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A Social Network Analysis of Occupational Segregation*

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Abstract

This paper proposes a simple social network model of occupational segregation, generated by the existence of inbreeding bias among individuals of the same social group. If network referrals are important in getting a job, then expected inbreeding bias in the social structure results in different career choices for individuals from different social groups, which further translates into stable occupational segregation equilibria within the labour market. Our framework can be regarded as complementary to existing discrimination or rational bias theories used to explain persistent observed occupational disparities between various social groups.

JEL codes: A14, J31, J70, Z13

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1 Introduction

Occupational segregation between various social groups is a persistent and pervasive phenomenon in the labour market. Richard Posner recently pointed out that "a glance of the composition of different occupations shows that in many of them, particularly racial, ethnic, and religious groups, along with one or the other sex and even groups defined by sexual orientation (heterosexual vs. homosexual), are disproportionately present or absent"[1]. There are countless

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[1]The quote is from an entry in "The Becker-Posner Blog", see http://www.becker-posner-blog.com. Posner also gives an example of gender occupational segregation: "a much higher percentage of biologists than of physicists are women, and at least one branch of biology, primatology, appears to be dominated by female scientists. It seems unlikely that all sex-related differences in occupational choice are due to discrimination..."

empirical studies that investigated the measurement and extent of occupational segregation, both in sociology and in economics[2], documenting its enduring relevance. Most studies analysing possible causes of occupational segregation agree that pure taste discrimination or theories based on market factors cannot explain alone occupational disparities. While a few alternative candidate theories have been considered [3], Arrow (1998) particularly indicated the concepts of direct social interaction and networks as most promising avenues for research in this context.

In this paper we apply social network theory to dynamically model occupational segregation in the labour market. Significant progress has been achieved in modeling labor market phenomena by means of social network analysis. Recent articles have investigated the effect of social networks on employment, wage inequality, labour market transitions, social welfare etc [4]. To our knowledge this is however the first application of social network analysis to modelling occupational segregation.

We construct a very simple three-stage model of occupational segregation between two homogeneous, exogenously defined, social groups acting in a 2-job labor market. In the first stage each individual chooses one of the two specialized educations to become a worker. In the second stage individuals randomly form friendship ties with other individuals, with a tendency to form more ties with individuals from the same social group, what we call inbreeding bias. In the third stage workers use their friendship contacts to search for jobs.

We show that with a positive inbreeding bias a complete polarization in terms of occupations across the two groups can arise as a stable equilibrium outcome. We extend our model by allowing for 'good' and 'bad' jobs, to analyze wage and unemployment inequality between the two social groups. We show that with large differences in job attraction a natural outcome of the model is that one group specializes in the good job, while the other group mixes over the jobs. Further, the group that specializes in the good job has a lower unemployment rate and a higher payoff. Thus our model is able to explain typical empirical patterns of gender, race or ethnical inequality.

In the remainder of this paper we review some empirical evidence on the existence of inbreeding bias and occupational segregation in Section 2, we describe our model of occupational segregation in Section 3, and we discuss the results

[2]Among the numerous studies documenting gender or racial occupational segregation, we mention a few that also have detailed literature review sections: Albeda (1986), Jacobs and Lim (1992), Blau, Simpson and Anderson (1998), Rich (1999)

[3]Theories advanced for explaining segregation in general and inequality among various social groups include a. pure discrimination theories, eg. Becker (1957), Arrow (1972a); b. various statistical discrimination theories, eg. Phelps (1972) , Arrow (1972b), Lazear and Rosen (1990), Coate and Loury (1993); c. theories based on intrinsic differences in ability or in attachment to the labour market, such as the human capital model by eg. Polachek (1981) ; d. theories based on group membership/ identity, adopted mainly from the residential segregation literature, eg. the review by Durlauf (2003) e. theories based on personal identity" developed by Akerlof and Kranton (2000)

[4]See for instance Montgomery (1991), Arrow and Borzekowski (2003), Calvó-Armengol and Jackson (2003, 2004), Bramoule and Saint-Paul (2004), Fontaine (2004), Lavezzi & Meccheri (2004).

on segregation in Section 4. We then derive results when jobs are not equally attractive in Section 5, and we summarize and conclude the paper in Section 6.

2 Evidence on Job Contact Networks and Inbreeding Bias

There is a well established set of stylized facts that show the importance of the informal job networks in searching and finding jobs. First, it is known that on average 50-60% of the workers obtain jobs through their personal contacts. Evidence in this sense comes both from sociology and economics, starting back in the 1960's and covering multiple countries, eg. Rees (1966), Granovetter (1995), Holzer (1987), Staiger (1990), Montgomery (1991), Topa (2001). A second empirical fact is that on average 40-50% of the employers use social networks of their current employees (ie. they hire recommended applicants) to fill their job openings: e.g. Holzer (1987). Third, the employee-employer matches obtained by making use of job contact networks appear to be of high quality: there is evidence that those who found jobs through personal contacts were less likely to quit, e.g. Datcher (1983), Devine and Kiefer (1991), and also had longer tenure on these jobs, e.g. Simon and Warner (1992)[5]. For a more detailed overview of studies on job information networks Ioannides and Datcher Loury (2004) is a very good reference. Job contact networks are thus documented to be very relevant in the labour dynamics process of matching employees to employers.

