An empirical analysis of euro cash payments

Jeanine Kippers
De Nederlandsche Bank, Amsterdam

Philip Hans Franses
Econometric Institute, Erasmus University Rotterdam

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Abstract

If the denominational structure of the euro is used in an optimal way, there should be no preferences for using certain coins and notes. In Kippers et al. (2003) it is documented that the Dutch public did have certain preferences concerning the Dutch guilder, in the sense that some notes and coins were used less often than expected. With the advent of the euro, it is now of interest to examine whether these preferences also exist for euro coins and notes, also as the denominational structure of the guilder was 1-2½-5, while for the euro it is now 1-2-5. In this paper we use two new and rather unique data sets for the Netherlands to empirically examine if the euro range is used in an optimal way. Using a statistical model, specifically designed for this purpose, we find that the Dutch public does not seem to have preferences for certain denominations. Hence, the task of paying with cash has become easier for Dutch consumers.

Keywords: euro cash, truncated Poisson regression model, payment behavior

1. Introduction

Cash payments are common and occur on great scale worldwide. To make cash payments possible, cash handling parties such as retailers and (central) banks make high costs. It is therefore relevant to study the way cash is used in order to optimize the payment system where it concerns cash. Also, as making cash payment is one of the many tasks consumers face, such a study reveals insights in consumer behavior.

Euro cash consists of seven banknotes with values 500, 200, 100, 50, 20, 10 and 5, and eight coins with values, 2, 1, 0.5, 0.2, 0.1, 0.05, 0.02 and 0.01. It is generally accepted that a 1-2-5 range, like the euro has, is theoretically the denominational structure that approaches the optimal one for a payment system, see also Boeschoten and Fase (1989). The question is of course if consumers behave according t this theoretical prediction.

Interestingly, at present there is no empirical study of how individuals actually handle euro cash payments. In our view this is an omission, and we believe that the scope of currency research on optimal denominational structures should be broadened from purely theoretical to empirical analysis. In theory, a denominational structure is considered as optimal if all denominations are employed to their full potential to ensure efficiency of the cash payment system. Obviously, there might be a theoretical optimal denominational range, but in practice people may behave otherwise. Indeed, the paying individuals themselves determine the way cash is used and exchanged. Hence, in order to assess the merits of a denominational range in use, one better study cash payment behavior in practice.

Additionally, we believe that cash payments concern an individual choice process, which is worthwhile to study. A consumer has a wallet with banknotes and coins, and s/he can choose any combination of these notes and coins, provided that it has a combined nominal value of at least the amount to be paid. Generally, in choice processes an individual chooses one item from a choice set on the basis of several attributes. In cash payments, the choice set is determined by the wallet content, but the ultimate choice is a combination of banknotes and coins. Computational effort is needed to make all possible combinations and to choose the optimal one. This is then also constrained by time. For example, according to a recent study by the Dutch retail association, the time between announcing the payment amount and the closure of the payment by handing over change and the receipt takes only 9 seconds on average. Apparently, individuals do not need much time, or do not get much time, to make their choice in cash payments. In sum, other than the question on which criterion the individual bases its optimal choice in cash payments, the choice process itself, because of its special features, constitutes an interesting research topic.

One of the drawbacks of empirical currency research at the individual level can be the difficulty of collecting data. As information on the use of denominations in cash payments is not readily available, one should collect it at the cash register, which is a time-consuming and expensive activity, as we now know by experience, see section 2 below.

In the empirical analysis in Kippers et al. (2003), a large dataset of cash payments in guilders was used. Facing the modeling challenges related to the special features of the cash payment process Kippers et al. (2003) developed an econometric model for describing cash payments. The model takes into account (i) the restrictive wallet content, as nobody has a wallet with multiple amounts of all coins and notes, (ii) the fact that the paying individual makes a simultaneous choice for all denominations, and (iii) the influence of the payment amount on the choices the individual can make. An important feature of that model is that the dataset unfortunately did not include information on the wallet contents. Hence, advanced simulation techniques are needed to estimate parameters. As we will discuss below, in the present study we aim to overcome this drawback.

Estimation of the model parameters provides a means to assess the use of the different denominations in relation to each other. For example, the conclusion from the analysis of guilder payments is that the 100-guilder note was preferred over others in cash payments while the 50-guilder note was less preferred than others, give a similar relevance for its use. An empirical study of cash payments in euro is a natural follow-up, and this is what we pursue here.

The analysis we present in this paper concerns a dataset of euro cash payments that was collected at two different retail locations in the Netherlands. We also have a second dataset, for validation of the results. Our aim is to study the payments in detail in order to have a better understanding of the choice process in cash payments. We intend to arrive at some conclusions on the actual use of the euro in cash payments. In Section 2, we describe the characteristics of this unique dataset. We also discuss the features of a second, survey-based, dataset. We study the data of euro cash payment in a two-step empirical analysis. In Section 3, we zoom in on the cash payments and categorize them according to a number of distinguishable payment principles that individuals seem to have applied in their choice of denominations for making cash payment. One of them is the ‘principle of least effort’, as was first presented by Cramer (1983). He developed a behavioral model for cash payments and defined efficient payments as those in which the number of notes and coins exchanged between payer and receiver is minimized. The question is whether in practice individuals also strive for efficiency in the Cramer sense. In Section 4, we move on to an econometric analysis that gives us the opportunity to draw conclusions on the use of euro denominations. To this end, we apply the econometric model that was developed in Kippers et al. (2003). We also use simulation techniques to support our analysis. All empirical results are summarized in Section 5. Finally, Section 6 concludes.

