Forecasting educational differences in life expectancy

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Abstract
Forecasts of life expectancy (LE) have fueled debates about the sustainability and solidarity of pension and health care systems. However, within populations there are large and growing inequalities in life expectancy between high and low educated groups that are relevant for these debates. In this paper, we present an approach to forecast LE for different educational groups within a population. As a basic framework we will use the Li-Lee model which has been developed as to coherently forecast mortality for different groups. We adapted the Li-Lee model to distinguish between overall, gender specific and education specific trends in mortality and extrapolated the time-trends in a flexible manner. We will illustrate our method for the population above age 65 in the Netherlands. Several data sources spanning different time windows were used to construct time series of mortality by gender, age and education. Our extended Li-Lee model was used to forecast mortality rates and translate them into estimates of LE. Results suggest that LE is likely to increase for all educational groups for both men and women in the Netherlands but that differences in LE between educational classes widen. Several sensitivity analyses illustrate advantages of our proposed methodology.

Keywords: healthy life expectancy, Lee Carter model, time-series
Background

Life expectancy has been increasing in most western countries and is expected to increase in the future (Bongaarts, 2006; Christensen, Dobhhammer, Rau, & Vaupel, 2009; Oeppen & Vaupel, 2002; Tuljapurkar, Li, & Boe, 2000; White, 2002). The rise in life expectancy has important implications for society as larger numbers of elderly people pose additional burdens on the health care and pension systems (Bongaarts, 2004; Christensen et al., 2009). This has led to political debates in many countries regarding the statutory retirement age and how to publicly finance the growing health care expenditures. For example, some Western-European countries have explicitly linked their retirement age to the increase in life expectancy (Ageing Working Group, 2012). Moreover, the life insurance and annuity industry incorporates the prospect of a continuing rise in life expectancy in their products leading to higher premiums (De Waegenaere, Melenberg, & Stevens, 2010; Pitacco, Denuit, Haberman, & Olivieri, 2008). However, such measures ignore the great differences in the length of life with respect to socio-economic status (SES) (Mackenbach et al., 2008; Van Kippersluis, O'Donnell, Van Doorslaer, & Van Ourti, 2010). Those with fewer years of education have much shorter lives and a growing number of studies report even a widening of inequalities in life expectancy between SES groups, so that the trend of life expectancy of the entire/overall male and female population becomes less informative over time (Mackenbach et al., 2003). Consequently, forecasts for socio-economic subgroups are required to inform the political debate adequately.

Since about the 1980s a growing number of approaches for forecasting life expectancy became available (Booth & Tickle, 2008). Although there have been exceptions, most approaches are based on time-series extrapolation models such as the Lee-Carter model (Lee, 2000). Lee-Carter based methods decompose time series of age specific mortality rates into a latent time trend and an interaction thereof with different age categories. The latent time trend is then forecasted using ARIMA modeling (mostly a random walk with drift) and serves as a basis to derive future age profiles of mortality rates and corresponding life expectancy projections. Until now, Lee-Carter based methods have been used to project mortality and life expectancy in the general population (often stratified by gender) but have not been used to project life expectancy for different SES groups. Although there have been population projections that accounted for the effect of changes in the educational distribution on mortality, these projections assumed only changes in projected overall mortality rates due to compositional changes while keeping educational differences in mortality fixed (KC & Lentzner, 2010; Samir et al., 2010). However, to date there are no forecasts of life expectancy (LE) stratified by level of education.
The goal of this paper is to develop an approach to forecast LE for different educational groups within a population. As a basic framework we will use the Li-Lee model which has been developed as an extension to the original Lee-Carter to coherently forecast mortality for different groups, e.g. countries or gender (Li & Lee, 2005). The rationale behind the Li-Lee model is that trends in mortality patterns will to some extent be similar in populations which have a lot in common such as for instance the health care system and the economic environment. Therefore, it is unlikely that in the future mortality patterns will diverge strongly in related populations. To date the Li-Lee model has not been applied for other purposes than for projecting LE coherently for different genders within a country (Li & Lee, 2005) or countries within a group of countries (Janssen, van Wissen, & Kunst, 2013; Li & Lee, 2005). Our paper extents this field offering a broader class of possible applications. For this purpose, we will extend the Li-Lee model in several manners. First, we made the model more flexible for incorporating more layers of group-specific time trends, while retaining the idea that the different groups share to a certain extent common trends. Second, we demonstrate how the different layers of the model could be used to integrate data of different quality and different time length allowing to combine shorter survey-based time series on mortality disaggregated by SES with longer register-based time series on general mortality. This solves an important problem in the current literature that avoids sub-group specific forecasts because mortality data on sub-groups is often of lesser quality. We will illustrate our method for the population above age 65 in the Netherlands and forecast LE by education for the years 2013-2042. Although in the Netherlands mortality by age and gender is routinely collected at the population level as part of the national vital statistics, the data on mortality by level of education is gathered from smaller and more selective surveys (Kulhánová, Hoffmann, Eikemo, Menvielle, & Mackenbach, 2014).