There is also extensive empirical evidence on the existence of inbreeding biases within social groups, ie. individuals are more likely to maintain ties to others within the same group, e.g. Doeringer and Piore (1971), Marsden (1987), Staiger (1990). Staiger (1990) documents for instance the existence of large inbreeding biases within gender groups[6]: over all occupations in a US sample, about 87% of the jobs obtained through contacts by men were based on information received from other men and 70% of the jobs obtained informally by women were based on information from other women. His results are very similar when looking at each occupation separately or by looking at all or each industry in part. Evidence from other fields such as social psychology indicates that in fact membership in exogeneously-defined (where the individual could not choose its group) group comes with strong intragroup solidarity, even when the

[5] There is however recent empirical evidence that jobs obtained through networks of personal contacts are not always better than those obtained through formal means. Using US and European data Bentolila, Michelacci and Suarez (2004) find that the use of social contacts, although helping in finding a job faster, can generate mismatches between a worker's occupational choice and his comparative productive advantage, leading to individual wage discounts of 5 to 7%. Pellizarri (2004) using European data finds that wage penalties and wage premiums of jobs obtained through contacts relative to jobs obtained formally are equally frequent accross countries and industries.

[6] Intuitively the inbreeding bias by gender is likely to be smaller than inbreeding biases by social groups differentiated along race or ethnicity, given the possible close-knit relationships between men and women.

groups are arbitrary categorized, as documented for instance in the "Robbers Cave" study by Sherif(1961)[7].

Using these stylized facts, we build a simple theoretical model based on social networks, able to explain stable occupational segregation equilibria without a need for other ingredients often used in this context. Our model should of course be seen as complementary to such existing theories in explaining the empirically observed occupational segregation patterns. While our study has common elements with theories concerned with group membership used on a large scale in sociology for explaining general segregation patterns (neighborhood segregation, school segregation, workplace segregation etc), it differs from these theories by explicitly modelling the dynamic network interaction[8].

3 A Model of Occupational Segregation

Consider the following model. A continuum of $2N$ workers is equally divided into two social groups, reds (R) and greens (G). They can work in two occupations, A or B. Both occupations require a thorough specialized education (training), hence a worker cannot work in an occupation if she is not qualified to do so by having followed one of the educational tracks corresponding to each of the two occupations. Therefore workers have to choose their education before they enter the labour market, in other words they have to make a decision regarding one of their two available career choices.

We consider the following timing:

1. Workers choose one education in order to specialize either in occupation A or in occupation B;

2. Workers randomly establish "friendship" relationships, thus forming a network of contacts;

3. Workers participate in the labour market and if they have a job they earn a wage w_A or w_B, respectively.

The choice of education in the first stage involves strategic behaviour and we therefore look for a Nash equilibrium in this stage. The expected payoff of

[7] A different literature stream looks at penalties associated with acting differently than according to the "behavioral prescriptions" of different social groups. Such studies investigating the (often negative) peer-pressure effect in one's social group are for instance the recent ones on "acting white" by Austen-Smith and Fryer (2005) and Fryer (2006). This theory can be seen as alternative to the intragroup inbreeding bias hypothesis we use in here. For our paper what matters is that a member of a certain social group is more likely to link ties with members of the same group and that this likelihood is not endogenously determined.

[8] The precursor of many such studies is the work by Schelling (1971) on emergence of segregated communities starting from a mild preference of individuals to be in an ethnic majority in their communities. In fact Schelling's model starts from similar assumptions as the ones we use here: two social groups, a stated mild preference in terms of neighbors (this is somewhat anologous to our inbreeding bias). The whole context and the dynamic modelling are however very much different.

a worker's educational choice given the choices of other workers is determined by the network formation process in the second stage and by the employment process in the third stage. We now make these two other stages more specific.

In the second stage the workers form a network of contacts. We assume this network to be random with an inbreeding bias (also known in the literature as assortative mixing). That is, we assume that the probability for two workers to create a tie is p when the two workers are from different social groups; however when two workers are from the same social group, the probability of creating a tie is λp with $\lambda > 1$. We will refer to two workers that created a tie as "friends"[9].

The third stage we envision is that of a dynamic labour process a la Jackson and Wolinsky (2004) or Bramoullé and Saint-Paul (2005), in which employed workers randomly lose their jobs while unemployed workers search for jobs. Unemployed workers receive job information either directly, or indirectly through their friends. The details of such a process are unimportant for our purposes. However, what is important is that we assume that unemployed workers have a higher propensity to receive job information when they have more friends with the *same* job background, that is, with the same choice of education. By this we implicitly assume that everybody has the same chance on the formal labour market, or in other words, that direct job search intensity is exogenously given for everybody. Since the details of the labour market process are not relevant, we ignore the precise dynamics and we simply assume that the probability that one is employed increases in the number of friends with the same education. Denote η_i^A as the number of friends of individual i that are A-educated, and denote η_i^B similarly for the number of friends of i that are B-educated. Further denote the probability that i becomes employed as s_i. Then $s_i = s(\eta_i^A)$ if i is A-educated and $s_i = s(\eta_i^B)$ if i is B-educated. We assume that $s(0) = s_0 > 0$ and $s'(x) > 0$ for all $x > 0$.

The eventual payoff of the workers depends on the employment status in each period and on the wage they receive. We assume that the wage rate decreases in the number of workers that choose a particular type of education. The assumption of a decreasing wage when the total number of employed workers increases can be explained with a simple classical model of a 2-goods economy with Cobb-Douglas utility functions, and a linear production function with labour as single input. Intuitively, when more workers are educated as A, more workers are employed as A. Thus the economy produces more A-products, which have to find their way to the consumer market. As the market price drops whenever production output increases, it follows that, in a competitive product and labour market, wages drop as well. Thus wages of A (B)-jobs are negatively related with the number of workers that choose an A (B)-education.

The assumption is formalized as follows.

Assumption 1 *Let* n_A *be the total number of workers that are educated as*

[9] We do not consider in our model the complication that individuals of one social group might have a higher probability of making contacts than members of the other group. There is evidence that men have a better access to contact networks than women, see for instance the introductory discussion in Petersen, Saporta and Seidel (2000) and the references therein.