2. Data

In this section we discuss the properties of the two datasets we use to empirically examine if there are preferences for certain euro coins or notes.

2.1 Dataset 1, collected at two retail stores

The first dataset, collected by three students, comprises a sample of cash payments that includes (i) the set of coins and banknotes in possession of the paying individual prior to the cash payment (wallet), (ii) the amount to be paid, (iii) the notes and coins selected by the payer to make the final payment, and (iv) possibly the notes and coins returned as change. As the use of money is a delicate and private subject, certainly for Dutch individuals, a method of data collection had to be chosen that minimized non-response. This concerns especially the collection of information on the wallet content, which, as mentioned, for privacy reasons was not present in de dataset used in Kippers et al. (2003). Furthermore, attention had to be paid to the timing of the survey as to avoid influencing the choice for notes and coins by the respondent.

The students posted at two different retail locations, a supermarket in an urban area and an appliance store (like “Home Depot”) in a rural area. During the cash payments, they registered the payment amount and the notes and coins used by the paying individual. Subsequently, the customer of the store that just made the payment was asked for co-operation in the research. If so, the customer was assisted in completing a survey. In the survey the respondent was also asked for the content of the wallet, that is, the wallet after the cash payment was done. Furthermore, the respondent was asked for other information regarding demographics (age and gender), possible personal experience as being a cashier and the number of times a customer goes shopping on average in a week. The latter two questions were included to check for any influence on cash payment behavior. Finally, the respondents were given a cognitive test, meant to measure computational skills, again with the purpose of testing the influence of these skills on cash payment behavior.

In four full days of posting, a total of 272 surveys were taken. In 40 cases the customer was asked by the cashier to add a small amount to make change easier. As this does not reflect the original intended cash payment behavior by the paying individual, these observations were corrected in order to represent the original intention of the paying individual. The surveys finally resulted in a dataset of 272 observations on cash payments and their matching wallets. Table 1 gives a few summary statistics of this first dataset.

2.2 Dataset 2, collected using a telephone survey

On the authority of the Nederlandsche Bank, a survey on payment behavior was held among Dutch respondents of 12 years and older in the week of April 3 through April 9, 2003. The respondents were selected from the TNS NIPO Capi@Home panel, a database of 45.000 households who regularly take part in surveys for TNS NIPO using their personal computer at home. The survey on payment behavior was distributed on seven different weekdays, resulting in completed surveys by 1.293 respondents about equally spread across the days of the week. The respondents were asked about their payment behavior on the day before.

Among several questions on payment behavior in general, the respondents were subsequently asked to recall the specifics of their recent cash payment. About 26% of the respondents indicated not to have made any cash payment the day before. Furthermore, 1% did not remember the specifics of their last cash payments. The resulting 952 respondents were able to indicate the payment amount and the notes and coins they used for payment, as well as the notes and coins they received as change. In order to relate the cash payments to the notes and coins they disposed of at the time of the payment, the respondents were asked to count the notes and coins in their wallet at the time of the survey. Subsequently they were asked if they made their recent cash payment from their wallet (and not from their pocket, for example) and if their wallet content had changed between the recent cash payment and the time of the survey. From these answers it is possible to construct the wallet content after payment and subsequently re-construct the wallet content before payment, in a similar way as we did for dataset 1. 63 respondents did not pay from their wallet and 42 respondents were not willing or able to give their wallet content.

What resulted is a dataset with all the specifics on the recent cash payment and matching wallet contents for 847 individuals. Incorrect entries and entries with extremes, such as unlikely high payment amounts and payments with 20 or more banknotes were removed from the dataset. Ultimately, the dataset contains 840 observations with information on the payment amount, the notes and coins used for payment by the paying individual and the notes and coins in their wallet prior to the payment.

As the first dataset concerns actual, and not stated, payment behavior, we will use this second dataset merely to check and validate the results for the first set of data.

3. Descriptive statistics

According to which principles do individuals make their cash payment choice? For our dataset 1, we search for such principles in euro payments. We start our search with the one most obvious, that is, the principle of least effort, for which Cramer (1983) developed a behavioral model. This model reflects rational behavior of individuals in cash payments. That is, one assumes that individuals are triggered to make a payment such that the smallest number of notes and coins is exchanged between payer and receiver, including change in case of overpayment.

