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TWO APPROACHES TO QUANTIFY THE CONCEPT OF EQUITABLE INCOME DISTRIBUTION

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I. ALTERNATIVE DEFINITIONS OF EQUITY

In sociopolitical discussions the phrase of equitable income distribution constitutes a central concept. It may even be maintained that the background of conflicts on incomes is the feeling that incomes show differences that surpass equitable differences. The solution of such conflicts might be furthered if more precise concepts than just feelings could be developed. Best of all would be, of course, the elaboration of measurable concepts and actual measurement. The present essay tries to clarify some recent contributions made to such an elaboration by the author, in collaboration with various colleagues (BERKOUWER *et al.*, 1978; BOUMA *et al.*, 1976; PEN, 1977; VAN PRAAG, 1978). It seems desirable to introduce the subject by the statement that there is no agreement among social scientists about the definition of an equitable distribution. In addition the reader should be reminded of the necessity not to consider incomes only, but at least also some characteristics of the source(s) of income, in particular the efforts that have to be made in order to obtain the income. So in what follows it will be assumed, often tacitly, that the distributions considered are those of income and efforts.

Disregarding the simplist thesis that to arrive at an equitable distribution we only have to eliminate income from capital, it seems that three definitions of an equitable distribution are most frequently defended.

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Definition I maintains that an equitable income distribution is characterized by equality of each person's income to the contribution she or he makes to national product. This may be called the liberalist definition. It implies that incomes from capital, from socio-economic status of parents and from exceptional inherited endowments with personality traits are all considered equitable. Especially the implication that nature is equitable deserves attention.

Definitions II and III may be dealt with by the introduction of the concept of admissible income differences, which is just another way of describing an equitable income distribution. The two definitions have in common that income differences compensating for differences in effort are acceptable and sometimes called compensatory differences. Definition II considers as admissible also differences due to the prevailing scarcity of productive personality traits. These differences are instrumental in allocating qualified individuals to the jobs where they are most productive. For the adherents to Definition II this is a reason to accept such differences as equitable. Adherents to Definition III do admit that 'scarcity rents' are useful for the most productive allocation of the individuals concerned, but do not agree that this makes such differences equitable. Their definition can only be presented in the form that an equitable income distribution equalizes welfare among all individuals: those who make greater efforts are compensated by income differences which in fact means that everybody is, then, equally 'happy'. This argument should be restricted to those factors of happiness that can be socially organized. It excludes differences in happiness due to purely personal elements such as religion, friendship, love or physiological handicaps of a serious nature[1].

This essay will be based on Definition III; this is a value judgement and should therefore be explicitly stated as such. It is useful to be aware, however, of the possibility that society develops in such a direction that Definitions II and III coincide (*cf.* TINBERGEN,

1. It may contribute to modesty for economists that, according to an inquiry by LEVY and GUTTMAN (1975), a happy family life, a satisfactory use of leisure, living within a satisfactory social group and good health contribute more than two-thirds to the variance in happiness and the two most important economic factors (income and job satisfaction) 13 per cent only. Of the variance explained by the six most important factors the contributions are, respectively, 84 and 16 per cent!

1977). Such a coincidence would be attained if, by a sufficient expansion of education and training all existing scarcities could be eliminated. This elimination would be reached if for every job to be filled in society a person can be found whose productive personality traits are identical to the traits required. Whether scarcity can be overcome again depends on the extent to which the relevant personality traits can be learned, given the inherited traits. The answer has to come from education research.

