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Spare part demand forecasting for consumer goods using installed base information

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Abstract

When stopping production, the manufacturer has to decide on the lot size in the final production run to cover spare part demand during the end-of-life phase. This decision can be supported by forecasting how much demand is expected in the future. Forecasts can be obtained from the installed base of the product, that is, the number of products still in use. Consumer decisions on whether or not to repair a malfunctioning product depend on the specific product and spare part. Further, consumers may differ in their decisions, for example, for products with fast innovations and changing social trends. Consumer behavior can be accounted for by using appropriate types of installed base, for example, full installed base for cheap but essential spare parts of expensive products, and warranty installed base for expensive spare parts of products with short lifecycle. The paper presents a general methodology for installed base forecasting of end-of-life spare part demand and formulates research hypotheses on which of four installed base types performs best under which conditions. The methodology is illustrated by case studies for eighteen spare parts of six products from a consumer electronics company. The research hypotheses are supported in the majority of cases, and forecasts obtained from installed base are substantially better than simple black box forecasts. Incorporating past sales via installed base supports final production decisions to satisfy future consumer demand for spare parts.

Keywords

Installed base forecast, end-of-life service, decision support, consumer goods, spare parts

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1 Introduction

Consumer satisfaction does not only depend on product quality, but also on the availability of spare parts to correct failures of products. Demand patterns of spare parts differ from those of finished goods for several reasons, such as warranty, perceived economic value of malfunctioning products, and reliability replacement cycles. Nowadays, many companies employ state-of-the-art forecast systems for demand of finished goods as part of their supply chain management. Such an approach is, however, much less widespread for spare part demand, possibly because companies tend to regard spare part issues as a liability rather than as a lucrative business. The lack of attention to spare part demand can easily lead to increasing dormant spare part stocks or to delayed replenishment and increasing repair delays. The main benefits of improved spare part demand forecasting and associated inventory policies will consist of lower inventory costs, better repair service, and higher consumer satisfaction.

This paper proposes the use of installed base concepts to forecast spare part demand for consumer products. Contrary to capital goods like planes and mainframe computers, where users often have service contracts with manufacturers, consumer goods manufacturers have much less or even no information on how many of their products are still in use that could cause purchases of spare parts for repair. Yet this number of products in use, referred to as the installed base, provides the key for demand for spare parts, and hence assumptions need to be made about its size. We frame these assumptions as installed base variables, and examples are lifetime base (number of products with age below the average lifetime), warranty base (number of products under warranty), and economic base (number of products with remaining value above repair costs). Spare part demand forecasts are especially important when the manufacturing of the product stops, that is, right at the start of the end-of-life (EOL) phase of the product. Here EOL is defined as the service period after ending production, when consumers still use the product and cause demand for spare parts when asking for repair. During the production phase, it may be relatively easy to cover demand for spare parts by simply increasing the lot size of the parts needed. During the EOL phase, however, it is often very costly or even impossible to produce or obtain extra items of spare parts that are used only in the ended product.

The main task considered in this paper is to forecast the demand for product-specific spare parts over the EOL phase. The general methodology is as follows. First, select an appropriate installed base variable. This choice can be guided by business insights in consumer behavior. Second, determine a model that relates spare part demand to the installed base variable using data during the production phase of the product, that is, before EOL. Third, extrapolate the installed base for the EOL phase. This extrapolation is relatively straightforward and involves choice of average lifetime (for lifetime base) or depreciation rate (for economic base). Finally, estimate demand for spare parts during EOL from the model and the extrapolated installed base.

Previous studies employing installed base concepts concern capital goods, where the manufacturer often has service contracts with the user and total numbers are relatively small. For example, aircraft manufacturers are even notified of the flying hours per plane. Our approach

for consumer goods may also be useful for relative cheap industrial goods, where manufacturers also have less information on the installed base. The problem is not so much to know how many items have been sold, as that is easily monitored, but to know how many are still in use and would qualify for purchase of parts to correct failures. Old products are often replaced by newer and often better products upon failure, and the original manufacturer is not informed.