If we apply the algorithm of Cramer to generate efficient cash payments for the amounts ranging from 0.01 to 100 euro, we find 36591 efficient payment schemes. For 28% of the amounts, only one efficient payment scheme exists. The other amounts can be paid efficiently with more than one payment scheme, with the absolute maximum being 54 different schemes for 8 different amounts between 36 and 37 euro, such as 36.67 euro. Furthermore, on average 5.8 notes and coins are exchanged in efficient payment schemes and no scheme in the given range involves more than 8 notes and coins if paid according to the efficient scheme. We refer to the appendix of Kippers et al. (2003) for a detailed description of the algorithm; see also Cramer (1983). The algorithm is based in the following components, that is

A	Payment amount
n(A)	Amount of notes and coins to be used for cash payment
d = 1,…, D	Denomination range
n(A,d)	Number of items of denomination d, used for paying the amount A
v(d)	Value of denomination d.

Solving

$$\min n(A) = \sum_{d=1}^{D} |n(A,d)|$$

$$s.t. \sum_{d=1}^{D} n(A,d)v(d) = A$$

can now generate efficient payment schemes.

The principle of least effort is theoretically appealing. However, for a number of reasons it might not be applied in practice. It requires some computational effort to determine the efficient payment scheme, especially as the scheme becomes more complex. Also, time pressure might force individuals to resort to an easier but sub-optimal payment scheme. The first step towards understanding cash payments involves an analysis of the extent to which individuals exhibit cash payment behavior according to the principle of least effort. For this purpose, we compare the payment schemes in our sample of actual payments with the efficient payments generated using the algorithm of Cramer. We concentrate on the payment scheme used by the paying individual. If this scheme coincides with the one calculated by Cramer for the same amount, we consider the payment to be efficient, irrelevant of the change given back by the cashier. The idea is that the paying individual did intend to make the payment as efficient as possible.

The Cramer model does not take the wallet into account. That is, in his model, it is assumed that all banknotes and coins needed to make the efficient payment are available to the payer. In our dataset 1, the wallet content prior to each of the payments is known, and hence we are interested in the payment that was efficient, given the wallet contents. If this individual has used the notes and coins available to him or her as efficient as possible, the cash payment still constitutes efficient behavior, but the wallet simply did not allow for a pure efficient payment scheme in the Cramer sense. We therefore expand the pure efficient payment schemes by the schemes that are efficient, given the restrictions of the wallet. For each payment in our dataset, we generate efficient payment schemes given the wallet restriction and the payment amount. The results of the comparison are presented in Table 2.

Table 2 shows a number of interesting results. First, 61% of the payments were done efficiently. This constitutes a statistically significant share of the individuals. It can therefore be concluded that individuals are more inclined to make an efficient payment than a non-efficient payment. Second, efficient versus non-efficient payments show similar results for the group of individuals with or without the appropriate wallet content to make a pure efficient payment. Statistically speaking, there is no significant difference. This means that individuals are equally inclined to make an efficient payment, irrelevant whether the wallet facilitates the pure efficient payment or not. These results indicate that indeed individuals do seem to strive for efficiency.

Still, 39% of the individuals in our sample did not pay efficiently. Apparently there was some reason for them not to employ the efficient payment. Apart from the reasons mentioned before (computational effort, time pressure) a number of alternative principles of payment could have been applied, conscious or unconscious from the viewpoint of the individual. A first check we perform is to see, by using a logit model, if the characteristics in Table 1 have explanatory value for paying "yes" or "no" efficiently. To save space, we summarize the results by stating that none of the covariates has a significant impact on paying efficiently, yes or no. Hence, for example, more cognitive skills do not lead to more payment efficiency.

Next, one can think that individuals feel a desire to empty the coins from their wallet, or feel a necessity to receive change for a subsequent payment. If all these would vary randomly, no pattern would be deducted. However, if there were preferences for certain denominations, there would be a structural deviation from efficient behavior. Then, it can be concluded that the available range of denominations employed is sub-optimal. In extreme cases of such preferences, it might be worthwhile for monetary institutions to change their denominational structure. We will check these possibilities in our model below and report the outcomes in Section 5.

We first carry out a preliminary search for patterns and payment principles by examining the cash payment behavior of the 53 individuals in our sample who did not pay efficiently while their wallet did include the pure efficient payment. This is simply a counting exercise to be executed manually. 22 individuals paid with only one token, usually being a banknote. These individuals applied a principle of least effort from their point of view, or simply wanted to 'break' the banknote to receive change. 10 paid with many coins and, considering their wallets, probably applied the principle of rebalancing their wallet content. For the other 21 individuals no logical payment principle can be deducted, although 8 of them seemed to have tried to make an efficient payment, but the true efficient combination in those cases was probably to difficult to come up with in a short period of time.

The counting exercise becomes a bit more involved when we want to assess the use of denominations as compared to others. About 35% of the non-efficient payments were paid with one denomination. One might now wonder if this is the obvious denomination given the wallet and payment amount? This question cannot be answered with this small sample, and not by means of such a simple counting exercise. In the next section we therefore proceed by following a modeling approach to compare the use of denominations in a given payment situation. Our intention is to draw statistical conclusions on the use of euro denominations in the Netherlands and to see if there are preferences for using some notes or coins.