Our paper is structured as follows. First, we present the different kinds of data available for estimating educational-specific mortality rates in the Netherlands. Then, we will describe our model specification and demonstrate how the education-specific mortality trends spanning over a shorter time frame could be combined with overall and gender-specific data from a longer time frame. Third, we estimate a base case forecast for gender-specific and SES-specific LE in the Netherlands using a specific set of key assumptions on the forecasted group-specific and common time trends. Finally, we demonstrate the sensitivity of our results to each of these key assumptions in four alternative scenarios.
Methods

Data

In the Netherlands there is no single data source that contains information on mortality stratified by gender, age and education. While deaths and births by age and gender have been recorded in the Netherlands since the 19th century, information on education level is still not available from vital statistics in the Netherlands. We could however create a time-series on mortality by education level for the years 1996-2012 using individual level data from the Dutch Labor Force Survey linked to the municipal population registries, since 1997 known as GBA (see Appendix A for details on how we constructed these time series). Educational attainment in our analysis was classified in three categories:

- Low: primary education (basisonderwijs);
- Middle: pre-vocational education (Vmbo, mbo 1, mavo);
- High: secondary education and tertiary education (Havo, Vwo, Mbo 2,3,4 , Hbo , Wo).

In this paper we focus on remaining life expectancy at age 65 because of two reasons. First of all, LE at age 65 has until 2012 been the official retirement age in the Netherlands (NB: also in many other Western countries official retirement ages are around 65) and most health care expenditures are centered in the elderly. Many countries have linked the pension-related income taxes to the increase in life expectancy and consider to raise retirement age in the future (Esping-Andersen, 2011). Secondly, by focusing on the 65+ we could more reliably estimate mortality trends by education given the concentration of deaths in the elderly. Table 1 shows estimates of life expectancy (LE) at age 65 calculated from the combined data. Life expectancy at age 65 has increased more for men than for women between 1996 and 2012 although LE is still higher for women. Educational differences at retirement age are more than 2.5 years for both men and women in 1996 and have widened since then for both men and women. This implies that lower educated enjoy less years in retirement than the higher educated. Overall LE increases over time probably also because the distribution of educational attainment has changed in a positive manner as the percentage enjoying a higher education has been increasing.
Table 1: Life expectancy (LE) at age 65 for men and women stratified by education in 1996 and 2012.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Education</th>
<th>1996</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>Overall</td>
<td>17.0</td>
<td>19.7</td>
</tr>
<tr>
<td>Men</td>
<td>Overall</td>
<td>15.1</td>
<td>18.3</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>16.2</td>
<td>19.2</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>14.5</td>
<td>17.7</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>13.5</td>
<td>15.9</td>
</tr>
<tr>
<td>Women</td>
<td>Overall</td>
<td>19.6</td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>21.1</td>
<td>22.9</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>20.0</td>
<td>21.8</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>18.5</td>
<td>19.6</td>
</tr>
</tbody>
</table>

The Lee-Carter model

The most popular model to forecast mortality rates and life expectancy is the Lee-Carter model (Lee and Carter 1992; Lee 2000; Koissi et al. 2006) which postulates that mortality rates can be modelled as a function of three sets of parameters: age-specific constants, a time-varying index and interaction terms between time and age (Lee and Carter 1992):

\[
\log[m(a,t)] = \alpha(a) + \beta_1(a) \times \kappa_1(t) + \varepsilon(a,t)
\]  

(1)

where \( m \) stands for mortality rate, \( a, t \) are indices for age and time (calendar year). The \( \alpha(a) \) parameters indicate the time-average log mortality rate stratified by age, \( \kappa_1(t) \) refers to an age-independent latent time trend in mortality that is shared by all ages, while \( \beta_1(a) \) can be interpreted as the interaction between each age category with the general time trend. The \( \beta_1(a) \) parameters tell us at which ages mortality declines or increases more rapidly or more slowly in response to changes in \( \kappa_1(t) \). Lee and Carter showed that the Singular Value Decomposition (SVD) method can be used to find the least square solution\(^1\). After fitting the model, one retains a series of time-dependent \( \kappa_1(t) \) values for each calendar year, which can be treated as a time series and forecasts of mortality rates can be made by forecasting \( \kappa_1(t) \) and substituting values of these forecasts of \( \kappa_1(t) \) into equation (1). Lee and Carter proposed that specifying a time series model of a random walk with drift parameter describes \( \kappa_1(t) \) best. Extensions of the Lee-Carter focused on alternative estimation techniques (Currie, Durban, & Eilers, 2004; Koissi, Shapiro, & Hognas, 2006) to select the

\(^1\) Since the model is underdetermined, the following constraints are usually put on the parameters so that a unique solution can be determined: \( \sum_a \beta_1(a) = 1, \sum_t \kappa_1(t) = 0 \).
optimal time-frame and to account for parameter uncertainty when forecasting mortality rates (Booth, Maindonald, & Smith, 2002; Booth & Tickle, 2008).

