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Private Equity Waves

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This study presents a dynamic model for the private equity market in which information revelation and uncertainty rationally explain the cyclical pattern of investment flows into private equity. The net benefit of private equity over public equity is i) uncertain and ii) agents have private information about the benefits of their investment. When these distinguishing characteristics determine investment behavior in private equity markets, rational investment waves may arise endogenously. Investment behavior reveals private information on the benefits of private equity financing and may trigger a cascade when investors jump on the bandwagon and invest irrespective of their private information content. We argue that the procyclical behavior of private equity volumes is strengthened by the revelation of information on the benefits of private equity investments. The occurrence and length of such waves in the market for private equity depend on the capabilities of agents.

Keywords: private equity, information economics

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I. INTRODUCTION

Private equity markets typically experience cycles with remarkable periods of private investment and wealth creation, followed by controversy and entrenchments. This study presents a model that rationally explains such behavior. Static models cannot explain the dynamic investment behavior and large fluctuations in investment volume observed in private equity markets. Unlike publicly traded securities, i) the net benefits of private equity are associated with substantial uncertainty and ii) cannot be observed by investing agents. Our dynamic model shows that, due to these specific characteristics inherent to private equity investments, waves of funds rationally allocated to private equity may arise endogenously when information on the benefits of private equity is revealed to others.

In our model of the private equity market, the demand curve is determined by the net expected benefit to companies that seek private financing, while the supply curve is determined by the return required by private investors on their committed capital. Obvious benefits of buyouts include tax shields and better control due to the smaller number of owners—as compared to public companies—and the resulting incentive structure. The supply of private funding requires the acceptance of illiquidity and a good deal of idiosyncratic risk. Public and private investment are in balance when private investment opportunities in the economy are financed until the expected net benefit from the marginal private investment is the same as the expected return on comparable public equity.

A distinguishing feature of this study is the extension of this approach to include private information, agents that differ in their capabilities to assess the value of an opportunity, and dynamic timing of investment. The procyclical nature of private equity waves depends not only on the uncertain evolution of the economy—where growth triggers investments—but also on the revelation of private information. Information economics and herding behavior strengthen the cyclical pattern of investment flows into private equity. Such a cascade of acquisitions is not simply a simultaneous investment where strategic bidders face a "dance of chairs" in the industry to find a partner before the music stops. A private equity wave results from investment behavior in previous periods that provides positive signals that incite new investments by private equity funds when agents realize that they have been slow with their investment, as described in Grenadier (1999). In our model, a limited pool of available high quality deals or a turn in the economy will end these waves. After the entrenchment period a new wave may arise.

Waves are observed in both venture capital and buyout markets. For instance, the high returns of buyout in the early 1980s can be attributed to information advantages of the early general partners, who "invented" leveraged buyouts as a vehicle to restructure inefficient and over-diversified businesses. It is plausible that observing the behavior of early investors was an important way of gaining information on that investment. Triggered by positive signals and returns in previous periods, commitment and investment levels of partnerships swelled. In particular, when current investments were driven partially by past investments, they tended to overwhelm the available opportunities when negative externalities arose. Negative externalities in demand and competition for a limited number of deals ended these waves. By 1991–1992 the private equity market in the United States had dried up.

The model presented here builds on information revelation to make predictions about the evolution of the excess rent and investment timing in private equity markets. We find that in a balance period the excess rent may build up until general partners of private equity funds start to invest unscrupulously in a wave. The excess rent then declines to its lowest level at the end of the wave. In private equity markets, the probability that waves will occur depends on the cycle, the heterogeneity of agents, and the reliability of their signals. The information advantage of established agents enables them to invest first and reveal signals to lesser-informed agents, who are more likely to invest at the end of the wave. When agents incorrectly infer their own information and are overly optimistic, waves arise sooner and have longer duration when rival agents are not aware of these unreliable signals.

Information flows and herding behavior of heterogeneous agents provide a plausible explanation why growth in private equity volume has not been uniform. Empirical research[1] in the literature confirms that private equity investment flows tend to increase after periods of good private equity performance and to decrease after poor performance. Kaplan and Schoar (2005) find that established and better-informed agents show persistence in their performance, while new agents entering the market in boom periods are more likely to show underperformance. Returns of established funds with good track records are much less affected by industry cycles and waves than returns of young funds. In our model, consistent with the empirical literature the interpretation of these market phenomena is based on heterogeneity of agents. The final returns from private equity investments can only be observed many years after the initial investments. Lesser informed investors may therefore rely on other agents' investment and market analyses and copy their investment decisions, leading to a rush in private equity investments.

This article is organized as follows. Section II relates our model to the relevant literature. Section III develops a dynamic model for the private equity market. Section IV discusses the implications of informational cascades and investment in recent buyout and venture capital waves. In Section V we consider extensions of irreversibility of investment and the implications of heterogeneity between agents. Section VI presents concluding remarks.

II. RELATED LITERATURE ON PRIVATE EQUITY CYCLES

Not only private equity investments, but also merger and acquisition activities cluster by industry in certain time periods and occur in waves (Mitchell and Mulherrin, 1996, Andrade, Mitchell, and Stafford, 2001). These waves are associated with high stock market valuations (Maksimovic and Phillips, 2001, Jovanovic and Rousseau, 2001). Two main theories explain the cause of merger waves. The neoclassical theory sees mergers as a response to industry shocks (Gort, 1962, Mitchell and Mulherrin, 1996), whereas the behavioral theory claims that stock market valuations drive merger waves (Shleifer and Vishny, 2003, and Rhodes-Kropf and Viswanathan, 2004). Empirical evidence shows that misvaluation drives waves (Rhodes-Kropf, Robinson and Viswanathan, 2005), but that shocks to the industry precede these waves (Harford, 2005). Private equity investors, however, are not able to use their overvalued stock as transaction currency, since most transactions are paid in cash. We propose that the procyclical nature of private equity volume is a result of the evolution in the economy and the herding behavior of agents.

Although private equity markets are an important part of the financial economy, there are not many theoretical models developed for them in the literature. Often, demand and supply of capital in the market for private equity are assumed to be in equilibrium when the required return equals the return on the global capital

[1] Gompers and Lerner (2000), Ljungqvist and Richardsson (2003a,b), Inderst and Müller (2004), and Diller and Kaserer (2005), Schmidt, Nowak and Knigge (2004).

markets. Gompers and Lerner (1998) conclude that the private equity market can be imbalanced, due to differences between sticky short-term and long-term supply curves. Supply in the market has to adapt to exogenous shocks,[2] but it takes 12 to 18 months before additional funds are raised. Supply may then overshoot demand, as private equity investors incorrectly infer the size of the investment opportunities or do not consider the costly adjustments associated with the growth in their activities.

The market mechanism presented here builds on information economics. Better-informed investors time their investment first and lesser-informed agents learn from them (Zhang, 1997, Grenadier, 1999). In our model waves or information cascades arise when agents ignore their own private information entirely, because the information revealed by previous agents' actions overwhelms the importance of their own information in their decisions (Banerjee, 1992, Bikhchandani, Hirshleifer and Welch, 1992, Grenadier, 1999). Persons and Warther (1997) model the adoption of financial innovations where agents decide to adopt after having observed the average profits of earlier adopters, which results in a boom and bust pattern. In private equity the return is only observed several years after the investment. However, investment behavior does provide signals on the expected return.

The information revelation mechanism of our model for private equity is closely related to the revelation model of Grenadier (1999). In that model, agents base their optimal investment decision on the state of the economy and on other agents' investment behavior and their revealed private information. However, in private equity markets, as a result of a declining demand function, negative externalities will affect the occurrence and duration of waves. As a consequence, waves arise and end endogenously resulting in a series of rational waves. The probability of such waves' arising depends on the heterogeneity of agents and the reliability of their signals, which alters the strategies of other agents and may deter herd formation. This study develops three main implications for the endogenous occurrence of private equity waves and the evolution of the excess rent of private equity compared to public sources, and may provide an incremental contribution in explaining previous observations in the empirical literature.