4. Statistical model

The analysis in the previous section involves a tedious counting exercise, while no conclusive statement can be made on the use of different denominations in relation to each other. We can however formalize our exercise with the purpose of spotting systematic preference or non-preference for denominations in cash payments, by applying an econometric model. This model can generally be used for any sample of cash payments with any given denominational range. The cash payment model was developed and appliedd in Kippers et al. (2003) for a sample of 2.000 payments in guilder coins and notes. It was found that the Dutch public showed a preference for paying with a 100-guilder banknote, while the 50-guilder note was generally less preferred than other denominations.

4.1 The cash payment model

The model focuses on the probability that an individual i selects a combination of denominations, represented by a D-dimensional discrete random variable Y_i, to pay amount A_i, given the wallet content w_i. D corresponds to the number of denominations in a denominational range. The realizations of this random variable are denoted by y_i.

For modeling purposes a hierarchical structure is imposed. According to this structure the choice of an individual is modeled separately for each denomination, starting from the high denomination to the lowest, where the choice for a certain denomination is conditioned on the given payment for previous denominations. Hence, we consider the following probability structure, that is,

$$\Pr[Y_i = y_i | w_i] = \Pr[Y_{D,i} = y_{D,i} | w_i] \Pr[Y_{D-1,i} = y_{D-1,i} | y_{D,i}, w_i] .. \Pr[Y_{1,i} = y_{1,i} | y_{D,i}, ..., y_{2,i}, w_i] \quad (1)$$

The choice for payment with $y_{d,i}$ tokens for denomination d is assumed to be truncated Poisson distributed where the choice set is restricted from below by a lower bound $lb_{d,\mathrm{i}}$ and from above by an upper bound $ub_{d,I}$, that is,

$$Y_{d,i} | y_{d+1,i}, ..., y_{D,i}, w_i \sim POI(\exp(x'_{d,i}\, \beta_d)) \times I[lb_{c,i}, ub_{d,i}] \quad (2)$$

where β_d is a parameter vector and $x_{d,i}$ contains explanatory variables for denomination d.

The upper bound represents the maximum number of tokens the individual can use for payment. In principle this upper bound is determined by the wallet content. Obviously, a paying individual cannot pay with more tokens than are available in the wallet. In some cases, however, the bound is determined by another criterion. It is assumed that individuals do not pay with a token if it is unnecessary, that is, if it is expected that the same token will be returned as change. The upper bound is determined by one of these criteria whichever is more restrictive. The lower bound represents the minimum number of tokens of a denomination the individual has to pay with in order to make the payment. This minimum is determined by the combination of the payment amount and the availability of other denominations in the wallet. If the wallet does not contain enough value of lower denominations to pay the amount, then the paying individual is forced to use a minimum number of tokens of the denomination under consideration. The upper and lower bounds are defined as follows

$$ub_{D,i} = \min\left(ceil\left(\frac{A_i}{v_D} \right), w_{D,i} \right) \tag{3}$$

$$ub_{d,i} = \min\left(ceil\left(\frac{A_i - amount(y_{d+1,i},\ldots,y_{D,i})}{v_d} \right), w_{d,i} \right) \qquad \text{for } d=D\text{-}1,\ldots,1 \tag{4}$$

$$lb_{D,i} = \max\left(ceil\left(\frac{A_i - amount(w_{1,i},\ldots,w_{D-1,i})}{v_d} \right), 0 \right) \tag{5}$$

$$lb_{d,i} = \max\left(ceil\left(\frac{A_i - amount(y_{d+1,i},\ldots,y_{D,i}) - amount(w_{1,i},\ldots,w_{d-1,i})}{v_d} \right), 0 \right) \; for\ d=D\text{-}1,\ldots 1 \tag{6}$$

with

$$amount(x_p,\ldots,x_q) = \sum_{k=p}^{q} v_k x_k \tag{7}$$

If $lb_{d,i}$=$ub_{d,i}$, the individual has no freedom of choice and hence

$$\Pr\left[Y_{d,i} = y_{d,i} = lb_{d,i} = ub_{d,i} \middle| y_{d+1,i},\ldots,y_{D,i}, w_i \right] = 1 \tag{8}$$

The main explanatory value is given by the payment amount. The cases of "no freedom of choice" are not informative, and in our empirical work we take care of these cases.

In addition to an intercept we include the explanatory variable ACORR, defined as the amount to be paid minus the value of the payments chosen for higher denominations, scaled to its face value, that is,

$$ACORR_{D,i} = \ln\left(\frac{A_i}{v_D} \right)$$

$$ACORR_{d,i} = \ln\left(\frac{A_i - amount(y_{D,i},\ldots y_{d+1},i)}{v_D} \right) \qquad for\ d=1,\ldots,D\text{-}1 \tag{9}$$

For a more elaborated description of the model, as well as for an outline of the estimation methods, we refer to Kippers et al. (2003).

We have written a program in Gauss for parameter estimation. However, one can also use Eviews. As a courtesy to the reader, we give the Eviews code in the appendix.