The Li-Lee model
A drawback of forecasting life expectancy for different but related populations (e.g. neighbouring countries) by fitting a separate Lee-Carter model for each population is that forecasts of life expectancy usually strongly diverge in the long run. This was recognized by Li-Lee who developed the Li-Lee model in response to this (Li & Lee, 2005). The central idea behind the Li-Lee model is that different groups within a population in the long run share a common time trend, but that there may be subgroup-specific deviations in the short run. Li-Lee did not specify a strict definition of what related population exactly mean but mention different genders within a country or different countries with similar levels of development. Li-Lee proposed to extend the Lee-Carter model in the following manner:

\[
\log[m(a, t, g)] = \alpha(a, g) + \beta_1(a) \times \kappa_1(t) + \beta_2(a, g) \times \kappa_2(t, g) + \varepsilon(a, t, g)
\]

Where \( g \) is an index for subgroups (for instance different countries or different genders), \( \kappa_2(t, g) \) indicate the subgroup specific time trends and \( \beta_2(a, g) \) the subgroup specific age interactions with the subgroup specific time trends. Similar as with the basic Lee-Carter model \( \alpha(a, g) \) equal the average log mortality rates by age and now also subgroup. \( \kappa_1(t) \) is the common time trend for all subgroups and \( \beta_1(a) \) the common age interactions with the common time trend. Li-Lee proposed to forecast values of \( \kappa_2(t, g) \) using a mean reverting process such as an AR(1) process. Forecasts can then be made for both the overall population and the population stratified by subgroup class that are coherent in the sense that they do not diverge or cross in the long run. Assuming a mean-reverting process for the subgroup-specific kappa parameter prevents a strong divergence of forecasts between subgroups as forecasts of \( \kappa_2(t, g) \) return to values of \( \kappa_2(t, g) \) as observed in the time-period used to fit the model. In case the subgroup specific time trends is difficult to characterize as a mean reverting process, Li-Lee advised to model mortality rates of each subgroup separately with a different Lee-Carter model. However, as noted by Li-Lee themselves, fitting a separate Lee-Carter model for each educational group might result in strong divergence in LE between educational groups when forecasting. On the other hand, a drawback of assuming a mean reverting process is that if widening mortality rates between subgroups have been observed in the data these will automatically become smaller in the future (and vice versa). Both options (modeling all subgroups separately or simultaneously but assuming mean reverting processes) thus have clear disadvantages. Note that Li-Lee proposed to use expert judgment rather than formal tests to decide whether the
subgroup-specific trends (i.e. \( \kappa_2(t, g) \)) should be modelled using a mean-reverting process or that each subgroup should be modeled separately.

**Our model specification**

To extrapolate mortality rates we used the Li-Lee model as starting point and extended it in several ways. First of all, we extended the Li-Lee model by distinguishing two different layers of subgroups instead of just one: gender and education. This means that there is common time trend shared by all groups \( \kappa_1(t) \), a time trend that is shared by all education classes within each gender \( \kappa_2(t, g) \), and a time trend that is specific for each educational class by gender \( \kappa_3(t, g, e) \). For each of these time trend parameters \( 1+2+2\times3=9 \) in total there is also a set of age-specific interaction terms, which leads to the following model specification:

\[
\log[m(a, t, g, e)] = \alpha(a, g, e) + \beta_1(a) \times \kappa_1(t) + \beta_2(a, g) \times \kappa_2(t, g) + \beta_3(a, g, e) \times \kappa_3(t, g, e) + \varepsilon(a, t, g, e)
\]

where \( g \) and \( e \) are indices for gender and education. The parameters \( \kappa_3(t, g, e) \) reflect the latent time trend per educational class and \( \beta_3(a, g, e) \) the education specific interactions with that time trend. Equation (3) can be estimated in a stepwise manner given that \( \log[m(a, t, g, e)] \) equal the time averaged log mortality rates by age, gender and education. First, to estimate \( \beta_1(a) \) and \( \kappa_1(t) \) the basic Lee-Carter mortality model from equation (1) is estimated for the total population not specified by gender and education. After estimating (1) \( \beta_2(a, g) \) and \( \kappa_2(t, g) \) can be estimated using the SVD by plugging in the estimates obtained in (1) in equation (2):

\[
\log[m(a, t, g)] - \alpha(a, g) - \beta_1(a) \times \kappa_1(t) = \beta_2(a, g) \times \kappa_2(t, g) + \varepsilon(a, g, t)
\]

To estimate (3) and (4) we used data spanning the period 1973-2012. We choose this period as from 1973 onwards, life expectancy for both men and women has been increasing. In the years preceding this year trends in life expectancy between men and women differed starkly. This choice of period is in line with previous research that indicated that the optimal time period for the Lee-Carter model using data from the Netherlands started in the seventies (Janssen et al., 2013; Stevens, De Waegenaere, & Melenberg, 2010). Furthermore, as our goal is to forecast LE 30 years ahead our choice of historical period is in concordance with a general recommendation that the historical period should be at least as long as the projection horizon (Janssen & Kunst, 2007).