First, we present a rational explanation for the waves in the market for private equity based on information revelation mechanisms. Empirical research shows that timing of investment is an important source of value (Ljungqvist and Richardson, 2003a,b, and Schmidt, Nowak and Knigge, 2004[3]). In boom periods, existing funds invest their capital rapidly and are also able to advance their exit, thereby improving their internal rate of return (Ljungqvist and Richardson, 2003b). In our model, agents with private information with a minor impact on value invest later in the cycle. Due to the private nature of private equity investments, investors are only able to infer the information held by other agents by observing their investment behavior. Cycles in private equity investments are caused by information revelation mechanisms, through which agents receive good news too late and rush to the market. While the cyclical nature of private equity results from information revelation, it is the accumulation of negative externalities that ends these waves, not the revelation of negative

[2] Exogenous shocks may cause fluctuations in private equity investments and fundraising. Most fluctuations in private equity fundraising are attributed to changes in the demand for private equity (Gompers and Lerner, 1998, Poterba, 1989).

[3] Schmidt et al. (2004) find that investment timing has an impact on venture capital funds' performance, but that the experience of the individual fund management creates value in buyout funds. Investment manager find it easier to value a mature business, whereas prices for startups are closely related to the climate of the stock exchange.

information. The irreversible or illiquid nature of private equity relative to public may introduce an additional option premium from optimal timing of private investment (Dixit and Pindyck, 1994). Whether this option premium exists depends on the irreversibility of private vs. public investments.[4]

Second, the dynamic model provides an explanation of how the return on private equity can exceed the return on public sources with similar risk characteristics. The excess rent advances the investment volume. The cyclical nature of venture capital investment and fundraising gives a positive signal on future investment returns and reinforces the cyclical nature of private equity investment and fundraising (Balboa and Martí, 2003, Jeng and Wells, 2000). Gompers and Lerner (2000) show that in overheated environments, too much money is chasing too few deals and investors earn poor returns. In our private equity model an excess rent can build up when in a sequence positive information is revealed on the value of new investments and reduces in an information cascade when investments are made without disclosing new information. In such a cascade the uncertainty remains the same, because no additional information is revealed.

Third, the model provides an explanation for the differences between established agents who persistently create value and agents with lesser capabilities.[5] Returns improve with partnership experience and follow-on funds appear to show improved performance (Kaplan and Schoar, 2005). Funds with lesser reputation established in boom periods realize lower internal rates of return (Kaplan and Schoar, 2005). In comparison with new fund managers established fund managers' performance is not negatively affected by these cycles. Our model provides a rational explanation for these observations. Inexperienced agents have an information disadvantage. In the Bayesian–Nash equilibrium better-informed agents invest before the excess rent builds up, while agents with lower-quality information invest later, when the cascade may have already started. The probability of cascades is further influenced by the reliability of signals.

III. AN INFORMATION MODEL FOR THE PRIVATE EQUITY MARKET

This section describes a dynamic model for the private equity market when the evolution of the economy is uncertain and agents have private information on the net benefits of private equity relative to public sources. First the basic model for the private equity market is discussed, followed by a description of the Bayesian–Nash equilibrium for two agents. The section proceeds with a discussion of the equilibrium of the model given an infinite number of private equity agents. The section closes with a statement of the results for investment behavior where rational waves in the private equity market may arise and end endogenously.

III A. ASSUMPTIONS OF A MODEL FOR THE PRIVATE EQUITY MARKET

The balance in the private equity market depends on the demand of firms or entrepreneurs for private equity, and the supply of capital that has access to other uses. The demand curve for a certain category of private equity is determined by the pool of available firms that seek private financing, each with different resources to deliver a certain expected return. The demand curve ranks the firm according to its

[4] With the development of secondary markets, the option premium from optimal timing is eroded.

[5] In the empirical literature, Kaplan and Schoar (2005) find average fund returns net of fees to be approximately equal to the return on the S&P 500, but a large degree of heterogeneity between funds exists.

profitability and for each additional firm the expected return of this marginal opportunity declines. In a competitive environment entrepreneurs are able to finance their firm at a cost equal to the required market return and to capture the excess value or surplus of the firm by retaining part of the equity capital. The marginal return declines.

The most competitive providers of capital expect to earn a return at least equal to the return in the global capital market with the same risk characteristics and possibly incorporates a liquidity premium. The supply curve of private equity (without segmentation and regulations) is likely to be very elastic, in the extreme horizontal, and its level depends on the balance in the global capital market.

In equilibrium the marginal return equals the required return. The static equilibrium implicitly assumes that the all agents have complete information and that there is a static balance between public and private equity.

However, we extend this static approach by taking into account the distinguishing nature of demand in the private equity market, that is: i) uncertainty in the economy and ii) private information on the net benefits of private equity investment. In the dynamic model the value of private equity opportunities varies over time and is uncertain due to overall changes in the economy, *X(t)*. The revelation of private information on the benefit of private investment gives the investment process an additional strategic dimension that affects the behavior of agents, the timing of private investments, and the size of the excess rent. Private equity investors that are aware that they are not yet fully informed (e.g., for specific sectors or emerging markets) will find it optimal to make a tradeoff between the benefits of immediate investment and the benefits of waiting and learning from the investment behavior of others.

The demand curve at a certain time *t* for a certain category of private equity is determined by the pool of available firms that seek private financing. The demand curve when written in terms of value instead of returns is given by.

$$V(t) = \theta X(t) - \kappa \sum_{\tau=0}^{t} I(\tau) \tag{1}$$

where $\sum_{\tau=0}^{t} I(\tau)$ denotes the total amount of capital invested in private equity until time *t*. The value of later investment opportunities decreases and the downward slope of the value demand curve is given by κ. The parameter θ denotes the net benefit of private equity over public equity. The stochastic demand depends on the evolution of the economy, *X(t),* and follows a geometric Brownian motion. We assume that this part of the value of private equity investment is publicly observable. For instance, for an application of the model in venture capital market *X(t)* can be represented by a portfolio of small NASDAQ stocks, while an application in the buyout market *X(t)* can be represented by a leveraged position in the S&P 500.

$$dX = \alpha X dt + \sigma X dz \tag{2}$$

where α is the instantaneous conditional expected percentage change in the (private equity) economy, *X(t)*, σ is the instantaneous conditional standard deviation, and *dz* is the increment of a standard Wiener process.

We assume a declining demand function in the market for private equity, because investment opportunities differ in their ability to deliver a return on capital ($\kappa > 0$ in Eq. 1). Later investors will experience accumulated negative externalities equal to

$$\kappa \sum_{\tau=0}^{t} I(\tau) = kN(t) \tag{3}$$

where k equals the net negative externality per (standard) investment I and $N(t)$ is the number of players that have invested up till time t. Therefore, the agent who invests first receives a payoff equal to $\theta X(t) - I$; and the i-th agent's payoff equals $\theta X(t) - I - (i-1)k$.

A distinguishing feature of our model relates to the revelation of private information in the market for private equity. Although the net benefit of private equity investment, θ, is not unambiguously observed by all private equity investors, agents have private information about the profitability of their investments. The sum of all signals represents the "net benefit" of private equity investment, or

$$\theta = \mu + S_1 + S_2 + ... \tag{4}$$

where μ is the expected value of θ and the signals S_i are independent, mean-zero random variables. We assume that agents differ in the quality of their private information and have different private information on the benefit of private equity relative to investments on financial markets. Consequently, the signals of agents will differ in size and hence in their impact on the overall value of the quality parameter, θ. Established agents are likely to have more private information than agents with lesser reputation. Also, investors who are experienced and specialized in buyouts in certain sectors or geographical locations have more private information than other agents from their own information-gathering process, unique skills, and experience. Consequently, the conditional uncertainty on the value of the net benefit of private equity differs per agent, measured by the accuracy of the expectation of the quality parameter θ. Thus, we assume that $var(S_1) > var(S_2) > ...$ and that the signals have a distribution of two points:

$$S_i = \begin{cases} -\varepsilon_i & with\ probability\ \frac{1}{2} \\ +\varepsilon_i & with\ probability\ \frac{1}{2} \end{cases}, \text{ for i = 1, 2,...} \tag{5}$$

To ensure that the first or previous player has a more precise estimate of θ than the next, it must hold that $\varepsilon_1 > \varepsilon_2 > ....$

The supply curve of our model consists of private equity funds willing to participate in a particular type of private equity transaction. We assume that each agent invests an identical amount I. Agents can time their investment, and thereby the supply of capital, as uncertainty on the benefits of private equity and the economy resolves over time. These transactions should at least match the return of an alternative use with a similar risk in financial markets (i.e., the net present value must be at least zero).