Consumer markets are often affected by heterogeneous behavior of consumers, as they differ in their sensitivity to product innovations. The installed base methodology is illustrated by case studies for three types of consumer products: refrigerators (white goods), and flat panel televisions and smartphones (consumer electronics). Although specific details of the findings for these products will partly be case-dependent, the methodology and general conclusions are of interest for all spare part supply chain managers. In order to apply the ideas presented here, these managers need the following information for each product of interest: past demand data for spare parts of interest (that is, specific for the product), past sales data of the product, average lifetime (for lifetime base), warranty period (for warranty base), and depreciation rate (for economic base). The choice of installed base type can be based on business insights or on statistical considerations like performance during the production period (before EOL). The methodology in this paper offers a method to forecast spare part demand during EOL that can be used, in combination with expert knowledge, to decide on the size of the final production run of each spare part to cover all future demand.

The remainder of this paper is structured as follows. Section 2 discusses some related literature. Section 3 presents various concepts of installed base and formulates the main research hypotheses. The methodology is described in Section 4, and it is illustrated in detail for a specific spare part in Section 5. Section 6 presents the results for a set of eighteen spare parts related to six products. Section 7 concludes and summarizes operational implications.

2 Background literature

The main topic of this paper is spare part demand forecasting for consumer products over their end-of-life phase, using the concept of installed base. Several recent publications are related to this research question. Kim and Park (2008) stress the importance of demand forecasting for the EOL phase to decide on final order sizes, and Chou et al. (2015) use the concept of installed base to forecast final orders of automobile parts. Their first finding is that production costs during EOL are higher than during the mature phase because of loss of economies of scale and of economies of scope. Second, they find that the optimal warranty period during EOL depends on product failure rates. In our application we have no information on failure rates, and the warranty period is given. Teunter and Fortuin (1998) optimize the final order size by minimizing costs. They do not evaluate actual forecast accuracy, and their method requires information on production costs, holding costs, and penalty costs that are unavailable in our case. Lengu et al. (2014) classify various order size distributions for a large set of spare parts for domestic appliances and

commercial airlines. They do not consider forecasting, and order sizes are not related to installed base. Romeijnders et al. (2012) forecast spare parts demand using information on component repairs in the aviation industry. Our focus is on spare part demand for consumer products during EOL, taking into account consumer behavior and sentiments.

Contributions on forecasting product demand (instead of spare part demand) are widespread, and we mention two recent publications. Van der Heijden and Iskandar (2013) study last time buy decisions for products sold under warranty. Their focus on demand forecasting at the start of the EOL phase is similar to ours, but they consider only simulated data and no real-world spare part data. Teunter et al. (2011) base their forecasts of sporadic demand on probabilities. Potential benefits are illustrated by simulation, not with real data. Tibben-Lemke and Amato (2001) predict demand for replacement parts from known failure ratios. Our products have unknown failure ratios, and demand also depends on heterogeneous consumer preferences. Islam and Meade (2000) analyze factors inducing replacement rather than repair, including socioeconomic factors and improved technology.

One of the first applications of installed base is Brockhoff and Rao (1993), who forecast new product adoption. Yamashina (1989) uses the concept of products still in use, which is similar to lifetime installed based. Auramo and Ala-Risku (2005) consider applications in service logistics, focusing mainly on how to obtain installed base information. Dekker et al. (2013) combine life cycles with installed base information to improve forecast performance. Their focus is on maintaining the lifetime installed base in industrial markets, whereas we consider also alternative base concepts (warranty, economic) for consumer markets. Jin and Tian (2012) use installed base in optimizing service parts logistics. They illustrate their method by simulations and do not consider forecasting. Bacchetti and Saccani (2012) provide an extensive overview of spare parts demand forecasting and investigate the currently still existing gap between research and practice in spare parts management. In terms of their classifications, our contribution falls in the category of regression methods (Section 4) for demand volumes with case study tests (Sections 5 and 6). None of the papers reviewed above makes a distinction between various installed base concepts (as we will do in the next section) nor is their performance tested with real data (as we will do in Sections 5 and 6).

3 Installed base

3.1 Installed base concepts

Traditionally, demand forecasting is based on simple extrapolation of historical data. Such so-called black-box methods are popular because of their simplicity, as the required information is limited to historical demand data. For time series of (weekly, monthly, quarterly, or yearly) data, these methods have become widespread in business since the work of Box and Jenkins (1976).