After estimating (4) \( \beta_3(a, g, e) \) and \( \kappa_3(t, g, e) \) can be estimated by plugging in the estimates obtained in (1) and (2) in equation (3):
\[
\log[m(a, t, g, e)] = \alpha(a, g, e) - \beta_1(a) \times \kappa_1(t) - \beta_2(a, g) \times \kappa_2(t, g) = \beta_3(a, g, e) \times \kappa_3(t, g, e) + \epsilon(a, g, e, t) \tag{5}
\]

As the estimation of the Li-Lee model is iterative in nature this allows to use time series of different lengths. In our case this meant we used longer time series (1973-2012) to model the overall trend and gender-specific trends while using shorter time series (1996-2012) to model deviations from these trends for the different education groups. Note that only values for the \( \kappa_1 \) and the \( \kappa_2 \) parameters for the period 1996-2012 were used in equation (5). After fitting the model in the steps described above, one retains 9 series of time-dependent \( \kappa_1(t), \kappa_2(t, g), \kappa_3(t, g, e) \) values. Forecasts of mortality rates can be made by forecasting \( \kappa_1(t), \kappa_2(t, g), \kappa_3(t, g, e) \) and substituting values of these forecasts into equation (3). Crucial for forecasting LE is the choice of a model how to extrapolate the different kappa parameters \( (\kappa_1(t), \kappa_2(t, g), \kappa_3(t, g, e)) \).

We think there is always a benefit in modelling common trends if there are theoretical reasons to assume common determinants of trends in mortality for different subgroups within the same population. If there are clear indications that subgroup specific time trends in equation (3) trends are not mean reverting, this should not imply that the mortality rates have nothing in common with the overall time trend. Even if the subgroup-specific kappa parameters would not be mean-reverting the influence of these subgroup-specific time trends will become less (and also the problem of divergence/convergence) if part of the time trend is jointly modelled. Also, when thinking in terms of uncertainty there is a clear benefit of modelling common trends as this generates a positive correlation between the forecasts for the different subgroups which makes sense. As we are interested in how educational differences in LE might develop in the future, this is especially relevant. In our specific application, modelling common trends also allows us to strengthen forecasts of LE by education by using longer time series for the overall and gender specific time trends. This mixture of a common time trend with potentially deviating subgroup-specific time trends follows a broader literature highlighting the importance of common unobserved factors in time series data of separate groups (Breitung & Pesaran, 2008). For the case of mortality time series the existence of a stable common long-run trend has been demonstrated in various seminal papers (Oeppen & Vaupel, 2002; Tuljapurkar et al., 2000; White, 2002). This strong common pattern has been attributed to an increasing similarity of the childhood disease environment and of dietary patterns as well as the general spread of the Western lifestyle due to the globalization (White, 2002). Moreover, the ongoing breakthroughs in health technology quickly diffuse among countries so that a common progress against fatal diseases has been achieved (Papageorgiou, Savvides, & Zachariadis, 2007).
To avoid more or less arbitrary expert judgments and to consider a broader category of time-series models to forecast all kappa parameters, we propose to use a criterion based approach to select optimal time-series models for all kappa parameters. Therefore, to forecast values for the kappa parameters of the different models we selected optimal ARIMA models by comparing the BIC values of different ARIMA models. We preferred this over assuming a random walk to model overall mortality and imposing that the gender and SES specific time trends would be mean reverting. Furthermore, as the time-series by education are rather short we preferred to select forecasting models this way, as small samples make it difficult to use almost all testing procedures. Note that to avoid jump-off bias we used the last observed mortality rates as a starting point for our forecasts.

**Sensitivity analysis**

To investigate the sensitivity of our forecasts with respect to several key assumptions and to illustrate the advantages of our proposed methodology we also forecasted LE in the following scenarios:

- **Scenario A** [assuming convergence]: in this scenario we imposed a random walk with drift for the common trend $k_1(t)$ and an AR(1) process for all gender and education specific time trends. This scenario is similar to original Li-Lee model specification in which all subgroup specific trends are mean reverting;

- **Scenario B** [no common trends]: in this scenario we fitted a Lee-Carter for each group separately. Similar as in the base case scenario we used data for the period 1973-2012 for overall and gender specific mortality and data for the years 1996-2012 for education specific mortality. For all Lee-Carter models we selected ARIMA models to extrapolate the kappa parameters by optimizing the BIC criterion;

- **Scenario C** [shorter historical period]: in this scenario we only used data for overall and gender specific mortality for the period 1996-2012. Everything else is the same as in our baseline model specification;

- **Scenario D** [shorter historical period without common trends]: in this scenario we forecasted mortality by fitting a Lee-Carter model for each education and gender group separately using data from the 1996-2012 only. To extrapolate the kappa parameters we again selected the optimal ARIMA models.

Scenario A mimics the original Li-Lee model by imposing a mean-reverting process to the subgroup specific trends. By also forecasting LE in scenarios B and D we can investigate the benefits of our proposed methodology as it allows comparing our base case forecasts to separate Lee-Carter forecasts for each subgroup. Scenario C allows us to investigate the added value of using a longer time series to model common trends. Scenario D is interesting to compare to scenario C as it allows a
straightforward comparison of separate Lee-Carter models and our modelling strategy in case time series for all groups are of equal length.
Results

Figure 1 displays estimates of the $\alpha$ parameters of equation (3) which are simply the time average log mortality rates (for the period 1996-2012) stratified by age, gender an education. From the two graphs it can be seen that there is a clear educational gradient in mortality rates for both genders converging over age.