An agent invests when the benefits from immediate investment are positive. At the Walrasian trigger, the state of the economy $X(t)$ has attained a level such that the net present value of the investment equals zero. We first assume that private equity transactions are as reversible as public shares. One argument is that there are secondary private equity markets where firms can be traded to other financial players. Also large blocks of shares on financial markets face price pressure. In Section IV A we leave this assumption and consider the real option effect of irreversibility on the timing of investment.

III B. BAYESIAN–NASH EQUILIBRIUM FOR TWO AGENTS

We denote as K_i the sum of negative externalities of players that have already invested, while θ_i is the public expectation of the net contribution of private equity after agent *i* has revealed his signal. These parameters are recursively defined by $K_i = K_{i-1} + k$ when agent *i* invests and by $\theta_i = \theta_{i-1} + S_i$ when agent *i* reveals its signal. For Agent 1, we set $K_0 = 0$ and $\theta_0 = \mu$.

Under complete information, the value of θ is public information, and so we assume that private equity investments are made when the investment meets the required return. Hence, agent *i* invests as soon as $X(t)$ has reached the Walrasian trigger, $X^*(\theta, K_i)$.

$$X^*(\theta, K_i) = \frac{I + K_i}{\theta} \tag{6}$$

For the first agent, the present value of the private equity investment equals the investment outlay *I.* At the next and higher trigger—due to the presence of negative externalities—the remaining agent will invest. (This process will be reiterated for multiple agents.)

When agents have different opinions about the value of θ, because of their private information, the level of the Walrasian trigger will be idiosyncratic. In general, an agent prefers to invest at a lower level of $X(t)$, when he has positive information. When an agent invests when the lower trigger is reached he reveals his positive information content, and when he waits, he reveals his negative information content. Since the Walrasian trigger is idiosyncratic, a specific agent's action of waiting is observed by its rivals.

Consider first the case of a market with only two agents and two investment opportunities. The better-informed agent, Agent 1, will always reveal his private information first, because his Walrasian trigger, evaluated by its own positive information, is reached first: $X^*(\mu + \varepsilon_1) < X^*(\mu + \varepsilon_2)$.

Hence, when Agent 1 has positive information, he invests when the state of the economy, $X(t)$, reaches the Walrasian trigger $X^*(\mu + \varepsilon_1)$. If he invests at this trigger, he signals that his private information is positive. When Agent 1 has negative information, he waits, signaling his negative information content. Agent 2 has now become the best-informed agent. Agent 1's private information has become public information, whereas Agent 2 still owns his private information.

If Agent 1 has signaled his positive information, Agent 2's action still depends on his own private information. If Agent 2 has positive information, he may invest immediately thereafter, when the impact of Agent 1's revealed signal on value exceeds the negative externalities, i.e., when $\frac{(\mu + \varepsilon_1 + \varepsilon_2)}{(\mu + \varepsilon_1)} \geq \frac{I + k}{I}$. When the negative externalities have a larger impact, Agent 2 waits until his Walrasian trigger has been reached, that is $X^*(\mu+\varepsilon_1+\varepsilon_2,k)=\dfrac{I+k}{(\mu+\varepsilon_1+\varepsilon_2)}$. If Agent 2 has negative information, he always waits until trigger $X^*(\mu+\varepsilon_1-\varepsilon_2,k)$ has been reached.

When Agent 1 has revealed his negative information, Agent 2 will reveal his information at trigger $X^*(\mu-\varepsilon_1+\varepsilon_2)$. When he has signaled its positive information, Agent 1 will invest completely informed at $X^*(\mu-\varepsilon_1+\varepsilon_2,k)$ and incurs only the negative externalities. When Agent 2 has signaled his negative information, one of the agents will invest at trigger $X^*(\mu-\varepsilon_1-\varepsilon_2)$ and the other at $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$.[6]

Compared to the case with complete information, when Agent 1 has positive information, he invests either too early or too late—when Agent 2 has negative or positive information respectively. Agent 1 with negative information invests accurately, when he will become informed about Agent 2's information. Agent 2 invests accurately, except when both agents have positive information and negative externalities are small. Agent 2 then creates value.

III C. Equilibrium for an Infinite Number of Private Equity Agents

Investment behavior is similar for a market structure with more agents and targets. The best-informed agent *i*—with the largest signal size—is the first agent to reveal his signal, because he is the only agent who does not prefer to "wait and see" other agents revealing their signals. After revealing his signal, agent *i+1* becomes the best-informed agent while agent *i* becomes the least-informed agent, similar to the 2-agents case.

A key difference between the 2-agents model and a model with an infinite number of agents relates to the impact of the accumulation of negative externalities and of previously revealed positive signals on the excess rent. When the excess rent has reached a certain size, the investment by a certain agent i^c does not reveal additional information, because he would also have invested when he had negative information. Agent i^c commences an information cascade in a private equity market when the current state of the economy, *X(t)*, exceeds his Walrasian trigger for a negative signal:

$$\frac{\left(I+K_{i^c-1}\right)}{\theta_{i^c-1}-\varepsilon_{i^c}}<X(t) \tag{7}$$

When an information cascade commences, agents rush to the market. Later agents cannot identify whether agent i^c had positive or negative information, because it is attractive for agent i^c to invest immediately in both cases. In such an information cascade these agents can only interpret the investment's signal on the basis of its expected value of zero. Also, later agents may be in a position to invest immediately irrespective of having positive or negative information, so their signals are not revealed to later agents. Agents jump on the bandwagon in a rational cascade in which they invest regardless of the content of their private information. In the resulting private equity cascade, total investments made at the same level of the state of the economy, *X(t)*, increase with each agent. No new private information is revealed.

However, each additional investment limits the pool of attractive remaining investment opportunities and a certain agent i^p will prefer to rely once again on its

[6] See Appendix A for the proof of the Bayesian-Nash equilibrium for two agents.

private information. This pivotal agent may therefore end or prolong a cascade. When his Walrasian trigger declines below the current state of the economy *X(t)*, he ends the private equity cascade by not investing. Alternatively, when this agent decides to invest immediately, he reveals a positive signal and a new cycle in the faltering cascade starts. A private equity wave can therefore consist of multiple cycles. Agent i^p is a pivotal player when he is the first agent whose Walrasian trigger for a negative signal exceeds the state of the economy at which the wave unfolds:

$$\frac{\left(I + K_{i^c-1} + k(i^{p-1} - i^c\right)}{\theta_{i^c-1} - \varepsilon_{i^p}} > X(t) \tag{8}$$

The pivotal agent will only prolong the faltering private equity cascade when he has positive private information about the benefits. Waiting reveals negative information and ends the cascade.

Proposition 1. Let $X(0) < I/(\mu + \varepsilon_1)$ *in order to prevent immediate investment by the best-informed player or revelation of his negative signal. The strategies for agent i = 1… that constitute the Bayesian–Nash equilibrium of a private equity wave are given by*

(i) *If* $i^c \le i < i^p$*, then invest immediately at the current trigger X(t)*

(ii) *If* $i^p < i < i^c$ *and* $S_i > 0$ *then invest at Walrasian trigger* $\left[\frac{I + K_{i-1}}{\theta_{i-1} + \varepsilon_i}\right]$

(iii) *If* $i^p \le i < i^c$ *and* $S_i < 0$ *then abandon investment*

(iv) *If* $i = i^p$*, and* $S_i > 0$*, then invest either immediately when* $\left[\frac{I + K_{i-1}}{\theta_{i^c-1} + \varepsilon_{i^p}}\right] < \left[\frac{I + K_{i^c}}{\theta_{i^c-1}}\right]$*, else invest at* $\left[\frac{I + K_{i-1}}{\theta_{i^c-1} + \varepsilon_{i^p}}\right]$.

