frac{90}{200}ga$ *then* (a) *for all* $\alpha \in [\frac{1}{10}, \frac{9}{10}]$, $C\&B \succ C\&A \succ C$; (b) *for all* $\alpha \in [\frac{9}{10}, 1]$, $C\&A \succ C\&B \succ C$.

Proof. A majority of the individuals in region C will prefer independence over a union with region A if the utility of the median voter is larger under independence than in a union with region A:

$$U_{\mathrm{I}}\left(\frac{1}{20} + \frac{1}{2}\alpha\right) = g + y - \frac{F}{\alpha - 1/10} > g\left(1 - a\left|\frac{1}{20}\right|\right) + y - \frac{F}{\alpha} = U_{C\&A}\left(\frac{1}{20} + \frac{1}{2}\alpha\right). \tag{A.15}$$

That is, if

$$\alpha^2 - \frac{1}{10}\alpha - \frac{2F}{ga} > 0. \tag{A.16}$$

This inequality is satisfied if $\alpha < \frac{1}{20} - \frac{1}{2}\sqrt{100 + 8F/ga}$ or if $\alpha > \frac{1}{20} + \frac{1}{2}\sqrt{100 + 8F/ga}$. Since $\frac{1}{20} - \frac{1}{2}\sqrt{100 + 8F/ga} < 0$ the relevant inequality is

$$\alpha > \frac{1}{20} + \frac{1}{2}\sqrt{100 + 8F/ga} = \alpha_1. \tag{A.17}$$

Similar comparisons of utility levels yield that a majority of individuals in region C will prefer independence over a union with region B if

$$\alpha > \frac{1}{20}\frac{F}{ga} + \frac{1}{10} = \alpha_2 \tag{A.18}$$

and a union with region A over a union with region C if

$$\alpha > \frac{1}{20}\frac{F}{ga} = \alpha_3. \tag{A.19}$$

To determine the ordering of $\frac{1}{10}$, $\frac{9}{10}$, 1, α_1, α_2 and α_3 we compare pairwise the six values and we get the following: For $0 < F < \frac{9}{200}ga$, $\frac{1}{10} < \alpha_2 < \alpha_1 < \frac{9}{10}$. For

$\frac{9}{200} < F < \frac{72}{200}ga$, $\frac{1}{10} < \alpha_3 < \alpha_2 < \alpha_1 < \frac{9}{10}$. For $\frac{72}{200} < F < \frac{81}{200}ga$, $\frac{1}{10} < \alpha_3 < \frac{9}{10} < \alpha_1 < \alpha_2 < 1$. For $\frac{81}{200} < F < \frac{90}{200}ga$, $\frac{1}{10} < \alpha_3 < \frac{9}{10} < \alpha_1 < \alpha_2 < 1$. The combination of these relations and of inequalities (A.17)–(A.19) gives us the result. □

Proof of Proposition 5.2. A majority in each region in favor of separation implies that there is a majority in the whole nation, so the proof of the if-part of the statement is immediate. The only if-part of the statement is equivalent to: *if there does not exist a majority in favor of separation in both regions then there does not exist a majority in favor of separation in the whole nation.* There are three different cases in which there does not exist a majority in favor of separation in both regions:

Case $\mathscr{A}$: There exist a majority in favor of separation in region A but not in region B.

Case $\mathscr{B}$: There exists a majority in favor of separation in region B but not in region A.

Case $\mathscr{C}$: There exists a majority in favor of separation in neither region B nor in region A.

In case $\mathscr{C}$ it is straightforward to see that there does not exist a majority in favor of separation in the whole nation.

