Market Integration Dynamics and Asymptotic Price Convergence in Distribution

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Revised: October 2015

EI2015-30

Abstract

This paper analyzes the market integration process of nominal prices, develops a model to analyze market integration, and presents a test of increasing market integration. A distinction is made between the economic concepts of price convergence in mean and variance. When both types of convergence occur, prices are said to converge in distribution. We present concepts and definitions related to the market integration process, link these to price convergence in distribution, argue that the Law of One Price is not a sufficient condition for market integration, and present a test of price convergence in distribution. We apply our methodology to two different cases, namely the integration of: i) the inland grains market in 19th Century USA, and ii) the Eurozone long-term bonds market after the euro entered circulation.

Keywords: Regional and global markets, Integration, Asymptotic price convergence, Mean, Variance.
JEL: C22, C32, N70, F15

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1 Introduction

The study of the development and integration of regional and global markets is highly topical in economics, with an emphasis on the relationship between market and economic development; see, recently, Schäfer and Steger (2014), Ke (2015) or Dobado et al. (2015). However, there are open questions about the determinants and effects of the market integration process, as well as the relationship between market integration and price volatilities. The main contribution of this paper is to fill the gap in the literature by defining and developing a model to analyze market integration using the volatilities of relative prices as a tool.

The basic idea focuses on measuring and quantifying market development and related concepts. Specifically, the paper highlights the distinction between two main concepts in the literature, namely the weak version of the Law of One Price (LOP) and the Market Integration Process. The former refers to the Extent of the Market problem, that is, whether the prices observed in different places arise from the same market (see Cournot, 1897; Stigler and Sherwin, 1985; Treadway, 2009), while the latter refers to the degree of relatedness of different market locations. This discussion is new to our knowledge. Throughout the paper, we will use the term LOP to denote the Law of One Price in the weak sense, unless stated otherwise.

Both concepts are closely related, but correspond to different economic situations. The LOP refers to a state of the market, while market integration refers to the internal dynamics of the market. In this paper we argue that Market Integration is a more general concept than LOP. Therefore, contrary to the common knowledge, LOP is a necessary but not sufficient condition for Market Integration.

It is traditional in economics to use cointegration analysis to conclude if the LOP is satisfied. A cointegrating relationship is expected under the LOP as it is a particular case of convergence, or more accurately, a state when two series have already converged. Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000) provide definitions and methods for what is called the "steady-state convergence". In this paper, we present a model for convergence that includes a transition path, that also represents the "catching-up convergence", or transition to the steady-state. So, this paper is also strongly related to the literature of (unit roots with) structural breaks; see Lanne et al. (2002) for a comparison of different methods.

Market integration studies based on cointegration are not well connected with the notion of the market integration process, as convergence as catching-up does not necessarily imply greater market integration. For instance, a structural break, such as a change in trade barriers, could imply a convergence to parity, but it does not necessarily imply a change in the internal dynamics of the market. In this paper we analyze the market integration process through the relative price distribution.

When the LOP is satisfied, both prices will have converged in mean, that is, in the first moment of the distribution. The LOP is not a matter of degree, but is a binary distinction. If two prices are statistically the same, that is, in the absence of any further shocks, and when all propagation (arising from any ARMA structure) has ended, then relative prices converge to a constant, so they might have originated in the same spread market. However, if two prices do not share at least one common non-stationary factor, it is not possible for them to come from the same market. Thus, the LOP is a matter of cointegration, not of correlation.

On the other hand, the relatedness of two different markets, or market locations, is a matter of degree and, hence, of contemporary and lagged cross covariances. This means that it is necessary to check at least the second moment of the price distribution to conduct an appropriate market integration analysis. In order to conclude positive market integration, it is necessary to have price convergence in distribution, and not just the LOP. Under normality, a necessary condition for market integration is convergence in both, mean and variance.

We use two different datasets to illustrate the methodology proposed.

The first empirical exercise presents the case of the inland grains market integration in 19th Century USA. We examine the historical prices of wheat in the USA, including coastal and inland cities. The empirical results show price convergence in mean and variance for this commodity in many cities in the sample, suggesting a strong market integration process occurred in 19th Century USA.

The second example studies whether the introduction of the euro fostered the convergence of the price of credit in the Eurozone. Our hypotheses are that: (i) the establishment of fixed exchange rates in 1999 (that removed the currency risk) and, (ii) the introduction of the common currency in 2002 (that reduced the transaction costs) could have triggered a convergence process of the long-term interest rates in the Eurozone. The analysis reveals that a convergence process as catching-up indeed occurred, but conclusively rejects the idea of convergence in mean and variance.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical framework, presents concepts and definitions related to the market integration processes, and links this concept with the notion of price convergence in distribution. Section 3 describes the model. In Section 4, the econometric representation and hypothesis testing are presented. Section 5 discusses the empirical results by applying the methodology to the USA wheat prices in the second half of 19th Century, and the long-term interest rates in the Eurozone after the euro entered in circulation. Finally, Section 6 gives some concluding remarks.

2 Theoretical Framework

In this section we describe the concepts and definitions relating to convergence, structure of markets, market integration and convergence of prices. We present the assumptions on the relationship between the prices of the goods to be analyzed. All prices are transformed into natural logarithms to induce linearity, and to avoid heteroskedasticity and non-normality. In what follows, $p_{i,t}$ is the log of a nominal price in place i at time t, $p_{i,t} = \log(P_{i,t})$, and $p_{j,t}$ is the log of a nominal price in place j at time t, $p_{j,t} = \log(P_{j,t})$.[1]

2.1 Market-related concepts

It is assumed that nominal prices need to be differenced once to be stationary. Based on economic theory, it is expected that any market-clearing nominal price follows a an integrated, I(1), process. This reflects the idea that some shifts in supply (for example, technological breakthroughs or changes in wages) or in demand (for example, changes in consumer preferences or population growth) imply price adjustments to clear the market in the long run.[2]

By definition, a stationary price level cannot change in the long run to clear the market precisely because stationarity implies a long term, constant value. However, as market conditions can change due to many factors, prices need to react to those changes in order to clear the market. Therefore, our analytical framework requires the log nominal price to be integrated of order one.

Our view of market efficiency follows that proposed by Lo (2004, 2005) for the Adaptive Market Hypothesis, in which transitory arbitrage situations in time and under uncertainty are allowed. Specifically, the (log) price series can be represented by an ARIMA$(p, 1, q)$ model, with $p > 0$ and/or $q > 0$. In this case, the persistent behaviour implied in such models represents transitory arbitrage possibilities. This clearing market condition is not implied for the Efficient Market Hypothesis, including "weak-form" efficiency, because a persistent component in prices is permitted.

In this framework, arbitrage possibilities have two dimensions, namely time and space, so the market efficiency notion used is also bi-dimensional. For instance, an extension of the Adaptive Market Hypothesis in the space domain can be used, which means that

[1] We use the term *price* in a very general way. For instance, $p_{i,t}$ can be the price of a good or service, but also the price of a market basket (price index), the price of credit (interest rate), and so on.

[2] There are simple models of price behavior that supports this assumption; see, for instance, Deaton (1999) or Shiue and Keller (2007).

transitory arbitrage possibilities in the space domain and under uncertainty, are allowed. In the sense of space efficiency, we say that a *spread market* exists when the prices of homogeneous goods traded in different locations come from the same market. In this case, a market clearing price is not a scalar, but a vector. Under market clearing conditions, relative prices in a spread market should follow a stationary process so that, in a strict sense, prices are expected to be the same in the long run.

In this paper we consider the prices of perfectly homogeneous or quasi-homogeneous goods or services. It is assumed that price similarities or dissimilarities (generated by quality, brand, and consumer perception) are time invariant. In such a case, Cournot's pioneering definition of *market* "an entire territory of which the parts are so united by the relations of unrestricted commerce that prices take the same level throughout with ease and rapidity" (Cournot, 1897, pp. 51-52), applies. Market integration is then the process experienced by two prices in different locations when tending to the previous definition by Cournot. Thus, we state the Definition 1 that follows directly from Cournot: there is Perfect Market Integration when prices take the same value "exactly and instantaneously":

Definition 1 *Perfect Market Integration (PMI) in a spread market occurs when m prices of the same product in m different locations are adjusted instantaneously, and clear the market, that is, when arbitrage opportunities are zero.*

Under Definition 1, for PMI the m observed time series prices would be exactly the same, if the transaction cost were zero.[3] Note that the relative prices in this case have a degenerate distribution at zero, and this probability distribution is constant over time, that is, there is no relative price dispersion.

Definition 1 coincides, therefore, with that of the Law of One Price in its strongest version, $p_{it} = p_{jt}$. That is, there is Perfect Market Integration between i and j if and only if $p_{it} = p_{jt}$. These concepts are useful for understanding the relationship between relative price dispersion and market integration, although they might appear to be utopian.

Obviously, the strongest version of the LOP is hard to find in practice. Instead, arbitrage opportunities should prevent prices from moving independently of each other in the absence of any further shocks and when all propagation (arising from any ARMA structure) has ended. Hence, a weaker version of the LOP can be defined as:

$$r_{ij,t} = p_{i,t} - p_{j,t} = \tau_{ij,t} + \varepsilon_{ij,t}, \tag{1}$$

where $\tau_{ij,t}$ denotes a deterministic component reflecting the trading barriers, transport costs and other transaction costs that may vary in time, and $\varepsilon_{ij,t}$ is a zero mean stationary

[3]For instance, the price vector is a singleton.

stochastic process. In short, if two prices arise from the same market, they should be cointegrated of order one, with cointegrating vector $[1, -1]$, once $\tau_{ij,t}$ has been removed.

