Evaluation of Postprocessing Dual-Energy Methods in Quantitative Computed Tomography

Part 1. Theoretical Considerations

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Five postprocessing methods for dual-energy quantitative computed tomography of the vertebral body were evaluated theoretically. The methods were compared by transforming the original sets of equations to a standard set. Only two of these methods produced optimal results, namely the basic approach of Goodsitt et al and the method of Nickoloff et al. The calibration approach of Goodsitt et al will produce optimal results only if calibration materials are available that mimic the anatomic constituents of the vertebral body better than those available currently. Theoretically, the methods of Cann et al and of Laval-Jeantet et al will not produce optimal results.

Key words: dual-energy; QCT; bone mineral assessment; fat content assessment.

THE ACCURACY OF bone mineral measurements of the vertebral body with single energy quantitative computed tomography (SEQCT) is influenced by the occurrence of intravertebral fat. Dual-energy quantitative computed tomography (DEQCT) has been proposed to improve the accuracy of bone mineral content determination[1-11] and to give additional information regarding the composition of the trabecular region of the vertebral body.[10,12-14]

DEQCT can be done using preprocessing[11] or postprocessing methods.[5-10] For preprocessing, special DEQCT hardware and software is required. However, postprocessing methods can be done easily on CT systems that allow a variable kVp selection. Various methods for postprocessing DEQCT have been proposed[5-10]; the authors' goal was to evaluate these methods and to establish their distinct value. In the current study, the methods are reported theoretically. In another report in the current issue of *Investigative Radiology* (1990;25:882-889), the authors discuss the practical aspects of using these methods.

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Theory

All quantitative CT methods in principle are based on the relation between the linear attenuation coefficient of a mixture of materials and the attenuation coefficients and concentrations of each of the materials (Equation 1):

$$\mu\{E\} = \sum_{i=1}^{n} (\mu_i\{E\} / r_i)\, c_i \qquad (1)$$

where, μ is the energy {E} dependent linear attenuation coefficient of the mixture and μ_i is that of the materials; r_i is the mass densities and c_i the concentrations. A list of abbreviations used for variables and subscripts is given in Appendix C.

The concentrations of the materials can be expressed in terms of their volumes and mass densities:

$$c_i = r_i V_i \qquad (2)$$

where, V_i is the fractional volumes of the materials. In computed tomography, the CT number (CT) is related to the attenuation coefficient by:

$$CT\{E\} = 1000\,(\mu\{E\} - \mu_w\{E\})/\mu_w\{E\} \qquad (3)$$

where, μ_w is the linear attenuation coefficient of water at energy E.

Equations 1 through 3 can be combined and rearranged to:

$$CT\{E\} = \Sigma\,(CT_i V_i\{E\}) \qquad (4)$$

where

$$1 = \Sigma V_i \quad (5)$$

See Appendix A for more details. Equation 4 states that the energy-dependent CT number of a mixture of materials (CT{E}) is the sum of the CT numbers of the pure materials (CT_i) multiplied by the fractions of volume (V_i) of the materials. The sum of the fractions of volume is 1 (Equation 5).

For understanding QCT of the trabecular region of the vertebral body, this region should be described in terms of Equation 4. The trabecular volume is composed of trabecular bone substance, water, red marrow, and fat. The trabecular bone substance itself is a mixture of the collagen matrix and bone mineral (calcium hydroxyapatite). Translating this anatomic description to Equation 4 yields:

$$CT_v\{E\} = CT_{bm}\{E\}V_{bm} + CT_c\{E\}V_c + CT_{rm}\{E\}V_{rm} + CT_f\{E\}V_f + CT_w\{E\}V_w \quad (6)$$

The subscripts v, bm, c, rm, f, and w indicate the trabecular region of the vertebral body, bone mineral, collagen, red marrow, fat, and water, respectively.

To understand the distinct features of the different postprocessing dual-energy QCT methods, these methods can be translated to Equation 6, which will be called the "basic formula."

Description of Methods

Postprocessing DEQCT methods were proposed initially by Rutherford et al,[1] Genant and Boyd,[2] and Brooks.[3] Their suggestions were followed by Cann et al,[5] who reported a method that is an extension of the single energy method.

