

# **Frontiers in Time Series and Financial Econometrics:**

## **An Overview\***

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## Abstract

Two of the fastest growing frontiers in econometrics and quantitative finance are time series and financial econometrics. Significant theoretical contributions to financial econometrics have been made by experts in statistics, econometrics, mathematics, and time series analysis. The purpose of this special issue of the journal on “Frontiers in Time Series and Financial Econometrics” is to highlight several areas of research by leading academics in which novel methods have contributed significantly to time series and financial econometrics, including forecasting co-volatilities via factor models with asymmetry and long memory in realized covariance, prediction of Lévy-driven CARMA processes, functional index coefficient models with variable selection, LASSO estimation of threshold autoregressive models, high dimensional stochastic regression with latent factors, endogeneity and nonlinearity, sign-based portmanteau test for ARCH-type models with heavy-tailed innovations, toward optimal model averaging in regression models with time series errors, high dimensional dynamic stochastic copula models, a misspecification test for multiplicative error models of non-negative time series processes, sample quantile analysis for long-memory stochastic volatility models, testing for independence between functional time series, statistical inference for panel dynamic simultaneous equations models, specification tests of calibrated option pricing models, asymptotic inference in multiple-threshold double autoregressive models, a new hyperbolic GARCH model, intraday value-at-risk: an asymmetric autoregressive conditional duration approach, refinements in maximum likelihood inference on spatial autocorrelation in panel data, statistical inference of conditional quantiles in nonlinear time series models, quasi-likelihood estimation of a threshold diffusion process, threshold models in time series analysis - some reflections, and generalized ARMA models with martingale difference errors.

**Keywords:** Time series, financial econometrics, threshold models, conditional volatility, stochastic volatility, copulas, conditional duration.

**JEL:** C22, C32, C58, G17, G32.

## **1. Introduction**

Statistics, econometrics and time series analysis have rich histories, with numerous accomplishments and associated journals and societies. Finance is a more recent discipline, as is financial econometrics. Two of the fastest growing frontiers in econometrics and quantitative finance are time series and financial econometrics. Significant theoretical contributions to financial econometrics have been made by experts in statistics, econometrics and time series analysis.

As a fairly recent discipline, it is easier to trace the origins of financial econometrics than of statistics and time series. The advent of threshold models, which are widely used in econometrics and financial econometrics, can be traced to Tong (1978) (for further details, as well as a number of new areas of exploration, see Tong (2015)), while the econometric analysis of time-varying volatility modelling has its origins in Engle (1982), which has led to a vast literature on dynamic volatility modelling and risk management, among others.

The direct connection between modelling thresholds and time series is obvious, given the time-varying nature of the thresholds and data. Furthermore, the connection between conditional volatility and time series models is also obvious, given that conditional volatility may be expressed as an ARMA process of the squared shocks to financial returns. The imaginative ideas developed in the papers by Tong (1978) and Engle (1982) have contributed significantly to the explosive growth in financial econometrics and time series models over the last few decades.

An early, and possibly the first, international conference that was focused solely on financial econometrics, specifically the “International Conference on Modelling and Forecasting Financial Volatility”, was held in Perth, Australia, in September 2001, with the keynote speakers being Clive Granger, Rob Engle and Neil Shephard. A special issue of the Journal of Applied Econometrics based on selected papers from the conference is overviewed in Franses and McAleer (2002), and a conference review is given in Oxley (2001). Such international conferences in financial econometrics are now widespread, especially with the establishment of the Journal of Financial Econometrics in 2003 and the Society for Financial Econometrics in 2007.

The special issue brings together the leading specialists in time series, financial econometrics, statistics and mathematics to present the latest theoretical developments in “Frontiers in Time Series and Financial Econometrics”. The novel and significant contributions to the special issue present an extensive range of theoretical papers, all of which can be used in practical applications, by the leading scholars in statistics, time series analysis and financial econometrics.