II. A PRECONDITION: MEASURABILITY OF WELFARE OR UTILITY

The application of Definition III of an equitable distribution crucially depends on the possibility to measure welfare (or, in older economic language, utility). In other words: quantification of the concept of equitable income distribution requires quantification of welfare. Whereas in PIGOU's opinion (PIGOU, 1920) such a quantification should not be excluded, most economists until recently followed PARETO (PARETO, 1896) in denying that possibility. Partly as a consequence of the general tendency of many sciences to expand beyond their traditional territory (TINBERGEN, 1980) some economists have reintroduced the measurement of welfare as part of the discipline. This essay deals with two approaches undertaken recently, admittedly both still in their childhood. As an introduction to the subject the present section discusses some elements appearing in a welfare (or utility) function. To the present author it seems appropriate to distinguish between variables, parameters and coefficients, with some important subdivisions. The main feature of the three elements is the degree of changeability. As the name indicates variables are changeable entities. Parameters, as usual in mathematical terminology, are constants for individual observations (in our case for human individuals or for households) but need not have the same constant value for all observations. Rather they characterize the observed individual. Coefficients, finally, are constants and characterize the impact of variables and parameters on utility. Ideally they would characterize the 'human race', but occasionally one may want to limit this characterization to parts of the human race or the human race in a certain cultural setting or stage.

As usual each attempt to classify elements has its limits or limitations. Thus, for practical purposes we may call some element a parameter, but at a closer look it may be variable provided we consider a long period of time. Family size provides an example.

This brings us to the practical aspect to mention by their concrete name a number of usually important elements. For our subject important examples clearly are income and job or occupation, the latter characterized by job evaluation variables which we will also indicate by the phrase intensities of required capabilities. Capabilities is a qualitative concept, such as intelligence, manual or physical strength or flexibility, but what matters in addition is the intensity or degree of that capability required to do the job 'properly'. What we will add to job evaluation is the 'learnability' of the required intensity. The philosophy behind this concept is that in order to attain a certain intensity of a qualification an individual starts from an innate component inherited from her or his parents and is able, during a learning process, to raise the intensity. The philosophy at stake is the tool that we hope will help us to integrate the educational aspect and the labour market aspect of a socio-economic analysis of income distribution or of a policy to change income distribution. The learnability has also a time aspect: capabilities that can easily be learned can also quickly be learned. This opens up the possibility that some capability intensities are variables, whereas other capability intensities are parameters, or, as we may also formulate it, are variables only in the long run.

Some of the most important parameters are the *innate components of productive capabilities*, such as intelligence, leadership, persistence, etc.; others can be health, creativity and family size.

III. UTILITY MEASUREMENT FROM BEHAVIOUR: 'REVEALED' UTILITY

In principle, at least two methods of observation of some entity are available, which may be called direct and indirect measurement. In the case of utility direct measurement so far has only been attempted by asking the individuals involved 'how happy' they were, clearly after some explanation of the measuring rod to be used. This method may be called an opinion poll, comparable to the numerous

public opinion polls held nowadays by Gallup institutes and their colleagues. For reasons of exposition of some consequences of utility measurement we will deal with the direct method in Section IV, even though this is slightly illogical. The indirect method does not measure the entity directly but derives it from observed behaviour. The result will be indicated by revealed utility, in line with well-know-similar expressions ('revealed preference', for instance). In cases such as our subject where a natural, generally accepted measuring rod does not (yet?) exist, both methods have a drawback which precisely is the reason why many members of the profession deny the possibility of measurement. The drawback of the indirect method, now to be discussed, is that some assumptions have to be made which need not be made for the application of the direct method. We may even add that alternatives of the method exist where with weaker assumptions only partial information is obtained. Thus in FRISCH's study (FRISCH, 1932) the weak assumption is made that sugar is a non-substitutable commodity, but FRISCH's measurement only aims at estimating the flexibility of marginal utility.

The approach offered in this section is more ambitious; it aims at defining an equitable income distribution, assumed identical to a state where all individuals considered have the same level of utility or welfare. Accordingly we have to start from a stronger assumption. It will be that utility does not depend on the values of the parameters in the utility function. Less strong forms of this assumption are (a) that utility depends only weakly on parameter values of (b) that our assumption only sets a limit to actual values of utility, namely that a person with higher parameter values, variables being equal, has a utility level not lower than a person with a lower parameter value. All that has been said in this paragraph is based on the further assumption that parameters are measured in such a way that rising values indicate characteristics usually considered attractive[2]. Among the examples of parameters mentioned at the end of Section II, only family size may have to be measured negatively.