4.2 Statistical test for preferences

If cash payment behavior of individuals reveals no preference for any denomination, then the probability of choosing a denomination for payment would be equal for all denominations in similar payment situations. In our cash payment model, similar payment situations are represented by equal lower and upper bounds, and by equal values for ACORR. Therefore, the question whether individuals are indifferent towards denominations can be answered statistically by testing if the parameter values (across intercepts and across the ACORR variables) in the cash payment model are equal across denominations, see also the appendix.

The hypothesis of equal parameters can formally be tested with a likelihood ratio (LR) test. Two models are compared, one with the restriction that the parameters are equal across denominations, and a second with unrestricted parameters as in (1)-(2). The LR statistic is a function of the resulting likelihood values of the estimated models where L_{UR} represents the likelihood of the unrestricted model and L_R represents the likelihood of the restricted model. The LR-statistic is defined as $-2ln(L_R/L_{UR})$ and it follows a chi-square distribution with the number of restrictions as degrees of freedom. Rejection of the null hypothesis suggests that the parameters significantly differ across denominations. That is, there is then no indifference towards denominations by individuals, and certain denominations are (more of less) preferred.

4.3 Empirical properties of the LR test

We use simulation techniques to examine the distributional properties of the LR statistic. Also, we examine with which amount of observations the test has reasonable power.

Starting point for our simulations is the set of pure efficient payment schemes for amounts 0.01 to 100 euro as generated using the Cramer algorithm. Pure efficient payments, in the Cramer sense, are by definition based only on the face value of denominations and therefore constitute examples of payments with indifference towards denominations. If the cash payment model is estimated using a dataset of pure efficient payments, the LR test of equal parameters should follow the asymptotic distribution. The following simulation scheme is executed.

1. For all payments, matching wallets are generated by assuming that the wallet contains the payment, with three tokens of each denomination added.
2. A sample of size 1.000 is randomly drawn. The cash payment model is estimated with the restriction that parameters β_d are equal across six denominations, that is, 1 through 50 euro, as well as without restrictions. The resulting likelihood values for the restricted and unrestricted models are used for a LR test with a null hypothesis of equal parameters, which is assumed to be chi-square distributed with 10 degrees of freedom (5 intercepts and 5 ACORR parameters)

Step 2 is repeated 200 times. The resulting empirical size turns out to be 6%, which is rather close to the nominal size of 5%. Given this finding, we can safely rely on the asymptotic chi-square distribution under the null hypothesis.

Next, we examine the power of this test, also to evaluate the size of our datasets. In other words, do our samples suffice to conclude on significant differences between parameters across denominations, if there are any? To estimate the power of the LR test for small samples, we also use simulation techniques.

The starting point for this simulation is again the set of pure efficient payment schemes for amounts 0.01 to 100 euro as generated using the Cramer algorithm. The following simulation scheme is executed to examine the power of the LR test for small samples, that is,

1. For a fraction α of these schemes, payments with one token of denomination d are replaced by payment with two tokens of denominations $d+1$, provided that denomination d has a face value of two times the denomination $d+1$.
2. For all payments, matching wallets are generated by assuming that the wallet contains the payment, with three tokens of each denomination added.
3. A sample of size n is randomly drawn. The cash payment model parameters are estimated with the restriction that the parameters are equal across denominations 1

through 50 euro, as well as unrestrictedly. The resulting likelihood values are used for the LR test.

4. Step 2 is repeated *N* times. The resulting percentage of rejected tests measures the empirical power.

By the replacement of payments, as executed in step 1 of the simulation scheme, we introduce a preference in the data. We use a replacement of a 20-euro note by two 10-euro notes for simulating a preference for the 10-euro note.

Now, we should say a few words about the effective sample size. The sample size n is not simply set to 272, which is the sample size of our euro cash payment data set. This is due to the fact that for these data there are about 75 effective observations with “free choice”, that is, with upper bounds exceeding lower bounds. Hence, a sample size of $n = 75$ in the simulation scheme is about comparable to the sample size of our cash payment dataset, where it concerns observations of free choice. The results of the simulations for different replacement rates are shown in Table 3.

From this table we observe that the empirical power of the test rapidly increases with larger replacement fractions. Also, note that this is already achieved for only 75 effective data points.

5. Empirical results

We estimate the parameters of the cash payment model with the sample data of 272 euro-cash payments using Maximum Likelihood with heteroskedasticity-consistent standard errors. Added payments are excluded from the dataset, as the cash payment model cannot use these for estimation. The reduced dataset contains 240 observations. Furthermore, given the specification of our model, parameter estimates are only based on those observations for which the upper bound in (3)-(4) exceeds the lower bound in (5)-(6). That is, if the lower bound equals the upper bound, the paying individual has no free choice and pays with the number of tokens determined by the coinciding bounds. These observations do not contribute to the likelihood. Given the hierarchical structure of our model, the number of observations in which free choice can happen decreases as the denomination becomes lower. Ultimately, no parameters for the lowest denomination, the 1-euro cent, can be estimated, as the lower bound equals the upper bound for all observations. As the number of observations with free choice becomes quite low for the denominations smaller than 1 euro, we focus on the denominations 1, 2, 5, 10, 20, 50 and 100 euro. The highest denominations, 200 and 500 euro, are excluded from analysis as for these too little payments were registered.