The broad theme of the special issue covers several exciting topics in time series and financial econometrics, including estimation, specification testing and prediction, analysis of multiple-threshold models, adaptive compositional models, high-dimensional stochastic regression, latent factors, asymmetric conditional duration models, heavy tails, calibrated option pricing models, maximum likelihood inference on spatial autocorrelation, high-dimensional functional coefficient time series, tests of independence when the observations are curves, functional index coefficient models with variable selection, quantile analysis for long memory stochastic volatility models, quasi-likelihood estimation of a threshold diffusion process, statistical

inference of conditional quantiles in nonlinear time series, hyperbolic memory, optimal model averaging, realized co-volatilities with asymmetry, leverage and long memory, statistical inference for panel dynamic simultaneous equations model for large  $N$  and large  $T$ , and stationary Lévy-driven CARMA processes.

The purpose of the special issue is to highlight a number of areas of research in which novel statistical, financial econometric and time series methods have contributed significantly to frontiers in time series and financial econometrics, specifically forecasting co-volatilities via factor models with asymmetry and long memory in realized covariance (Asai and McAleer (2015)), prediction of Lévy-driven CARMA processes (Brockwell and Lindner (2015)), functional index coefficient models with variable selection (Cai, Juhl and Yang (2015)), LASSO estimation of threshold autoregressive models (Chan, Yau and Zhang (2015)), high dimensional stochastic regression with latent factors, endogeneity and nonlinearity (Chang, Guo and Yao (2015)), sign-based portmanteau test for ARCH-type models with heavy-tailed innovations (Chen and Zhu (2015)), toward optimal model averaging in regression models with time series errors (Cheng, Ing and Yu (2015)), high dimensional dynamic stochastic copula models (Creal and Tsay (2015)), a misspecification test for multiplicative error models of non-negative time series processes (Gao, Kim and Saart (2015)), sample quantile analysis for long-memory stochastic volatility models (Ho (2015)), testing for independence between functional time series (Horvath and Rice, (2015)), statistical inference for panel dynamic simultaneous equations models (Hsiao and Zhou (2015)), specification tests of calibrated option pricing models (Jarrow and Kwok (2015)), asymptotic inference in multiple-threshold double autoregressive models (Li, Ling and Zakoian (2015)), a new hyperbolic GARCH model (Li, Li and Li (2015)), intraday value-at-risk: an asymmetric autoregressive conditional duration

approach (Liu and Tse (2015)), refinements in maximum likelihood inference on spatial autocorrelation in panel data (Robinson and Rossi (2015)), statistical inference of conditional quantiles in nonlinear time series models (So and Chung (2015)), quasi-likelihood estimation of a threshold diffusion process (Su and Chan (2015)), threshold models in time series analysis - some reflections (Tong (2015)), and generalized ARMA models with martingale difference errors (Zheng, Xiao and Chen (2015)).

The interesting, timely and novel contributions to this special issue should highlight and encourage innovative research in a variety of challenging areas associated with time series analysis and financial econometrics. It is exciting that many of the leading international scholars in statistics, mathematics, econometrics, time series analysis and financial econometrics have contributed to an innovative special issue on “Frontiers in Time Series and Financial Econometrics”.

The plan of the remainder of the paper is as follows. An overview of the 21 papers in the special issue is presented in Section 2. Some final remarks are given in Section 3.

## **2. Overview**

The first paper is “Forecasting co-volatilities via factor models with asymmetry and long memory in realized covariance”, by Manabu Asai (Faculty of Economics, Soka University, Japan) and Michael McAleer (Department of Quantitative Finance, National Tsing Hua University, Taiwan). Modelling covariance structures is known to suffer from the curse of dimensionality. In order to avoid this problem for forecasting, the authors propose a new factor multivariate stochastic volatility (fMSV) model for realized covariance measures that accommodates asymmetry and long memory. Using

the basic structure of the fMSV model, the authors extend the dynamic correlation MSV model, the conditional and stochastic Wishart autoregressive models, the matrix-exponential MSV model, and the Cholesky MSV model. The empirical results for 7 financial asset returns for US stock returns indicate that the new fMSV models outperform existing dynamic conditional correlation models for forecasting future covariances. Regarding the forecasting performance for one-day, five-day and ten-day horizons, the recommended specification among the new fMSV models is the stochastic Wishart autoregressive specification with asymmetric effects for the periods during and after the global financial crisis (GFC), while the Cholesky fMSV model with long memory and asymmetry displays the best performance for periods outside the GFC.