Application of the method requires the availability of an earnings equation expressing earnings in terms of a number of variables and

2. Admittedly this way of measuring parameters makes the assumption as formulated under (b) very close to a tautology, but it remains meaningful.

parameters[3]. Notwithstanding the considerable volume of research done on earnings equations, not many examples can be found which satisfy this condition. The main reason is that in the more successful results of estimating earnings equation so many dummies are used of which it is not clear whether they represent a variable or a parameter. More generally it is the lack of direct measurements available of what are the real parameters.

One of the best examples known to us is to be found in the revised version of a recent study by BERKHOUWER *et al.* (1978, p. 9) where earnings y of *AKZO* higher personnel before harmonization[4] are expressed in terms of:

x_1 general education required
x_2 specialized education required
x_3 experience required
x_4 leadership capability
x_5 capability to establish external contacts
x_1' actual general education of individual
x_2' actual specialized education of individual
x_3' actual experience of individual

Only two non-cognitive capabilities were included since factor analysis carried out by J. HARTOG (1978) seemed to show that two such factors are relevant. *AKZO*'s practice is to use some seven of them, because their denominations are more satisfactory to the persons involved; there remains the clear possibility of a considerable degree of overlapping between these seemingly different denominations.

Of the eight independent variables (in the statistical significance of that phrase) mentioned we consider as the variables in the terminology of this essay x_1, x_2 and x_3: they represent the job chosen. In contradistinction, x_1', x_2' and x_3' can be considered as parameters characterizing, certainly in the short run, the individuals' intel-

3. This implies that we restrict ourselves to labour incomes. In developed countries those cover an overwhelming part of total primary income.

4. The situation before harmonization of the salary scales used in the sixteen enterprises of which *AKZO* was composed seems to be a better representation of a free market.

lectual capabilities, both the innate components and the (largely innate) learning capabilities. We are inclined also to consider x_4 and x_5 as largely innate capabilities, although this is not certain and would need further investigation. The material available does not permit a decomposition of these variables into a variable component and a parameter component, however.

The earnings equation runs:

$$\begin{aligned} y = {} & \underset{(2.97)}{0.125\,x_1} + \underset{(6.14)}{0.313\,x_2} + \underset{(6.27)}{0.177\,x_3} + \underset{(8.30)}{0.218\,x_4} + \underset{(9.67)}{0.247\,x_5} \\ & + \underset{(2.58)}{0.073\,x_1'} + \underset{(3.59)}{0.138\,x_2'} - \underset{(0.41)}{0.010\,x_3'} \qquad (1) \\ & \bar{R}^2 = 0.861 \end{aligned}$$

where all variables have been normalized (average = 0 and standard deviation = 1), and t-values have been given in parentheses. Since the individuals observed have been free to choose their job and the corresponding income it follows that the coefficients in front of the variables x_1, x_2 and x_3 constitute the trade-offs of these variables against income y. Therefore an equitable distribution (or a system of acceptable income differences) will be defined by

$$y_E = 0.125\,y_1 + 0.313\,x_2 + 0.177\,x_3 \qquad (2)$$

In order to judge whether the actual income distribution (1) is or is not equitable we compare the standard deviations of both distribution (1) and distribution (2). The variance of y_E can be calculated from (2) if we know the correlation coefficients between x_1, x_2 and x_3, which are: $r_{12} = 0.842$, $r_{13} = 0.219$ and $r_{23} = 0.423$. The standard deviation σ_E of y_E turns out to be 0.517. This is about half of the standard deviation of y, which is unity, as observed. Changing the strong assumption that parameters do not affect utility into a weaker one, where parameters do (positively) affect utility, we arrive at the conclusion that an equitable distribution is considerably less unequal than the actual distribution; at least 40 per cent.

In order to avoid misunderstanding we want to list once again the assumption underlying our example:

(i) we defined equity to prevail when welfare or utility is equalized among the individuals considered;
(ii) we assumed that parameters do not affect welfare negatively (or, in the case of family size, positively);
(iii) we assumed that all relevant parameters and variables were included in our analysis;
(iv) we restricted our exercise to the sample of *AKZO* employees, which does not include less qualified employees usually called workers.