The estimation results are presented in Table 4. The parameters for ACORR are all significantly different from zero, and obtain the expected positive sign.

We can visualize the estimation results for the different denominations by plotting expected values for each denomination on the basis of the estimated parameters, and for a range of values for ACORR. We use the following simulation scheme to generate a large sample of expected values for a range of values of ACORR_d, that is,

1. For each denomination *d*, we draw 50 different grid-arranged parameters $\beta_d^{(n)}$ from a normal distribution with as mean the estimated parameters and as covariance matrix the estimated heteroskedasticity-consistent covariance matrix for $n=1,\ldots.N$, with N is 10000.
2. For a given value of ACORR_d we compute the choice probabilities between the upper bounds and lower bound 0 and construct the expected value of Y_d.
3. The average value of these simulated expected values over N replications provides the expected value of the expectation of Y_d.

Various resulting curves are presented in figure 1. The curves show a stepwise pattern. The jumps constitute an increase of the upper bound by 1. Given that the upper bound is dependent on the value of ACORR, its value changes with the increase of ACORR.

If there were no preferences, the resulting curves would coincide for all denominations. Indeed, Figure 1 shows curves that are relatively close to each other. The figure only shows a difference in expected value for the 20-euro banknote and the 50-euro banknote for most values of exp (ACORR). The expected value for 20 euro is generally lower and for 50 euro generally higher than those of other denominations. We use an example to illustrate this phenomenon. We view the curves on one point on the horizontal axis, and take the value of 1.5 for exp (ACORR). In this situation, the payer faces a choice between three options, that is, (i) not paying with a given denomination, (ii) paying with one token, and (iii) paying with two tokens. The amount left has a value that is one and a half times the nominal value of the denomination. As ACORR is scaled with the face value of the relevant denomination, we can directly compare the data points on the curves for each denomination. They represent the expected value for payment with the given denomination in this particular situation, which is equal for all denominations. In this example, the expected value for the 50-euro notes is higher than for the other denominations, while the expected value is lowest for the 20-euro notes.

To give an idea of how parameter uncertainty propagates in the expected values, we also plot a point-wise 95% confidence interval of the N simulated expected values given the range of ACORR in Figure 1. Obviously, the confidence interval would become smaller as the parameters of the model are estimated for a larger sample of cash payments. From Figure 2, it can be seen that the difference between the curves for the 50-euro and 20-euro note does not seem significant.

To formally examine if there are differences in preference, we use the LR test. It obtains a value of 5.03, which does not exceed the 5% critical value of 18.31 of the chi-square distribution with 10 degrees of freedom. We therefore conclude that there are no differences in preferences for one or more denominations.

To validate this finding for dataset 1, we now turn to the second dataset. We report the estimation results in Table 5. We observe that the parameter estimates bear strong similarities with those in Table 4. The LR test statistic obtains a value of 10.85, which again is far from the relevant 5% critical value.

6. Conclusion

In this paper we examined whether the Dutch public has preferences for using certain euro coins and notes. An important motivation concerned the results in Kippers et al. (2003), who documented that such preference rankings existed for the Dutch guilder, which had the 1-2½-5 based denomination range. Using two new and unique datasets, and a specifically designed model and a statistical test with good properties, we found that for the euro no such differences appear to exist.

Our analysis suggests two potentially fruitful avenues for further research. A first is to see if our current findings for the Netherlands also hold for other European countries. A second is to examine which coins or notes are considered for replacement in case one denomination is not available. The last topic entails that a natural experiment is not possible, so other data collection methods should be considered.

Appendix Computer code in Eviews (for the relevant notes and coins for dataset 1)

```
@logl logl1

@temp c1 c2 c3 c4 c5 c6 c7  l1 l2 l3 l4 l5 l6 l7

lambda1=@exp(c(11)+c(12)*acorr100)*(ub100>lb100)+3*(ub100=lb100)
lambda2=@exp(c(21)+c(22)*acorr50)*(ub50>lb50)+3*(ub50=lb50)
lambda3=@exp(c(31)+c(32)*acorr20)*(ub20>lb20)+3*(ub20=lb20)
lambda4=@exp(c(41)+c(42)*acorr10)*(ub10>lb10)+3*(ub10=lb10)
lambda5=@exp(c(51)+c(52)*acorr5)*(ub5>lb5)+3*(ub5=lb5)
lambda6=@exp(c(61)+c(62)*acorr2)*(ub2>lb2)+3*(ub2=lb2)
lambda7=@exp(c(71)+c(72)*acorr1)*(ub1>lb1)+3*(ub1=lb1)


c1=@cpoisson(ub100,lambda1)-@cpoisson(lb100-1, lambda1)*(lb100>0)
c2=@cpoisson(ub50,lambda2)-@cpoisson(lb50-1, lambda2)*(lb50>0)
c3=@cpoisson(ub20,lambda3)-@cpoisson(lb20-1, lambda3)*(lb20>0)
c4=@cpoisson(ub10,lambda4)-@cpoisson(lb10-1, lambda4)*(lb10>0)
c5=@cpoisson(ub5,lambda5)-@cpoisson(lb5-1, lambda5)*(lb5>0)
c6=@cpoisson(ub2,lambda6)-@cpoisson(lb2-1, lambda6)*(lb2>0)
c7=@cpoisson(ub1,lambda7)-@cpoisson(lb1-1, lambda7)*(lb1>0)

l1=(ub100-lb100>0)*(-lambda1+b100*@log(lambda1)-@log(c1)-@log(@fact(b100)))
l2=(ub50-lb50>0)*(-lambda2+b50*@log(lambda2)-@log(c2)-@log(@fact(b50)))
l3=(ub20-lb20>0)*(-lambda3+b20*@log(lambda3)-@log(c3)-@log(@fact(b20)))
l4=(ub10-lb10>0)*(-lambda4+b10*@log(lambda4)-@log(c4)-@log(@fact(b10)))
l5=(ub5-lb5>0)*(-lambda5+b5*@log(lambda5)-@log(c5)-@log(@fact(b5)))
l6=(ub2-lb2>0)*(-lambda6+b2*@log(lambda6)-@log(c6)-@log(@fact(b2)))
l7=(ub1-lb1>0)*(-lambda7+b1*@log(lambda7)-@log(c7)-@log(@fact(b1)))



logl1 = l1+l2+l3+l4+l5+l6+l7
```