The statistical properties of the random variable, $\varepsilon_{ij,t}$, are closely related to the market efficiency concept. For example, under the efficient market hypothesis, $\varepsilon_{ij,t}$ fulfills $\text{cov}(\varepsilon_{ij,t}, \varepsilon_{ij,t-k}) = 0$ for $k = 1, 2, \ldots$, and has zero mean and finite variance, $\sigma^2_{\varepsilon_{ij}}$. On the contrary, if the adaptive market hypothesis is assumed, $\varepsilon_{ij,t}$ possesses a predictable structure. A proxy for market efficiency, the speed of adjustment, is usually measured with autoregressive polynomials, but it is inevitably related to the frequency of the data. For example, with annual data, an autoregressive in $\varepsilon_{ij,t}$ measures the inter-annual arbitrage possibilities, a measure of inter-annual market efficiency. However, the intra-annual efficiency of the market is included in the variance of the shocks, $\sigma^2_{\varepsilon_{ij}}$, reflecting the idea that the smaller is the variance, the smaller are the arbitrage possibilities within a year. To better understand this idea, let us think about a monthly first-order autoregressive process with decreasing persistency in time and constant error variance. If we only observed the annual average of this process, we might not be able to capture the autoregressive behavior but, instead, observe a decreasing error variance. This can be demonstrated by using the exact aggregation method by Casals et al. (2009).[4]

Fama (1970) noted that a market is (totally) efficient when there are no arbitrage opportunities from exploitation of some information. According to this, a totally (in time and space) efficient market is characterized by: (i) no predictable structure, and (ii) $\sigma^2_{\varepsilon_{ij}} = 0$. Therefore, $p_{it} = p_{jt} + \tau_{ij,t}$ reflects Fama's idea. In contrast, a totally inefficient market is characterized by $\sigma^2_{\varepsilon_{ij}} = \infty$, which means that there is no long run relationship between the nominal prices. In such a case, the information does not flow at all between the markets, and so the LOP, even the weakest version defined in (1), is rejected.[5]

Hence, there are only two fundamental ways in which two markets can become more integrated. Both ways are related to the reduction of arbitrage possibilities, which implies moving from the weakest to the strongest version of the LOP; from $p_{it} = p_{jt} + \tau_{ij,t} + \varepsilon_{ijt}$ to $p_{it} = p_{jt}$. The first way is through a reduction toward zero of the transaction costs (*e.g.*, those originated by technological improvements, the repeal of some protectionist laws, etc) which will produce an abrupt or smooth shift in the component $\tau_{ij,t}$. This means prices will be closer to each other, but will not necessarily imply a higher synchronization. The second way is through an increase in the inter- or intra-period efficiency, namely a

[4]For instance, the monthly AR(1) process: $(1 - .8B)z_t = a_t$, with $\text{var}(a_t) = 1$ returns the exact aggregated annual model $(1 - .07B)z^A_t = (1 - .19B)a^A_t$, with $\text{var}(a^A_t) = 184.3$, while the less persistent monthly AR(1) process: $(1 - .4B)z_t = a_t$, with $\text{var}(a_t) = 1$ returns the exact aggregated annual model $z^A_t = (1 - .04B)a^A_t$, with $\text{var}(a^A_t) = 30.6$. So, the error variance of the aggregated models falls considerably.

[5]Note that $\sigma^2_\varepsilon = \infty$ implies the non-stationarity of ε_{ijt}, which is incompatible with any version of the LOP.

reduction in the persistence of the autoregressive structure (if any), or in the variance of the shocks affecting the relative price. This clearly increases the synchronization of nominal prices. With this in mind, we define the term *Market Integration Process*:

Definition 2 *A Market Integration Process towards PMI occurs when arbitrage opportunities decrease continuously to zero.*

This definition means that greater market efficiency in time or space implies greater market integration. The main assumption in this case is that there is a transitive relationship between arbitrage opportunities, relative price dispersion, and increasing market integration. In other words, if in a certain spread market the relative price dispersion is decreasing continuously, then there is evidence that market integration is increasing for this specific market. Therefore, in a Market Integration Process, the variable $\varepsilon_{ij,t}$ in equation (1) converges to a degenerate distribution equal to zero.

2.2 Price convergence in distribution

Given our conceptual model of market integration, we have a definition of price convergence in distribution that is consistent with the notion of the Market Integration Process. Price convergence in distribution is more general than a simple notion of relative price dispersion.

Based on the relation between arbitrage and market integration, and following the stochastic definitions of convergence in output presented by Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000), we have the following definition, where $\mathcal{F}_t$ denotes the information set of the agents at time t:

Definition 3 *For Asymptotic Price Convergence in Distribution (APCD), prices of goods i and j converge asymptotically in distribution if:*

$$\lim_{k\to\infty} \mathbb{E}\big[(p_{i,t+k} - p_{j,t+k})^p | \mathcal{F}_t\big] = 0, \, \forall p = 1, 2, ..., n.$$

Corollary 1 *Assuming the normality and the asymptotic expected variance of the relative prices, $p_{i,t}$ and $p_{j,t}$ converge asymptotically in distribution under Definition 3 for $p = 1, 2$.*

Thus, it is necessary to check the evolution of the first two moments of the distribution of relative prices to conclude convergence in distribution under the Gaussian assumption. In contrast, market integration studies based on cointegration analysis concentrate only on the first moment when two nominal prices began to comove so the log differential converged in mean, and subsequently follows this steady-state. For example, a change in trade barriers, such as tariffs, could imply a level change in relative prices (convergence in mean). However, this could happen together with an increase in the residual relative price dispersion (that is, divergence in variance).

For the first moment condition, when $p = 1$, Definition 3 should be interpreted as the weakest form of the LOP presented in equation (1). Additionally, Definition 3 implies $\lim_{t\to\infty} \tau_{ij} = 0$ and then the long term forecast of the (log) price differential, the relative price, is a zero mean stationary process.

For the second moment condition, when $p = 2$, note that $\mathbb{E}\big[(p_{i,t} - p_{j,t})^2 | \mathcal{F}_t\big] = \operatorname{var}(\varepsilon_{ij,t}|\mathcal{F}_t)$, the unconditional variance. Thus, $p_{i,t}$ and $p_{j,t}$ will converge asymptotically in distribution if $\lim_{t\to\infty} \operatorname{var}(\varepsilon_{ij,t}|\mathcal{F}_t) = 0$. Therefore, in order to test the implications of Definition 3, we should relax the assumption of a constant variance and make it depend on t. The notion behind this requirement is that the fluctuations around a constant mean of a relative price series can be considered as the net idiosyncratic shocks in the two market locations. Market integration enhances the ability of the two market locations to cope with shocks in nominal prices. Therefore, one would expect that, as integration in a spread market increases, the dispersion of those transitory shocks would decrease.

In summary, assuming the conditions given in Corollary 1, there is convergence in distribution and, therefore, convergence to the strongest version of the LOP, $p_{it} = p_{jt}$, when: (i) $\tau_{ij,t}$ tends to zero, and (ii) $\varepsilon_{ij,t}$ converges in probability to zero.

3 Model

This section introduces the model for representing the convergence process. From equation (1), the model for the (log) price differential may be written as:

$$\begin{aligned} r_{ij,t} &= \tau_{ij,t} + \varepsilon_{ij,t}, \\ \tau_{ij,t} &= \mu_{ij} + \nu_{ij}(B)\xi_t^{t^*_{ij}}, \\ \phi_{ij,p}(B)\varepsilon_{ij,t} &= \theta_{ij,q}(B)a_{ij,t}, \end{aligned} \tag{2}$$

where B is the backshift (lag) operator, such that $Bp_t = p_{t-1}$, and the relative price, $r_{ij,t} = \log(P_{i,t}/P_{j,t}) = p_{i,t} - p_{j,t}$, has an additive decomposition between a deterministic component, $\tau_{ij,t}$, and stochastic component, $\varepsilon_{ij,t}$. In the deterministic component, μ_{ij} is a constant mean, $\nu_{ij}(B)$ is the convergence operator, and $\xi_t^{t^*_{ij}}$ represents an event that lasts permanently after time t^*_{ij}, as one whenever $t > t^*_{ij}$, and zero otherwise. The stochastic component follows a zero mean, strictly stationary and invertible (that is, the autoregressive and moving average polynomials have all their zeros lying outside the unit circle) ARMA(p,q) process, and $a_{ij,t}$ is a weak white noise stochastic process.[6]

[6]We assume a *weak* white noise process to permit non-constant variance.

In Model (2), the transition path is represented by a combination of the convergence operator with the deterministic variable, $\xi_t^{t^*_{ij}}$:

$$\nu_{ij}(B)\xi_t^{t^*_{ij}} := \frac{\omega_s(B)}{\delta_r(B)}B^b\xi_t^{t^*_{ij}}, \tag{3}$$

where $\omega_s(B) = \omega_0 - \omega_1 B - ... - \omega_s B^s$, $\delta_r(B) = 1 - \delta_1 B - ... - \delta_r B^r$, there are no common factors between $\omega_s(B)$ and $\delta_r(B)$, and s, r, b are non-negative integers. The concept of convergence is closely linked to stability, so that $\delta_r(B)$ should be stable. The transition path will be stable when the roots of the characteristic equation, $\delta_r(B) = 0$, lie outside the unit circle.

Note that the long run gain of the transition path can be estimated as a function of the estimated parameters in (3). The steady-state gain, g_{ij}, for a stable convergence process, is $g_{ij} := \sum_{k=0}^{\infty}\nu_{ij,k} = \nu_{ij}(1) < \infty$. The estimated steady-state gain will be used for testing asymptotic convergence in mean when a transition path is needed. When the initial condition, represented by the constant mean, μ_{ij}, and the long run gain, g_{ij}, have the same value with opposite sign, there is evidence of convergence in mean.

4 Representation and Hypothesis Testing

Under normality, Model (2) could be used for testing convergence in the sense of the APCD in Definition 3. For the requirement of convergence in mean (when $p = 1$ in Definition 3), both prices converge asymptotically if $p_{i,t} - p_{j,t}$ is a stationary process and $\tau_{ij,t}$ tends to zero as t grows. For the requirement of convergence in variance (when $p = 2$ in Definition 3), asymptotic convergence in distribution arises if, additionally, ε_{ijt} converges in probability to zero. In the subsections below, we describe the procedures proposed for testing asymptotic price convergence in both, mean and variance.