IV. DIRECT MEASUREMENT OF UTILITY

As announced in Section III we are now going to discuss the other method of utility measurement, the essence of which is that individuals are asked to express their degree of satisfaction or welfare in terms of a number of verbal classifications, customary in public opinion polls of many types. We mentioned social indicators as a similar method whose application has spread widely in the last decade or so (*cf.* THIERRY *et al.*, 1977). The authors just mentioned join us in making a distinction between actual and required indicators, comparable to our distinction between supply and demand intensities of qualifications. MCKENNELL (1978) joins us in making a distinction between cognitive and non-cognitive components of well-being.

VAN PRAAG and his colleagues have concentrated on the impact of income on satisfaction or utility and collected a huge amount of information on how the individuals approached think they would evaluate incomes different from their actual income. They extend their questionnaire over nine different states of satisfaction, defined verbally, from 'very badly off' to 'very well off'. The use of words to characterize intensities of satisfaction implies the possibility that the same word may not have an identical meaning to different persons. But since human language constitutes the most important means of communication between human beings, among other purposes to implement socio-economic policies, the approach is the best available. Moreover the use of nine different verbal indications reduces the possibility of misunderstanding. Another danger may

be the one of simulation; this danger is minimized by replacing the individual's name by a number and by guaranteeing confidentiality. Whenever similar questioning were applied by official institutions, the danger of simulation may be reduced by announcing the use of the material for at least two different purposes in which opposite interests play a role[5]. For the present purpose these problems are irrelevant, however.

An important next step in the method now under discussion is the quantification of the answers by the scientists who organized the data collection. In this point a divergence of some importance exists between VAN PRAAG and this author. VAN PRAAG has chosen in favour of the cumulated lognormal probability distribution. This implies, first of all, that all utility levels are situated between 0 and 1. Moreover, it implies that each of the nine income figures corresponding with the nine deciles between 0 and 1, *i.e.* with equidistant figures from 0.1 through 0.9. Finally, it facilitates the mathematics of a number of interesting further uses of the utility function carried out by VAN PRAAG and his collaborators. The present author prefers a logarithmic function of income corrected for the impacts of a series of parameters and variables, hence $a \ln (y-b)$ where a is a constant, y income and b an agglomerate of relevant parameters and variables as defined before.

Research done by VAN HERWAARDEN and KAPTEYN (1979) shows that among the two-parameter functions tested the two alternatives just mentioned show the best fit. An economic argument in favour of the logarithmic function is that it shows decreasing marginal utility throughout, whereas the cumulated lognormal distribution does not. In a few applications made by the present author the former function is more convenient (TINBERGEN, 1979).

VAN PRAAG's function, often written as $\mathcal{N}[(\log X - \mu)/\sigma;\ 0, 1]$ admits a satisfactory interpretation of μ with the aid of actual net income, and family size. It was used to define an equitable income distribution in the following way (BOUMA *et al.*, 1976). Two tests were

5. Thus, an inquiry into the number of rooms available to each of the households in a Dutch city in war time was announced to be undertaken with two purposes: (1) for the rationing of coal and (2) for the obligatory quartering of people whose houses had been destroyed by bombing.

taken with the aid of the 2663 observations taken from members of the Dutch Consumer Union in 1971. One consisted of testing a theory of utility

$$\omega\,[E\,(\log X - \mu)\,\sigma] = \mathrm{F}\,(X, s, v, w, t)$$

where:

X = income after tax
s = occupation
v = years of schooling
w = capability to take independent decisions
t = age.

The material on X, v and t was of satisfactory quality. The material used for s and w was very crude, for lack of better data. The variable s was measured as one of the three quartiles of the v-material for each of ten occupational groups into which the sample was subdivided. The parameter w was taken equal to 1 for wage and salary earners, to 2 for lower and middle executives, teachers, professional experts and agrarians, and 3 for the professions and for commercial occupations. Two forms of functions were taken for F.