![Lee-Carter $\alpha$ parameter estimates by age, gender and education](image)

*Figure 1:* Lee-Carter $\alpha$ parameter estimates by age, gender and education

Figure 2 displays estimates of the kappa and beta parameters as described in equation (3). From the upper left graph in figure 2 we can see a clear downward trend in overall mortality over time as illustrated by the decreasing $\kappa_1$ values. The deviations of the different genders with the overall time trend are also displayed in the same graph. For men the increasing $\kappa_2$ values for from 1973 to about 2000 shows that mortality has been decreasing at a less rapid pace than overall while the reverse is true for women. However, from about 2000 onwards this pattern has reversed. The interaction of different age groups with the time trends as displayed in the upper right graph are sometimes less straightforward to interpret but indicate that changes in the age profile have also been gender specific. At ages 65 to about 77 men have negative $\beta_2$ values while ages above 82 have positive $\beta_2$ values indicating that at these ages mortality rates have been changing less than average mortality at those ages. The middle left graph in figure 2 shows that the overall decline in mortality has been slower for the lower educated men as the $\kappa_3$ values increased over time. It should be noted that changes in the $\kappa_2(t,g)$ and the $\kappa_3(t,g,e)$ values over time are much smaller than changes in $\kappa_1(t)$ over time as much of the changes over time in mortality have already been captured by the common trend.
Figure 2: Lee Carter kappa (all graphs on the right) and (all graphs on the right) beta parameters of the model.
Table 2 displays the optimal ARIMA models selected using the BIC criterion used to forecast values for the different kappa parameters. From this table we can observe that the overall time trend $\kappa_1(t)$ is, similar as in previous studies, best modeled using a random walk with drift. Although the gender specific trends $\kappa_2(t, g)$ do not contain a drift term, both the time trend for men and women is not mean reverting. All education specific time trends are also not mean reverting. While for women both the high and middle educated time trend is modeled best as a random walk without drift, the lower educated time trend does contain a drift term. For men, the time trends for all educational groups contain a drift term. However, it should be kept in mind that changes in the $\kappa_3(t, g, e)$ values over time are much smaller than changes in $\kappa_2(t, g)$ and $\kappa_1(t)$ values over time so that also the ‘amount of drift’ is much smaller.

**Table 2: optimal ARIMA models for the different kappa’s**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gender</th>
<th>Education</th>
<th>ARIMA model Mortality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1(t)$</td>
<td>Men &amp; women</td>
<td></td>
<td>(0,1,0) with drift</td>
</tr>
<tr>
<td>$\kappa_2(t, men)$</td>
<td>Men</td>
<td></td>
<td>(0,1,2)</td>
</tr>
<tr>
<td>$\kappa_2(t, women)$</td>
<td>Women</td>
<td></td>
<td>(0,1,0)</td>
</tr>
<tr>
<td>$\kappa_3(t, men, high)$</td>
<td>Men</td>
<td>High</td>
<td>(2,1,0) with drift</td>
</tr>
<tr>
<td>$\kappa_3(t, men, middle)$</td>
<td>Men</td>
<td>Middle</td>
<td>(0,1,1) with drift</td>
</tr>
<tr>
<td>$\kappa_3(t, men, low)$</td>
<td>Men</td>
<td>Low</td>
<td>(1,1,0) with drift</td>
</tr>
<tr>
<td>$\kappa_3(t, women, high)$</td>
<td>Women</td>
<td>High</td>
<td>(0,1,0)</td>
</tr>
<tr>
<td>$\kappa_3(t, women, middle)$</td>
<td>Women</td>
<td>Middle</td>
<td>(0,1,0)</td>
</tr>
<tr>
<td>$\kappa_3(t, women, low)$</td>
<td>Women</td>
<td>Low</td>
<td>(0,1,0) with drift</td>
</tr>
</tbody>
</table>

Figure 3 displays trends and forecasts of LE (left graphs) and differences in LE between different subgroups (right graphs). From this figure it can be seen that LE is predicted to increase for all educational classes for both men and women but that LE increases less for the lower educated. The difference in LE between the high and low educated increases at the same pace as observed in the period 1996-2012 for both men and women. Furthermore, although differences in LE between men and women are expected to decrease, the rate of this decrease is slower than has been observed in the last decade. Also noteworthy are the prediction intervals that increase over time and the fact that the trends between the subgroups are rather similar as a result of modelling the common time trends.
Figure 3: Forecasts of LE at age 65 for overall population and different subgroups including 95% prediction intervals and forecasts differences in LE at age 55 between different subgroups.
Table 3 displays estimates of life expectancy in 2042 in the different scenarios and Table 4 displays differences in LE between different groups in 2040. If we compare predictions of the scenarios in the sensitivity analysis to base case analyses we can observe several things. First of all, predictions of overall and gender specific LE in scenario C and D which are based on the period 1996-2012 are higher than in the base scenario. This is due to the fact that in this period LE has been increasing rather sharply. In scenario A in which we imposed mean reversion we can see that differences between in LE men and women and between educational classes decline as a result thereof.