In the second paper, “Prediction of Lévy-driven CARMA processes”, by Peter Brockwell (Department of Statistics, Colorado State University, USA) and Alexander Lindner (Institut für Mathematische Stochastik, TU Braunschweig, Germany), the authors consider the problem of determining the conditional expectations,  $E(Y(h)|Y(u); u < 0)$  and  $E(Y(h)|Y(u); -M < u < 0)$ , where  $h > 0$ ,  $0 < M < \infty$ , and  $Y(t)$  is a strictly stationary continuous-time ARMA (CARMA) process driven by a Lévy process  $L$ . If the driving Lévy process satisfies  $E(L(1)^2) < \infty$ , then these are the minimum mean-squared error predictors of  $Y(h)$  given  $(Y(t)), t < 0$ , and  $(Y(t)), -M < t < 0$ , respectively. Conditions are established under which the sample path of  $L$  can be recovered from that of  $Y$ , when  $Y$  is causal and strictly stationary and also without these assumptions when  $L$  is a pure jump Lévy process. When  $E(L(1)^2) < \infty$ , and  $Y$  is causal and strictly stationary, the best linear predictors are determined, the latter yielding a simple algorithm for determining the parameters of the ARMA process obtained by sampling the CARMA process at regular intervals.

The third paper by Zongwu Cai (Department of Economics, University of Kansas, USA), Ted Juhl (Department of Economics, University of Kansas, USA), and Bingduo Yang (School of Finance, Jiangxi University of Finance and Economics, China) is on “Functional index coefficient models with variable selection”. The paper considers variable selection in a semi-parametric time series model with functional coefficients. Variable selection in the semi-parametric model needs to account for the fact that the parametric part is estimated with faster convergence rate than is the nonparametric component. The new variable selection procedures use a smoothly-clipped absolute deviation penalty function and consist of two steps: the first step selects covariates with functional coefficients that enter the semi-parametric model, while the second step performs variable selection with parametric coefficients. Asymptotic properties, such as consistency, sparsity and the oracle property of the two-step estimators, are established. A Monte Carlo simulation study is conducted to examine the finite sample performance of the proposed estimators and variable selection procedures. An empirical example exploring the predictability of asset returns demonstrates the practical application of the proposed functional index coefficient autoregressive models and variable selection procedures.

“LASSO estimation of threshold autoregressive models”, the fourth paper, is by Ngai Hang Chan (Department of Statistics, Chinese University of Hong Kong, China), Chun Yip Yau (Department of Statistics, Chinese University of Hong Kong, China), and Rong-Mao Zhang (Department of Mathematics, Zhejiang University, China). The paper develops a novel approach for estimating a threshold autoregressive (TAR) model with multiple regimes, and establishes its large sample properties. By reframing the problem in a regression variable selection context, a least absolute shrinkage and



selection operator (LASSO) procedure is proposed by the authors to estimate a TAR model with an unknown number of thresholds, where the computation can be performed efficiently. It is further shown that the number of thresholds and the location of the thresholds can be estimated consistently. A near optimal convergence rate of the threshold parameters is also established. Simulation studies are conducted to assess the performance in finite samples, and the results are illustrated with an application to quarterly U.S. real GNP data over the period 1947-2009.

The fifth paper by Jinyuan Chang (Department of Mathematics and Statistics, University of Melbourne, Australia), Bin Guo (Guanghua School of Management, Peking University, China), and Qiwei Yao (Department of Statistics, London School of Economics, UK) is on “High dimensional stochastic regression with latent factors, endogeneity and nonlinearity”. The paper considers a multivariate time series model which represents a high dimensional vector process as a sum of three terms: a linear regression of some observed regressors, a linear combination of some latent and serially correlated factors, and a vector white noise. The authors investigate inference without imposing stationary conditions on the target multivariate time series, the regressors and the underlying factors. Furthermore, they deal with the endogeneity that there exist correlations between the observed regressors and the unobserved factors. The authors also consider the model with a nonlinear regression term which can be approximated by a linear regression function with a large number of regressors. The convergence rates for the estimators of the regression coefficients, the number of factors, factor loading space and factors are established when the dimension of the time series and the number of regressors may both tend to infinity together with the sample size. The proposed method is illustrated with both simulated and real data examples.