The second test consisted of the establishment of an income equation (similar to an earnings equation), expressing X or its log in terms of s, v, w and t or their logs. In both types of tests either the difference $s - v$ or the log ratio $\log s/v$, both squared, were added for reasons irrelevant to the present article[6].

In all, seven type I and twenty type II tests were taken. The R^2 (coinciding practically with the $\bar{R}^2$ because of the large number of observations) were not impressive: they varied between 0.198 and 0.258 for Type I and between 0.315 and 0.400 for Type II. Most of the σ_b-values are satisfactory. For the actual income equations they have been added in *Table 1*. For the equitable income equations they are similar. Out of these equations five pairs were available showing the same independent variables expressing (i) equitable incomes and (ii) actual incomes as the dependent variable, or their logarithms. These pairs are reproduced in *Table 1*.

6. The objective here was to test an aspect of the utility function, hypothesized in earlier work of the author but rejected by the overwhelming majority of tests (the 'tension theory').

Table 1

Equitable (E) and actual (A) incomes or their logarithms in the five comparable pairs

Cases 1/11	$\log X_E$	$= 0.27 \log v + 0.10 \log w + 0.50 \log t$			
	$\log X_A$	$= 0.44 \log v + 0.24 \log w + 0.71 \log t$			
		(0.04)	(0.02)	(0.02)	
Cases 2/12	$\log X_E$	$= 0.14 \log s + 0.12 \log w + 0.47 \log t$			
	$\log X_A$	$= 0.38 \log s + 0.23 \log w + 0.67 \log t$			
		(0.04)	(0.03)	(0.03)	
Cases 6/16	$\log X_E$	$= 0.245 \log s + 0.28 \log (s/v)^2 + 0.10 \log w + 0.48 \log t$			
	$\log X_A$	$= 0.54 \log s + 0.46 \log (s/v)^2 + 0.12 \log w + 0.67 \log t$			
		(0.04)	(0.09)	(0.02)	(0.02)
Cases 4′/34′	$0.001\, X_E$	$= 0.23\, v + 0.00\, (s - v)^2 + 1.28\, w + 4.38\, t$			
	$0.001\, X_A$	$= 0.54\, v + 0.39\, (s - v)^2 + 2.85\, w + 7.31\, t$			
		(0.06)	(0.10)	(0.35)	(0.29)
Cases 9/39	$0.001\, X_E$	$= 0.24\, v$		$+ 1.28\, w + 4.38\, t$	
	$0.001\, X_A$	$= 0.67\, v$		$+ 3.63\, w + 7.39\, t$	
		(0.05)		(0.26)	(0.30)

If we forget for a while the shaky quality of our independent variables, one feature of *Table 1* deserves mention. The ratio between the coefficients of a given independent variable appearing in the X_E equation to that appearing in the X_A equation is not, in the average, very different between v, w and t. In our terminology schooling v was assumed to be closer to a variable – something open to choice – whereas ability to take independent decisions was thought to be closer to a parameter – something innate or nearly so. In principle the latter should be clearly more scarce than the former. This appears not to be so. That finding hints into the direction that w is to a lesser degree innate than often thought. The conclusion we draw is that it is highly desirable to collect better material on the variables and parameters in order to use it in the future samples of the VAN PRAAG type; material of the quality of the *AKZO* material discussed in Section III.

As a finishing observation we want to repeat that this article is an attempt to illustrate two approaches to quantifying the concept

of equitable income distribution and stress the word 'illustrate', hoping that we may have wetted the appetite for the collection of better and more extensive data on the phenomena discussed.

REFERENCES

BERKOUWER J., HARTOG J. and TINBERGEN J.: *Alternative specifications of earnings equations*, Institute for Economic Research discussion paper series, 7804/G, Erasmus University, Rotterdam 1978.

BOUMA N., VAN PRAAG B.M.S., and TINBERGEN J.: 'Testing and Applying a Theory of Utility', *European Economic Review*, 8 (1976), pp. 181–191.

FRISCH R.: *New Methods of Measuring Marginal Utility*, Tübingen 1932.