Proof: Appendix B

III D. Numerical Example of Private Equity Model

Figure 1 shows a numerical example of the model. The parameter values and assumptions are included in the legend. Suppose a traded index *X(t)* is representative for part of the value of ventures and follows a geometric Brownian motion as shown in Panel A. This index represents the publicly available information on the ventures' value.

Agents are heterogeneous in the quality of their private information. The first agent to reveal its information is the best-informed one. When he invests, the public expectation of the net benefit of private equity is increased from 0.90 to 1.15. This information is revealed to other agents when the index reaches the first-time level 87 = 100/1.15 at time 1.3. Agent 2 has negative private information on his venture and does not follow Agent 1 immediately, thereby signaling his information content. When the index reaches level 91 at time 1.4, the third agent signals his positive information to the market with its investment.

Agents 4 and 5 also invest at time 1.4, and the sequential revelation of positive information raises the excess rent of private equity so much that Agent 6 would invest irrespective of his private information. Later agents incorporate the expected value of

Agent 6's signal to be 0 in their assessment of θ. The wave has commenced. Negative externalities accumulate with each investment until Agent 9 becomes the pivotal agent, but he prolongs the cascade with another cycle. The next pivotal agent is Agent 14, who does not invest and ends the cascade. A new wave may commence at a later time.

Panel B of Figure 1 depicts the typical pattern of the excess rent of private equity in our model. The rents build up first and reach their highest level just before the cascade starts. The excess rent increases because the sudden revelation of positive information makes private investment opportunities more attractive and agents rush their investment. Within the cascade the excess rent declines with each agent's investment to an extent equal to the size of the negative externalities.

[Insert Figure 1]

III E. RESULTS FOR INVESTMENT BEHAVIOR

Buyout markets have known waves of investment volume, as shown in Figure 2. It is plausible that the first wave in the 1980s resulted from signals that value could be created by imposing efficient production processes, reducing agency considerations, restructuring cumbersome conglomerates, and using the financing opportunities created in the development of the junk bond market. When public markets improved again after 1988, many agents revealed positive signals, which resulted in a sharp increase in the number and popularity of leveraged buyouts, until at the end of the 80s investments overwhelmed the available deals. The wave of private equity in 1990 focused on the added value of growth that relied more on the state of the economy, $X(t)$ [7]. The ability to create value through financial engineering was a distinguishing capability in the 1980s, but now these skills have become a commodity.

The Bayesian-Nash equilibrium of a private equity wave in proposition 1 is consistent with this behavior and recognizes the following equilibrium regions:

In a **balance period** ($i^p < i < i^c$) the revelation of positive information raises the expected rent from private equity over public investment, while negative externalities or deferment of investment by an agent reduce the excess rent. Information on the excess rent is revealed only when the economy is growing. Investment timing and the occurrence of such waves depend on the evolution of the economy $X(t)$ and are therefore procyclical. A private equity wave commences due to the repeated revelation of positive information, which indicates that certain agents have actually been too late with their investment.

In the **cascade** ($i^c \leq i < i^p$) there is no revelation of information. As soon as it becomes publicly known that the excess rent in private equity as compared with public sources, or $X(t)$ of private equity has reached a critical level, investments in private equity become so attractive that even negative private information cannot deter an agent from going forward. Each investment then decreases the excess rent with the size of the negative externalities, and the excess rent reaches its lowest level

[7] For instance, the public information variable $X(t)$ could be seen as a weighted portfolio of value drivers, while the value of LBOs depends on the state of the economy as reflected by the GDP and public market index. The value that derives from leverage is reflected by interest rate related variables like inflation and term- and credit-spread. The weight given to each driver is likely to be time dependent. The value of restructuring companies in the 1980s has other fundamentals of value than the wave of growth in the 1990s. The index for venture capital will have different value drivers. Private information about the additional benefits of private equity is the second source of value of private equity, represented by θ.

with the pivotal agent, when the pool of attractive deals gets exhausted. The pivotal agent can prolong the faltering private equity wave when he has positive information or may distort the cascade when he reveals a negative signal.

When the cascade ends, a **balance period** starts again ($i^p < i < i^c$) and the expected rents from private equity have been completely eroded. The impact of private information on the expected rents has been reduced, because the better-informed agents have already revealed their signals and the adverse impact of negative externalities on value gets more important. When the state of the economy declines as well, the market for private equity dries up.

Uncertainty on the net benefits of private equity in our model depends on the size of all signals that have not yet been revealed. When the uncertainty is high—as for buyouts in the early 1980s—private information plays an important role and waves are likely to arise more often. The occurrence of waves also depends on heterogeneity of agents and the reliability of signals. In our model, agents with less experience will invest at later stages of the game. Agents that have received negative information are only able to invest in private equity when an investment cascade unfolds. For those agents, the level of the excess rent declines, as no information has been revealed.

[Insert Figure 2]

IV. Extensions: Optimal Timing and Heterogeneous Capabilities

This model can be extended further in several directions. An important one is the situation in which agents time their investment optimally when private equity investment is considered less reversible than public sources. Other more realistic extensions include the heterogeneous capabilities of agents, and uncertainty in the reliability of signals.

IV A. Optimal Timing of Irreversible Investment and Intensity of Competition.

We assumed that private equity investments are not more costly to reverse than investment in publicly traded securities.[8] The development of a secondary market for private equity and widely available capital at private equity funds makes it easy to trade firms. By contrast, in the 1980s the leveraged buyout market was less transparent, so less-competitive agents had preferential access to deals and were consequently better able to time their investments optimally.

When private equity is considered "costly to reverse" investment requires an option premium to compensate for the loss in flexibility not to invest if market conditions turn out to be unfavorable (McDonald and Siegel, 1986, Dixit and Pindyck, 1994). When agents have the ability to perfectly time their investments well, the value of the future investment then equals

$$W(t) = \max_{\tau \geq t} e^{-r\tau} \left[\theta X(\tau) - I, 0\right] \tag{9}$$

Where τ is the optimal stopping time. Appendix C presents the value of the option *W(t)* under full information on parameter *θ*.

[8] One could argue that illiquidity also occurs when trading large blocks of firms on financial markets.

The Bayesian–Nash equilibrium[9] for two and more agents when they can optimally time their irreversible investment is similar to the assumption of reversible investment.[10] An important difference is the higher level of the investment triggers, which are now defined as

$$\overline{X}(\theta) = X^{*}(\theta)\frac{\gamma}{\gamma - 1} \tag{10}$$

where γ is the positive root of the fundamental quadratic equation (see appendix C). The difference between the investment triggers in Eq. 6 and in Eq. 10 is the factor $\frac{\gamma}{\gamma-1}$ representing the flexibility value of the option to "wait and see." With irreversible investments, a wave takes off with agent i^c

$$\frac{\left(I + K_{i^c-1}\right)}{\theta_{i^c-1} - \varepsilon_{i^c}}\frac{\gamma}{\gamma - 1} < X(t) \tag{11}$$

In contrast to the case without negative externalities (Grenadier, 1999), the negative externalities hinder the formation of a cascade, and more than two consecutive positive signals may be needed to start an information cascade. The excess rent is raised by each sequential investment that signals positive information. The formula is analogous to Eq. 8 except for the option value factor—the pivotal player is defined by the agent i^p for whom the following condition holds:

$$\frac{\left(I + K_{i^c-1} + K(i^{p-1} - i^c\right)}{\theta_{i^c-1} - \varepsilon_{i^p}}\frac{\gamma}{\gamma - 1} < X(t) \tag{12}$$

IV B. CAPABILITIES AND REPUTATION OF AGENTS

Agents with better information capabilities are characterized by higher signal sizes ε_i. The presence of many capable agents can either advance or hinder the start of a private equity wave. It is the trade-off between the impact of previous agents and that of the current agent that determines the commencement of a cascade.