4.1 Testing asymptotic price convergence in mean

Testing APCD for $p = 1$ or Asymptotic Price Convegence in Mean, hereafter APCM, requires the log price differential, corrected by the transition path if needed, to be stationary. Moreover, it requires that $\lim_{t\to\infty}\tau_{ij,t} = g_{ij} + \mu_{ij} = 0$, if a transition path is present. Steady-state convergence arises when a transition path is not needed for a stationary representation of $\varepsilon_{ij,t}$. In contrast, we say that there is catching-up convergence if the transition path is required and $\tau_{ij,t}$ tends to zero as t grows.

As the goods whose prices are analyzed are assumed to be homogeneous, univariate analysis could be used to examine convergence in pairs. As $r_{ij,t} = p_{i,t} - p_{j,t}$, model (2), can easily be estimated as the univariate model of the relative price, $r_{ij,t}$. This not only makes the analysis simpler, but also has gains in terms of the power of the unit root tests. When a transition path is required to obtain a stationary representation, the problem belongs to the literature of unit roots tests with structural breaks; see Lanne et al. (2002) and Lanne and Lütkepohl (2002) for more details. For our case, Saikkonen and Lütkepohl (2002), hereafter SL-GLS, present a test for a unit root with different level shifts that includes the transition path given in (3). Specifically, the authors show that the convergence parameters in $\nu_{ij}(B)$, or the time at which the convergence begins, t^*_{ij}, do not affect the limiting distribution of the non-stationarity test, which is very convenient for our application. Furthermore, the Shin and Fuller (1998) test, hereafter SF, which is more powerful than ADF-type tests in the case of ARMA structures, can also be used.

When the non-stationarity hypothesis is rejected in the univariate version of model (2) all the parameters can be jointly estimated and the estimates are asymptotically normally distributed, so that standard inference can be applied. We use this representation because it has two remarkable advantages for our purposes: (i) simplicity in specification and estimation; and (ii) maximum likelihood statistical properties permit us to use standard asymptotic tests when testing APCM.

In order to test the null hypothesis $\lim_{t\to\infty} \tau_{ij} = g_{ij} + \mu_{ij} = 0$, we propose two procedures. As $\hat{g}_{ij}$ and $\hat{\mu}_{ij}$ are consistent and asymptotically normal estimators of g_{ij} and μ_{ij}, respectively, then $\sqrt{T}[\hat{g}_{ij} + \hat{\mu}_{ij} - (g_{ij} + \mu_{ij})]/\hat{\sigma}_{g\mu} \xrightarrow{d} N(0,1)$, where $\hat{\sigma}_{g\mu}$ is calculated using the Delta method (see, for example, Cramér, 1946). On the other hand, the likelihood ratio statistic $-2\log l(\Theta_2|p_{1,t}, p_{2,t}, \xi_t^{t^*})/l(\Theta_1|p_{1,t}, p_{2,t}, \xi_t^{t^*})$, which follows a χ^2 distribution asymptotically with 1 degree of freedom and where $\Theta_2 = \{\alpha, \omega_0, ..., \omega_s, \delta_1, ..., \delta_r, \phi_{1,ii}, ..., \phi_{p,ii}, \theta_{1,ii}, ..., \theta_{q,ii}, \theta_{ij}\}$, can be applied for the same purpose. Independently of the test used, when $p_{i,t}$ and $p_{j,t}$ are cointegrated and $\lim_{t\to\infty} \tau_{ij} = g_{ij} + \mu_{ij} = 0$ cannot be rejected, then $p_{i,t}$ and $p_{j,t}$ are said to converge asymptotically in mean. We will then conclude APCD for $p = 1$.

4.2 Testing asymptotic price convergence in variance

Assuming the conditions in Corollary 1, testing the APCD now requires testing whether the residual variance in model (2) tends to zero. We propose using the well-known Lagrange multiplier test of Breusch and Pagan (1979), which tests whether the estimated variance of the residuals is unconditionally homoscedastic. We regress the squared residuals on an exogenous variable. The test statistic, LM, is the product of the coefficient of determination (R^2) from this regression and the sample size n, namely $LM = nR^2$, where LM is the Lagrange multiplier statistic. The statistic is asymptotically distributed as $\chi^2(1)$ under the null hypothesis of homoscedasticity.

If the null hypothesis is not rejected, there is no evidence in favor of the APCD as the variance of ε_{ijt} is considered constant over time. In that case, APCD and increasing market integration (through increasing market efficiency) is rejected. When the null hypothesis is rejected, unfortunately, we cannot directly conclude that there is APCD, as APCD implies heteroscedasticity, but the reverse is not always true. In that case, we could have growing integration, disintegration, or both in different periods.

In these inconclusive case, we suggest observing the residual standard deviation calculated with a rolling window. We could then use this visual aid diagnostic to decide on the evolution of the variance, the APCD and, finally, the market integration process. We will focus on how to do this in the empirical section below.

5 Empirical Results on Price Convergence

This section illustrates the methodology by applying it to two different datasets. First, we focus on the international market integration of commodities or *globalization*, a field of international economics and economic history that has aroused great interest in the last years; some recent examples are Clark (2015), Dobado et al. (2012, 2015) and Brunt and Cannon (2014). Second, we accommodate the methodology to analyze the convergence of the price of credit in the Eurozone, the long-term interest rate, since the euro enters circulation.

5.1 Wheat price convergence in 19th Century inland US

The empirical analysis in this section considers the historical annual series of wheat prices in seven cities in the USA, namely New York (NY), Philadelphia (P), Alexandria (A), Cincinnati (CI), Chicago (CH), Indianapolis (I), and San Francisco (SF). All of these cover a common period in the second part of the 19th Century.[7] The goal of this empirical analysis is to characterize the market integration of the Midwest and the East and West Coast of the United States.

Nominal prices are annual averages, and are expressed in US dollars. The selection of the markets is based on data availability and geographical representativeness. Markets in the coastal zones and inland territories in the 19th Century USA are represented. All the information on the sources of this dataset is given in the Appendix A. The series of nominal prices are shown in Figure 1, and their relative prices are given in Figure 2.

In Figure 1, the nominal series clearly wander, showing little or no affinity for a constant mean value. Looking at the latest observations in the sample, many of the

[7] NY covers 1800-1913, P covers 1800-1896, CI covers 1816-1913, CH and I cover 1841-1896, and SF covers 1852-1916.

series present a similar level and the cross-sectional dispersion is much lower than in the rest of the sample. This is confirmed in the corresponding relative price graphs in Figure 2.

Note that there is visual evidence in Figure 2 that the relative price series show a strong affinity for a constant parity value, close or equal to one. Note also that the convergence paths are similar in all cases. In the case of Chicago and Indianapolis, it seems that the prices for wheat are cheaper than in the other cities, and this holds for all of the 19th Century.

Figures 1 and 2 here

All nominal prices show similar statistical properties, namely they: (i) are integrated of order one; (ii) need to be transformed to natural logs to avoid heteroskedasticity, non-normality and non-linearity; (iii) fit a zero mean ARIMA(2,1,1) model; and (iv) have a small number of impulse interventions due to the American Civil War.[8] The AR(2) structures have two conjugate imaginary roots, leading to damped oscillations with a period of 5-13 years. A damping factor of around 0.5 represents quasi-cyclical behaviour, where the period describes the time elapsed (in years) from peak to trough. There is no evidence of over-differentiation in the univariate models of nominal prices as the null hypothesis of MA(1) noninvertibility is clearly rejected by the Generalized Likelihood Ratio (GLR) test of Davis et al. (1995). Moreover, the Shin-Fuller test does not reject the null hypothesis of non-stationarity in an alternative ARIMA(3,0,1) model. Consequently, I(1) is confirmed in all cases. These results are summarized in Table 1.

Table 1 and Figure 2 here

On the contrary, relative prices do not seem to be stationary, especially at the beginning of each sample (see Figure 2). In all cases, adding a deterministic convergence component seems to be enough to represent the transition path and having a stationary representation. The estimates for relative prices are reported in Table 2 for New York and Chicago as separate numeraires. The rest of the estimates are reported in the Table 3.[9]

[8] All the interventions are of an impulse type and do not significantly affect the results.

[9] All the estimated parameters are statistically different from zero, including the steady-state gains, g, and the convergence operator is stable in most cases. Q statistics by Ljung and Box (1978) show no signs of poor fit, except in a few cases, where an AR(2) operator might fit better. For simplicity, only AR(1) models are shown. The conclusions do not change significantly if a second-order representation is used. The initial specification for the stochastic part is according to the correlogram, AIC (Akaike, 1974) and HQ (Hannan and Quinn, 1979), which agree on the same initial specifications.

The model identified and estimated in each case is relatively simple: i) an AR(1) process for the stochastic part; ii) a mean, μ; and iii) a gradual and monotone convergence path, $\omega_0/(1-\delta_1 B)$, for the deterministic component. The estimated parameters, with their standard deviations and some diagnostic tools, are also presented. The estimation results for the A/NY prices are not included in Table 2 as it was not possible to find a stable convergence path in this case. The most likely reason is that this convergence process was so slow that the appropriate representation is close to a straight ramp with positive slope.

Tables 2 and 3 here

In all the analyses reported in Tables 2 and 3, we fix t^*_{ij} optimally as the year in which the convergence could have started. In each case we estimate t^*_{ij}, searching from the beginning to the end of the sample, as the value that maximizes the log-likelihood function. Table 4 shows the estimated starting date for the convergence path, t^*_{ij}, for every case. The earliest starting dates of the convergence processes are the pairs A/P and NY/P in 1836 and 1839, respectively. Two important factors played in favor of this earlier market integration. They are: i) markets with easy access to the Atlantic, and ii) relatively close to each other. Often, a shorter distance between cities implies an earlier market integration, although this is not always true. The size of the markets, which is not analyzed here, is certainly another important factor to explain the starting dates of the integration processes. For the remaining cities, the convergence process starts at around the American Civil War. A special case is San Francisco, whose convergence process starts at the beginning of the sample in 1853.