We now take up cases $\mathscr{A}$ and $\mathscr{B}$. The following formulae are useful in what follows:

$$U_{\mathrm{II}}\left(\frac{1}{2}\right) = g - \frac{1}{2}ga\alpha + y - \frac{F}{1-\alpha} < g + y - F = U_{\mathrm{I}}\left(\frac{1}{2}\right). \tag{A.20}$$

From Proposition 3.1, we know that case $\mathscr{A}$ implies that

$$U_{\mathrm{II}}\left(\frac{1+\alpha}{2}\right) < U_{\mathrm{I}}\left(\frac{1+\alpha}{2}\right). \tag{A.21}$$

We also know that

$$\frac{\partial U_{\mathrm{I}}(i)}{\partial i} = -ga \quad \forall i \in \left[\frac{1}{2}, 1\right], \tag{A.22}$$

which implies that $U_{\mathrm{I}}(i)$ is decreasing in $i \in [\frac{1}{2}, (1+\alpha)/2]$. Furthermore,

$$\frac{\partial U_{\mathrm{II}}(i)}{\partial i} = ga \quad \forall i \in \left[\alpha, \frac{1+\alpha}{2}\right], \tag{A.23}$$

which implies that $U_{\mathrm{II}}(i)$ is increasing in $i \in [\frac{1}{2}, (1+\alpha)/2]$. Finally, we note that,

$$\frac{\partial U_{\mathrm{II}}(i)}{\partial i} = -ga \quad \forall i \in \left[\frac{1+\alpha}{2}, 1\right]. \tag{A.24}$$

This implies that $U_{\mathrm{II}}(i)$ is decreasing in $i \in [(1+\alpha)/2, 1]$.

From expressions (A.21)–(A.23) it follows that $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [\frac{1}{2}, (1+\alpha)/2]$ and from expressions (A.21), (A.22) and (A.24) it follows that $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [(1+\alpha)/2, 1]$, so $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [\frac{1}{2}, 1]$. Inequality (A.20) and the continuity of the utility function in i imply that the individuals sufficiently close to $\frac{1}{2}$ prefer unification. Hence, there does not exist a majority in favor of separation in case $\mathscr{A}$.

Using Proposition 3.1, we know that case $\mathscr{B}$ implies that

$$U_{\mathrm{II}}\left(\frac{\alpha}{2}\right) < U_{\mathrm{I}}\left(\frac{\alpha}{2}\right). \tag{A.25}$$

We also know that

$$\frac{\partial U_{\mathrm{I}}(i)}{\partial i} = ga \quad \forall i \in \left[0, \frac{1}{2}\right], \tag{A.26}$$

which implies that $U_{\mathrm{I}}(i)$ is increasing in $i \in [0, \frac{1}{2}]$. Furthermore,

$$\frac{\partial U_{\mathrm{II}}(i)}{\partial i} = ga \quad \forall i \in \left[0, \frac{\alpha}{2}\right], \tag{A.27}$$

so $U_{\mathrm{II}}(i)$ is increasing in $i \in [0, \alpha/2]$. We next note that

$$\frac{\partial U_{\mathrm{II}}(i)}{\partial i} = -ga \quad \forall i \in \left[\frac{\alpha}{2}, \alpha\right], \tag{A.28}$$

so $U_{\mathrm{II}}(i)$ is decreasing in $i \in [\alpha/2, \alpha]$. Finally, note that

$$\frac{\partial U_{\mathrm{II}}(i)}{\partial i} = ga \quad \forall i \in \left[\alpha, \frac{\alpha + 1}{2}\right] \tag{A.29}$$

so $U_{\mathrm{II}}(i)$ is increasing in $i \in [\alpha, \frac{1}{2}]$.