The presence of many capable agents (ε_i is high) favors the start of an investment wave, since a sequence of positive information creates an excess rent while the impact of negative externalities is relatively small and the excess rent easily overwhelms the private information of any single agent. Also, when agents differ to a greater extent in their capabilities a lower excess rent is sufficient to overwhelm private information.

We define heterogeneity in our context, Ψ_i, as the difference in subsequent agents' impact on value due to their private information. Heterogeneity at a given point in time is a result of differences in information content and size. When a series of signal sizes remain large, there is more heterogeneity because agents differ in their private information content. If signal sizes of subsequent agents decrease fast, there is

[9] Waves arise from information revelation, not pre-emption, i.e. assuming that k = 0 as in Grenadier (1999).

[10] This is an extension of the general model of Grenadier (1999) with negative externalities.

heterogeneity between agents, because agents differ in their impact on value. We measure heterogeneity, Ψ_i, by comparing the capabilities of an individual agent with those of previous agents. The weight given to early revealed information is reduced by a factor of ½, because the expected impact on the excess rent declines. The heterogeneity measure for the i^{th} agent equals

$$\Psi_i = \sum_{j=1}^{i-1} \varepsilon_j \tfrac{1}{2}^{i-1-j} - \varepsilon_i \tag{13}$$

Figure 3 depicts a scatterplot for the base case of the Monte Carlo simulation of trajectories of *X(t)*, of the signal sizes and of their signs, in which we show the size of a cascade measured by the total number of agents in a wave (y-axis). The x-axis shows the degree of heterogeneity. Agents with a high degree of heterogeneity are presented on the right side and homogenous agents on the left. The two lines give the average number of agents in a wave for a heterogeneity level for high negative externalities (base case in scatterplot) and low negative externalities (no scatterplot depicted). We observe that for a higher degree of heterogeneity, more agents are able to participate in a wave. When agents are heterogeneous, the excess rent can build up to a higher level. The cascade will contain more agents, because more negative externalities can accumulate before the excess rent is eroded. Furthermore, the excess rent can decrease to a lower level before private information again matters in the investment decision, because the impact of later agents' private information decreases under heterogeneity. The curve for low negative externalities lies above the one for the base case.

[Insert Figure 3]

Figure 4 depicts the probability of a wave's occurring and the average ranking of agents that start the wave in the Monte Carlo simulation (of the base case). The rising curve shows the probability that a wave will occur. The declining curve in Figure 4 shows the average rank of the agent that starts a wave for different heterogeneity measures. When agents are homogenous, it requires many subsequent agents to start a cascade, and the probability of a wave is low. When agents are heterogeneous, the excess rent builds up fast and a single agent's private information is less likely to deter a wave from starting. Therefore, the probability of a wave's starting increases with the degree of heterogeneity. For a degree of heterogeneity above 0.07, on average the fifth agent starts the private equity wave.

[Insert Figure 4]

IV C. Reliability of Signals and Capability of Agents

Besides differences in the impact of agents' private information on value, agents may differ in their ability to interpret their information and may mistake their negative private information for positive.[11] Agents with good track records provide a reliable signal and act according to the information they actually receive. Incapable

[11] Questioning the reliability of signals by agents is legitimate. The establishment of private equity funds has created a new agency relationship between general partners and limited partners. Also, without agency problems it is plausible that inexperienced general partners will be more inaccurate or overly optimistic in estimating the value of an opportunity.

(or inexperienced) agents may give unreliable signals and act as if they have positive private information.

We introduce in our model two types of agents, capable and incapable, where the probability that an agent is capable is denoted by *f*. When all agents are unaware of the presence of unreliable signals they assume that *f* equals 1. Then, incapable agents raise the excess rent falsely. When all agents are aware that some agents may reveal unreliable signals—but cannot identify them—all agents will give a lower weight to all revealed positive signals in their update of the net benefit of the private equity parameter, θ_{i-1}. As a consequence $\theta_i = \theta_{i-1} + \frac{f}{2-f}\varepsilon_i$ if agent i invests, and $\theta_i = \theta_{i-1} - \varepsilon_i$ if agent i does not invest. In an investment cascade, agents cannot identify the signals of rivals and they expect a value of $\theta_i = \theta_{i-1}$ as before.

Figure 5 exhibits the probability that a wave will start for various heterogeneity levels between agents for $f = 0.8$ and 1. Whether the presence of incapable agents increases or reduces the probability of a wave depends on whether they are aware of the unreliability of signals. Obviously, if agents are not aware that other agents may exhibit unreliable signals, waves occur sooner and more often. The correction for revealed positive information is not made, as agents interpret *f* to be equal to 1. Because the (expected) excess rent is higher when a wave commences, more agents participate.

The agents' awareness of the unreliability of some signals substantially reduces both the probability of a cascade and its length. The excess rent rises more slowly in the run-up period, because investment signals only partially reliable information and agents will be more cautious in their investment decisions, as the revelation of negative information has a larger impact. Therefore, a longer sequence of agents revealing positive information by investment actions is required to raise the expected excess rent enough to start a wave. Unreliable pivotal agents raise the rent incorrectly and prolong the wave. The proportion of incapable agents investing in a cascade is higher than in the balance stage.

In the limiting case all agents are unreliable. When all agents are aware of this fact, investment does not reveal any information and all signals is ignored. Then no waves can occur and all agents invest at their Walrasian trigger with positive information, $(I + K_{i-1})/(\mu + \varepsilon_i)$. When agents are not aware of unreliability of signals, in the limiting case a sequence of waves arise, until the impact on value of negative externalities exceeds the impact of positive information.

[Insert Figure 5]

VI. CONCLUDING REMARKS

Private equity investments tend to evolve in waves and have at various times created or destroyed a tremendous amount of wealth. The buyout wave of the 1980's was facilitated by the junk bond market and the restructuring of cumbersome conglomerates, but ended suddenly in 1989 when companies experienced financial distress. In 1992, a second period of high growth for buyouts took off. As with venture capital, the extremely favorable economic conditions and stock market climate during the late 1990s and the exit route via initial public offerings resulted in a wave. Although private equity volumes obviously depend on the expected net benefits, at times they tend to overshoot the evolution of the economy.

Many have attributed the pronounced excessive growth stages of these waves in buyouts and venture capital to irrational over-investment that ignores the evolution of the economy and the decreasing pool of high quality deals. An information cascade is said to occur when agents ignore their own private information and instead emulate the behavior of previous actors. An informational cascade is not simply a simultaneous investment, by which merger waves of strategic players are explained as a "dance of chairs." The distinctive characteristic is that the investors will ignore their private information about the future profitability of private equity investments and mimic the action of the competitors, which is a rational choice.

In our model for the private equity market the expected benefit of private equity financing is, in equilibrium at the margin, the same as the return for comparable public equity sources. However, in contrast to the trading of listed firms on efficient financial markets, in which information on the value of investments is instantly available to all agents, private companies tend to be less transparent, and the benefits of private equity cannot be instantly observed by all agents. The assessment of the investment quality, θ, depends on the private signals that other agents who operate in other, possibly closely related, sectors and areas reveal by their investment behavior. In our model a dynamic Bayesian equilibrium incorporates this distinguishing feature of imperfect information in private equity vs. public investment. The incremental contribution of such a dynamic model for the private equity market depends on the insights the results give us for investment behavior:

Empirical research in the literature confirms that investment flows into the industry tend to increase after periods of good private equity performance and to decrease after poor performance. Information economics and herding behavior provide a plausible explanation why growth in private equity volume has not been uniform. The model shows that waves may arise endogenously when investment behavior reveals private information, and investors jump on the bandwagon; this continues until negative externalities from over-investment end these waves. Cycles in private equity investments are caused by information revelation mechanisms, by which agents receive good news very late and rush to the market. These waves arise procyclically, as more information is revealed when the economy booms. The accumulation of negative externalities due to declining demand, rather than the revelation of negative information, ends these waves. In particular, in an investment cascade the number of investors increases rapidly, leading to the end of a wave.