**Table 3**: Forecasts of life expectancy in 2042 (LE) at age 65 in several scenarios with 95% prediction intervals between brackets

<table>
<thead>
<tr>
<th>Gender</th>
<th>Education</th>
<th>Base case</th>
<th>Scenario A</th>
<th>Scenario B</th>
<th>Scenario C</th>
<th>Scenario D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined</td>
<td>Overall</td>
<td>22.3 (21.1/23.4)</td>
<td>22.3 (21.1/23.4)</td>
<td>22.3 (21.1/23.4)</td>
<td>24.1 (23.1/25)</td>
<td>24.1 (23.1/25)</td>
</tr>
<tr>
<td>Men</td>
<td>Overall</td>
<td>21.1 (19.7/22.4)</td>
<td>21.1 (19.8/22.3)</td>
<td>19.9 (15.6/23.3)</td>
<td>22.8 (21.8/23.8)</td>
<td>23.3 (22.3/24.3)</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>22.8 (21.4/24.4)</td>
<td>21.8 (20.5/23)</td>
<td>24.3 (23.1/25.5)</td>
<td>24.4 (23.2/25.5)</td>
<td>24.3 (23.1/25.5)</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>21.5 (20.1/22.9)</td>
<td>20.4 (19/21.7)</td>
<td>23.1 (21.9/24.2)</td>
<td>22.9 (21.7/24.1)</td>
<td>23.1 (21.9/24.1)</td>
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<tr>
<td></td>
<td>Low</td>
<td>17.9 (16.6/19.2)</td>
<td>19 (17.5/20.4)</td>
<td>19.5 (18.9/20.1)</td>
<td>19 (17.8/20.2)</td>
<td>19.5 (18.9/20.1)</td>
</tr>
<tr>
<td>Women</td>
<td>Overall</td>
<td>23.8 (22.6/24.8)</td>
<td>23.8 (22.6/24.8)</td>
<td>24.1 (22.7/25.4)</td>
<td>24.9 (23.8/25.9)</td>
<td>24.7 (23.4/26.1)</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>25.2 (23.9/26.6)</td>
<td>25.1 (24/26)</td>
<td>26.4 (24.9/27.9)</td>
<td>26.3 (25.3/27.3)</td>
<td>26.4 (24.8/27.9)</td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>24.2 (22.9/25.4)</td>
<td>24 (22.9/25.1)</td>
<td>25.3 (23.6/27.1)</td>
<td>25.3 (24.2/26.5)</td>
<td>25.3 (23.6/27)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>20.7 (19.1/22.2)</td>
<td>22.4 (20.9/23.7)</td>
<td>21.5 (20.1/22.8)</td>
<td>21.4 (20.1/22.6)</td>
<td>21.5 (20.1/22.8)</td>
</tr>
</tbody>
</table>

**Table 4**: Differences in life expectancy in 2042 (LE) at age 65 in several scenarios

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>Scenario A</th>
<th>Scenario B</th>
<th>Scenario C</th>
<th>Scenario D</th>
</tr>
</thead>
<tbody>
<tr>
<td>men vs. women</td>
<td>2.7 (2.1/3.3)</td>
<td>2.7 (2.3/3.1)</td>
<td>4.2 (0.5/8.7)</td>
<td>2.1 (1.8/2.4)</td>
<td>1.4 (-0.2/3.1)</td>
</tr>
<tr>
<td>high vs. low educated men</td>
<td>4.9 (4.3/5.7)</td>
<td>2.8 (2.2/3.4)</td>
<td>4.8 (3.4/6.1)</td>
<td>5.4 (4.8/6.0)</td>
<td>4.8 (3.4/6.2)</td>
</tr>
<tr>
<td>high vs. low educated women</td>
<td>4.5 (3.3/5.8)</td>
<td>2.7 (2/3.4)</td>
<td>4.9 (2.9/7.0)</td>
<td>4.9 (4.5/5.4)</td>
<td>4.9 (2.9/6.9)</td>
</tr>
</tbody>
</table>
From Table 4 we can see that prediction intervals of differences in LE between subgroups increase if we model them without common trends as is done in scenarios B and D. A big advantage of using the Li-Lee approach is that the correlation between predictions of LE for different subgroups is taken into account by modeling a common trend. This results in much smaller variation in predicted differences in LE between subgroups of the base case scenario and scenario C compared to scenarios B and D in which we estimated a separate Lee-Carter model for each subgroup. The 95% prediction interval of the difference in LE between men and women includes 0 if we model no common trend in scenario D, which seems implausible. Also noteworthy from table 3 and 4 is that by modelling common trends predictions of differences in LE between educational groups are fairly similar while the levels of the LE predictions may change as different time periods are chosen to model the common trends.

To better understand the consequences of the key assumptions for the forecasts we compare scenario A and B in figure 4 and 5. Figure 4 clearly illustrates that assuming convergence (scenario A) for modeling education specific time trends would imply a clear trend break in those educational differences, which seems implausible. Figure 5 shows that if separate Lee-Carter models are used (no common time trends), the forecasts of the different groups do not seem coherent with the forecasts of the overall group in which they all are part. This is illustrated most prominently by forecasts of LE for men. Thus, even in case we have diverging gender and/or education-specific time trends there is still benefit in modelling to some extent common underlying trends.
Figure 4: Forecasts of LE at age 55 in scenario A. Overall population including 95% prediction intervals and forecasts of LE at age 55 for the different educational groups. Forecasts of differences in LE at age 55 between men and women (upper graph), high and low educated men (middle graph), high and low educated women (bottom graph) including 95% prediction intervals.
Figure 5: Forecasts of LE at age 55 in scenario B. Overall population including 95% prediction intervals and forecasts of LE at age 55 for the different educational groups. Forecasts of differences in LE at age 55 between men and women (upper graph), high and low educated men (middle graph), high and low educated women (bottom graph) including 95% prediction intervals.
Discussion