Restriction corresponding with indifference

```
c(11)=c(21)=c(31)=c(41)=c(51)=c(61)=c(71)
c(12)=c(22)=c(32)=c(42)=c(52)=c(62)=c(72)
```

Table 1: Characteristics of respondents in Dataset 1, where "Gamma" is the appliance store and "Plusmarkt" is a supermarket for FMCGs.

	"Gamma"	"Plusmarkt"	All observations
Age (average)	47.4	49.9	48.1
Gender (1 = male)	64.0 %	16.2 %	50.9%
# weekly shopping trips	3.4	3.6	3.5
Experience as a cashier? (1 = yes)	22.2 %	24.3 %	22.8%
Computing skills series a, % correct answers	90.4 %	93.2 %	91.2%
Computing skills series b, % correct answers	62.1 %	47.3 %	58.1%
Computing skills series c, % correct answers	81.2 %	64.9 %	76.8%
Computing skills series d, % correct answers	46.0 %	32.4 %	42.3%
Score, computing skills	6.2	4.9	5.9

Table 2:

Efficient versus non-efficient payment schemes in our cash payment dataset 1, across individuals with wallets that facilitated a pure efficient payment (Cramer) and those for which the wallet was too restrictive to make a pure efficient payment.

	Efficient payment		Non-efficient payment		Total
Wallet content facilitates pure efficient payment	80		53		133
Wallet content does not facilitate pure efficient payment	85		54		139
Total	165	(61%)	107	(39%)	272

Table 3:

Empirical power of LR test for varying replacement rates

Replacement rate	25%	50%	75%
Rejection frequency	0.01	0.37	0.96
# Replications	500	200	200

Notes: The replacement rate represents a preference and is defined as the percentage of efficient payments in a sample in which payment with one 20-euro note is randomly replaced with two 10-euro notes. The empirical power is defined as the percentage of runs in which the null hypothesis of equal parameters is rejected.

Table 4:

Parameter estimates with heteroskedasticity consistent standard errors for the cash payment model. The sample size of this dataset 1 is 240.

	Intercept		ACORR		Effective observations (free choice)
Denomination	Parameter	se	Parameter	Se	
100	0.855	0.829	2.174 a	0.465	10
50	0.519	0.435	1.413 a	0.363	76
20	0.122	0.162	1.019 a	0.187	76
10	0.332	0.161 a	1.188 a	0.257	66
5	0.139	0.263	1.754 a	0.576	48
2	0.142	0.262	1.633 a	0.390	46
1	-0.032	0.218	1.780 a	0.376	39

a Significant at the 5% level

Table 5:

Parameter estimates with heteroskedasticity consistent standard errors for the cash payment model. The sample size of dataset 2 is 840.

	Intercept		ACORR		Effective observations
Denomination	Parameter	se	Parameter	se	(free choice)
100	-1.789	2.755	4.744	3.083	9
50	-0.245	0.330	1.379 a	0.326	211
20	-0.509 a	0.141	1.949 a	0.207	330
10	-0.156 a	0.091	1.991 a	0.173	387
5	-0.293 a	0.099	1.937 a	0.199	302
2	-0.240 a	0.095	1.725 a	0.146	288
1	-0.230 a	0.095	1.575 a	0.171	227