From expressions (A.25)–(A.27) it follows that $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [0, \alpha/2]$, from expressions (A.25), (A.26) and (A.28) it follows that $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [\alpha/2, \alpha]$ and from expressions (A.20), (A.26) and (A.29) it follows that $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [\alpha, \frac{1}{2}]$, so $U_{\mathrm{II}}(i) < U_{\mathrm{I}}(i)$ for all $i \in [0, \frac{1}{2}]$. Inequality (A.20) and the continuity of the utility function in i imply that the individuals sufficiently close to the individual located at $\frac{1}{2}$ prefer unification. Hence there does not exist a majority in favor of separation in the whole nation, which proves case $\mathscr{B}$. □

Appendix B

It is widely argued that wealth and income differences are one of the main factors behind regional movements which seek secession. One of the assumptions in the basic model is that the initial endowment per individual is equal in both regions. To study wealth differences across regions we write the initial endowment of the individuals in region A and in region B as y_A and y_B, respectively. We suppose that these incomes differ by a factor θ, $\theta > 0$, and we write $y_B = \theta y_A$. It can be verified that wealth differences do not matter when the public good is financed by lump sum taxes. We therefore change the system of taxation to proportional taxes. When both regions separate there are different tax levels in each region and when there is union we have just one tax level to finance the public good. One justification for this assumption is that a difference in tax-levels between the regions is not sustainable when the subject of taxation (e.g. capital or labor) is mobile in a union. If there exists a tax difference between the regions in a union then the capital or the labor will be located in the region with the lowest tax level. It is also possible that the legislation of a union allows just one tax rate. This leads to three proportional tax levels: t_A denotes the tax level under separation in region A, t_B the same in region B and t denotes the proportional tax level in a union. Recall that Proposition 3.2 tells us that there exists an α_A such that region A prefers unification if and only if $\alpha \in (0, \alpha_A)$ and there exists an α_B such that region B prefers unification if and only if $\alpha \in (\alpha_B, \frac{1}{2})$.

Proposition B.1. *When θ increases, α_A and α_B will increase. For $\theta > 8F/(8F - ga)$ region A always prefers union and for $\theta < (8F - ga)/8F$ region B always prefers union.*

This Proposition is in line with the idea that it is more attractive to unite with a rich region than with a poor region: An increase in θ implies that the individuals in region B become relatively richer compared to the individuals in region A. Union becomes therefore more attractive for region A and less attractive for the individuals in region B, which is reflected by the increases in α_A and α_B, respectively.

Proof of Proposition B.1. Note that we can use Proposition 3.1 in this proof. We will prove that an increase in θ leads to an increase in α_A. The proof that an increase in θ leads to an increase in α_B has the same structure as in the α_A-case and it is therefore not given. Recall that we restricted α to values between 0 and $\frac{1}{2}$. The utility of individual $\alpha/2$ in a union is

$$U_{\mathrm{I}}\left(\frac{\alpha}{2}\right) = g - \frac{ga}{2} + \frac{ga\alpha}{2} + y_A - \frac{F}{(\alpha + (1-\alpha)\theta)y_A} y_A \tag{B.1}$$

and under separation

$$U_{\mathrm{II}}\left(\frac{\alpha}{2}\right) = g + y_A - \frac{F}{\alpha}. \tag{B.2}$$

Let $U_{\mathrm{I}}^{\theta}(\alpha)$ and $U_{\mathrm{II}}^{\theta}(\alpha)$ be the utility of individual $\alpha/2$ under union and separation, respectively. Like in the standard model, define $\alpha_A(\theta)$ as

$$\begin{aligned} &U_{\mathrm{I}}^{\theta}(\alpha) < U_{\mathrm{II}}^{\theta}(\alpha) \quad \text{for } \alpha < \alpha_A(\theta), \\ &U_{\mathrm{I}}^{\theta}(\alpha) = U_{\mathrm{II}}^{\theta}(\alpha) \quad \text{for } \alpha = \alpha_A(\theta), \\ &U_{\mathrm{I}}^{\theta}(\alpha) > U_{\mathrm{II}}^{\theta}(\alpha) \quad \text{for } \alpha > \alpha_A(\theta). \end{aligned} \tag{B.3}$$