In SEQCT, a calibration device that contains different solutions of a material that mimicks bone (usually K_2HPO_4 in water, or calcium hydroxyapatite in a water equivalent plastic) is scanned simultaneously with the patient. The calibration device is used to generate a bone equivalent calibration line that relates the mean CT number ($CT\#_{bc}$) of the different solutions to g/cm³ K_2HPO_4 (or calcium hydroxyapatite):

$$CT\#_{bc} = a \times Beq + b \quad (7)$$

in which, "a" is the slope and "b" is the intercept of the calibration line; "Beq" is the bone mineral equivalent value (usually called bone mineral content) in g/cm³; and subscript bc stands for bone-equivalent calibration. Throughout this paper the "#" after "CT" indicates mean CT numbers of objects that are measured.

The mean CT number ($CT\#_v$) of the vertebral body is converted to g/cm³ Beq with the help of the calibration line:

$$Beq = (CT\#_v - b) / a \quad (8)$$

The single energy method describes the anatomic multicomponent reality of the vertebral body in terms of a two-component model, eg, bone mineral in water. Translating this description (see Appendix B for more details) to the basic formula (Equation 6) means that slope "a" represents $(CT_{bm}\{E\} - CT_x\{E\})/r_{bm}$ and intercept "b" represents $CT_x\{E\}$, where CT_x is the CT number of the mixture of the nonmineral components of the trabecular body (collagen, red marrow, water, and fat). This results in: $CT_xV_x = CT_cV_c + CT_{rm}V_{rm} + CT_fV_f + CT_wV_w$. The assumption that this mixture has the attenuation characteristics of water leads to the limited accuracy of bone mineral measurements with single energy. It is assumed that $CT_{bm}\{E\}$ can be approximated by $CT\#_{bc}\{E\}$. Currently, dipotassium hydrogenphosphate (K_2HPO_4) is the most widely used bone-mimicking material for calibration purposes. It has attenuation characteristics similar to those of calcium hydroxyapatite.

Cann et al[5] assume for their postprocessing dual-energy method that the difference in the mean CT number of the trabecular portion of the vertebral body, determined at two different energies, is due to the mineral content only. This means that the influence of the fat content on the CT number should be the same at both energies. However, this is not the case. A bone equivalent calibration line is generated for both the scanning energies. The bone mineral equivalent value is computed subsequently using the equation:

$$Beq = \frac{(CT\#_v\{E1\} - CT\#_v\{E2\}) - (b\{E1\} - b\{E2\})}{(a\{E1\} - a\{E2\})} \quad (9)$$

{E1} and {E2} indicate the two different scanning energies. In terms of the basic formula (Equation 6), this means that slope "a" represents $(CT_{bm}\{E\} - CT_y\{E\})/r_{bm}$ and intercept "b" represents $(1 - V_f)CT_y\{E\}$. CT_y is the CT number of the mixture of the nonmineral and nonfat components of the vertebral body. This is: $CT_yV_y = CT_cV_c + CT_{rm}V_{rm} + CT_wV_w$. Using the calibration technique $b\{E\} \approx CT\#_w\{E\}$, where the subscript w indicates water-equivalent, it follows that $CT\#_w\{E\}$ should be equal to $(1 - V_f)CT_y\{E\}$. CT_w is zero according to Equation 3. On one hand, the condition can be fulfilled if $V_f = 1$, which means that the trabecular region contains fat only; this is not an anatomic reality. However, the condition can be fulfilled if $CT_y\{E\}$ is zero, which is impossible for the two scanning energies. Therefore, this method will not give optimal results in bone mineral content determination.