A and let n_B be defined analogously. Define $\phi_a = n_a/2N$ and $\phi_b = n_b/2N$ as the proportion of workers A or respectively B-educated. Then the wage of an A-job, $w_A(\phi_A)$, (and a B-job, $w_B(\phi_B)$) is decreasing in ϕ_A (or ϕ_B), and $\forall x: \ w_A(x) = w_B(x)$. Further, w_A and w_B are continuous and

$$\lim_{\phi_A \downarrow 0} w_A(\phi_A) = \lim_{\phi_B \downarrow 0} w_B(\phi_B) = \infty.$$

Note that we initially assume that wages for A and B jobs are equal if there are as many A-educated workers as B-educated workers. In Section 5 we relax this assumption.

We can now define the payoff of a worker. First, define μ_R and μ_G as the fraction of workers in R (G) that choose education A. Next, denote $S(x, y)$ as the expected employment probability of i, when a fraction x of i's own social group has the same education as i and a fraction y of the other social group has the same education as i. That is,

$$S(x, y) = \sum_{k=0}^{\infty} s(k) f(k|x, y),$$

where $f(k|x, y)$ is the probability that a worker i has k friends of the same education, given that a fraction x of i's social group and a fraction y of the other social group choose the same education as i.

Then the payoff function of a worker i who is from group R and A-educated is

$$\Pi_i^R(A; \mu_R, \mu_G) = w_A \left(\frac{\mu_R + \mu_G}{2} \right) S(\mu_R, \mu_G). \tag{1}$$

Similarly,

$$\Pi_i^R(B; \mu_R, \mu_G) = w_B \left(1 - \frac{\mu_R + \mu_G}{2} \right) S(1 - \mu_R, 1 - \mu_G). \tag{2}$$

$$\Pi_i^G(A; \mu_R, \mu_G) = w_A \left(\frac{\mu_R + \mu_G}{2} \right) S(\mu_G, \mu_R). \tag{3}$$

$$\Pi_i^G(B; \mu_R, \mu_G) = w_B \left(1 - \frac{\mu_R + \mu_G}{2} \right) S(1 - \mu_G, 1 - \mu_R). \tag{4}$$

We note that if there is an inbreeding bias in the social network ($\lambda > 1$) and $s'(\eta) > 0$ for $\eta > 0$, then for all $x > y$

$$S(x, y) > S(y, x),$$

since a worker is more likely to form a friendship relation with a worker from its own group, than with a worker from the other group.

3.1 Equilibrium

We would like to characterize the Nash equilibria in the model above. We are, in particular, interested in those equilibria in which there is segregation. We define segregation as follows:

Definition 1 *Let μ_X, $X \in \{R, G\}$ be the fraction of workers in social group X that choose education A. There is* complete segregation *if $\mu_R = 0$ and $\mu_G = 1$, or, vice versa, $\mu_R = 1$ and $\mu_G = 0$. There is* partial segregation *if for $X \in \{R, G\}$ and $Y \in \{R, G\}$, $Y \neq X$: $\mu_X = 0$ but $\mu_Y < 1$, or, vice versa, $\mu_X = 1$ but $\mu_Y > 0$.*

In a Nash equilibrium each worker chooses the education that gives the highest payoff given the education choices of all other workers. Since workers of the same group are homogenous, a Nash equilibrium implies that if *some* worker in a group chooses education A (B), then no other worker in the same group should prefer education B (A). With this idea in mind we reformulate the equilibrium concept in a way that turns out to be useful.

Definition 2 *Let μ_X, $X \in \{R, G\}$ be the fraction of workers in social group X that choose education A. A pair (μ_R, μ_G) is an* equilibrium *if and only if, for $X \in \{R, G\}$, the following hold*

$$\Pi_i^X(A; \mu_R, \mu_G) \leq \Pi_i^X(B; \mu_R, \mu_G) \text{ if } \mu_X = 0 \quad (5)$$

$$\Pi_i^X(A; \mu_R, \mu_G) = \Pi_i^X(B; \mu_R, \mu_G) \text{ if } 0 < \mu_X < 1 \quad (6)$$

$$\Pi_i^X(A; \mu_R, \mu_G) \geq \Pi_i^X(B; \mu_R, \mu_G) \text{ if } \mu_X = 1 \quad (7)$$

In our initial analysis we often find multiple equilibria. However, some of these equilibria are not dynamically stable. We therefore use a simple stability concept based on a standard myopic adjustment process of strategies. That is, we think of the equilibrium as the outcome of an adjustment process in which more and more workers revise their education choice if it is profitable to do so given the choice of the other workers. To be concrete, we consider stationary points of a dynamic system in which

$$\frac{d\mu_X}{dt} = k\left(\Pi_i^X(A; \mu_R(t), \mu_G(t)) - \Pi_i^X(B; \mu_R(t), \mu_G(t))\right).$$

The stability properties of stationary points in such dynamic systems are well-known, see e.g. Chiang (1984, p.641-645). We base our definition on these properties, taking into account that the process might converge to a segregation equilibrium, thus to the boundaries of the solution space.