LEVY S. and GUTTMAN L.: 'On the multivariate structure of well-being', *Social Indicators Research*, 2 (1975), pp. 361–388.

MCKENNELL A.C.: 'Cognition and affect in perceptions of well-being', *Social Indicators Research*, 5 (1978), pp. 389–426.

PARETO V.: *Cours d'économie politique*, Paris 1896.

PEN J. and TINBERGEN J.: *Naar een rechtvaardiger inkomensverdeling*, Amsterdam/Brussel, 1977.

PIGOU A.C.: *The Economics of Welfare*, London 1920.

THIERRY Hk., ZANDERS H.L.G., KOOPMANS E., and DE WOLFF Ch.J.: *Sociale indicatoren in beweging*, Deventer 1977.

TINBERGEN J.: 'Gerechtigkeit als gesellschaftliches Ziel', in: *Soziale Probleme der modernen Industriegesellschaft*, Verhandlungen auf der Arbeitstagung des Vereins für Sozialpolitik in Augsburg 1976, Berlin 1977.

TINBERGEN J.: 'Optimal Education, Occupation and Income Distribution in a Simplist Model, NL 1962', forthcoming (1979).

TINBERGEN J.: 'Überdeckung wissenschaftlicher Gebiete als Ausgangspunkt einer Integration der Wissenschaften', forthcoming (1980).

VAN HERWAARDEN F.G. and KAPTEYN A.: private correspondence (1979).

VAN PRAAG B.M.S.: *De verdeling van inkomen en macht*, Leiden 1975 (where previous English publications are quoted).

SUMMARY

Three alternative definitions of equity are presented, which coincide under special conditions only. The author prefers the definition that equity means equality of welfare. This presupposes measurability of welfare (or utility). Two methods of measurement are discussed, called revealed or indirect and direct measurement. Concrete applications of both are shown, together with their shortcomings. In both cases equitable distribution is found to be less unequal than actual distribution in the Netherlands. Further research needed, on additional relevant determinants of utility and on learnability of some capabilities are formulated.

ZUSAMMENFASSUNG

Drei verschiedene Definitionen einer gerechten Verteilung (von Einkommen und Berufen) werden diskutiert, die nur in gewissen Situationen zusammenfallen. Der Verfasser zieht diejenige vor, wo Gerechtigkeit als Gleichheit der Befriedigung (des Nutzens im breiten Sinne) definiert wird. Das setzt jedoch voraus, dass die Befriedigung messbar sei. Zwei Methoden einer Messung, als indirekte und direkte bezeichnet, werden besprochen. Die erste benutzt Verhaltensweisen als Quelle, die zweite Befragungen. Nachteil der ersten Methode ist, dass eine weitere Arbeitshypothese herangezogen werden muss; Nachteil der zweiten ist, dass man die Antworten der Befragten als glaubenswert betrachtet. Einige Ergebnisse der beiden Methoden mit niederländischem Material zeigen, dass die gerechte Verteilung deutlich weniger ungleich ist als die wirkliche. Die Notwendigkeit weiterer Untersuchungen nach relevanten Bestimmungsgründen des Nutzens und nach der Erlernbarkeit gewisser Eigenschaften wird betont.

RÉSUMÉ

L'auteur discute trois définitions alternatives d'une distribution équitable (ou juste) des occupations et des revenus. Sous certaines conditions ces définitions coïncident. L'auteur préfère la définition d'égalité du bien-être (de l'utilité), ce qui implique comme condition préalable que le bien-être puisse être mesuré. On discute deux méthodes de mesurage, indiquées comme indirecte et directe. La première se base sur le comportement observé, la seconde sur les réponses à un questionnaire. Le désavantage de la première méthode consiste en la nécessité d'introduire une hypothèse additionelle, celui de la seconde méthode en la nécessité qu'il n'y ait pas de biais dans les réponses. Quelques résultats obtenus à l'aide de matériel néerlandais montrent que la distribution juste est clairement moins inégale que la distribution actuelle. L'auteur recommande des recherches plus poussées sur des déterminants du bien-être et sur la mesure dans laquelle certaines capabilités peuvent être apprises.