In 1984, another dual-energy approach was reported by Laval-Jeantet et al.[6] This approach takes into account the energy dependency of the fat influence. Apart from bone equivalent calibration lines, which yield slope a{E}

and intercept b{E}, fat equivalent calibration lines are generated for each energy using the CT number of 0 g/cm³ bone equivalent (0% fat) and the CT number ($CT\#_f$) of a fat equivalent material (100% fat). The slopes ($\alpha\{E\}$) and intercepts ($\beta\{E\}$) of these fat equivalent calibration lines are used in the following equations:

$$CT\#\{E1\} = a\{E1\} \times Beq + b\{E1\} + \alpha\{E1\} \times F + \beta\{E1\} \quad (10)$$

$$CT\#\{E2\} = a\{E2\} \times Beq + b\{E2\} + \alpha\{E2\} \times F + \beta\{E2\} \quad (11)$$

in which F is the percentage of fat by volume in the vertebral body. These equations are solved to obtain the bone material equivalent value and the percentage of fat by volume.

Transforming these equations to the basic formula (Equation 6) shows that slope "a" of the bone-equivalent calibration line represents $(CT_{b\,m}\{E\} - CT_y\{E\})/r_{b\,m}$. The slope "$\alpha$" of the fat equivalent calibration line represents $(CT_f\{E\} - CT_y\{E\})/100$. This method assumes that $CT\#_f\{E\} = CT_f\{E\}$; this is only true if the fat-equivalent used for calibration purposes has exactly the same attenuation characteristics as the intravertebral fat tissue. The sum of the intercepts "$b + \beta$" should represent $CT_y\{E\}$. However, when using the calibration technique, b and β are both determined by the 0 g/cm³ sample, which simulates $CT_y\{E\}$. So $b + \beta$ is 2 times $CT_y\{E\}$. Therefore, this method will cause inaccuracies in determination of both the bone mineral content and fat content.

In 1987, Goodsitt et al[8] proposed two new dual-energy methods. The first approach uses the same bone equivalent calibration lines as the approaches of Laval-Jeantet et al and Cann et al. In addition, the CT numbers of fat equivalent ($CT\#_f\{E\}$) and soft tissue equivalent ($CT\#_s\{E\}$) materials are used. This leads to the following equations:

$$CT\{E1\}N = a\{E1\} \times Beq + b\{E1\} \quad (12)$$

$$CT\{E2\}N = a\{E2\} \times Beq + b\{E2\} \quad (13)$$

$$CT\#\{E1\} = CT\{E1\}N + V_f \times (CT\#_f\{E1\} - CT\#_s\{E1\}) \quad (14)$$

$$CT\#\{E2\} = CT\{E2\}N + V_f \times (CT\#_f\{E2\} - CT\#_s\{E2\}) \quad (15)$$

$CT\#_s$ is the CT number of the 0 g/cm³ sample in the calibration device. CT{E}N is the calculated estimate of what the mean CT number of trabecular bone would be if the spongiosa contained no fat. These equations are solved for the bone mineral equivalent value and the volume fraction of fat.

Transforming these equations to the basic formula (Equation 6) shows that slope "a" represents $(CT_{b\,m}\{E\} - CT_y\{E\})/r_{b\,m}$; intercept "b" represents $CT_y\{E\}$; and $(CT\#_f\{E\} - CT\#_s\{E\})$ should be equal to $(CT_f\{E\} - CT_y\{E\})$. As with the method of Laval-Jeantet et al, it is assumed that $CT\#_f\{E\}$ is equal to $CT_f\{E\}$. Therefore, a fat-equivalent material used for calibration purposes should have exactly the same attenuation characteristics of the intravertebral fat. Furthermore, this method assumes that $CT\#_s\{E\}$ equals $CT_y\{E\}$. Because CT_y is the CT number of a mixture of materials (collagen and red marrow and water) for which the fractions of volume will vary interindividually, this condition cannot be fulfilled. However, this problem could be avoided partly if calibration materials were available that simulate trabecular bone substance (calcium hydroxyapatite within a collagen matrix) diluted in a red marrow environment. Then, the assumption should be made that there is a fixed mineralization of the collagen matrix. If such calibration materials were available, slope "a" would represent $(CT_{b\,s}\{E\} - CT_{r\,m}\{E\})/r_{b\,s}$; intercept "b" would represent $CT_{r\,m}\{E\}$; and $(CT\#_f\{E\} - CT\#_s\{E\})$ should be equal to $(CT_f\{E\} - CT_{r\,m}\{E\})$. The subscript bs indicates bone substance. The water compartment of the trabecular region should then be combined with the red marrow compartment.