Definition 3 *Let (μ_R^*, μ_G^*) be an equilibrium and define*

$$G = \begin{bmatrix} \frac{\partial(\Pi_i^R(A;\mu_R^*,\mu_G^*) - \Pi_i^R(B;\mu_R^*,\mu_G^*))}{\partial\mu_R} & \frac{\partial(\Pi_i^R(A;\mu_R^*,\mu_G^*) - \Pi_i^R(B;\mu_R^*,\mu_G^*))}{\partial\mu_G} \\ \frac{\partial(\Pi_i^G(A;\mu_R^*,\mu_G^*) - \Pi_i^G(B;\mu_R^*,\mu_G^*))}{\partial\mu_R} & \frac{\partial(\Pi_i^G(A;\mu_R^*,\mu_G^*) - \Pi_i^G(B;\mu_R^*,\mu_G^*))}{\partial\mu_G}. \end{bmatrix} \quad (8)$$

The equilibrium is stable under the following conditions for $X, Y \in \{R, G\}$, $X \neq Y$*;*

(i) if $\mu_X^* = 0$*, then* $\Pi_i^X(A; \mu_R^*, \mu_G^*) < \Pi_i^X(B; \mu_R^*, \mu_G^*)$*;*

(ii) if $\mu_X^* = 1$*, then* $\Pi_i^X(A; \mu_R^*, \mu_G^*) > \Pi_i^X(B; \mu_R^*, \mu_G^*)$*;*

(iii) if $\mu_X^* = 0$ *or* 1*, and* $\mu_Y^* \in (0, 1)$*, then*

$$\frac{\partial(\Pi_i^Y(A; \mu_R^*, \mu_G^*) - \Pi_i^Y(B; \mu_R^*, \mu_G^*))}{\partial \mu_Y} < 0;$$

(iv) if $\mu_R^* \in (0, 1)$ *and* $\mu_G^* \in (0, 1)$*, then*

$$\mathit{trace}(G) < 0 \text{ and } \mathit{det}(G) > 0.$$

The equilibrium is weakly stable *if the above conditions only hold with weak inequality signs.*

Conditions (i), (ii) and (iii) are applied to segregation equilibria, while condition (iv) is applied to non-segregation equilibria. If an equilibrium is stable, then the dynamic system converges back to the equilibrium after any small perturbation. This is not necessarily true for a weak equilibrium.

4 Occupational segregation

We next characterize equilibria for three cases. In the benchmark case network effects are nonexistent. In the second case network effects are important, but there is no inbreeding bias in the social network, that is, $\lambda = 1$. In the third case, we consider the full model including network effects and an inbreeding bias.

4.1 A market without network effects

We first consider a labour market in which the probability to get a job does not depend on a worker's social network. That is $s(\eta) = s_0$. We obtain a standard result

Proposition 1 *Suppose* $s(\eta) = s_0 \in (0, 1]$*.* (μ_R^*, μ_G^*) *is a weakly stable equilibrium if and only if*

$$w_A\left(\frac{\mu_R^* + \mu_G^*}{2}\right) = w_B\left(1 - \frac{\mu_R^* + \mu_G^*}{2}\right). \quad (9)$$

Proof. If $s(\eta) = s_0$, then equation (9) is equivalent to

$$\Pi_i^X(A; \mu_R^*, \mu_G^*) = \Pi_i^X(B; \mu_R^*, \mu_G^*)$$

for $X \in \{R, G\}$. Clearly, if (9) holds then (μ_R^*, μ_G^*) is an equilibrium.

Further, since $w_A(x)$ and $w_B(x)$ are decreasing in x, it is easy to see that

$$\frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_X} = \frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_Y} = \frac{s_0}{2}(w'_A(\cdot) + w'_B(\cdot)) < 0$$

for $X, Y \in \{R, G\}$, $X \neq Y$. This implies that for G defined as (8) we have trace$(G) < 0$ and $\det(G) = 0$. Hence, all conditions for *weak* stability are satisfied.

Moreover any (μ'_R, μ'_G) for which $w_A > w_B$ cannot be equilibria since then

$$\Pi_i^X(A; \mu'_R, \mu'_G) > \Pi_i^X(B; \mu'_R, \mu'_G).$$

This would only be consistent with an equilibrium if all workers chose A as education, that is $\mu_R = \mu_G = 1$. However, in Assumption 1 we have assumed that $w_B \to \infty$ if $(\mu_R + \mu_G)/2 \to 1$. Hence, if $(\mu_R + \mu_G)/2 = 1$, then $w_A < w_B$, and we have a contradiction.

Similarly in an equilibrium it is not possible that $w_A(\cdot) < w_B(\cdot)$. ■

This proposition simply restates the classical view that the price of labor, wages, are equal to the value of the marginal product of labor. Since workers are homogenous with respect to their productivity, everyone earns the same wage and occupational segregation or social inequality does not occur [10]. Note that Proposition 1 does not give a unique equilibrium, but a (convex) set of equilibria (μ_R^*, μ_G^*) for which $w_A(\cdot) = w_B(\cdot)$. This is also the reason why each equilibrium is only *weakly* stable. After a small perturbation to an equilibrium, a best response dynamic process as the one described above converges back to the set for which $w_A = w_B$. However, the process does not converge to the 'starting' equilibrium that was originally perturbed. Thus the equilibria cannot be strongly stable.

4.2 A labour market with network effects, but without inbreeding in the social network

We further assume that having a social network is important to get a job. However, if we do not introduce an inbreeding bias in the network of friendship relations, the result of Proposition 1 hardly changes. That is, all workers receive the same payoff.

[10] Since all workers are indifferent between education A or B, a partial segregation equilibrium does exist. However, this equilibrium would be excluded if workers in the same social group are slightly heterogeneous with respect to their education preferences or their productivity.

Proposition 2 *Suppose $s'(\eta) > 0$, but $\lambda = 1$. Then (μ_R^*, μ_G^*) is a weakly stable equilibrium if and only if*

$$w_A\left(\frac{\mu_R^* + \mu_G^*}{2}\right) S(\mu_R^*, \mu_G^*) = w_B\left(1 - \frac{\mu_R^* + \mu_G^*}{2}\right) S(1-\mu_R^*, 1-\mu_G^*). \quad (10)$$

We omit the proof as it is similar to that of Proposition 1.

If we compare Propositions 1 and 2, we notice that the network effects allow for a difference in the probability to become employed. This also implies that in the equilibrium there might be a wage difference between A and B-workers. Thus, while the wage of A-workers might be lower than that of B-workers, this is compensated by a higher probability of getting a job for A-educated workers.