For spare parts demand, the past sales of products that contain the part are evidently also relevant. The *installed base* of the product is the number of sold products that can lead to demand

of its spare parts. More precisely, for each time period (week, month, quarter, or year), the *lifetime installed base* grows with the quantity that leaves the warehouse and it declines with the number of returned products and with the number of products exceeding the expected lifetime. Consumer (electronic) products are typically sold through independent retailers who do not share sales information with manufacturers. For the electronics company in our case study, it generally takes between two and three weeks after leaving the warehouse before the products are sold and enter the customer base. The sales and demand data are typically available on a weekly basis. Let S(t) be the product sales to customers in week t, let R(t) be the returns from customers of that week, and let L denote the average lifetime of the product, including second-hand usage. Then lifetime installed base (IBL) at the end of week t is defined as follows

$$IBL(t) = \sum_{i=t-L+1}^{t} (S(i) - R(i))$$

(where sales start in period 1 and S(i) and R(i) are defined as 0 for times i < 1). Inderfurth and Mukherjee (2008) discuss the typical shape of IBL, as shown in Figure 1. This shape is characterized by three phases in the product life cycle. First is the initial phase with growing sales, followed by a mature phase where sales gradually fall back. Finally comes the end-of-life phase, where the product is no longer sold. Demand for spare parts may be expected to be relatively low in the initial phase, as most products are relatively young and will generally function well. This demand is expected to rise during the mature phase and possibly also during early parts of the EOL phase, whereas later on, demand will gradually diminish as more products reach their lifetime.

Black box methods forecast the demand for the EOL phase by extrapolating demand that has been observed during the initial and mature phases. Such methods will tend to over-estimate actual demand, because the increasing demand trends during the mature phase will break down somewhere during the EOL phase, because customers discard the products. Pince and Dekker (2011) report similar problems for forecasts based on exponential smoothing.

Figure 1 shows also the typical shape of warranty installed base. Consumers may determine their demand decision for repair based on product warranty regulations. The warranty period is the maximum period for which the company supports sustainability of the product at its own expense. After this period, customers have to carry costs of repair and logistics by themselves, which may lead them to purchase a new product instead of asking repair of the old one. For a warranty period of W periods, the warranty installed base (IBW) is

$$\text{IBW}(t) = \sum_{i=t-W+1}^{t} (S(i) - R(i)).$$

(again with S(i) = R(i) = 0 for i < 1). As the warranty period is smaller than the average lifetime of the product, IBW is identical to IBL during the first W sales periods and it is smaller afterwards.

After expiration of the warranty period, consumers may still generate demand for spare parts. Such behavior is rational if the remaining economic value of the product after repair exceeds

the repair costs. For period t, let c(t) be the repair costs, let $v_i(t)$ be the remaining value of the product bought in week i, and let the economic decision for repair be denoted by $E_i(t) = 1$ if $v_i(t) > c(t)$ and $E_i(t) = 0$ if $v_i(t) \le c(t)$. Then the *economic installed base* (IBE) is defined as the part of the lifetime installed base for which repair is economical, that is,

$$IBE(t) = \sum_{i=t-L+1}^{t} E_i(t) \times (S(i) - R(i)).$$

(with S(i) = R(i) = 0 for i < 1). To make this base operational, values of repair costs and remaining value are needed. In our case study, we will take the price of the spare part as the repair cost. We do not incorporate handling and labor costs, as these cost figures are not available and vary per customer location and per type of failure. This choice implies under-estimation of actual costs, and hence over-estimation of the actual economic installed base. In other applications, more accurate values of IBE can be obtained if more accurate cost information is available. The remaining value $v_i(t)$ is determined by assuming exponential value decay and final unity value at the end of average lifetime. Let p_i (>1) be the price of the product sold in period i, then the decay rate a_i for products sold in that period is obtained from the condition that $1 = p_i \times \exp(a_i \times L)$, so that $a_i = -\ln(p_i)/L$. The remaining value is equal to $v_i(t) = p_i \times \exp(a_i \times (t-i))$. Figure 2 shows the ingredients used to determine IBE. Usually, the remaining value exceeds repair costs through all of the warranty period, so that IBE will take values between IBW and IBL. After the warranty period has ended, it is still economical for some time to demand spare parts for repair. This interim period is longer for longer lifetimes and lower repair costs, as it pays to replace cheap spare parts for prolonged product use.