The dynamic model provides an explanation for how the return on private equity can exceed the return on public funding sources with the same characteristics. The model presented here builds on information revelation to make predictions about the evolution of the excess rent and investment timing of private equity. We find that in a balance period the better-informed agents build up excess rent until lesser-informed agents' mimicking investments start a cascade. The excess rent then declines to its lowest level at the end of the wave.

Consistent with the empirical literature, in our model the interpretation of these market phenomena is based on heterogeneity of agents. Private information about the value creation is likely to flow from the best-informed investor to the least-informed investor. Established agents have a larger information advantage that enables them to invest first, before the excess rent has built up. Less-informed agents invest later and are thus more likely to enter at the end of the wave, when the level of the excess rent is associated with higher uncertainty. The model provides a rational explanation based on information economics for the differences between agents who persistently create value and agents who fail to deliver. When agents are inexperienced (e.g. unable to correctly assess the value of an investment or invest from agency considerations), unreliable signals may affect the occurrence and size of waves.

We believe that incorporating uncertainty, private information, and herding behavior in a dynamic model for private equity can help provide an explanation of the empirical observations in the literature.

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APPENDIX A. PROOF OF PROPOSITION A.1

Proposition A.1

Agent 1 invests at Walrasian trigger $X^*(\mu+\varepsilon_1)$ *when he has a positive signal. When he has a negative signal, he waits until trigger* $X^*(\mu-\varepsilon_1+\varepsilon_2)$ *to observe the action of Agent 2. If Agent 2 reveals positive information by investment, Agent 1 invests at* $X^*(\mu-\varepsilon_1+\varepsilon_2,k)$. *If Agent 2 waits, Agent 1 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2)$ *and Agent 2 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$, *or Agent 2 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2)$ *and Agent 1 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$.

Agent 2 waits until trigger $X^*(\mu+\varepsilon_1)$ *to observe whether Agent 1 either invests and reveals positive information or waits and reveals negative information. If Agent 1 reveals positive information, Agent 2 invests at* $\max\left(X^*(\mu+\varepsilon_1),X^*(\mu+\varepsilon_1+\varepsilon_2,k)\right)$ *when he has positive information, and at* $X^*(\mu+\varepsilon_1-\varepsilon_2,k)$ *when he has negative information. If Agent 1 has revealed negative information and Agent 2 has positive information, Agent 2 will invest at trigger* $X^*(\mu-\varepsilon_1+\varepsilon_2)$. *When both Agents have negative information, Agent 1 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2)$ *and Agent 2 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$, *or Agent 2 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2)$ *and Agent 1 invests at trigger* $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$.

Proof of Proposition A.1

Let us first take Agent 1's strategy as given.

Agent 2 cannot improve his strategy. Suppose that Agent 2 has a positive signal.

When Agent 1 invests at trigger $X^*(\mu+\varepsilon_1)$ he reveals a positive signal ε_1. If Agent 2 had invested before Agent 1 he could have improved his payoff. However, had he invested before Agent 1, his expectation of θ would have been $\mu + \varepsilon_2$. Contingent on this expectation it would be optimal for Agent 2 to wait for the Walrasian trigger $X^*(\mu+\varepsilon_2)$ to invest. Investment at a lower trigger would yield a negative Net Present Value. When a positive signal is revealed, Agent 2 can do no better than investing at or above his Walrasian trigger $\max\left(X^*(\mu+\varepsilon_1),X^*(\mu+\varepsilon_1+\varepsilon_2,k)\right)$. Investment at a lower state of the economy, *X(t)* either is not possible or yields a negative NPV.

When Agent 1 does not invest at trigger $X^*(\mu+\varepsilon_1)$, his negative signal $-\varepsilon_1$ is established. Agent 2 is now fully informed and invests at Walrasian trigger $X^*(\mu-\varepsilon_1+\varepsilon_2)$. Investing earlier would yield a negative NPV.

Suppose that Agent 2 has received a negative signal.

While it is not optimal for Agent 2 with positive information to invest before Agent 1 has revealed its information, it is certainly so when Agent 2 has negative information. The same arguments hold. If Agent 1 has revealed his positive signal at $X^*(\mu+\varepsilon_1)$, Agent 2 is then fully informed and will invest at trigger $X^*(\mu+\varepsilon_1-\varepsilon_2,k)$. Investing earlier would yield a negative NPV. When Agent

1 has revealed his negative signal, Agent 2 reveals his negative signal at $X^*(\mu-\varepsilon_1+\varepsilon_2)$ by abstaining from investment. Both agents are completely informed. Agent 2 invests with probability ½ at trigger $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$ and with probability ½ at trigger $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$, whereas Agent 1 invests at the other Walrasian trigger. Investing earlier would yield a negative NPV.

Let us now take Agent 2's strategy as given.

Agent 1 cannot improve his strategy. Suppose that Agent 1 has a positive signal.
Agent 1 does not prefer to invest before his trigger $X^*(\mu+\varepsilon_1)$. His expectation of θ equals μ + ε_1. Investing earlier would yield a negative NPV. If he waits, Agent 2 assumes negative information revelation, reveals his own information content at trigger $X^*(\mu-\varepsilon_1+\varepsilon_2)$ and Agent 1 will then be completely informed. If Agent 2 reveals positive information, Agent 1 would have preferred to invest at the earlier Walrasian trigger $X^*(\mu+\varepsilon_1+\varepsilon_2)$. However, Agent 2 could also reveal negative information, in which case Agent 1 will also be late with its investment, as the Walrasian trigger $X^*(\mu+\varepsilon_1-\varepsilon_2)$ has been exceeded.

Suppose that Agent 1 has a negative signal.
When Agent 1 does not know the signal of Agent 2, it is not optimal for Agent 1 to invest before $X^*(\mu-\varepsilon_1+\varepsilon_2)$. Based on his expected value of $\theta=\mu-\varepsilon_1$, investment would yield a negative NPV. When Agent 2 reveals positive information, Agent 1 is fully informed and can do no better than investing at trigger $X^*(\mu-\varepsilon_1+\varepsilon_2,k)$. When Agent 2 reveals his negative information, both agents are fully informed. Agent 1 invests with probability ½ at trigger $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$ and with probability ½ at trigger $X^*(\mu-\varepsilon_1-\varepsilon_2,k)$, whereas Agent 2 invests at the other trigger. Investing earlier would yield a negative NPV.

APPENDIX B. PROOF OF PROPOSITION 1

Proposition B.1

Let $X(0) < I/(\mu + \varepsilon_1)$ *in order to prevent immediate invest by the best-informed player or revelation of his negative signal. The strategies for agent i = 1… that constitute the Bayesian–Nash equilibrium are given by*

(i) *If* $i^c \le i < i^p$*, then invest immediately at the current trigger X(t)*

(ii) *If* $i^p < i < i^c$ *and* $S_i > 0$ *then invest at Walrasian trigger* $\left[\dfrac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_i}\right]$ or at *X(t)*, when *X(t)* > $\left[\dfrac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_i}\right]$.

(iii) *If* $i^p \le i < i^c$ *and* $S_i < 0$ *then abandon investment*

(iv) *If* $i = i^p$*, and* $S_i > 0$*, then invest either immediately when* $\left[\dfrac{I+K_{i-1}}{\theta_{i^c-1}+\varepsilon_{i^p}}\right] < \left[\dfrac{I+K_{i^c}}{\theta_{i^c-1}}\right]$*, else invest at* $\left[\dfrac{I+K_{i-1}}{\theta_{i^c-1}+\varepsilon_{i^p}}\right]$.

Proof of proposition B.1

The game is defined recursively. A best-informed agent *i, for* $i^p < i < i^c$ and second-best-informed agent *i+1* interact. The agent that reveals its negative information becomes the least-informed agent and will be out of the game. An agent that invests cannot pursue other actions.