Table 4 here

Perfect homogeneity of wheat across markets is assumed as it simplifies the analysis, improves the performance of the unit root tests, and seems a realistic assumption. Tables 2 and 3 (last column) show the results of the unit root tests, using the Shin-Fuller test. In all cases, non-stationarity is clearly rejected when a transition term is introduced from t^*_{ij}. However, the same tests generally do not reject non-stationarity at any standard level when there is no convergence path in the model. This reveals evidence of potential asymptotic convergence in mean as catching-up.

These empirical results are self-evident in the graphs of nominal prices (Figure 1), and are confirmed in the relative price graphs (Figure 2). As the relative prices are transition-stationary, which fulfills the first requirement for asymptotic mean convergence, we performed the formal test for APCM using the models presented in Tables 2 and 3.

The results of the tests for APCM (H_0: $g_{ij} + \mu_{ij} = 0$) are presented in Table 5. Both the student-t and LR tests strongly confirm that the wheat price series converge in APCM for one half of the possible pairs in our data set. In the pairs with New York as the numeraire, prices converge in this APCM in strong sense, except for prices in San Francisco. One reason could be that these cities are on opposite sides of the country, and the Panama Canal was only available in 1914. Surprisingly, relative prices between San Francisco and Chicago have converged, in the APCM sense, at the end of the sample. The rail connection between these two cities might have made this possible. However, the Chicago, Cincinnati and Indiana prices had not converged until towards the end of the sample, in the APCM sense, even though the price gaps are very small.

Table 5 here

In order to examine APCD when $p = 2$, we use the Breusch-Pagan test and graphical analysis, as explained in Section 4.2. The residuals series are obtained from the models presented in Tables 2 and 3.

The results of the test are reported in Table 6.[10] The statistics for the joint null hypothesis that the residuals are homoscedastic are rejected in most cases at the 5% level. Therefore, asymptotic price convergence in variance cannot be rejected. The only exception is the case for relative prices A/P. In that case, the relative price dispersion, corrected for autocorrelation, is around 4%, and is the lowest in our sample. This means that the level of market integration between Alexandria and Philadelphia was already close to the maximum possible by that time and, therefore, no integration process existed during that period. Alexandria and Pennsylvania had probably already converged in variance.

For the rest of the cases, we draw the evolution of the residual standard deviations of the natural log of relative prices in order to see whether the heterosckedasticity detected is generated by a decreasing variance. The residual standard deviations are calculated using rolling windows with a span of $t = 35$, and are shown in Figure 3.[11] The figure clearly suggests that the standard deviations decrease over time, with both growing market efficiency and integration, although it is not so clear that they tend to zero in this sample. However as most of the standard deviations are converging to the same level, we assume that this 4% level was the maximum degree of integration that could be attained

[10]The parameter estimates of this model are available from the authors upon request.

[11]Different values of the size of the rolling window were tried. Obviously, the higher are the values, the smoother are the series. However, this does not change the trend of the standard deviations.

by that time due to technological limitations. Therefore, reaching this level can be interpreted as the fulfillment of the APCV.

Table 6 and Figure 3 here

Another additional result of our empirical analysis is the relation found between distance and: i) the estimated transaction costs at the beginning of the sample, $\hat{\mu}$, and ii) the evolution of the market integration (see Figure 3). Considering the former (i), distances between most of the cities are available in the legend of Figure 3. The values of $\hat{\mu}$ in Tables 2 and 3 confirm what was expected: the transaction costs at the beginning of the convergence process are positively correlated with the distance. For instance, by sorting the cities by distances with respect to NY we have: San Francisco, Chicago, Indianapolis, Cincinnati and Philadelphia, whose estimated absolute values for μ are $1.4, 0.45, 0.46, 0.21$ and 0.17, respectively. This relation holds for all the cities as numeraires. Regarding the latter (ii), when looking at Figure 3, we observe that the distance between markets was inversely related with the degree of market integration at the beginning of the convergence process. Hence, distance was a good predictor for the integration level between markets in the USA sample around 1870. However, this does not hold from around 1900, when the importance of the distance vanishes, except for the case of San Francisco which seems to be *too* far away from the rest.

In summary, the results show that during the 19th Century in USA there was: (i) a unique wheat spread market; and (ii) a powerful market integration process, in the sense of convergence in distribution. Regarding the mean, one-half of the examined pairs come to parity at the end of the 19th Century. In the rest of the cases, the gap is very small, although statistically different from zero. With respect to the variance, A/P did not improve its market integration degree as it was already very high at the beginning of the sample. Instead, the rest of the relative prices show a market integration process during this period. All of them but San Francisco, as a natural consequence of the distance, seem to converge to what could be assumed as the maximum integration level by that time.

5.2 Interest Rates Convergence in the Eurozone

This section studies the convergence of monthly data on several Eurozone long-term interest rates (average yields for 10 years government bonds); the source of the dataset is given in Appendix A. The exercise analyzes whether the elimination of the currency risk and the reduction of the transaction costs, following the establishment of fixed exchange rates in 1999 and the introduction of the common currency in 2002, could trigger a convergence process of the long-run interest rates in the Eurozone. Therefore, the example attempts to answer central questions about the market integration in the EMU long-term bonds markets. Did the interest rates converge in mean and/or variance after the euro was established? Can we find convergence clubs in the long-term interest rates? Notice that the Maastricht treaty requires relative convergence both, in prices and long-term interest rates for all of the Eurozone members. Here, however, we only focus on long-term interest rates.

We choose these variables because: i) it shows a somehow unusual application with no goods or services prices but credit prices; and ii) these are the interest rates commonly used to analyze the movements in the yield spread and the country risk premiun for sovereign bonds in the Eurozone.

We consider the interest rates in nine countries belonging to the EMU, namely Austria (AS), Belgium (BL), France (FR), Germany (GR), Greece (GC), the Netherlands (NT), Italy (IT), Portugal (PT) and Spain (SP). Therefore, the analysis covers more than 90% of the EMU economy in terms of GDP, and allows us to conduct a comparative study between core and peripheral countries. The data includes 108 monthly observations from 1/1999 to 12/2007 except for Greece, whose series starts from 1/2001 (84 monthly observations).[12] Data after 2007 are removed from the analysis due to the partial disintegration in the long-term bonds market caused by the subprime mortgage crisis and the subsequent financial crisis. We do this to avoid the potential bias against the convergence hypothesis that the inclusion of data after 2007 could cause.[13] We will come back to this idea at the end of the section.

As in the previous example, all the nominal prices are expressed in logarithms. This time the price (of credit) is written as $p_{i,t} = \log(1+yield_{i,t})$. The relative prices, commonly known as *sovereign risk premia*, are also expressed in logarithms and are defined as in (1). We only analyze the prices relative to Germany's 10 years bond interest rate as it is the common measure for the risk premium in the Eurozone. During this section, we will use indistinctively the terms nominal prices or interest rates, and relative prices or risk premia.

The levels of the interest rates studied are depicted in Figure 4. The cross-sectional dispersion across rates seems higher at the beginning of the sample than after 2003, suggesting that a convergence process may have occurred. The preliminary analysis

[12]The exchange rates of all the EMU members in the analysis, but Greece, were locked at fixed rates against each other on 1 January 1999. However, the conversion rate for the Greek drachma was effective on 1 January 2001.

[13]See Figure 4 in which the year 2008 is also depicted. The figure shows a breakdown in the convergence process around the beginning of 2008.

shows that all the nominal prices have similar statistical properties, as they: (i) are integrated of order one; and (ii) have an three-order autoregressive structure and a zero mean.[14] These results are summarized in Table 7. The estimated parameters, and some diagnostic tools are also reported. All the parameters are significantly different from zero, and there is no evidence of poor fit. In all cases, the SF test rejects the null hypothesis of nonstationarity and no sign of an invertible representation; when a second difference and a MA(1) operator to control over-differentiation are added, is found. Consequently, I(1) is confirmed in all the interest rates analyzed.

Figure 4 and Table 7 here

Following the methodology proposed, we now study the behavior of the relative prices. Figure 4 shows the risk premium with Germany for all the countries in our dataset. In this figure all but Greece, Italy and Portugal interest rates seem to converge in mean to that of Germany, although a deeper analysis is required to test this hypothesis.[15] The risk premia decreased, at least partially, because of and from the establishment of the common currency. For instance, the Austrian, Belgian and Spanish risk premia were significantly higher at the beginning of the sample, and practically converged to parity between 2005-07 (see Figure 4). In the cases of France and the Netherlands, the initial gap is lower but a convergence process also seems apparent.

Technically speaking, when looking at Figure 4, risk premia series do not seem to be stationary. At least a deterministic convergence component should be included to obtain a stationary representation. To do so, we set t^*_{ij} to December 2001 for all i, being j Germany in all the cases, as the euro entered circulation, physically, the first day of 2002. In this way, we test wether this event had an effect on the long-term interest rate convergence in the EMU. In any case, despite the historical or empirical reasons that justify the use of this month as the initial point for the convergence process, we perform a thorough search seeking alternative starting dates within the sample. We do this by comparing the value of the likelihood function for several models with different convergence operators and initial points. None of these attempts yielded superior results.

Hence, we fit model (2), with t^*_{ij} fixed to 12/2001, to every risk premium series and report the estimation results in Table 8. The model identified is relatively simple: (i) an AR(1) process for the stochastic part (although an AR(2) fits better in the Portugal case); (ii) a mean, μ; and (iii) a gradual and monotone convergence path, $\omega_0/(1-\delta_1 B)$. In all the cases, adding such a convergence component appears to be sufficient to represent

[14]The initial specification is according to *pacf* values, AIC and H-Q criteria. All three criteria are in accordance with the same specification.

[15]Contrary to the example in Section 5.1, the parity line is equal to zero as a relative price is expressed as $r_{ij,t} = \log[(1 + yield_{i,t})/(1 + yield_{j,t})]$.

the transition path and induce a stationary representation. Table 8 shows for all the cases that the estimates are statistically different from zero, the convergence operator is estimated to be stable and the diagnostic statistics reveal no sign of poor fit.