“Sign-based portmanteau test for ARCH-type models with heavy-tailed innovations” is the sixth paper, by Min Chen (Institute of Applied Mathematics, Chinese Academy of Sciences, Beijing, China) and Ke Zhu (Institute of Applied Mathematics, Chinese Academy of Sciences, Beijing, China). The paper proposes a sign-based portmanteau test for diagnostic checking of ARCH-type models estimated by the least absolute deviation approach. Under the strict stationarity condition, the asymptotic distribution is obtained. The new test is applicable for very heavy-tailed innovations with only finite fractional moments. Simulations are undertaken to assess the performance of the sign-based test, as well as a comparison with two other portmanteau tests. A real empirical example for exchange rates is given to illustrate the practical usefulness of the test.

Tzu-Chang Cheng (Department of Economics, University of Illinois at Urbana-Champaign, USA), Ching-Kang Ing (Institute of Statistical Science, Academia Sinica and Department of Economics, National Taiwan University, Taiwan), and Shu-Hui Yu (Institute of Statistics, National University of Kaohsiung, Taiwan) present the seventh paper, namely “Toward optimal model averaging in regression models with time series errors”. The paper considers a regression model with infinitely many parameters and time series errors. The authors choose weights for averaging across generalized least squares estimators (GLS) obtained from a set of approximating models. However, as the GLS estimators are usually infeasible, depending on the unknown inverse covariance matrix of the errors, the authors construct feasible generalized least squares (FGLS) estimators using a consistent estimator of the unknown inverse matrix. Based on this inverse covariance matrix estimator and FGLS estimators, they develop a feasible autocovariance-corrected Mallows model averaging criterion to select weights, thereby providing an FGLS model averaging estimator of the true regression function. The authors show that the generalized squared error loss

of the averaging estimator is asymptotically equivalent to the minimum one among those of GLS model averaging estimators, with the weight vectors belonging to a continuous set that includes a discrete weight set as a proper subset.

The eighth paper, “High dimensional dynamic stochastic copula models”, is by Drew D. Creal (Booth School of Business, University of Chicago, USA) and Ruey S. Tsay (Booth School of Business, University of Chicago, USA). The paper builds a class of copula models that captures time-varying dependence across large panels of financial assets. The new models nest Gaussian, Student’s  $t$ , grouped Student’s  $t$ , and generalized hyperbolic copulas with time-varying correlations matrices, as special cases. They introduce time variation into the densities by writing them as factor models with stochastic loadings. The proposed copula models have flexible dynamics and heavy tails, yet remain tractable in high dimensions due to their factor structure. The Bayesian estimation approach leverages a recent advance in sequential Monte Carlo methods, known as particle Gibbs sampling, which can draw large blocks of latent variables efficiently and in parallel. The authors use this framework to model an unbalanced, 200-dimensional panel consisting of credit default swaps and equities for 100 U.S. corporations. Their analysis shows that the grouped Student’s  $t$  stochastic copula is preferred over seven competing models.

“A misspecification test for multiplicative error models of non-negative time series processes” is the ninth paper, by Jiti Gao (Department of Econometrics and Business Statistics, Monash University, Australia), Nam Hyun Kim (Department of Economics, University of Konstanz, Germany), and Patrick W. Saart (Department of Mathematics and Statistics, University of Canterbury, New Zealand). In recent years, analysis of financial time series has focused largely on data related to market trading activity. Apart

from modelling the conditional variance of returns within the generalized autoregressive conditional heteroskedasticity (GARCH) family of models, attention has also been devoted to other market variables, especially volumes, number of trades and durations. The financial econometrics literature has focused attention on Multiplicative Error Models (MEMs), which are considered particularly suited for modelling non-negative time series processes. The paper establishes an alternative misspecification test for the MEMs of non-negative time series processes. In the literature, although several hypothesis testing procedures are available for MEMs, the proposed testing procedures are particularly useful in the context of MEMs of waiting times between financial events as the outcomes have important implications regarding the fundamental concept of point processes. The paper makes a number of statistical contributions, especially for nonparametric hypothesis testing of unobservable variables.