a Significant at the 5% level

Figure 1: Expected value of Y_d for denominations 1 euro through 50 euro

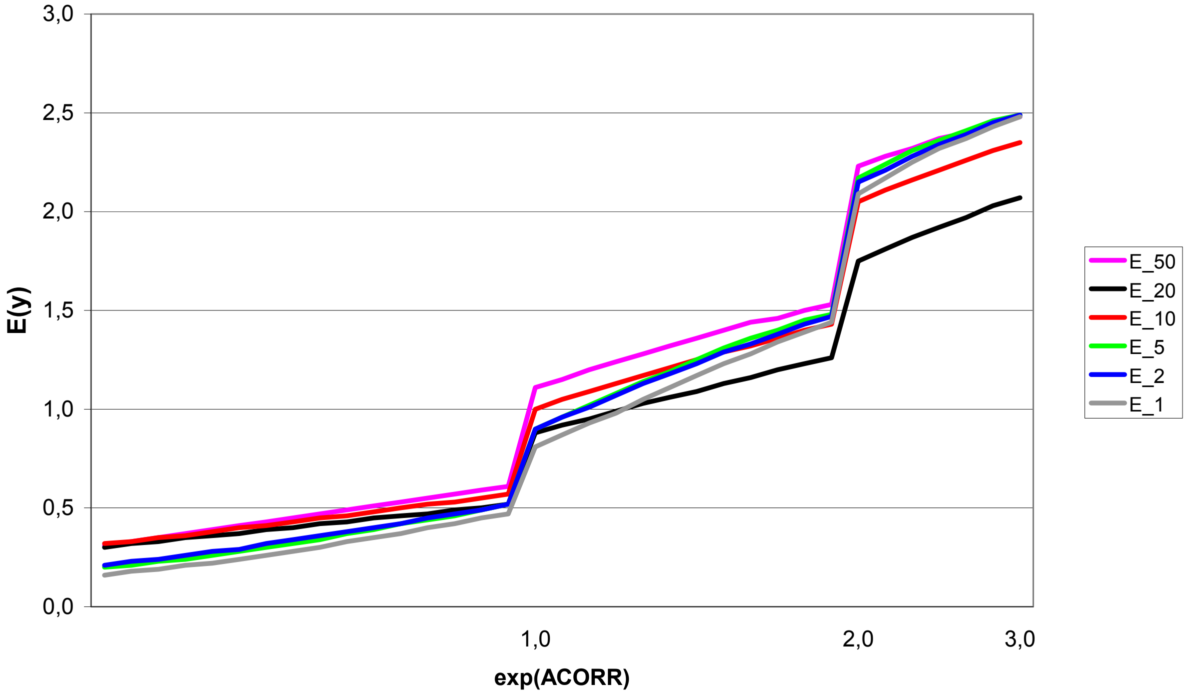
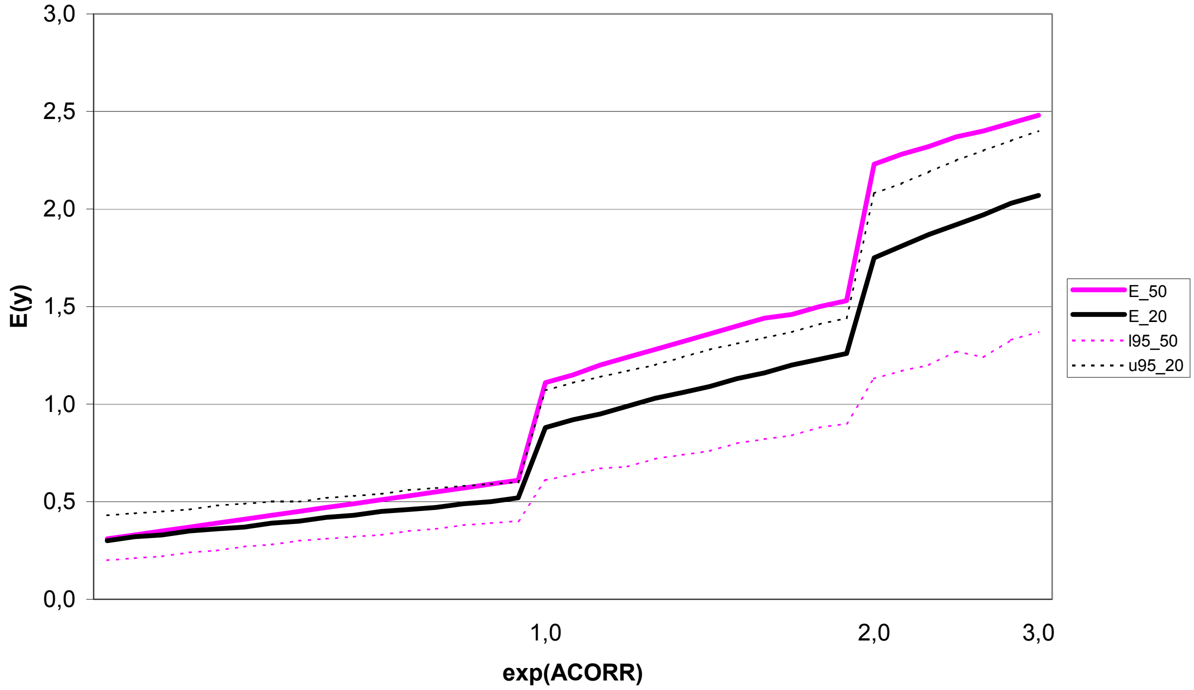


Figure 2: 95%-confidence intervals around E[Yd] for denominations 20 and 50 euro



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