Asymmetry and Leverage in Conditional Volatility Models*

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Abstract

The three most popular univariate conditional volatility models are the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten, Jagannathan and Runkle (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991). The underlying stochastic specification to obtain GARCH was demonstrated by Tsay (1987), and that of EGARCH was shown recently in McAleer and Hafner (2014). These models are important in estimating and forecasting volatility, as well as capturing asymmetry, which is the different effects on conditional volatility of positive and negative effects of equal magnitude, and leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility. As there seems to be some confusion in the literature between asymmetry and leverage, as well as which asymmetric models are purported to be able to capture leverage, the purpose of the paper is two-fold, namely: (1) to derive the GJR model from a random coefficient autoregressive process, with appropriate regularity conditions; and (2) to show that leverage is not possible in these univariate conditional volatility models.

Keywords: Conditional volatility models, random coefficient autoregressive processes, random coefficient complex nonlinear moving average process, asymmetry, leverage.

JEL classifications: C22, C52, C58, G32.

1. Introduction

The three most popular univariate conditional volatility models are the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten, Jagannathan and Runkle (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991). The underlying stochastic specification to obtain GARCH was demonstrated by Tsay (1987), and that of EGARCH was shown recently in McAleer and Hafner (2014).

These models are important in estimating and forecasting volatility, in capturing asymmetry, which is the different effects on conditional volatility of positive and negative effects of equal magnitude, and (possibly) in capturing leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility. The purpose of the paper is two-fold, namely: (1) to derive the GJR model from a random coefficient autoregressive process, with appropriate regularity conditions; and (2) to show that leverage is not possible in these univariate conditional volatility models.

The derivation of three well known conditional volatility models, namely GARCH, GJR and EGARCH, from their respective underlying stochastic processes raises two important issues: (1) the regularity conditions for each conditional volatility model can be derived in a straightforward manner; and (2) the GJR and EGARCH models can be shown to capture asymmetry, but they can also be shown to be unable to capture leverage.

The paper organized is as follows. In Section 2, the GARCH, GJR and EGARCH models are derived from different stochastic processes, the first two from random coefficient autoregressive processes and the third from a random coefficient complex nonlinear moving average process. It is shown that asymmetry is possible for GJR and EGARCH, but that leverage is not possible. Some concluding comments are given in Section 3.

2. Stochastic Processes for Conditional Volatility Models

2.1 Random Coefficient Autoregressive Process and GARCH

Consider the conditional mean of financial returns as in the following:

$$y_t = E(y_t \mid I_{t-1}) + \varepsilon_t \tag{1}$$

where the returns, $y_t = \Delta \log P_t$, represent the log-difference in stock prices (P_t), I_{t-1} is the information set at time *t-1*, and ε_t is conditionally heteroskedastic. In order to derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks, ε_t.

Consider the following random coefficient autoregressive process of order one:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \tag{2}$$

where

$$\phi_t \sim iid(0, \alpha),$$
$$\eta_t \sim iid(0, \omega).$$

Tsay (1987) showed that the ARCH(1) model of Engle (1982) could be derived from equation (2) as:

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2. \tag{3}$$

where h_t is conditional volatility, and I_{t-1} is the information set at time *t-1*. The use of an infinite lag length for the random coefficient autoregressive process in equation (2), with appropriate restrictions on the random coefficients, can be shown to lead to the GARCH model of Bollerslev (1986).

As the ARCH and GARCH models are symmetric, in that positive and negative shocks of equal magnitude have identical effects on conditional volatility, there is no asymmetry, and hence also no leverage, whereby negative shocks increase conditional volatility and positive shocks decrease conditional volatility (see Black (1976)).

It is worth noting that at least one of ω or α must be positive for conditional volatility to be positive. From the specification of equation (2), it is clear that both ω and α should be positive as they are the variances of two different stochastic processes.

2.2 Random Coefficient Autoregressive Process and GJR

The GJR model of Glosten, Jagannathan and Runkle (1992) can be derived as a simple extension of the random coefficient autoregressive process in equation (2), with an indicator variable $I(\varepsilon_{t-1})$ that distinguishes between the different effects of positive and negative returns shocks on conditional volatility, namely:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \psi_t I(\varepsilon_{t-1}) \varepsilon_{t-1} + \eta_t \tag{4}$$

where

$\phi_t \sim iid(0, \alpha)$,

$\psi_t \sim iid(0, \gamma)$,

$\eta_t \sim iid(0, \omega)$,

$I(\varepsilon_{t-1}) = 1$ when $\varepsilon_{t-1} < 0$,

$I(\varepsilon_{t-1}) = 0$ when $\varepsilon_{t-1} \geq 0$.