This paper demonstrates a novel approach to combine mortality trends measured at different layers in a population (overall, gender, education) available for time frames of different length to forecast life expectancy. By allowing for a high degree of flexibility while ensuring the necessary degree of coherence our model overcomes problems of the commonly applied models that either estimate the mortality trends at the different layers either completely pooled or completely separate. We demonstrated that even if sub-group specific trends in mortality appear to be diverging modelling a common trend can have benefits. This did not only have an impact of the mean forecasts but also on the prediction intervals of the forecasts and the correlation between forecasts of different educational groups. We have illustrated the usefulness of the approach by projecting LE by level of education in the Netherlands up until 2042. Our base scenario projected a general increase at all levels with a continuing convergence of male and female life expectancy but divergence of life expectancy between the educational classes, which was slightly stronger in men than in women. In our case study we combined data from a long time series to reliably estimate the time trend on the overall group with shorter time-series of education-specific data. In that sense, shorter time series borrowed information from the longer time series. This is an important advantage as data on mortality by education is in many countries (including the Netherlands) of lesser quality than data on overall mortality.

This study represents the very first approach to forecast life expectancy by level of education/SES. Therefore, it is impossible to compare our results to previous forecasts. However, we can compare our forecasts of overall LE and LE by gender to previous forecasts. The most recent official projection of Statistics Netherlands (CBS) projects life expectancy at age 65 to be 22.2 years in men and 24.3 years in women in 2042 (van Duin, Janssen, & Stoeldraijer,) which is a bit higher than our projections where we estimated 21.1 years in men and 23.8 years in women in 2042. Given that Statistics Netherlands used a similar historical period (data from 1970-2011) the differences can be explained by the fact that they included an additional layer in their variant of the Li-Lee model – the experience of other countries of Western Europe. Compared to the Netherlands the mortality improvement was much more positive in the other countries over the whole historical period 1970—2011. Hence, adding a shared trend among all countries in the Li-Lee model produced together with the assumption of mean reversion a more positive trend in Dutch LE than without this additional level. We believe that this is a meaningful assumption given the strong interdependencies of the countries in terms of economic prosperity, technological progress and lifestyles. Given the flexibility of our
model, such higher layers could of course be included but we focused in this paper mainly on the layer of SES differentials for the purpose of illustration.

A drawback of the original Lee-Carter model as well as the Li-Lee model and our model is that the age-time interactions are assumed constant. We checked whether a changing age profile could be incorporated by adding the second factor obtained from the singular value decomposition. However, there was not a strong trend over time, and including the second factors in the forecasts only slightly increased prediction intervals but did not change mean predictions. We also forecasted life expectancy assuming there were no education specific time trends (this is equivalent to setting $\kappa_3(t, g, e)$ equal to zero in equation (3)) which led to a narrowing of inequalities in LE by education. This is due to the fact the models are fitted on the log scale and that absolute decreases in mortality are bigger when mortality rates are higher. Furthermore, we also predicted LE in a scenario in which we selected optimal ARMA models for the $\kappa_2(t, g)$, $\kappa_3(t, g, e)$ parameters instead of optimal ARIMA model. In terms of differences in LE between subgroups results were similar as in scenario A in which we also assumed mean-reverting processes for the subgroup specific trends. A specific merit of the approach we propose is its simplicity allowing a broad range of applications with minor computational effort and relatively modest data requirements as we do not include determinants of mortality in our model. However, the latter can also be seen as a drawback of our approach.

Separate modeling of smoking-associated and non-smoking associated mortality in the Netherlands revealed that in the short-run a further convergence of male and female mortality is likely (Janssen et al., 2013). A logical next step would be to investigate possibilities to include determinants when forecasting LE by education.

The results of our forecasts indicated diverging trends of mortality among the high and low educated subgroups, which was stronger in men than in women. A recent study on trends in socio-economic inequalities in mortality reports first signs of a narrowing of inequalities in men several Western countries, while inequalities in women continued it’s widening (Mackenbach et al., 2014). Although this analysis did not include the Netherlands and targeted at another age-range (30-74), we must admit that ignoring underlying determinants of SES differentials such as smoking or alcohol consumption may have affected our forecasts. One could speculate whether the widening we found for inequalities in LE for men were too pessimistic and actually a narrowing appears more plausible. In countries with better data on education-specific life expectancy and its determinants one may could test such a hypothesis in more detail. In our data we did not find signals for such a narrowing. Generally, educational attainment is related to health through a variety of mechanisms running from education to health but also vice-versa (Cutler & Lleras-Muney, 2010; Smith, 1999). Nevertheless, a
A clear and causal effect of more education on lower mortality has been demonstrated convincingly in a series of analyses of natural experiments, mostly compulsory schooling reforms (Clark & Roayer, 2013; Van Kippersluis, O'Donnell, & van Doorslaer, 2011). Important channels through which education influences health are life-style related risk factors such as smoking, alcohol consumption, dietary patterns and physical inactivity but also financial resources, housing and work conditions and access to care. Despite great advances in medical treatment, a decrease in smoking prevalence and programs to tackle health inequalities, the large differentials in life expectancy between SES groups persisted and even widened up until today, suggesting that more fundamental societal forces drive these inequalities (Meara, Richards, & Cutler, 2008; Olshansky et al., 2012; Phelan, Link, & Tehranifar, 2010). Therefore, it is likely that SES disparities will endure in the future even if the precise mechanisms explaining the differentials change over time.