Hwai-Chung Ho (Institute of Statistical Science, Academia Sinica, and National Taiwan University, Taiwan) presents the tenth paper on “Sample quantile analysis for long-memory stochastic volatility models”. The paper investigates the asymptotic properties of sample quantile estimates in the context of long memory stochastic volatility models in which the latent volatility component is an exponential transformation of a linear long memory time series. The author focuses on the least absolute deviation quantile estimator. It is shown that, although the underlying process is a sequence of stationary martingale differences, the estimation errors are asymptotically normal with convergence rate slower than  $\sqrt{n}$  and determined by the dependence parameter of the volatility sequence. A non-parametric resampling method is used to estimate the normalizing constants by which the confidence intervals are constructed. The author conducts a simulation study as well as an empirical analysis of the Value-at-Risk estimate of the S&P 500 daily returns. Both are consistent with the

theoretical findings, and provide clear evidence that the coverage probabilities of confidence intervals for the quantile estimates are severely biased if the strong dependence of the unobserved volatility sequence is ignored.

The eleventh paper on “Testing for independence between functional time series” is by Lajos Horvath (Department of Mathematics, University of Utah, USA) and Gregory Rice (Department of Mathematics, University of Utah, USA). There is frequent interest in verifying a relationship between two or more time series. Such analysis is typically carried out by causality and/or independence tests which have been well analyzed when the data are univariate or multivariate. Modern data, especially in empirical finance, are increasingly of a high dimensional or functional nature, for which finite-dimensional methods are not suitable. The authors develop an approach to check that the data obtained from two functional time series are independent. The new procedure is based on the norms of empirical cross-covariance operators and is asymptotically validated when the underlying populations are assumed to be in a class of weakly dependent random functions, which include the functional ARMA, ARCH and GARCH processes.

Cheng Hsiao (Department of Economics, University of Southern California, USA) and Qiankun Zhou (Department of Economics, University of Southern California, USA) analyze “Statistical inference for panel dynamic simultaneous equations models” in the twelfth paper. The authors examine the identification and estimation of panel dynamic simultaneous equations models. They show that the presence of time-persistent individual-specific effects does not lead to changes in the identification conditions of traditional Cowles Commission dynamic simultaneous equations models. However, the limiting properties of the estimators depend on how the cross-section dimension,  $N$ , or the time series dimension,  $T$ , goes to infinity. They propose three

limited information estimators, namely panel simple instrumental variables (PIV), panel generalized two stage least squares (PG2SLS), and panel limited information maximum likelihood (PLIML) estimators. It is shown that the three estimators are all asymptotically unbiased, independently of how  $N$  or  $T$  tends to infinity. Monte Carlo studies are conducted to compare the performance of the PLIML, PIV, PG2SLS, Arellano-Bond-type generalized method of moments, and the Akashi-Kunitomo least variance ratio estimator, and to demonstrate whether the reliability of statistical inference depends critically on whether an estimator is asymptotically unbiased.

The thirteenth paper, “Specification tests of calibrated option pricing models”, is by Robert Jarrow (Cornell University - Samuel Curtis Johnson Graduate School of Management, Cornell University, USA) and Simon Kwok (School of Economics, University of Sydney, Australia). In spite of the popularity of model calibration in finance, empirical researchers have placed greater emphasis on model estimation than on the equally important goodness-of-fit problem, due partly to the ignorance of modellers, but even more so to the ability of existing statistical tests to detect specification errors. In practice, models are often calibrated by minimizing the sum of squared differences between the modelled and actual observations. It is challenging to disentangle model error from estimation error in the residual series. In order to circumvent this difficulty, the authors examine an alternative estimation of the model by exact calibration. Unlike the error minimization approach, all information about dynamic misspecification is channelled to the parameter estimation residuals under exact calibration. In the context of option pricing, the authors illustrate that standard time series tests are powerful in detecting various kinds of dynamic misspecification. The simulation results show that, compared with the error minimization approach, exact

calibration of the Black-Scholes model delivers a more accurate hedging performance that is robust to both gradual and abrupt structural shifts of state variables.

“Asymptotic inference in multiple-threshold double autoregressive models” is the fourteenth paper, by Dong Li (Mathematical Sciences Center, Tsinghua University, China), Shiqing Ling (Department of Mathematics, Hong Kong University of Science and Technology, Hong Kong, China), and Jean-Michel Zakoian (University Lille 3 and CREST, France). The paper investigates a class of multiple-threshold models, called Multiple Threshold Double AR (MTDAR) models. A sufficient condition is obtained for the existence and uniqueness of a strictly stationary and ergodic solution to the first-order MTDAR model. The authors examine the Quasi-Maximum Likelihood Estimator (QMLE) of the MTDAR model. The estimated thresholds are shown to be  $n$ -consistent, asymptotically independent, and to converge weakly to the smallest minimizer of a two-sided compound Poisson process. The remaining parameters are  $\sqrt{n}$ -consistent and asymptotically multivariate normal. In particular, these results apply to the multiple threshold ARCH model, with or without an AR component, and to the multiple threshold AR models with ARCH errors. A score-based test is also presented to determine the number of thresholds in MTDAR models. The limiting distribution is shown to be distribution-free and is easy to implement in practice. Simulation studies are conducted to assess the performance of the QMLE and the new score-based test in finite samples. The results are illustrated with an application to quarterly U.S. real GNP data over the period 1947–2013.