Table 8 here

Some simple conclusions can already be drawn from the estimates reported in Table 8. First: Greece, Portugal and Italy show the highest initial risk premia with 48, 30 and 30 basis points, respectively; while France and the Netherlands present the lowest with 12, and 14 basis points, respectively. Second: Greece, Belgium and Portugal show the highest estimated catching-up gains during the period with 24, 17 and 16 basis points, respectively; while France and Italy present the lowest reduction in the gap with only 6 and 7 basis points. While the reduction in the gap of Greece and Portugal, or the lack of it in France, can be explained by the size of the initial risk premium, the case of Italy is striking as its catching-up gain was clearly lower than expected, given its initial level.

Now we test the hypothesis of APCM in the EMU and describe the convergence process to an hypothetical unique interest rate level that could have emerged at some point between the years 2001 and 2007, before the collapse of the long-term bonds market began. Testing the APCM requires a stationary process or a transition-stationary process in which the convergence path is stable. The latter condition holds for all the risk premia (see Table 8). Regarding stationarity, SF test rejects nonstationarity in all cases at a 10% level when the transition term is introduced from t^*_{ij}. However it is not rejected at the 5% level for Belgium, Italy and Spain. The same test does not reject nonstationarity at any standard level for the risk premia when the transition path is not included in the model. This reveals some evidence of asymptotic convergence as "catching-up," and implicitly rejects convergence as "steady-state". Therefore, we can now proceed to analyze the long-term remaining gap, τ_{ij}.

The results of the tests for APCM using the student-t and the LR statistics are presented in Table 9. Both tests coincide confirming that the interest rate levels in Germany and the rest of the countries included in the analysis do not fulfill the APCM in strict sense as the remaining gap is statistically different from zero. However, according to the values of the student-t and the LR statistics, nominal interest rates in Austria, Belgium and France were closer to converge in mean to that of Germany than those of Greece, Italy, Portugal or Spain. The Netherlands is somehow in between these two groups.

Table 9 here

The grouping produced by the analysis is not unexpected and fits the prediction of the economic theory. Austria, Belgium and France (the core countries) have relatively similar economies and their common border with Germany facilitates commercial activities and influence. On the other hand, Greece, Italy, Portugal and Spain (the peripheral countries) have much more in common with each other and the mere physical distance with Germany could reduce its influence on the evolution of the nominal interest rates.

Finally, we perform the formal test for APCV for all the relative prices. As explained in Section 4.2, we first use the BP test to conclude whether the variance of the unexpected shocks affecting the risk premia could have fallen during this period. These shocks are estimated by the residuals obtained from the models presented in the Table 9.

The results of the BP test for the risk premia are reported in the last column of Table 9. The statistics for the null hypothesis that the residuals are homoscedastic are not rejected in most cases. Homoscedasticity is rejected only for the Italy premiun risk at the 5% level. But in this case, the potential heteroscedasticity detected is generated by a increasing variance. The evolution over time of the residual standard deviation, calculated using rolling windows with a span of $t = 12$, indicates that the standard deviation is increasing. As a consequence, APCV can be rejected in all the cases.

Now we briefly discuss the results of this empirical exercise. As it was noticed in Section 2, one of the assumptions for market integration is that prices represent homogenous or quasi-homogenous products. Assuming this here implies that, after the implementation of EMU, the capability of the countries to repay the debt was quasi-homogenous or, maybe more likely, the fact that a potential bailout was implicit for members in case of default made the risk quasi-homogenous. The rejection of the convergence in mean hypothesis could then be related with the deviation of this assumption. Although the quality dissimilarities of the debt could keep constant from 2002 for a while, clearly the fictitious degree of homogeneity was broken by the precarious situation of some state members at some time. Interestingly, although not shown here, one could use the methodology proposed to estimate the time when this segmentation took place. It suffices to sequentially add observations and conduct the unit root test to each risk premium (with the transition path), and determine when the test does not reject nonstationarity. This way, one may obtain an estimate of when the disintegration of the nominal interest rate of each country with respect to that of Germany started.

6 Concluding Remarks

The paper examines an interesting and timely question: in a world where globalization is such a buzzword, a procedure to test whether markets are integrated or not is suggested. The standard measure is the Law of One Price, however the main contribution of the paper is to offer a model illustrating other aspects of convergence and identifying their statistical properties. The approach is based on cointegration and conditional variance analyses, but its flexibility makes it compatible with either steady-state or catching-up convergence. The paper showed the equivalence between the strict Law of One Price (when two prices are strictly the same), perfect market integration, and price convergence in distribution.

Our procedure has five advantages over the alternatives presented in the literature: 1) It requires a specific form of cointegration (the stationarity of the relative prices) and a reduction in the residual's volatility of the relative prices' ARMA model. Hence, it is much more demanding than mere cointegration, and reduces the type II error when testing the null hypothesis of market integration. 2) The nonstationarity (Shin and Fuller, 1998) and noninvertibility (Davis et al., 1995) tests used have better statistical properties than their alternatives, making the results more reliable and robust. 3) The measure of market integration produced allows comparisons over time and across space (see Figure 3). 4) It is not affected by region-specific factors because these are removed when filtering the stationary relative prices by ARMA models. 5) It is simple, because it only requires some tests and univariate ARMA models, which have a clear advantage with respect to VAR, VARMA or alternative multivariate representations in terms of computational stability, and simplicity in the specification and interpretation.

The definitions and methods presented here are useful in areas besides applied microeconomics, international economics and economic history. Macroeconomic aggregates, such as monetary markets or price levels, can be analyzed in the same way regarding economic integration. Our methods could also be helpful to understand related macro-market and economic integration problems, and can also be extended to the study of convergence clubs, in the sense of Quah (1997).

Two empirical analysis show how to use the proposed methodology, leading to interesting conclusions. The first exercise reveals that most of the USA inland grain markets became more integrated during the second half of the 19th Century. Specifically, they showed asymptotic price convergence in mean and variance (therefore in distribution), with few exceptions. The second application demonstrates that the EMU long-term interest rates converge as catching-up during the period 2002-2007, although they failed to converge in mean and variance and hence, in distribution.

7 Acknowledgements

The authors wish to thank Maria T. González-Pérez, Viktor Todorov and a reviewer for very helpful comments and suggestions. Garcia-Hiernaux gratefully acknowledges financial support from Ministerio de Ciencia e Innovación, ref. ECO2011-23972 and

Fundación Ramón Areces. McAleer is most grateful to the Australian Research Council and National Science Council, Taiwan for financial support.

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Appendix

A Data sources

A.1 US wheat prices

New York City, 1800-1913 Cole, A.H., 1938. Wholesale Commodity Prices in the United States, 1700-1861: Statistical Supplement. Harvard University Press, Cambridge; Rubinow, I.M., 1908. Russian Wheat and Wheat Flour in European Markets. USDA Bureau of Statistics Bulletin No. 66, Washington: GPO; Vierteljahrshefte zur Statistik, various years.

Philadelphia, 1800-1896 Bezanson, A. et al., 1937. Wholesale Prices in Philadelphia, 1784-1861, Part II. University of Pennsylvania Press, Philadelphia; Bezanson, A. et al., 1954. Wholesale Prices in Philadelphia 1852-1896. University of Pennsylvania Press, Philadelphia; and Cole, A.H., 1938. Wholesale Commodity Prices in the United States, 1700-1861: Statistical Supplement. Harvard University Press, Cambridge.

Alexandria, 1801-1913 Peterson, A.G., 1929. Historical Study of Prices Received by Producers of Farm Products in Virginia, 1801-1927. Technical Bulletin of the Virginia Polytechnic Institute.

Cincinnati, 1816-1913 Berry, T.S., 1943. Western Prices Before 1861: A Study of the Cincinnati Market. Harvard University Press, Cambridge; Cincinnati Price Current, various years; Cole, A.H., 1938. Wholesale Commodity Prices in the United States, 1700-1861: Statistical Supplement. Harvard University Press, Cambridge; and White, H.E., 1935. An Economic Study of Wholesale Prices at Cincinnati, 1844-1914. Cornell University, Ph.D. dissertation.

Chicago, 1841-1913 NBER Macrohistory Database.

Indianapolis, 1841-1913 Houk, H.J., 1942. A Century of Indiana Farm Prices, 1841 to 1941. Purdue University, Ph.D. dissertation.

San Francisco, 1852-1913 Annual Report of the Chamber of Commerce of San Francisco. Neal Publishing Company. San Francisco, various years; Annual Report of the San Francisco Merchants Exchange. Commercial News Publishing, San Francisco, various years; Annual Report of the San Francisco Produce Exchange. Commercial Publishing Company, San Francisco, various years; Berry, T.S., 1984.

Early California: Gold, Prices, Trade.The Bostwick Press, Richmond; Davis, H., 1894. Appendix I: Tables Relating to California Breadstuffs, The Journal of Political Economy, 2 (4) 600-612; Sacramento Union, various years; Transactions of the California State Agricultural Society. O.M. Clayes, Sacramento, various years.