As we focused on the 65+ our LE forecasts have a clear relevance for the debate regarding retirement age and the demand for health care. As in the Netherlands current policy is to couple retirement age to LE (van Duin, 2013), our forecasts suggest that the lower educated will experience a decrease in the number of years in retirement as their forecasted increase in LE is below the average increase in LE. With respect to a possible increase in the demand for health care due to increased longevity our results also suggest that this additional demand may be caused more by the higher educated than the lower educated. If financing of health care and pension schemes will not change these differential changes in LE implies a redistribution of wealth from the lower to the higher educated.

Concluding, we think that our extended Li-Lee model provides a good method to forecast LE by education and to make optimal use of available information. Our forecasts suggest that differences between educational groups in LE are not likely to disappear but seem to widen. Therefore, these differences should be taken into account in political decisions that affect solidarity issues between these groups.
Appendix: estimation of mortality rates by education

Mortality rates for different education classes for the years 1996-2012 were estimated by first estimating age and calendar year specific relative risks on mortality (denoted $RR(a,t,e)$ which equals the mortality rate of educational class $e$, age $a$, year $t$ divided by the mortality rate of the reference educational class age $a$, year $t$). These relative risks were then used to decompose mortality rates from the total population by exploiting the following relationship:

$$m(a, t, e) = RR(a, t, e) \times \frac{m(a, t)}{\sum_e RR(a, t, e) \times p(a, t, e)} \quad (A1)$$

Equation (A1) states that mortality rates in a particular year at a particular age are the weighted average of the mortality rates of the different educational subgroups ($p(a, t, e)$ denotes the proportion of a particular age, year, education group) and that the ratio of mortality rates between different subgroups can be expressed in relative risks. Estimates of $RR(a, t, e)$ were made using data from Labour Force Survey (LFS) linked to the death registry. The LFS is a rotating panel survey from Statistics Netherlands that exists from 1987 onwards. The LFS is the largest data source in which information on educational attainment is collected in the Netherlands and consists of a sample of more than 60,000 households annually. From 1996 onwards it is possible to link persons that have participated in the LFS to the death registry. This makes it possible to quantify the relation between educational attainment and mortality. Values of $p(a, t, e)$ values were taken directly from the LFS.

Regarding mortality, we constructed a panel where the annual number of deaths of all persons ever interviewed in LFS is obtained from the death registry and the number of exposures is estimated as the sum of the people surveyed in a particular year and the survivors from the previous year. To estimate $RR(a, t, e)$ we fitted a Poisson regression model with the exposure as offset and the expected number of deaths by year, age, education class and year as outcome variable:

$$E(D|a, e, t, y) = \exp(\theta'X) \quad (A2)$$

Where $y$ denotes, $X$ a vector of predictor variables and $\theta$ the vector of coefficients that need need to be estimated. Predictor variables were dummy variables indicating educational class and interactions thereof with age and calendar year (both as continuous variables). To control for confounding a set of dummy variables for each year and age were added to the model. Furthermore, a variable measuring the length of follow-up time in the LFS and an interaction thereof with age were added to the model. This is intended to control for selection effects into the LFS registry. From the regression
model we calculated $RR(a,t,e)$. Table A1 displays estimates of exponentiated coefficients of the regression model (coefficients for the year and age dummies are not shown).

**Table A1: estimated coefficients for the regression models (low educated are the reference category)**

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle</td>
<td>0.677**</td>
<td>0.620**</td>
</tr>
<tr>
<td>High</td>
<td>0.497**</td>
<td>0.487**</td>
</tr>
<tr>
<td>Middle x age</td>
<td>1.055**</td>
<td>1.059**</td>
</tr>
<tr>
<td>High x age</td>
<td>1.081**</td>
<td>1.081**</td>
</tr>
<tr>
<td>Low x year</td>
<td>0.950**</td>
<td>1.111**</td>
</tr>
<tr>
<td>Middle x year</td>
<td>0.956**</td>
<td>1.113**</td>
</tr>
<tr>
<td>High x year</td>
<td>0.961**</td>
<td>1.108**</td>
</tr>
<tr>
<td>Low x year x age</td>
<td>1.011**</td>
<td>1.009**</td>
</tr>
<tr>
<td>Middle x year x age</td>
<td>1.007**</td>
<td>1.007**</td>
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<tr>
<td>High x year x age</td>
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<tr>
<td>followuptime</td>
<td>0.978**</td>
<td>1.008</td>
</tr>
<tr>
<td>followuptime x age</td>
<td>0.999</td>
<td>0.991**</td>
</tr>
</tbody>
</table>

* significant at 0.05; ** significant at 0.01
References


van Duin, C. (2013). Indexation of the pension age to projected remaining life expectancy in the netherlands.