Hence, since $U_{\mathrm{I}}^{\theta}(\alpha)$ and $U_{\mathrm{II}}^{\theta}(\alpha)$ are differentiable in α for $\alpha \in (0, \frac{1}{2})$,

$$\frac{\partial[U_{\mathrm{I}}^{\theta}(\alpha_A(\theta)) - U_{\mathrm{II}}^{\theta}(\alpha_A(\theta))]}{\partial\alpha} < 0. \tag{B.4}$$

Next note that at $\alpha_A(\theta)$, $U_{\mathrm{I}}^{\theta}(\alpha(\theta)) - U_{\mathrm{II}}^{\theta}(\alpha(\theta)) = 0$. Hence,

$$\frac{\partial[U_{\mathrm{I}}(\alpha_A(\theta)) - U_{\mathrm{II}}(\alpha_A(\theta))]}{\partial\alpha}\,\mathrm{d}\alpha + \frac{\partial[U_{\mathrm{I}}(\alpha_A(\theta)) - U_{\mathrm{II}}(\alpha_A(\theta))]}{\partial\theta}\,\mathrm{d}\theta = 0. \tag{B.5}$$

This implies that

$$\frac{\mathrm{d}\alpha_A(\theta)}{\mathrm{d}\theta} = \frac{-\partial[U_{\mathrm{I}}^{\theta}(\alpha_A(\theta)) - U_{\mathrm{II}}^{\theta}(\alpha_A(\theta))]/\partial\theta}{\partial[U_{\mathrm{I}}^{\theta}(\alpha_A(\theta)) - U_{\mathrm{II}}^{\theta}(\alpha_A(\theta))]/\partial\alpha}. \tag{B.6}$$

Note that $\partial U_{\mathrm{II}}^{\theta}(\alpha_A(\theta))/\partial\theta = 0$,

$$\frac{\partial U_{\mathrm{I}}^{\theta}(\alpha)}{\partial\theta} = \frac{F(1-\alpha)}{(\alpha+(1-\alpha)\theta)^2} > 0 \tag{B.7}$$

and recall that

$$\frac{\partial[U_{\mathrm{I}}^{\theta}(\alpha_A(\theta)) - U_{\mathrm{II}}^{\theta}(\alpha_A(\theta))]}{\partial\alpha} < 0. \tag{B.8}$$

Hence $\mathrm{d}\alpha_A(\theta)/\mathrm{d}\theta > 0$. This completes the proof. □

Proposition B.2. *The socially desirable outcome does not change when the endowments across regions vary.*

Proof. In a social optimum the sum of all individual utilities is maximized. The utility under union is

$$U_{\mathrm{I}} = \int_0^1 U_{\mathrm{I}}(i)\,\mathrm{d}i = \sum_{x=A,B} s_x[g(1 - a\mathrm{E}(d_i|l)) + y_x - t_x y_x]. \tag{B.9}$$

This implies that

$$U_{\mathrm{I}} = g(1 - a\mathrm{E}(d_i|l)) + y - \sum_{x=A,B} s_x t_x y_x = g\left(1 - \frac{a}{4}\right) + y - F. \tag{B.10}$$

The utility under separation is

$$U_{\mathrm{II}} = \int_0^1 U_{\mathrm{II}}(i)\,\mathrm{d}i = \sum_{x=A,B} s_x[g(1 - a\mathrm{E}(d_i|l)) + y_x - t_x y_x]. \tag{B.11}$$

So total utility under separation can be written as

$$U_{\mathrm{II}} = \alpha\left[g\left(1 - a\frac{\alpha}{4}\right)\right] + (1-\alpha)\left[g\left(1 - a\frac{1-\alpha}{4}\right)\right] + y - 2F. \tag{B.12}$$

These utilities are equal to the utilities of equations 16 and 17. We can therefore apply the same analysis as in the standard model. □

This Proposition implies that differences in initial endowments across regions are irrelevant for the socially optimal outcome in which aggregate utility is maximized. Moreover, the possibility of choosing different taxation systems for compensation or for wealth transfers does not influence the socially desirable outcome.

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