The only difference between the approaches of Goodsitt et al[8] and Laval-Jeantet et al is the intercept β used by the latter. The difference (D) between the bone mineral equivalent value calculated according to Laval-Jeantet et al and the value calculated according to Goodsitt et al can be derived easily:

$$D = \frac{(\beta\{E1\} \times \alpha\{E2\}) - (\beta\{E2\} \times \alpha\{E1\})}{(\alpha\{E1\} \times a\{E2\}) - (\alpha\{E2\} \times a\{E1\})} \quad (16)$$

A similar relation for the fat content can be derived.

The second approach of Goodsitt et al[8] is a direct derivation of the basic formula (Equation 6):

$$CT\#\{E1\} = V_{b\,s} \times CT\#_{b\,s}\{E1\} + V_f \times CT\#_f\{E1\} + V_s \times CT\#_s\{E1\} \quad (17)$$

$$CT\#\{E2\} = V_{b\,s} \times CT\#_{b\,s}\{E2\} + V_f \times CT\#_f\{E2\} + V_s \times CT\#_s\{E2\} \quad (18)$$

$$1 = V_{b\,s} + V_f + V_s \quad (19)$$

$CT\#_{b\,s}\{E\}$ is the CT number of pure bone substance (bone mineral in collagen matrix); $V_{b\,s}$ is the fraction of volume of bone substance. This method will be called the "basic approach" to avoid confusion with the approach of Goodsitt et al reported previously in the current study.

In the original study by Goodsitt et al,[8] $CT\#_{b\,s}\{E\}$ was estimated by scanning a sample of femoral cortex at the two energies separately from the patient, and determining the maximum CT number in the cortical region of the midshaft. This sample was scanned separately

to avoid imaging artifacts. Ethylalcohol 100% was used to determine $CT\#_f\{E\}$; $CT\#_s\{E\}$ was defined as 0.

$CT\#_{bs}\{E\}$ represents the CT number of the mixture of bone mineral ($CT_{bm}\{E\}$) and collagen ($CT_c\{E\}$). The combination of these two materials is justified by assuming a constant mineralization of the collagen matrix. The basic approach should give good results if $CT\#_f\{E\}$ is equal to $CT_f\{E\}$ and $CT\#_s\{E\}V_s$ is equal to $CT_{rm}\{E\}V_{rm} + CT_w\{E\}V_w$, and if $CT\#_{bs}\{E\} \times V_{bs}$ equals $CT_{bm}\{E\} \times V_{bm} + CT_c\{E\} \times V_c$.

The last method reported in the current study is an approach reported by Nickoloff and Feldman in 1985[7] and presented in more detail in 1988.[10] Their approach is a direct derivation of the basic formula:

$$CT\#\{E1\} = \Omega\{E1\} \times c_b + \Theta\{E1\} \times c_f + \sigma\{E1\} \times c_s + \delta + \pi\{E1\} \quad (20)$$

$$CT\#\{E2\} = \Omega\{E2\} \times c_b + \Theta\{E2\} \times c_f + \sigma\{E2\} \times c_s + \delta + \pi\{E2\} \quad (21)$$

$\Omega\{E\}$, $\Theta\{E\}$ and $\sigma\{E\}$ are the energy-dependent and material-specific coefficients calculated from the linear attenuation coefficients of the different materials. C_b, C_f, and C_s are the concentrations of bone substance, adipose tissue, and soft tissue, respectively. δ is -1000. $\pi\{E\}$ is the offset value for water. The coefficient can be described in terms of the CT number of the pure materials: $(CT_i\{E\} + 1000)/r_i$. Combining this with Equation 2, this approach can be rewritten to the basic approach of Goodsitt et al discussed earlier. Only one difference between the two methods remains; the water offset value used by Nickoloff. This water offset does not originate from the basic formula (Equation 6), because the soft tissue compartment incorporates water and red marrow.[10] Therefore, the water offset value is an empirical correction factor for CT number scale drift.