A.2 EMU long-term interest rates

The source of the data for this example is Eurostat. The data is available in the website: `http://ec.europa.eu/eurostat/en/web/products-datasets/-/IRT_LT_MCBY_M`

B Figures and Tables

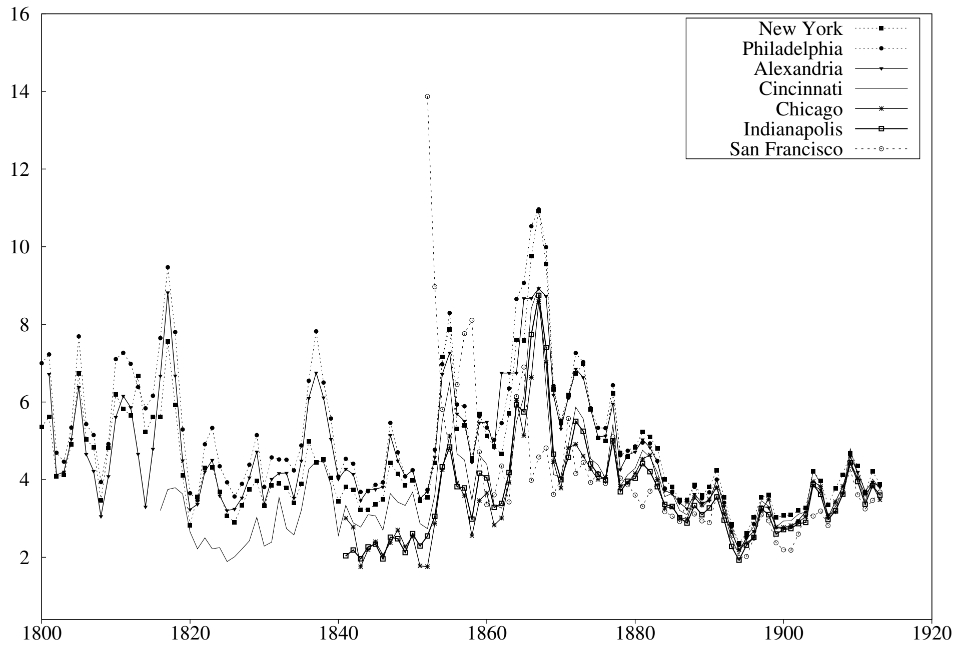


Figure 1: Nominal Prices of Wheat in 19th Century USA

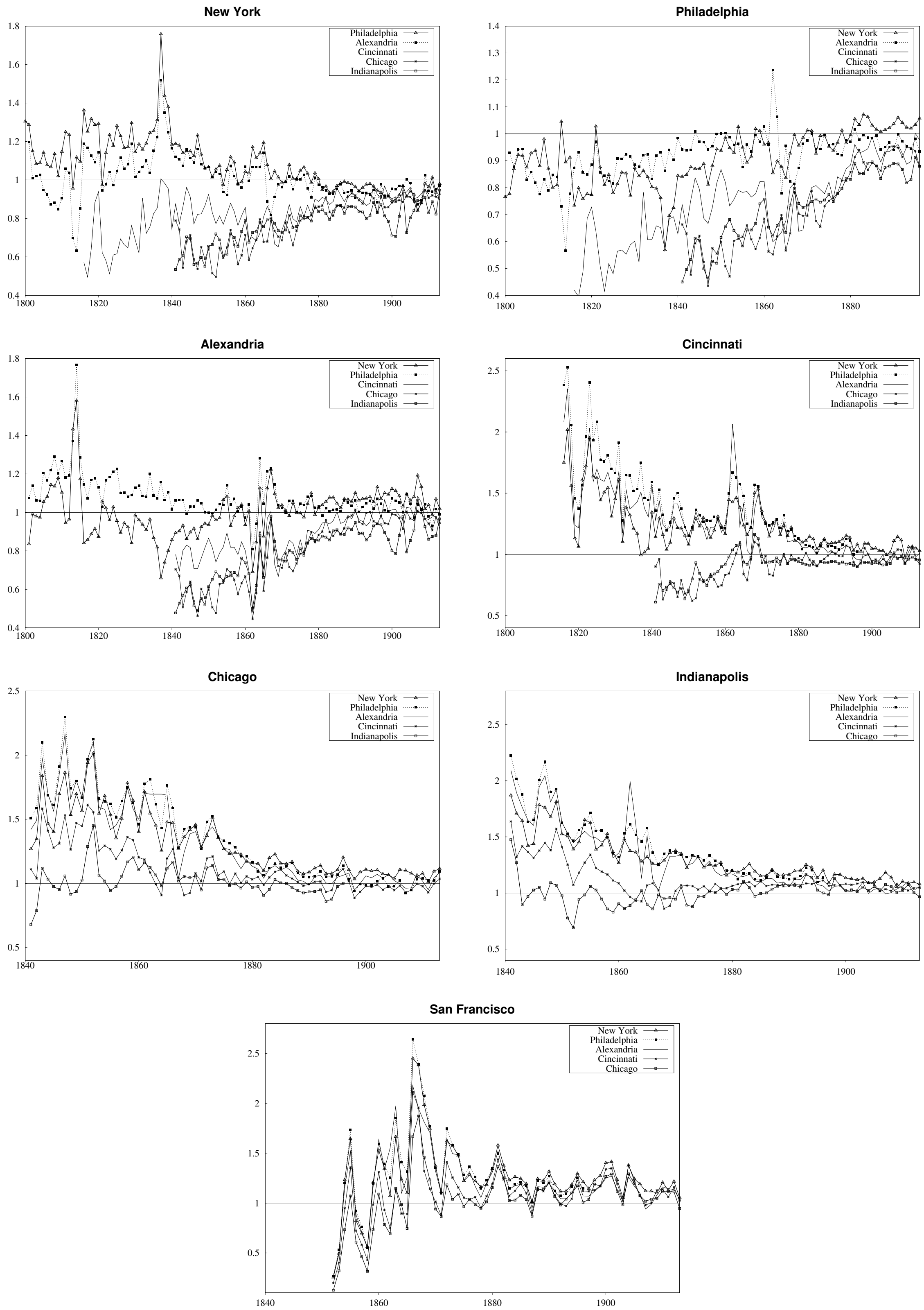
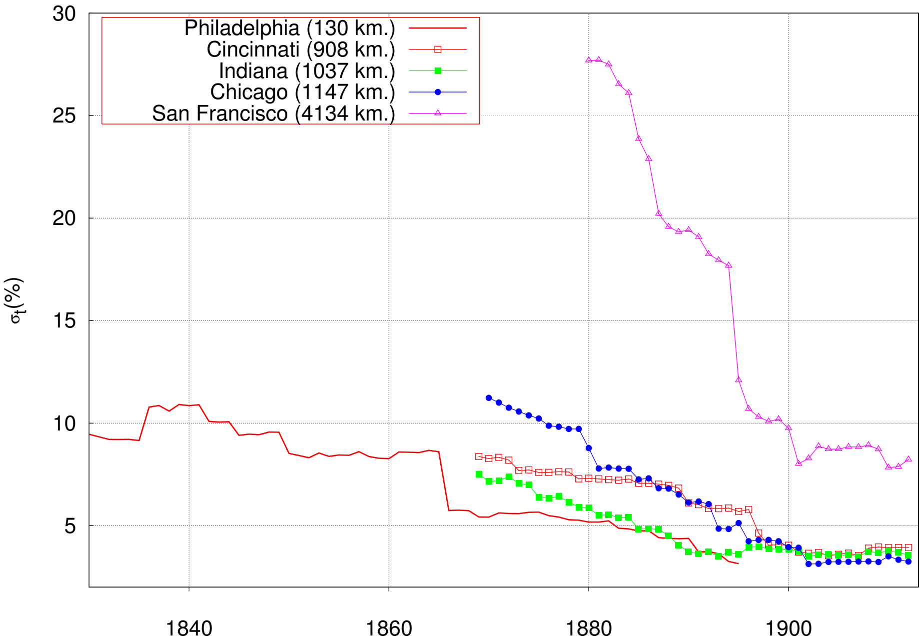


Figure 2: Relative Prices of Wheat in 19th Century USA



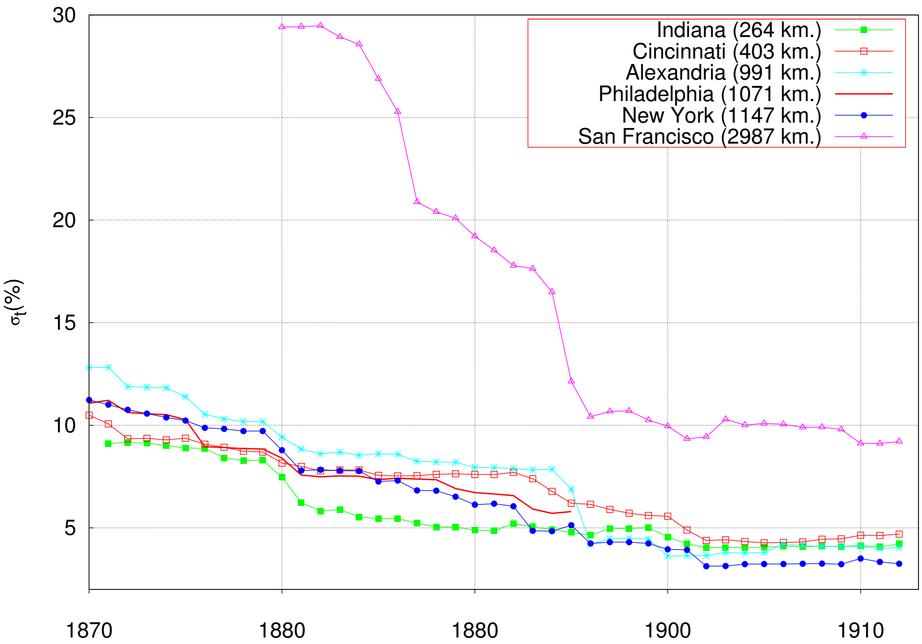
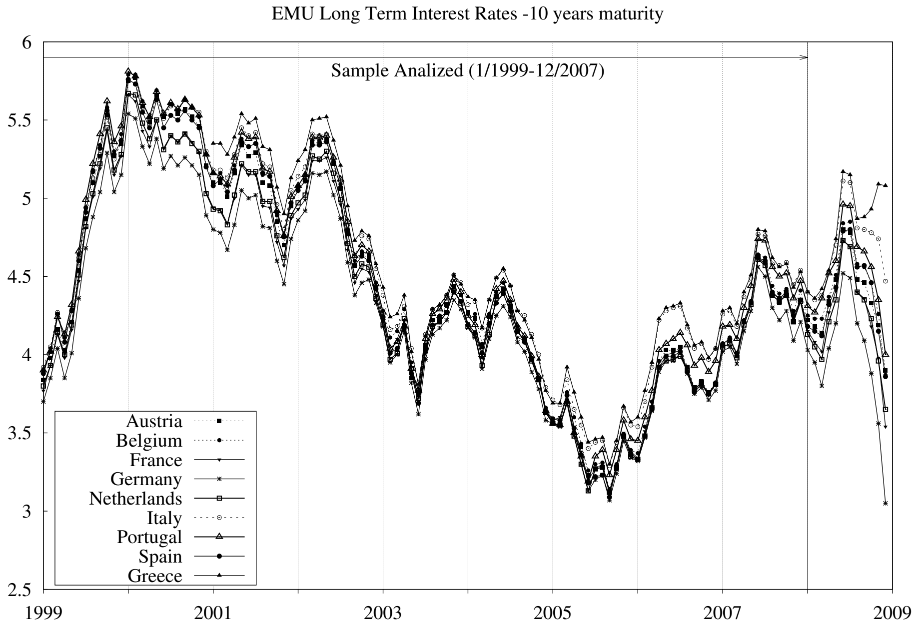


Figure 3: Market Integration in 19th Century USA.The series are the residual standard deviations of the natural log of relative prices calculated using rolling windows with a span of $t = 35$. Top plot: New York as numeraire. Bottom plot: Chicago as numeraire.