Muyi Li (Wang Yanan Institute for Studies in Economics, Xiamen University, China), Wai Keung Li (Department of Statistics and Actuarial Science, University of Hong Kong, China), and Guodong Li (Department of Statistics and Actuarial Science,

University of Hong Kong, China) present “A new hyperbolic GARCH model” in the fifteenth paper. There are two commonly used hyperbolic GARCH processes, the FIGARCH and HYGARCH processes, in modeling the long-range dependence in volatility. However, the FIGARCH process always has infinite variance, and the HYGARCH model has a more complicated form. This paper builds a simple bridge between a common GARCH model and an integrated GARCH model, and hence a new hyperbolic GARCH model along the lines of FIGARCH models. The new model remedies the drawback of FIGARCH processes by allowing the existence of a finite variance, as in HYGARCH models, while it has a form nearly as simple as the FIGARCH model. Two inference tools, including the Gaussian QMLE and a portmanteau test for the adequacy of the fitted model, are derived, and an easily implemented test for hyperbolic memory is also constructed. The finite sample performance of the new procedures is evaluated by simulation experiments, and an empirical example gives further support to the new model.

Shouwei Liu (School of Economics, Singapore Management University, Singapore) and Yiu-Kuen Tse (School of Economics, Singapore Management University, Singapore) analyze “Intraday value-at-risk: An asymmetric autoregressive conditional duration approach” in the sixteenth paper. The paper estimates the intraday Value-at-Risk (IVaR) for stocks using real-time transaction data. Tick-by-tick data filtered by price duration are modelled using a two-state asymmetric autoregressive conditional duration (AACD) model, and the IVaR is calculated using Monte Carlo simulation based on the estimated AACD model. Backtesting results of the New York Stock Exchange (NYSE) show that the IVaR calculated using the AACD method outperforms those using several currently available alternative methods.



The seventeenth paper, “Refinements in maximum likelihood inference on spatial autocorrelation in panel data”, is by Peter M. Robinson (London School of Economics, UK) and Francesca Rossi (University of Southampton, UK). In a panel data model with fixed effects, possible cross-sectional dependence is investigated in a spatial autoregressive setting. An Edgeworth expansion is developed for the maximum likelihood estimate of the spatial correlation coefficient. The expansion is used to develop more accurate interval estimates for the coefficient, and tests for cross-sectional independence that have better size properties, than corresponding rules of statistical inference based on first-order asymptotic theory. Comparisons of finite sample performance are carried out using Monte Carlo simulations.

“Statistical inference of conditional quantiles in nonlinear time series models” is the eighteenth paper, by Mike K.P. So (Department of Information System, Business Statistics and Operations Management, Hong Kong University of Science and Technology, China) and Ray S.W. Chung (Division of Environment, Hong Kong University of Science and Technology, China). The paper analyzes the statistical properties of a two-step conditional quantile estimator in nonlinear time series models with unspecified error distribution. The asymptotic distributions of the quasi-maximum likelihood estimators and the filtered empirical percentiles are derived. Three applications of the asymptotic results are considered. First, the authors construct an interval estimator of the conditional quantile without any distributional assumptions. Second, they develop a specification test for the error distribution. Finally, using the specification test, they propose methods for estimating the tail index of the error distribution that would support the construction of a new estimator for the conditional quantile at the extreme tail. The asymptotic results and their applications are illustrated

by simulations and real data analyses, in which the proposed methods for analyzing daily and intraday financial return series have been adopted.