To use this method, a determination of the effective energy is required. To achieve this, a device with compartments containing different concentrations of calcium hydroxyapatite is scanned simultaneously with the patient, and the effective energy is computed from the slope of the linear regression fit of the measured CT numbers of the compartments versus the concentrations. This effective energy estimation may be in error because the effective energies at the site of the calibration device are different from those at the vertebral body.

The material-specific coefficients are calculated using knowledge of the elemental composition and the mass density of bone substance (again, it is assumed that there is a constant mineralization of the collagen matrix), intravertebral fat, and red marrow, including the water compartment. Instead of finding suitable materials that mimick tissue for calibrating purposes, as required in the methods reported previously, this method requires an exact knowledge of the chemical and physical properties of the anatomic components of the vertebral body.

Discussion

Four of the five methods discussed in the current study use materials that mimick tissue for calibration purposes. It is assumed that CT_{bm} can be simulated by $CT\#_{bc}$. This is only true if the material that mimicks bone has the same attenuation characteristics as the real bone mineral. For instance, when dipotassium hydrogenphosphate (K_2HPO_4) is chosen as calibration material, an error in bone mineral estimation will occur due to the (slight) difference in attenuation characteristics between K_2HPO_4 and calcium hydroxyapatite. This error was discussed by Gluër et al[15] and a correction factor was calculated. Furthermore, when using K_2HPO_4 solutions for calibration, additional errors can occur due to the so-called displacement effect. The errors arising from the use of K_2HPO_4 are discussed extensively by Rao et al[16] and Crawley et al.[17]

For the postprocessing method of Cann et al, it is first assumed that CT_f is independent of energy. Secondly, the collagen compartment is combined with the red marrow and water compartments. It is assumed that this "soft tissue" compartment has the same attenuation characteristics as water (-equivalent). Both assumptions will lead to errors, as reported by Rao et al.[16]

For the method of Laval-Jeantet et al, CT_f is approximated by $CT\#_f$. This means that the fat equivalent material should have the same attenuation characteristics as the intravertebral fat for a correct determination of the bone mineral content and the fat content. However, this method shows a methodologic problem due to the double intercept, which will lead to inaccuracies. Apart from the intercept problem, the calibration method of Goodsitt et al[8] is essentially the same as that of Laval-Jeantet et al. The same can be said about fat calibration. The combination of the collagen compartment, red marrow compartment, and water compartment to one soft tissue compartment can be a source of error. This combination of compartments is used by Cann et al and Laval-Jeantet et al. Authors[5,6,8] justify this combination by assuming that the attenuation characteristics of "soft tissue" are the same as those of water.

This "soft tissue problem" could be avoided partly by rearranging the compartments. Then, the assumption should be made that there is a fixed mineralization of the collagen matrix. The collagen and mineral compartments can then be combined to a bone substance compartment. In that case, the "soft tissue" compartment contains red marrow and water only. However, calibration materials that mimick bone substance within a red marrow environment are not available currently. The assumption that red marrow could be mimicked by water for all

energies is an approximation with limited value. The CT numbers for red marrow, calculated for the elemental composition as specified by Woodard and White,[18] would vary from −34 Hounsfield Units (HU) at 40 keV to +13 HU at 80 keV. The CT number of water is 0 for all energies due to the Hounsfield scale definition (Equation 3).

Ignoring the collagen matrix as an attenuating component when using materials that mimick bone mineral in a water-equivalent environment inevitably will cause inaccuracies. This was shown in an experimental setup by Goodsitt et al.[19]

The basic approach of Goodsitt et al avoids, as well as possible, the "soft tissue problem." The collagen and bone mineral compartments are combined to a bone substance compartment. Therefore, the calibration materials used for this method should have the same attenuation characteristics as trabecular bone substance, intravertebral fat, and red marrow/water. If so, this method will give good results. However, an accurate determination of $CT_{bs}\{E\}$ will be difficult due to beam hardening.