Cases (ii), (iii) and (iv)

Let us first take the strategy of the second-best-informed agent *i+1* as given. The best-informed agent with positive information invests either at his Walrasian trigger $X^*(\theta_{i-1}+\varepsilon_1) = \dfrac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_1}$, or at the current state of the economy X(t), when $X^*(\theta_{i-1}+\varepsilon_1) < X(t)$, at which level the previous agent has revealed positive information. The best-informed agent with negative information abstains from investment and signals his negative information. Investment prior to the Walrasian trigger would result in a negative NPV. Investment before the current state of the economy, *X(t)*, is reached is not possible by definition.

The best-informed agent also does not want to postpone its investment until less-informed agents have revealed their information. When they reveal positive information, his investment at his Walrasian trigger $X^*(\theta_{i-1}+\varepsilon_1) = \dfrac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_1}$ would yield a positive NPV. When they reveal negative information at a trigger incorporating the best-informed agent's negative information, investment prior to this trigger would also been preferred, analogous to the 2-agents case.

Let us now take the strategy of agent *i* as given. The second-best-informed Agent *i+1* does not want to invest before the best-informed agent *i* has revealed its signal. The second-best-informed agent would base his decisions on θ_{i-1}. If Agent *i+1* has positive information, the impact on value is smaller than for the best-informed agent *i*. Hence, when Agent *i+1* invests before Agent *i* reveals its signal, it will cause a negative NPV decision, $\frac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_i} < \frac{I+K_{i-1}}{\theta_{i-1}+\varepsilon_{i+1}}$. When Agent *i+1* has negative information the same reasoning holds.

Case (i)

When Agent i with $i^c \le i < i^p$ has negative information, immediate investment is preferred. When he waits and does not invest during the cascade, he will receive a payoff of 0, which is less than the expected payoff of investing during the cascade. The same holds for an agent with positive information.

APPENDIX C. DERIVATION OF FULL-INFORMATION EXERCISE TRIGGER WITHOUT NEGATIVE EXTERNALITIES

Let *W(X; θ)* denote the value of each agent's option in a world where θ is a known parameter. Using standard arguments (Dixit and Pindyck, 1994), *W(X; θ)* must solve the following equilibrium differential equation:

$$\frac{1}{2}\sigma^2 X^2 W_{XX} + \alpha X W_X - rW = 0 \tag{C.1}$$

Equation (A.1) must be solved subject to appropriate boundary conditions. These boundary conditions serve to ensure that an optimal exercise strategy is chosen:

$$\begin{gathered} W[X^*(\theta),\theta] = \theta X^*(\theta) - I \\ W_X[X^*(\theta),\theta] = \theta \end{gathered} \tag{C.2}$$

The value of the perfect information option strategy can be written as

$$W(X;\theta) = \begin{cases} \left(\dfrac{I}{\gamma-1}\right)^{1-\gamma}\left(\dfrac{\theta}{\gamma}\right)^{\gamma} X^{\gamma} & \text{for X} < \text{X}^*(\theta) \\ \theta X - I & \text{for X} > \text{X}^*(\theta) \end{cases} \tag{C.3}$$

where

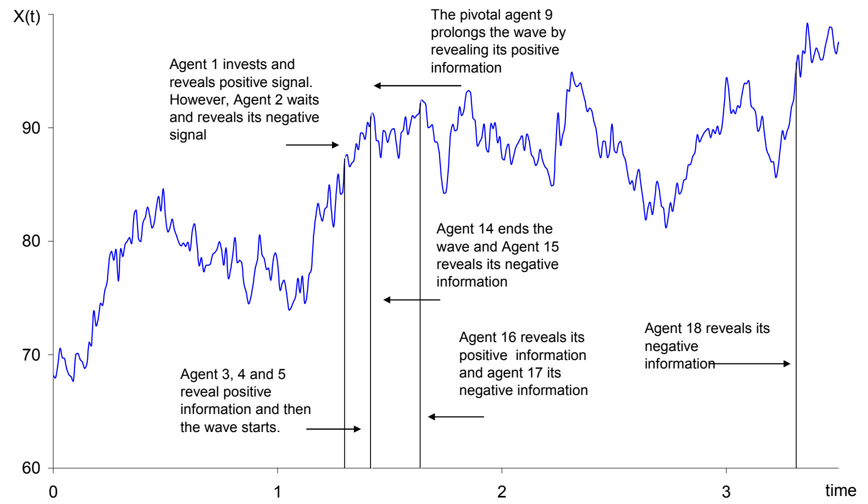
$$X^*(\theta) = \frac{\gamma}{\gamma-1}\cdot\frac{I}{\theta} \tag{C.4}$$

$$\gamma = \frac{-(\alpha-\sigma^2/2)+\sqrt{(\alpha-\sigma^2/2)^2+2r\sigma^2}}{\sigma^2} > 1 \tag{C.5}$$

and where $\alpha < r$ to ensure convergence.

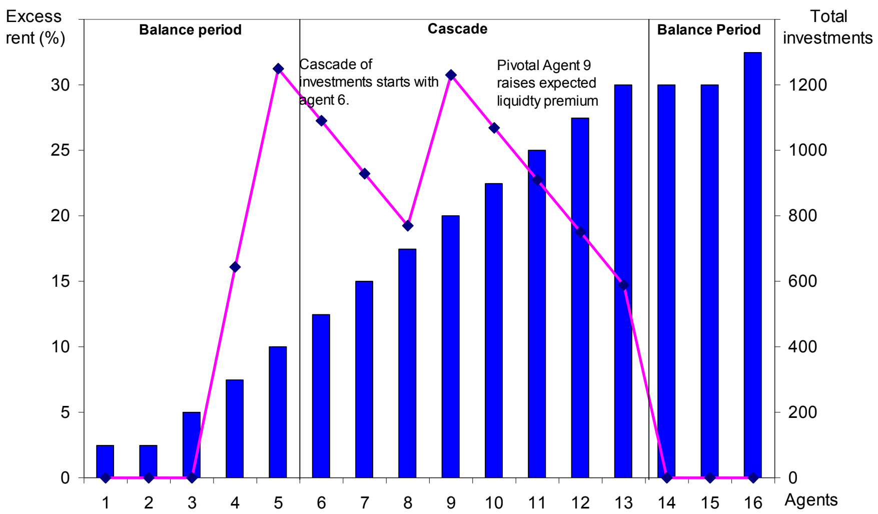
Figure 1. Numerical Example of a Private Equity Wave

Panel A. The Evolution of the Economy and Investment by Agents



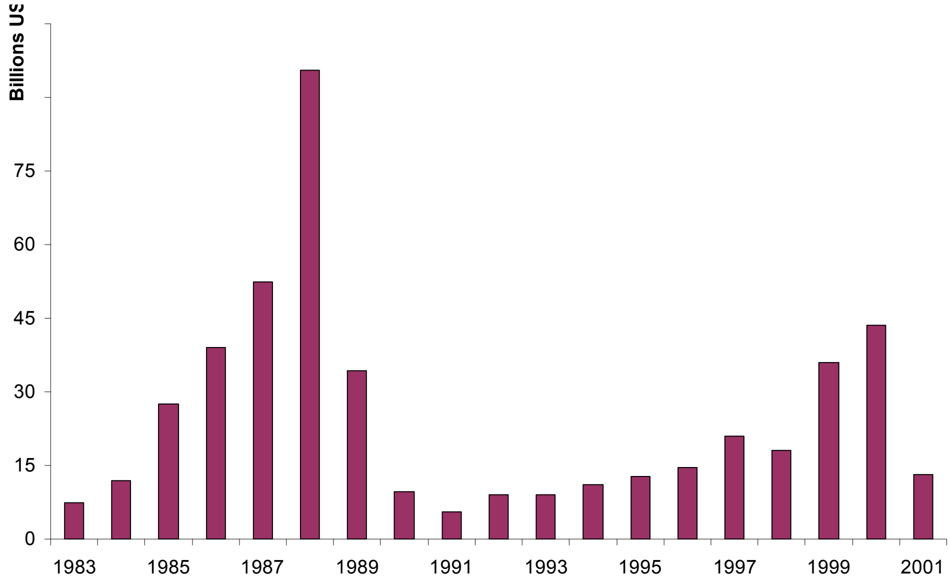
Agent	1	2	3	4	5	6	7	9	14	15	16	18
Private signal	0.25	-0.24	0.23	0.22	0.21	-0.20	0.19	0.17	-0.12	-0.11	0.10	-0.08
Pos. Walrasian trigger	87	75	91	79	71	66	68	74	80	85	92	95
Neg. Walrasian trigger	154	114	152	117	97	85	87	92	92	98	105	105
Public θ	1	1.15	0.91	1.14	1.36	1.57	1.57	1.57	1.74	1.62	1.51	1.52
Neg. ex	0	4	4	8	12	16	20	28	48	48	48	52
Current X(t)	87	87	91	91	91	91	91	91	91	91	92	95
Time	1.3	1.3	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.5	3.31