While there might be wage differences between A and B-workers, segregation by occupation is not a likely outcome if there is no inbreeding bias in the social network. Although there exists a weakly stable equilibrium with segregation, this equilibrium would be ruled out if we would introduce a slight heterogeneity in the worker's education preferences or in their productivity.

Perhaps it is slightly surprising that the network effects do not directly result in segregation. One has to remember that if there is no inbreeding bias in the social network these network effects, as well as the wages, are group-independent. Thus the value of an A-education or B-education only depends on the total number of other workers that choose education A or B, not on the number of workers choosing A or B in each group. It should then be clear that there is no reason to expect segregation as the group identity does not matter in making an education choice.

4.3 A labour market with network effects and a social network with inbreeding

We now consider the unrestricted version of our labour market model. Our first observation is that the equilibrium changes drastically, even with a small amount of inbreeding bias.

Proposition 3 *Suppose $s'(\eta) > 0$ and $\lambda > 1$. A weakly stable equilibrium (μ_R^*, μ_G^*), in which $0 < \mu_R^* < 1$ and $0 < \mu_G^* < 1$, does not exist.*

Proof. Suppose (μ_R^*, μ_G^*) is a stable equilibrium, and $\mu_R^* \in (0,1)$ and $\mu_G^* \in (0,1)$. By condition (6)

$$\Pi_i^R(A; \mu_R^*, \mu_G^*) = \Pi_i^R(B; \mu_R^*, \mu_G^*) \text{ and } \Pi_i^G(A; \mu_R^*, \mu_G^*) = \Pi_i^G(B; \mu_R^*, \mu_G^*) \quad (11)$$

Substituting (1)-(4) into (11) and rewriting these equations become

$$\frac{w_A(\frac{\mu_R^*+\mu_G^*}{2})}{w_B(1-\frac{\mu_R^*+\mu_G^*}{2})} = \frac{S(1-\mu_R^*, 1-\mu_G^*)}{S(\mu_R^*, \mu_G^*)} = \frac{S(1-\mu_G^*, 1-\mu_R^*)}{S(\mu_G^*, \mu_R^*)}. \quad (12)$$

Since $\lambda > 1$, $x > y$ implies $S(x, y) > S(y, x)$. But this means that if $\mu_R^* > \mu_G^*$, then

$$\frac{S(1-\mu_R^*, 1-\mu_G^*)}{S(\mu_R^*, \mu_G^*)} < \frac{S(1-\mu_G^*, 1-\mu_R^*)}{S(\mu_G^*, \mu_R^*)},$$

which contradicts (12). The same reasoning holds for $\mu_R^* < \mu_G^*$. Hence, it must be that $\mu_R^* = \mu_G^*$.

However (μ_R^*, μ_G^*) with $\mu_R^* = \mu_G^*$ cannot be a weakly stable equilibrium. To see this, suppose that (μ^*, μ^*) with $\mu^* \in (0, 1)$ is a weakly stable equilibrium. Hence $\Pi_i^X(A; \mu^*, \mu^*) = \Pi_i^X(B; \mu^*, \mu^*)$ for $X \in \{R, G\}$ and $\text{trace}(G) \leq 0$ and $\det(G) \geq 0$, where G is defined in (8). Now for $X \in \{R, G\}$

$$\begin{aligned}\frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_X} &= \frac{1}{2}w_A'(\mu^*)S(\mu^*, \mu^*) + w_A(\mu^*)S_1(\mu^*, \mu^*) \\ &+ \frac{1}{2}w_B'(\mu^*)S(1-\mu^*, 1-\mu^*) + w_B(1-\mu^*)S_1(1-\mu^*, 1-\mu^*),\end{aligned}$$

where $S_j(\mu, \mu) = \left.\frac{\partial S(x_1, x_2)}{\partial x_i j}\right|_{x_1=\mu, x_2=\mu}$ for $j = 1, 2$. The cross effect is

$$\begin{aligned}\frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_Y} &= \frac{1}{2}w_A'(\mu^*)S(\mu^*, \mu^*) + w_A(\mu^*)S_2(\mu^*, \mu^*) \\ &+ \frac{1}{2}w_B'(\mu^*)S(1-\mu^*, 1-\mu^*) + w_B(1-\mu^*)S_2(1-\mu^*, 1-\mu^*),\end{aligned}$$

for $Y \in \{R, G\}$, $Y \neq X$.

Since $\lambda > 1$, it must be that $S_1(\mu, \mu) > S_2(\mu, \mu)$ for all $\mu \in (0, 1)$. Therefore,

$$\frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_X} > \frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_Y}$$

for $X, Y \in \{R, G\}$, $X \neq Y$.

Because $\text{trace}(G) \leq 0$, it must be that

$$\frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_Y} < \frac{\partial(\Pi_i^X(A;\cdot) - \Pi_i^X(B;\cdot))}{\partial \mu_X} \leq 0.$$

But then it is easy to see that $\det(G) < 0$. This contradicts weak stability. ■

This proposition shows that even with a small inbreeding bias, segregation by occupation is a natural outcome. At least one social group specializes fully in one type of occupation. The intuition is that an inbreeding bias in the social network creates a group-dependent network effect. Thus if in group R slightly more workers choose A than in group G, then the value of an A-education is higher in group R than in group G, while the value of a B-education is lower in group R. Positive feedback then ensures that the initially small differences in education choices between the two groups widen and widen until at least one group segregates into one type of education.

While we have shown that in a labour market model with network effects and inbreeding bias segregation is a natural outcome, the question remains what the segregation equilibria look like and whether there can be sustained wage differences between groups. Depending on the functional form of $w_A(\cdot)$ and $w_B(\cdot)$ and $s(\cdot)$, there could be many equilibria. However, *complete* segregation is the most prominent outcome.