The conditional expectation of the squared returns shocks in (3), which is typically referred to as the GJR (or threshold GARCH), can be shown to be an extension of equation (3), as follows:

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I(\varepsilon_{t-1}) \varepsilon_{t-1}^2 \ . \qquad (5)$$

The use of an infinite lag length for the random coefficient autoregressive process in equation (4), with appropriate restrictions on the random coefficients, can be shown to lead to the standard GJR model with lagged conditional volatility.

It is worth noting that at least one of (ω, α, γ) must be positive for conditional volatility to be positive. From the specification of equation (4), it is clear that all three parameters should be positive as they are the variances of three different stochastic processes.

The GJR model is asymmetric, in that positive and negative shocks of equal magnitude have different effects on conditional volatility. Therefore, asymmetry exists for GJR if:

Asymmetry for GJR: $\gamma > 0$ **.**

A special case of asymmetry is leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility. The conditions for leverage in the GJR model in equation (5) are:

Leverage for GJR: $\alpha < 0$ **and** $\alpha + \gamma > 0$ **.**

It is clear that leverage is not possible for GJR as both α and γ, which are the variances of two stochastic processes, must be positive.

2.3 Random Coefficient Complex Nonlinear Moving Average Process and EGARCH

Another conditional volatility model that can accommodate asymmetry is the EGARCH model of Nelson (1990, 1991). McAleer and Hafner (2014) showed that EGARCH could be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, as follows:

$$\varepsilon_t = \phi_t \sqrt{|\eta_{t-1}|} + \psi_t \sqrt{\eta_{t-1}} + \eta_t \tag{6}$$

where

$\phi_t \sim iid(0,\alpha)$,

$\psi_t \sim iid(0,\gamma)$,

$\eta_t \sim iid(0,\omega)$,

$\sqrt{\eta_{t-1}}$ is a complex-valued function of η_{t-1}.

The conditional variance of the squared returns shocks in equation (6) is given as:

$$h_t = E(\varepsilon_t^2 \mid I_{t-1}) = \omega + \alpha |\eta_{t-1}| + \gamma \eta_{t-1}. \tag{7}$$

It is worth noting that the transformation of h_t in equation (7) is not logarithmic, but the approximation given by:

$$\log h_t = \log(1 + (h_t - 1)) \approx h_t - 1$$

can be used to replace h_t in equation (7) with 1 + $\log h_t$. The use of an infinite lag for the RCCNMA process in equation (6) would yield the standard EGARCH model with lagged conditional volatility.

EGARCH differs from GARCH and GJR in that, given the logarithmic transformation, no sign restrictions on (ω, α, γ) are necessary for conditional volatility to be positive. However, it is clear from the RCCNMA process in equation (6) that all three parameters should be positive as they are the variances of three different stochastic processes. Therefore, asymmetry exists for EGARCH if:

Asymmetry for EGARCH: $\gamma > 0$ **.**

The conditions for leverage in the EGARCH model in equation (7) are:

Leverage for EGARCH: $\gamma < 0$ **and** $\gamma < \alpha < -\gamma$ **.**

As acknowledged in McAleer and Hafner (2014), leverage is not possible as both α and γ, which are the variances of two stochastic processes, must be positive.

3. Concluding Remarks

The paper was concerned with the three most widely-used univariate conditional volatility models, namely the GARCH, GJR (or threshold GARCH) and EGARCH models. These models are important in estimating and forecasting volatility, as well as in capturing asymmetry, which is the different effects on conditional volatility of positive and negative effects of equal magnitude, and in capturing leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility.

As there seems to be some confusion in the literature between asymmetry and leverage, as well as which asymmetric models are purported to be able to capture leverage, the purpose of the paper was two-fold, namely: (1) to derive the GJR model from a random coefficient autoregressive process, with appropriate regularity conditions; and (2) to show the GJR and EGARCH models are able to capture asymmetry, but are unable to capture leverage.

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