Fei Su (ISO Innovative Analytics, San Francisco, USA) and Kung-Sik Chan (Department of Statistics and Actuarial Science, University of Iowa, USA) present “Quasi-likelihood estimation of a threshold diffusion process” in the nineteenth paper. The threshold diffusion process of Tong (1990) is a continuous time process satisfying a stochastic differential equation with a piecewise linear drift term and a piecewise smooth diffusion term, such as a piecewise constant function or a piecewise power function. The paper considers the estimation of the (drift) parameters, indexing the drift term of a threshold diffusion process with continuous time observations. Maximum likelihood estimation of the drift parameters requires prior knowledge of the functional form of the diffusion term, which is often unavailable. The authors propose a quasi-likelihood approach for estimating the drift parameters of a two-regime threshold diffusion process that does not require prior knowledge of the functional form of the diffusion term, and show that, under mild regularity conditions, the quasi-likelihood estimators of the drift parameters are consistent. Moreover, the estimator of the threshold parameter is super consistent and weakly converges to some non-Gaussian continuous distribution. The autoregressive parameter estimates in the drift term are shown to be jointly asymptotic normal, with the same distribution as when the threshold is known. The empirical properties of the quasi-likelihood estimator are studied by simulation. The authors apply the threshold model to estimate the term structure of a long time series of US interest rates. The proposed approach and asymptotic results can be extended to the case of a multi-regime threshold diffusion process.

The twentieth and penultimate paper by Howell Tong (Emeritus Professor, Department

of Statistics, London School of Economics), is entitled “Threshold models in time series analysis - some reflections”. The author reflects on the developments of the threshold model in time series analysis since its birth in 1978, with particular reference to econometrics. The paper highlights one implementation of the principle in the form of the threshold autoregressive model, while commenting on others, and reviews the historical development, including the original motivation and impact. Topics covered include all-step-ahead prediction, asymmetry; Bayesian decision, business cycles, catastrophe, conditionally heteroscedastic autoregressive models with thresholds, jump resonance, misspecified models, non-likelihood approach, nonlinear unit root, non-stationarity, open-loop system, panel threshold model, smooth threshold autoregressive models, splines, structural breaks, threshold autoregressive models, threshold moving average models, threshold principle, threshold unit roots, and volatility.

The twenty-first and final paper, entitled “Generalized ARMA models with martingale difference errors”, is by Tingguo Zheng (Wang Yanan Institute for Studies in Economics, Xiamen University, China), Han Xiao (Department of Statistics and Biostatistics, Rutgers University, USA), and Rong Chen (Department of Statistics and Biostatistics, Rutgers University, USA). The analysis of non-Gaussian time series has been studied extensively and has many applications. Many successful models can be viewed as special cases or variations of the generalized autoregressive moving average (GARMA) models, where a link function similar to that used in generalized linear models is introduced and the conditional mean, under the link function, assumes an ARMA structure. Under such a model, the ‘transformed’ time series, under the same link function, also assumes an ARMA form. Unfortunately, unless the link function is an identity function, the error sequence defined in the transformed ARMA model is usually not a martingale difference sequence. In this paper, the authors extend the

GARMA model so that the resulting ARMA model in the transformed space has a martingale difference sequence for the errors. The benefits of such an extension are four-fold, namely there are easily verifiable conditions for stationarity and ergodicity, the Gaussian pseudo-likelihood estimator is consistent, standard time series model building tools are readily available, and the asymptotic distribution of the maximum likelihood estimator can be established. The authors also propose two new classes of non-Gaussian time series models under the new framework. The performance of the proposed models is demonstrated with simulated and real examples.

### **3. Final remarks**

The collection of novel, interesting, topical and technical papers in this special issue on “Frontiers in Time Series and Financial Econometrics” by the leading experts in mathematics, statistics, econometrics, time series analysis and financial econometrics is likely to highlight and enhance further innovative research in a variety of challenging areas associated with the rapidly expanding frontiers in time series analysis and financial econometrics. The papers in the special issue are highly likely to be widely read and used in practice in a range of disciplines, and also be well cited in the literature.

It is our pleasure to acknowledge all the contributors for preparing their invaluable, novel, interesting, topical and technical papers on “Frontiers in Time Series and Financial Econometrics” in a timely manner, and for their willingness to participate in the rigorous editorial review process. The Guest Co-editors offer special thanks to the Editors of the Journal of Econometrics for their sustained support and encouragement, and to the numerous referees for their very helpful comments and suggestions on the excellent papers in the special issue.

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