Proposition 4 *Suppose* $s'(\eta) > 0$ *and* $\lambda > 1$. *Then*

(i) $(\mu_R, \mu_G) = (1, 0)$ *and* $(\mu_R, \mu_G) = (0, 1)$ *are stable equilibria.*

(ii) if for all $x \in [0, 1]$

$$\frac{w_A(x/2)}{w_B(1 - x/2)} > \frac{S(1 - x, 1)}{S(x, 0)}, \tag{13}$$

and

$$\frac{w_A((1 + x)/2)}{w_B((1 - x)/2)} < \frac{S(1 - x, 0)}{S(x, 1)}, \tag{14}$$

then $(\mu_R, \mu_G) = (1, 0)$ *and* $(\mu_R, \mu_G) = (0, 1)$ *are the only two stable equilibria.*

Proof. (i) If (15) holds, then

$$\Pi_i^R(A; 1, 0) > \Pi_i^R(B; 1, 0) \text{ and } \Pi_i^G(A; 1, 0) < \Pi_i^G(B; 1, 0)$$

for $(\mu_R, \mu_G) = (1, 0)$. This is clearly a stable equilibrium. The same is true for $(\mu_R, \mu_G) = (0, 1)$.

(ii) Note that (13) and (14) do not contradict (15). Hence, $(\mu_R, \mu_G) = (1, 0)$ and $(\mu_R, \mu_G) = (0, 1)$ are both stable equilibria. We only have to show that there are no other stable equilibria. From Proposition 3 we already know that (μ_R, μ_G) with $0 < \mu_R < 1$ and $0 < \mu_G < 1$ cannot be a stable equilibrium. So consider $\mu_R = 0$. If $\mu_G < 1$, then (μ_R, μ_G) can be an equilibrium only if $\Pi^G(A; 0, \mu_G) \leq \Pi^G(B; 0, \mu_G)$. However, this is excluded by condition (13). Similarly, if $\mu_R = 1$, then $\mu_G > 0$ is excluded as an equilibrium by condition (14). ■

Part (i) shows that complete segregation is always an equilibrium outcome. That is, one social group specializes in one occupation, and the other group in the other occupation. Part (ii) establishes sufficient conditions for uniqueness of a fully segregated equilibrium. To understand these conditions, note that if all workers would choose A as education, $(\mu_R, \mu_G) = (1, 1)$, then everyone has an incentive to choose education B, as the wage of B-jobs is infinitely higher. The conditions (13) and (14) then say that if all green workers would choose A, then the red workers *always* have an incentive to choose education B, either because the wage of B-jobs is in equilibrium higher than that of A-jobs, or the prospects of finding a B-job are much higher due to the social network effects.

5 Social Inequality

The discussion above ignores differences in wages and unemployment. In fact, since we have assumed that A and B jobs are equally attractive, it is easily seen that under complete segregation there is no wage and unemployment inequality. However, not only is this in sharp contrast to observed gender and race gaps in wages and unemployment, it is also not obvious how our result of complete segregation can be sustained when there are large wage-induced incentives. That is: why would someone stick to the education choice of her social group when the wage benefits of choosing the other education are very large? This motivates us to extend our framework in order to look at the robustness of our results under wage and employment inequality. We do this by making the following assumption on the wage function

Assumption 2 *For every $x \in (0,1)$, $w_A(x) > w_B(x)$.*

Thus if there are as many A-educated workers as B-educated workers, then the A-educated workers earn a higher wage. The implicit assumption behind Assumption 2 is that the marginal utility consumers derive from product A is larger than the marginal utility from product B. This is a natural assumption as there is no reason to expect that different products are equally liked.

We derive results on wage and unemployment inequality under Assumption 2. Our first observation is that the proof of Proposition 3 does not depend on the fact that $w_A(x) = w_B(x)$ for every x. Hence, this proposition also holds under Assumption 2.

Proposition 5 *Suppose $s'(\eta) > 0$, $\lambda > 1$ and Assumption 2 holds. A weakly stable equilibrium (μ_R^*, μ_G^*), in which $0 < \mu_R^* < 1$ and $0 < \mu_G^* < 1$, does not exist.*

We next characterize the segregation equilibria. We consider two cases; either the difference between A and B-jobs is relatively small compared to the social network effect, or the difference is relatively large. We first consider the case in which the job difference is relatively small. In this case, *complete* segregation is the most prominent outcome.

Proposition 6 *Suppose $s'(\eta) > 0$, $\lambda > 1$, Assumption 2 holds and*

$$\frac{w_A(1/2)}{w_B(1/2)} < \frac{S(1,0)}{S(0,1)}. \tag{15}$$

Then $(\mu_R, \mu_G) = (1,0)$ and $(\mu_R, \mu_G) = (0,1)$ are stable equilibria. In these equilibria,

$$w_A(\cdot) > w_B(\cdot),$$

and, if $\mu_X = 1$ and $\mu_Y = 0$ for $X, Y \in \{R, G\}$, $X \neq Y$, then

$$\Pi_i^X(A;\cdot) > \Pi_i^Y(B;\cdot) > \Pi_i^Y(A;\cdot) > \Pi_i^X(B;\cdot). \tag{16}$$

Proof. If (15) holds, then

$$\Pi_i^R(A;1,0) > \Pi_i^R(B;1,0) \text{ and } \Pi_i^G(A;1,0) < \Pi_i^G(B;1,0)$$

for $(\mu_R, \mu_G) = (1,0)$. This is clearly a stable equilibrium. Further, as $(\mu_R + \mu_G)/2 = 1 - (\mu_R + \mu_G)/2 = 1/2$, it holds that $w_A > w_B$. Finally

$$S(1,0)w_A(1/2) > S(1,0)w_B(1/2) > S(0,1)w_A(1/2) > S(0,1)w_B(1/2),$$

and this is equivalent to (16).