The method of Nickoloff et al[10] does not use calibration materials, so uncertainties due to the choice of calibration materials can be avoided. Instead, it uses material-specific coefficients that can be derived if the physical and chemical properties of the various constituents are known and if a reliable estimation can be made of the effective scanning energy at the place of the vertebral body. If so, this method will produce good results.

The authors of the current study intended to evaluate theoretically five postprocessing dual-energy methods for quantitative CT. Sources of error were indicated. In Part 2 of this report (1990;25:882–889), the practical aspects of these methods will be discussed and will focus on the following items: (1) choice of the DEQCT method; (2) the influence of the choice of tissue-equivalent materials for calibration purposes; (3) the difference between simultaneous peripheral calibration and nonsimultaneous central calibration for those methods using tissue-equivalent calibration lines; and (4) the difference between effective energy estimation at the place of the calibration device and at the place of the vertebral body for the method of Nickoloff et al.[10]

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Appendix A

Derivation of equation (4): Equation (2) substituted in equation (1) gives:

$$\mu\{E\} = \sum_{i=1}^{n} (\mu_i\{E\}V_i) \qquad \text{(AA.1)}$$

According to equation (3):

$$\mu\{E\} = \mu_w(1 + CT\{E\}/1000) \qquad \text{(AA.2)}$$

Substitution of (AA.2) in (AA.1) gives:

$$\mu_w(1 + CT\{E\}/1000) = \sum_{i=1}^{n} (V_i\,\mu_w(1 + CT_i\{E\}/1000) \qquad \text{(AA.3)}$$

After division by μ_w and after cancelling the left side 1, by the sum of the fractions of volume (= 1 according to equation [5]) on the right side, and after multiplication by 1000, (AA.3) can be reduced to equation (4).

Appendix B

Translation of equation (8) into the basic formula (equation [6]). Theoretically, Beq is the concentration of bone mineral: c_{bm} Then, equation (8) can be rewritten to:

$$CT\#_v = (a \times c_{bm}) + b \qquad (AB.1)$$

Equation (6) can be rewritten to:

$$CT_v = CT_{bm} \times V_{bm} + CT_x \times V_x \qquad (AB.2)$$

with,

$$CT_x \times V_x = CT_c \times V_c + CT_{rm} \times V_{rm} + CT_f \times V_f + CT_w \times V_w$$

Using equation (2) in (AB.2) gives:

$$CT_v = CT_{bm} \times c_{bm}/r_{bm} + CT_x \times V_x \qquad (AB.3)$$

$$V_x = 1 - V_{bm} = 1 - c_{bm}/r_{bm} \qquad (AB.4)$$

(AB.4) substituted in (AB.3) gives:

$$CT_v = c_{bm}/r_{bm}\,(CT_{bm} - CT_x) + CT_x \qquad (AB.5)$$

Comparing (AB.1) to (AB.5), gives:

$$a = (CT_{bm} - CT_x)/r_{bm},$$

and

$$b = CT_x$$

Appendix C

Legend of variables and subscripts used in this paper:

Variables

μ:	linear attenuation coefficient (cm^{-1})
E:	energy (keV)
r:	mass density (g/cm^3)
c:	concentration (g/cm^3)
V:	Volume fraction
CT:	CT-number
CT#:	mean CT-number measured on objects
a:	slope of bone equivalent calibration line
b:	intercept of bone equivalent calibration line
Beq:	bone equivalent value (g/cm^3)
α:	slope of fat equivalent calibration line
β:	intercept of fat equivalent calibration line
F:	percentage of fat by volume
Ω:	material specific coefficient for bone substance
Θ:	material specific coefficient for fat
σ:	material specific coefficient for soft tissue
δ:	−1000
π:	water offset value

Subscripts

$_i$:	any material
$_w$:	water
$_v$:	trabecular region of vertebral body
$_{bm}$:	bone mineral
$_c$:	collagen
$_{rm}$:	red marrow
$_f$:	fat
$_{bc}$:	bone equivalent calibration
$_s$:	soft tissue
$_x$:	combination of collagen, red marrow, fat and water
$_y$:	combination of collagen, red marrow and water
$_{bs}$:	bone substance: combination of bone mineral and collagen