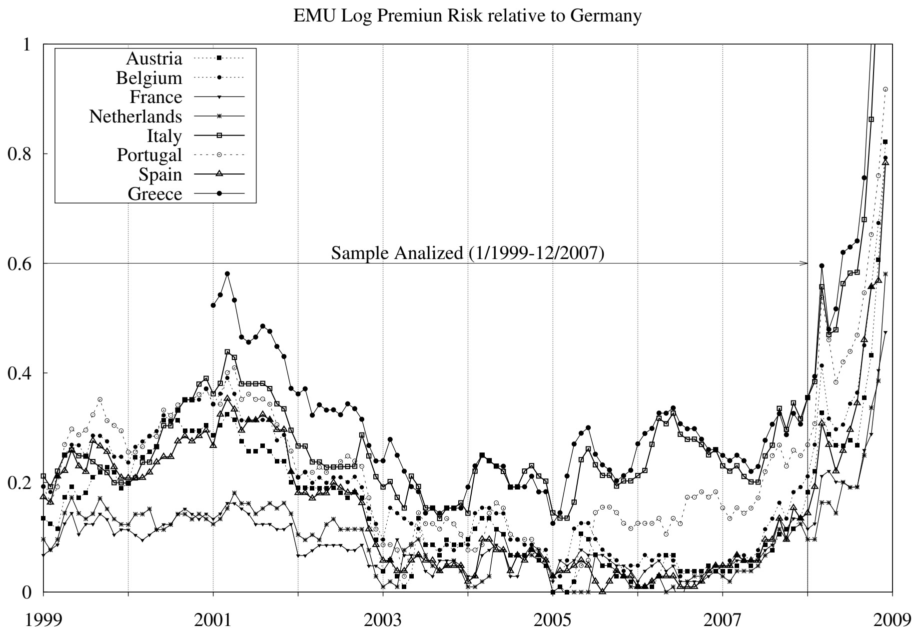


Figure 4: Market integration in the Eurozone bonds 1999-2008. Top plot: EMU long-term nominal interest rates in logs. Bottom plot: Risk premia with Germany in logs.

Table 1: Estimated Univariate Models of Wheat Prices in Log Differences(1) (Prices in Ag./liter of grain)

Sample	Variable (Mnemonics)	AR(2) $\hat{\phi}_1$ (s.e.)	$\hat{\phi}_2$ (s.e.)	MA(1) $\hat{\theta}$ (s.e.)	Resid. Std.Dev. (%)	ACF(2) $Q_{(9)}$	SF(3) $H_0 : \phi = 1$	GLR(4) $H_0 : \theta = 1$
1800-1913	New York	0.63	-0.34	0.62	15.7	7.1	1.28	6.85
	(NY)	(0.14)	0.09	0.13				
1841-1913	Chicago	0.69	-0.31	0.70	18.0	3.3	0.6	2.4
	(CH)	0.18	0.12	0.16				
1800-1896	Philadelhia	0.78	-0.38	0.67	17.0	6.3	0.7	5.0
	(P)	0.15	0.10	0.14				
1801-1913	Alexandria	0.71	-0.44	0.64	16.1	10.9	0.7	12.0
	(A)	0.12	0.09	0.11				
1816-1913	Cincinatti	0.72	-0.26	0.78	17.5	3.5	0.8	3.5
	(CI)	0.15	0.11	0.13				
1841-1913	Indianapolis	0.62	-0.23	0.62	16.1	7.1	0.1	2.4
	(IN)	0.22	0.12	0.21				
1852-1913	San Francisco	0.49	-0.38	0.58	18.5	17.0	0.0	10.1
	(SF)	0.18	0.13	0.15				

Notes: (1) Eighteenth and Nineteenth Century yearly prices in gr.Ag./liter. (2) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (3) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary against an alternative ARIMA(3,0,1) model. (4) GLR: Generalized Likelihood Ratio (GLR) test of Davis, Chen and Duismuir (1995) for the null hypothesis of noninvertibility of an MA(1) operator.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 2: Models of Relative Prices Including a Convergence Path

Sample	Variable (Mnemonics)	AR(1) $\hat{\phi}_1$ (s.e.)	Convergence Paramenters $\hat{\omega}_0$ (s.e.)	$\hat{\delta}_1$ (s.e.)	$\hat{l}$ (s.e.)	$\hat{g}$ (s.e.)	Mean $\hat{\mu}$ (s.e.)	Resid. Std.Dev. (%)	ACF$^{(1)}$ $Q_{(9)}$	SF$^{(2)}$ $H_0 : \phi = 1$
Panel A: Relative prices with New York as Numeraire										
1841-1914	Chicago	0.20	0.026	0.94	16.6	0.46	-0.45	7.7	8.9	37.5**
	(CH/NY)	(0.11)	(0.007)	(0.03)	(8.6)	(0.11)	(0.01)			
1800-1896	Philadelphia	0.48	-0.076	0.52	17.2	-0.16	0.17	6.3	7.1	15.2**
	(P/NY)	(0.09)	(0.025)	(0.14)	(12.8)	(0.03)	(0.01)			
1801-1913	Alexandria	-		-		-		-	-	-
	(A/NY)									
1816-1914	Cincinatti	0.44	0.016	0.91	10.5	0.18	-0.21	6.1	13.2	21.2**
	(CI/NY)	(0.10)	(0.011)	(0.08)	(11.0)	(0.06)	(0.01)			
	Indianapolis	0.45	0.023	0.94	17.2	0.42	-0.46	5.6	8.4	15.2
	(IN/NY)	(0.11)	(0.009)	(0.03)	(12.8)	(0.13)	(0.02)			
1852-1914	San Francisco	0.46	-0.84	0.47	0.92	-1.6	1.4	20.0	7.5	16.6**
	(SF/NY)	(0.10)	(0.21)	(0.10)	(0.40)	(0.2)	(0.2)			
Panel B: Relative prices with Chicago as Numeraire										
1800-1896	Philadelphia	0.25	-0.039	0.92	12.6	-0.54	0.53	8.6	17.2	23.2**
	(P/CH)	0.13)	(0.010)	(0.03)	(5.8)	(0.10)	(0.02)			
1801-1913	Alexandria	0.60	-0.080	0.81	4.4	-0.43	0.45	8.6	6.9	13.8**
	(A/CH)	(0.09)	(0.039)	(0.09)	(2.9)	(0.06)	(0.04)			
1816-1914	Cincinatti	0.28	-0.10	0.59	1.5	-0.25	0.29	8.0	9.4	29.0**
	(CI/CH)	(0.11)	(0.04)	(0.19)	(1.1)	(0.03)	(0.02)			
	Indianapolis	0.39	-0.021	0.80	4.2	-0.11	0.067	6.6	17.3	23.0**
	(IN/CH)	(0.11)	(0.017)	(0.16)	(4.5)	(0.03)	(0.019)			
1852-1914	San Francisco	0.51	-0.97	0.53	1.2	-2.1	2.1	21.2	7.6	14.3**
	(SF/CH)	(0.11)	(0.26)	(0.11)	(0.6)	(0.2)	(0.2)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 3: Models of Relative Prices Including Convergence Path

Sample	Variable (Mnemonics)	AR(1) $\hat{\phi}_1$ (s.e.)	Convergence $\hat{\omega}_0$ (s.e.)	Paramenters $\hat{\delta}_1$ (s.e.)	$\hat{l}$ (s.e.)	$\hat{g}$ (s.e.)	Mean $\hat{\mu}$ (s.e.)	Resid. Std.Dev. (%)	ACF$^{(1)}$ $Q_{(9)}$	SF$^{(2)}$ $H_0 : \phi = 1$
Panel C: Relative prices with Philadelphia as Numeraire										
1801-1913	Alexandria	0.39	0.024	0.73	2.8	0.09	-0.13	4.3	5.0	32.2**
	(A/P)	(0.09)	(0.015)	(0.16)	(2.3)	(0.02)	(0.01)			
1816-1914	Cincinatti	0.51	0.046	0.81	4.4	0.25	-0.30	6.9	23.8	13.1**
	(CI/P)	(0.11)	(0.024)	(0.12)	(3.5)	(0.06)	(0.02)			
	Indianapolis	0.83	0.19	0.41	0.70	0.25	-0.52	6.2	16.2	1.5*
	(IN/P)	(0.09)	(0.06)	(0.25)	(0.74)	(0.06)	(0.02)			
1852-1914	San Francisco	0.52	-1.7	0.48	0.91	-3.3	3.1	22.4	4.9	9.6**
	(SF/P)	(0.12)	(0.9)	(0.13)	(0.46)	(1.0)	(1.0)			
Panel D: Relative prices with Alexandria as Numeraire										
1816-1914	Cincinatti	0.38	0.030	0.89	8.1	0.28	-0.25	8.6	13.6	24.6**
	(CI/A)	(0.10)	(0.016)	(0.07)	(5.8)	(0.06)	(0.02)			
	Indianapolis	0.67	0.043	0.89	8.1	0.41	-0.44	8.8	7.0	9.0**
	(IN/A)	(0.09)	(0.028)	(0.09)	(5.8)	(0.10)	(0.05)			
1852-1914	San Francisco	0.51	-0.80	0.47	0.90	-1.5	1.3	18.9	10.2	14.5**
	(SF/A)	(0.10)	(0.19)	(0.11)	(0.04)	(0.2)	(0.2)			
Panel E: Relative prices with Cincinatti as Numeraire										
1816-1914	Indianapolis	0.57	0.096	0.64	1.8	0.27	-0.31	5.4	13.2	10.8**
	(IN/CI)	(0.10)	(0.033)	(0.12)	(1.0)	(0.03)	(0.03)			
1852-1914	San Francisco	0.40	-0.84	0.50	1.0	-1.7	1.6	20.8	13.4	19.4**
	(SF/CI)	(0.12)	(0.23)	(0.12)	(0.5)	(0.2)	(0.2)			
Panel F: Relative prices with Indianapolis as Numeraire										
1852-1914	San Francisco	0.40	-0.54	0.68	2.1	-1.7	1.6	20.5	12.5	10.8
	(SF/P)	(0.12)	(0.21)	(0.10)	(1.0)	(0.2)	(0.2)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 4: Optimal Starting Time t^*_{ij} for the Convergence Path