The same is true for $(\mu_R, \mu_G) = (0,1)$. ■

This proposition states that if the difference in wages is not too large, complete segregation is always an equilibrium outcome. Thus one social group specializes in one occupation, and the other group in the other occupation. Since the social groups are of equal size, the employment probabilities in the two social groups are the same. However, since the wage of A is higher in the equilibrium, the social group that specializes in occupation A obtains a higher payoff than the other group. Hence, social inequality is a natural outcome of this model.

Interestingly, if some workers make mistakes in their education choice, then the workers that are the worst off are from the same social group as the workers that are the best off. Thus, if $\mu_R = 1$ and $\mu_G = 0$, then the red workers that choose A receive the highest wage and have the best employment probabilities. However, if some of the red workers choose B by mistake, then these red B-worker are the most disadvantaged, as they earn the lowest wage and have the lowest employment chances.

We turn next to the case in which wage differentials are large. We have the following proposition.

Proposition 7 *Suppose $s'(\eta) > 0$, $\lambda > 1$, Assumption 2 holds and*

$$\frac{w_A(1/2)}{w_B(1/2)} > \frac{S(1,0)}{S(0,1)}. \tag{17}$$

(i) There is no equilibrium with complete segregation.

(ii) There are at least two stable equilibria with partial segregation, in which either $\mu_R = 1$ or $\mu_G = 1$. If $\mu_X = 1$ for $X \in \{R, G\}$, then for $Y \in \{R, G\}$, $Y \neq X$

$$\Pi_i^X(A;\cdot) > \Pi_i^Y(B;\cdot) = \Pi_i^Y(A;\cdot) > \Pi_i^X(B;\cdot). \tag{18}$$

Proof. (i) If (17) is true, then for $(\mu_R, \mu_G) = (1,0)$

$$\Pi_i^G(A;1,0) > \Pi_i^G(B;1,0). \tag{19}$$

Thus G-workers would like to deviate by choosing education A, and therefore $(\mu_R, \mu_G) = (1,0)$ cannot be an equilibrium. The same holds for $(\mu_R, \mu_G) = (0,1)$.

(ii) As $\Pi_i^X(\cdot)$ is continuous in μ_G, it follows from equation (19) and Assumption 1 that there must be a μ^*, such that

$$\Pi_i^G(A;1,\mu^*) = \Pi_i^G(B;1,\mu^*),$$

and

$$\frac{\partial(\Pi_i^G(A;1,\mu^*) - \Pi_i^G(B;1,\mu^*))}{\partial \mu_G} < 0.$$

Moreover, $S(1,\mu^*) > S(\mu^*,1)$ and $S(1-\mu^*,0) > S(0,1-\mu^*)$, so we have

$$S(1,\mu^*)w_A(\cdot) > S(\mu^*,1)w_A(\cdot) = S(1-\mu^*,0)w_B(\cdot) > S(0,1-\mu^*)w_B(\cdot),$$

and this is equivalent to (18) for $X = R$ and $Y = G$. As

$$\Pi_i^R(A;1,\mu^*) > \Pi_i^R(B;1,\mu^*),$$

it is also clear that $(\mu_R, \mu_G) = (1, \mu_G^*)$ is a stable equilibrium. The same is true for $(\mu_R, \mu_G) = (\mu^*, 1)$. ■

The proposition makes clear that complete segregation cannot be sustained if the wage differential is too large. Starting from complete segregation, a large wage differential gives incentives to the group specialized in B-jobs to switch to A-jobs. Interestingly, the unsustainable complete segregation equilibrium is then replaced by a partial equilibrium in which one group specializes in job A, while the other group has both A and B-workers. As in the previous case of small wage differentials, the workers of the group specializing in A-jobs receive the highest payoffs, hence we have again a social inequality outcome. However in this case the wages of A-workers in the equilibrium are not necessarily higher than that of B-workers. It is the higher employment rate of the group specializing in A that makes the difference. The employment rate of the group specializing in A-jobs is given by $S(1,x)$ where x is the fraction of A workers in the group that does not specialize. On the other hand the employment rate of the group that does not specialize is $xS(x,1) + (1-x)S(x,0)$. Thus the group that specializes in the A-job has a lower unemployment rate than the other group.

6 Summary and Conclusions

We have investigated in this paper a simple social network framework where jobs are obtained through a network of contacts formed stochastically. We have shown that even with a very small amount of inbreeding bias within each social group, dynamically stable occupational segregation equilibria will arise. If the wage differential across the occupations is not too large, complete segregation will always be sustainable. If the wage differential is large, complete segregation cannot be sustained, but a partial segregation equilibrium in which one of the group fully specializes in one type of educations while the other group mixes, is sustainable. Furthermore, this model is able to explain sustained wage and unemployment differences between the social groups.

While our oversimplified model is able to describe patterns of occupational segregation and inequality, we neither claim nor think that our explanation should be seen in isolation as the ideal candidate to explain occupational segregation and inequality. Other factors are very likely (and documented) to be relevant. What is important is to consider our model as complementary to other frameworks and as possibly accounting for part of the persistent occupational segregation observed in practice. To this end it is of course important to empirically document in future research how relevant are the mechanisms described in this paper. Another avenue for future research is to extend the framework to other very relevant issues, such as the position of minority vs. majority groups, by looking at the interaction in this context of social groups of different sizes.