Panel B. The Evolution of Net Benefits during a Private Equity Wave



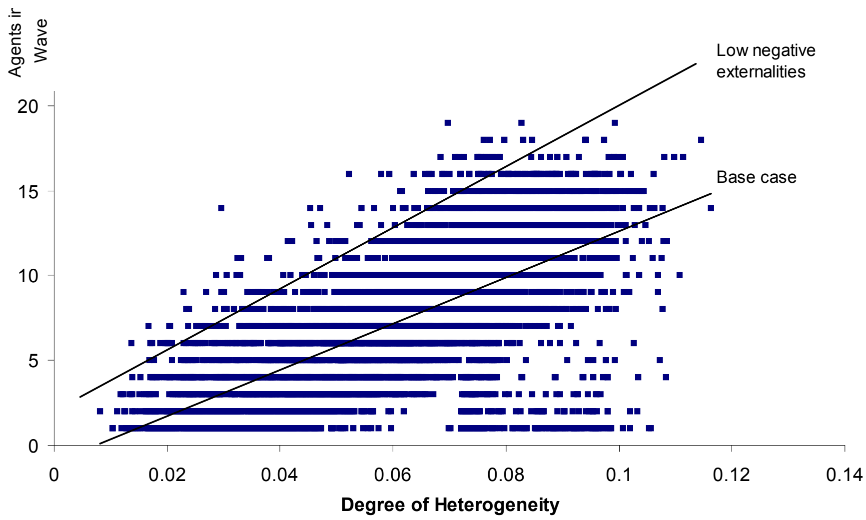
Parameter values: the index *X(t)* follows a geometric Brownian motion. Each agent can invest an amount I = 100m in ventures in this sector; demand declines with k = 4 for each transaction. The initial expected net benefit of private equity over public investment, θ, equals 90%.

Figure 2. Total Investment in Leveraged Buyouts per Year.



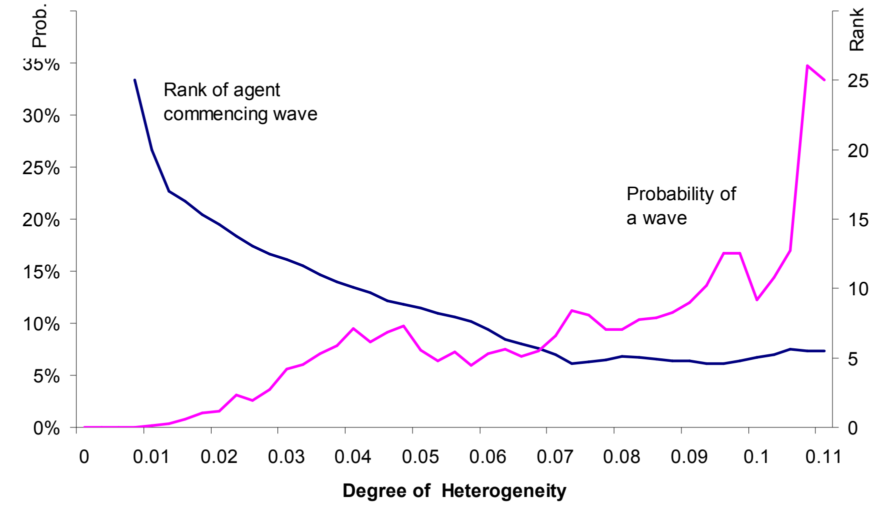
Source: VentureEconomics. Completed leveraged buyout transactions in US$ billions per announcement date. The value includes net debt of target.

Figure 3. Heterogeneity between Agents Increases the Size of the Wave (Number of Agents).



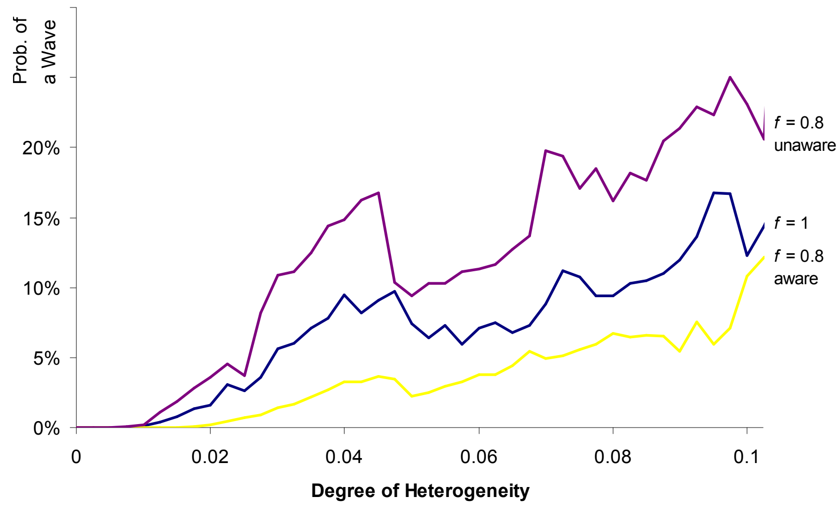
The Monte Carlo simulation takes 10.000 draws of the trajectory of $X(t)$. The initial expectation of θ or μ_0 equals 1. The first signal, ε_1, equals 0.1. Later signals ε_i are a ratio of the previous signal ε_{i-1}. This ratio is uniformly distributed between 0.7 and 1. the investment I is 100. The size of the negative externalities, k, equals 1. In the scatterplot of the base case, for each agent in the simulation that started a wave we depicted its heterogeneity measure and the total number of agents in a wave. The two curves give the average number of agents in a wave for a heterogeneity level for high (k = 1) and low negative externalities (k = ½) (no scatterplot depicted).

Figure 4. Heterogeneity of Agents Increases the Probability of Occurrence of a Wave.



The Monte Carlo simulation takes 10.000 draws of the trajectory of *X(t)*. The initial expectation of θ or μ_0 equals 1. The first signal, $\varepsilon_{I,}$ equals 0.1. Later signals ε_i are a ratio of the previous signal ε_{i-1}. This ratio is uniformly distributed between 0.7 and 1. The investment *I* is 100. The size of the negative externalities, *k*, equals 1. For each heterogeneity interval of size 0.025 we divided the total number of instances at which a wave started by the total number of instances where an agent had this heterogeneity measure to determine the probability of a wave.

Figure 5. Reliability of Signals Affects the Probability of Occurrence of a Wave



The Monte Carlo simulation takes 10.000 draws of the trajectory of *X(t)*. The initial expectation of θ or μ_0 equals 1. The first signal, ε_I equals 0.1. Later signals ε_i are a ratio of the previous signal ε_{i-1}. This ratio is uniformly distributed between 0.7 and 1. The investment *I* is 100. The size of the negative externalities, *k*, equals 1. For each heterogeneity interval of size 0.025 we divided the total number of instances at which a wave started by the total number of instances where an agent had this heterogeneity measure to determine the probability of a wave. The proportion of reliable agents is 100 and 80%. Agent are either aware or unaware of the unreliability in the market.