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	1866	1839	–	1874	1856	1853
Chicago	1866	X	1866	1874	1860	1863	1853
Philadelphia	1839	1866	X	1836	1875	1866	1853
Alexandria	–	1874	1836	X	1874	1870	1853
Cincinnati	1874	1860	1875	1874	X	1856	1853
Indianapolis	1856	1863	1866	1870	1856	X	1853
San Francisco	1853	1853	1853	1853	1853	1853	X

Table 5: Testing Asymptotic Price Convergence in Mean by Pairs

Panel A: Long Run Gap Estimation Results and t-student test for convergence in mean[1]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	0.01	0.015	–	0.05	-0.05	-0.24**
Chicago	0.01	X	-0.01	0.017	0.034**	-0.043**	-0.043
Philadelphia	0.015	-0.01	X	-0.042**	-0.045	-0.21	-0.043
Alexandria	–	0.017	-0.042**	X	0.028	-0.034	-0.20**
Cincinnati	0.05	0.034**	-0.045	0.028	X	-0.047**	-0.097**
Indiana	-0.05	-0.043**	-0.21	-0.034	-0.047**	X	-0.061
San Francisco	-0.24**	-0.043	-0.043	-0.20**	-0.097**	-0.061**	X

Panel B: LR test for convergence in mean[2]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	0.02	0.27	–	0.18	0.09	10.8**
Chicago	0.02	X	0.01	0.12	3.9**	4.1**	0.50
Philadelphia	0.27	0.01	X	4.4**	0.45	4.8**	7.1**
Alexandria	–	0.12	4.4**	X	0.40	0.01	8.1**
Cincinnati	0.18	3.9**	0.45	0.40	X	4.4**	3.8*
Indiana	0.09	4.1**	4.8**	0.01	4.4**	X	1.33
San Francisco	10.8**	0.50	7.1**	8.1**	3.8*	1.33	X

Notes: (1) In the student-t test of Asymptotic Price Convergence in Mean, H_0: $g_{ij} + \mu_{ij} = 0$ is that the long run gap between nominal prices is zero. (2) Likelihood Ratio (LR) test of Asymptotic Price Convergence in Mean, where H_0 is the same as above.

*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 6: Testing Asymptotic Price Convergence in Variance by Pairs[1]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	14.2**	4.3**	–	10.2**	12.0**	12.3**
Chicago	14.2**	X	14.0**	6.6**	9.6**	5.45**	11.6**
Philadelphia	4.3**	14.0**	X	0.2	4.6**	5.7**	8.3**
Alexandria	-	6.6**	0.2	X	3.4*	6.3**	16.5**
Cincinnati	10.2**	9.6**	4.6**	3.4*	X	8.6**	11.2**
Indiana	12.0**	5.45**	5.7**	6.3**	8.6**	X	12.2**
San Francisco	12.3**	11.6**	8.3**	16.5**	11.2**	12.2**	X

Notes: (1) Breusch-Pagan test is a Likelihood Ratio test of Asymptotic Price Convergence in Variance, where H_0 is homoscedasticity. If the null hypothesis is rejected, there is conditional heteroscedasticity, with variance decreasing with time starting at t^*.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 7: Estimated Univariate Models of Long term Interest Rates in Log Differences[1]

Sample	Variable (Mnemonics)	AR(1) $\hat{\phi}$ (s.e.)	AR(2) $\hat{\phi}_{11}$ (s.e.)	$\hat{\phi}_{2}1$ (s.e.)	Resid. Std.Dev. (%)	ACF[2] $Q_{(9)}$	SF[3] $H_0 : \phi = 1$
1999-2007	Austria	0.57	-0.27	0.28	0.15	25.9	18.7
	(AS)	(0.13)	(0.14)	(0.12)			
	Belgium	0.54	-0.24	-0.27	0.14	25.3	19.9
	(BL)	(0.14)	(0.15)	(0.13)			
	France	0.52	-0.23	-0.30	0.15	28.4	20.1
	(FR)	(0.15)	(0.16)	(0.13)			
	Germany	0.51	-0.24	-0.28	0.15	28.6	21.3
	(GR)	(0.15)	(0.16)	(0.13)			
	Netherlands	0.54	-0.25	-0.32	0.15	29.1	20.1
	(NT)	(0.13)	(0.14)	(0.12)			
	Italy	0.54	-0.26	-0.28	0.14	26.4	20.0
	(IT)	(0.14)	(0.15)	(0.13)			
	Portugal	0.56	-0.23	-0.28	0.14	26.3	18.9
	(PT)	0.18	0.13	0.15			
	Spain	0.54	-0.24	-0.29	0.15	25.1	19.7
	(SP)	(0.14)	(0.15)	(0.12)			
2001-2007	Greece	0.49	-0.15	-0.28	0.13	29.9	17.3
	(GC)	0.18	0.19	0.14			

Notes: (1) Average yields for 10 years government bonds. (2) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first 36 lags. (3) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.

Table 8: Models of the Log Premiun Risk Series Including a Convergence Path: Relative Interest Rates with Germany as Numeraire

Sample	Variable	AR(2)		Convergence Paramenters				Mean	Resid.	ACF[(1)]	SF[(2)]
	(Mnemonics)	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\omega}_0$	$\hat{\delta}_1$	$\hat{l}$	$\hat{g}$	$\hat{\mu}$	Std.Dev.	$Q_{(9)}$	$H_0 : \phi = 1$
		(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(%)		
1999-2007	Austria	0.88		-0.014	0.92	10.9	-0.16	0.23	0.023	37.5	3.1**
		(0.05)		(0.009)	(0.06)	(8.4)	(0.04)	(0.03)			
	Belgium	0.91		-0.011	0.94	15.0	-0.17	0.26	0.021	30.5	1.6*
		(0.05)		(0.007)	(0.05)	(11.8)	(0.06)	(0.03)			
	France	0.77		-0.013	0.77	3.4	-0.06	0.12	0.017	46.7	8.2**
		(0.07)		(0.012)	(0.22)	(4.3)	(0.01)	(0.01)			
	Netherlands	0.76		-0.0092	0.90	9.2	-0.09	0.14	0.021	52.7	8.7**
		(0.07)		(0.0058)	(0.07)	(6.7)	(0.02)	(0.01)			
	Italy	0.91		-0.034	0.50	1.0	-0.07	0.30	0.026	27.8	1.8*
		(0.04)		(0.022)	(0.24)	(1.0)	(0.02)	(0.04)			
	Portugal	1.13	-0.26	-0.026	0.84	5.2	-0.16	0.30	0.026	45.7	4.6**
		(0.09)	(0.09)	(0.018)	(0.12)	(4.5)	(0.04)	(0.03)			
	Spain	0.93		-0.081	0.43	0.74	-0.14	0.23	0.021	38.0	1.1*
		0.04)		(0.019)	(0.14)	(0.22)	(0.04)	(0.03)			
2001-2007	Greece	0.87		-0.035	0.85	5.7	-0.24	0.48	0.026	33.8	3.1**
		(0.05)		(0.016)	(0.07)	(3.0)	(0.05)	(0.04)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first 36 lags. (2) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.

Table 9: Convergence Tests for Interest Rates

Relative Interest Rates	$\hat{\tau}_{ij}$	s.e.	T-student[1]	LR[2]	Breusch-Pagan[3]
Austria	0.063	0.03	2.0**	2.1**	0.2
Belgium	0.088	0.04	2.0**	1.6*	0.0
France	0.062	0.01	6.2**	2.1**	0.7
Netherlands	0.044	0.01	2.9**	14.3**	0.3
Italy	0.230	0.03	7.7**	2.5**	2.8*
Portugal	0.140	0.03	4.7**	5.3**	0.0
Spain	0.086	0.03	3.1**	8.1**	0.0
Greece	0.250	0.03	8.3**	2.0**	0.2

Notes: (1) In the student-t test of Asymptotic Price Convergence in Mean, H_0: $\hat{\tau}_{ij} = g_{ij} + \mu_{ij} = 0$ is that the long run gap between nominal prices is zero. (2) Likelihood Ratio (LR) test of Asymptotic Price Convergence in Mean, where H_0 is the same as above.
(3) Breusch-Pagan test is a Likelihood Ratio test of Asymptotic Price Convergence in Variance, where H_0 is homoscedasticity. If the null hypothesis is rejected, there is conditional heteroscedasticity, with variance decreasing with time starting at t^*.
*(**)Rejects the null hypothesis at the 10% (5%) level of significance.