References

[1] Akerlof, G. A and R. E. Kranton (2000), Economics and identity, *Quarterly Journal of Economics*, 115(3), pp. 715-753

[2] Albeda, R.P. (1986), Occupational segregation by race and gender, *Industrial and Labor Relations Review*, 39(3), pp. 404-411

[3] Arrow, K.J (1972a), Models of job discrimination in A.H. Pascal, ed., *Discrimination in economic life*, Lexington, MA

[4] Arrow, K. J. (1972b), The theory of discrimination, in. O. Ashenfelter, A. Rees (Eds.), *Discrimination in Labor Markets*, Princeton University Press, Princeton, NJ

[5] Arrow, K.J. (1998), What has economics to say about racial discrimination?, *Journal of Economic Perspectives*, 12(2), pp. 91-100.

[6] Arrow, K.J. and R. Borzekowski (2004), Limited network connections and the distribution of wages, *mimeo*, Federal Reserve Board.

[7] Becker, G. (1957), *The Economics of Discrimination*, Chicago: Chicago University Press

[8] Austen-Smith, D. and R.G. Fryer (2005), *Quarterly Journal of Economics*, 120(2), pp. 551-583

[9] Bentolila, S. , C. Michellachi and J. Suarez (2004), Social contacts and educational choice, *mimeo*, CEMFI

[10] Blau, F.D., P. Simpson and D. Anderson (1998), Continuing progress? Trends in occupational segregation in the United States over the 1970's and 1980's, *NBER Working Paper* No. 6716

[11] Bramoullé, Y. and G. Saint-Paul (2004), Social networks and labor market transitions, *mimeo*, Université Laval and Université Toulouse.

[12] Calvó-Armengol, A. and M.O. Jackson (2004), The effects of social networks on employment and inequality, *American Economic Review*, 94(3), pp. 426-454.

[13] Calvó-Armengol, A. and M.O. Jackson (2005), Networks in labor markets: Wage and unemployment dynamics and inequality, *Journal of Economic Theory*, forthcoming.

[14] Chiang A.C. (1984), *Fundamental Methods of Mathematical Economics*, 3rd edition, New York: McGraw-Hill.

[15] Coate, S. and G.C. Loury (1993), Will affirmative-action policies eliminate negative stereotypes?, *American Economic Review*, 83(5), 1220-1240

[16] Datcher, L. (1983), The impact of informal networks on quit behavior, *Review of Economics and Statistics*, 65(3), 491-495.

[17] Devine, T.J. and N.M. Kiefer (1991), *Empirical Labor Economics: The Search Approach*, New York: Oxford University Press.

[18] Doeringer P.B. and M.J. Piore (1971), *Internal Labor Markets and Manpower Analysis*, Lexington: Heath Lexington Books.

[19] Durlauf S. N. (2003), Groups, Social Influences and Inequality: A Membership Theory Perspective on Poverty Traps, *mimeo*, University of Wisconsin

[20] Fontaine, F. (2004), Do workers really benefit from their social networks?, *IZA Discussion Paper* No. 1282.

[21] Fryer, R.G. (2006), "Acting White", *Education Next*

[22] Granovetter, M.S. (1995), *Getting a Job: A Study of Contacts and Careers*, 2nd edition, Chicago: University of Chicago Press.

[23] Holzer, H.J. (1987), Informal job search and black youth unemployment, *American Economic Review*, 77(3), 446-452.

[24] Jacobs, J. A. and S.T. Lim (1992), Trends in Occupational and Industrial Sex Segregation in 56 Countries, 1960-1980, *Work and Occupations*, 19(4), 450-486

[25] Lavezzi, A.M. and N. Meccheri (2005), Social networks in labor markets: The effects of symmetry, randomness and exclusion on output and inequality, *mimeo*, Università di Pisa.

[26] Lazear, E. P and S. Rosen (1990), Male-female wage differentials in job ladders, *Journal of Labor Economics*, VIII, S106-S123

[27] Ioannides, Y.M. and L. Datcher Loury (2004), Job information networks, neighborhood effects, and inequality, *Journal of Economic Literature*, XLII, pp. 1056-1093

[28] Marsden, P.V. (1987), Core discussion networks of americans, *American Sociological Review*, 52(1), 122-131.

[29] Montgomery, J.D. (1991a), Social networks and labor-market outcomes: Toward an economic analysis, *American Economic Review*, 81 (5), 1408-1418.

[30] Montgomery, J. D. (1998), Toward a Role-Theoretic Conception of Embeddedness, *American Journal of Sociology* ,104 (July), pp. 95-125.

[31] Pellizzari, M. (2004), "Do friends and relatives really help in getting a new job?", *mimeo*, CEP LSE

[32] Peterson, T., I. Saporta and M.L. Seidel (2000), Offering a job: meritocracy and social networks, American Journal of Sociology, 106, pp. 763-816

[33] Phelps, E. S. (1972), *The statistical theory of racism and sexism*, American Economic Review, pp. 659-61

[34] Polacheck, S. (1981), Occupational self-selection: A human capital approach to sex differences in occupational structure, *Review of Economics and Statistics* 58, pp 60-69

[35] Rees, A. (1966), Information networks in labor markets, *American Economic Review*, 56, 559-566.

[36] Rich, J. (1999), Gender segregation in the academic staff of universities in Great Britain, 1980/81-1993/94, *Economica*, 66(264), pp. 509-531

[37] Schelling, T. (1971), Dynamic models of segregation, *Journal of Mathematical Sociology*, 1, pp.- 143-186

[38] Sherif, M. (1961), *Intergroup Conflict and Cooperation: The Robbers Cave Experiment*, Norman: University Book Exchange.

[39] Simon, C.J. and J.T. Warner (1992), Matchmaker, matchmaker: The effect of old boy networks on job match quality, earnings, and tenure, *Journal of Labor Economics*, 10(3), 306-330.

[40] Staiger, D. (1990), The effect of connections on the wages and mobility of young workers, *mimeo*, MIT.

[41] Topa, G. (2001), Social interactions, local spillovers and unemployment, *Review of Economic Studies*, 68(2), 261-295.