In the construction of IBE, it is assumed that all consumers apply the same decay rate for remaining value of the product. Consumers may differ in their subjective evaluation of remaining value, for example, if they vary in their sensitivity for social trends and technological innovations. The decision on whether or not it is economical to repair the product will then depend on heterogeneous tastes. For this situation of mixed economic decisions, we define the mixed economic installed base (IBM) similar to IBE, but with varying perceived lifetimes and hence varying value decay rates. In our IBM applications, we follow Rogers (2003) and divide consumers in five adopter segments. Early adopters replace the product relatively fast, corresponding to a relatively short lifecycle. In line with Rogers, the consumers are distributed as follows over the segments (in parenthesis is the lifecycle within each segment, as fraction of the overall average): 2.5% innovators (0.6), 13.5% early adopters (0.7), 34% early majority (1.0), 34% late majority (1.05), and 16% laggards (1.3).

3.2 Research hypotheses

The main question in forecasting with installed base is which type of base gives the best information. The answer to this question is likely to depend on characteristics of the spare part and

of the product. We formulate a set of research hypotheses, which are tested empirically in Sections 5 and 6.

The *first hypothesis* is that forecasts from installed base improve upon simple black box methods that only use historical demand data. The *second hypothesis* is that lifetime installed base provides the best forecasts for essential and expensive spare parts of non-trendy products with long lifetime, whereas warranty installed base is better for products with short life cycle. The *third hypothesis* is that economic installed base works best for non-essential spare parts of products with considerably longer lifetime than warranty period. This situation corresponds to a relatively long interim period in Figure 2, where users can choose to repair non-essential parts only if remaining lifetime is long enough to compensate repair costs. Finally, the *fourth hypothesis* is that mixed economic installed base works best if consumers differ much in their acceptance of new products. If, for example, a considerable portion of the consumers switch to a new product before the old one has lost its function, then spare part demand falls below what is expected from a purely economic point of view.

4 Forecast methodology

4.1 Installed base and spare part demand

In order to set up our model, we first make a set of simplifying assumptions that will later be relaxed. We first assume that the product is sold only in period 0 and that the spare part is so cheap that it will be replaced when it breaks down. As time goes by, ageing of the product affects spare part demand in two ways, that is, wear-out and end-of-use. For a given customer, let T_1 be the (continuously measured) time of failure of the product requiring a spare part for repair, and let T_2 be the (continuously measured) time where this customer ends use of the product. Both times can be seen as random variables, with survival distributions $S_i(t) = P_i(T_i > t)$, i = 1,2. This customer will not demand the spare part if $T_1 \ge T_2$. The demand probability $p_d(t)$ in period t, that runs in continuous time from t-1 to t, is equal to the joint probability $P(t-1 < T_1 < t, T_2 > t)$. Assume that the probability of break-down does not depend on the decision of continued product use, that is, $P(t-1 < T_1 < t \mid T_2 > t) = P(t-1 < T_1 < t)$, then the demand probability is

$$p_d(t) = P(t-1 < T_1 < t, T_2 > t) = P(t-1 < T_1 < t) \times P(T_2 > t) = (S_1(t-1) - S_1(t)) \times S_2(t).$$
 (1)

If the hazard rates for product failure and product disuse are both assumed to be constant over time, so that the product and spare part do not age in that sense, then the survival functions are exponential, that is, $S_i(t) = \exp(-a_i t)$ with $a_i > 0$. Then (1) becomes

$$p_d(t) = (\exp(-a_1(t-1)) - \exp(-a_1t)) \times \exp(-a_2t) = (\exp(a_1) - 1) \times \exp(-(a_1 + a_2)t).$$
 (2)

This is the probability per customer of demand in period t. The expected total demand D(t) in period t, with remaining installed base IB(t), is equal to $p_d(t) \times IB(t)$. Let $b_0 = \ln(\exp(a_1) - 1)$ and $b_2 = -(a_1 + a_2)$, then (2) gives

$$\ln(D(t)) = b_0 + \ln(\text{IB}(t)) + b_2 \times t. \tag{3}$$

This relation has been obtained under various simplifying assumptions. The variable t in (3) denotes the age of the product, which depends on the moment it was bought. If we neglect product disuse, as this information is usually not available to the manufacturer, then installed base in period t is $\mathrm{IB}(t) = \sum_{s=1}^t X(t-s)$, where X(t-s) are the sales in the period s time units before the current one. Then expected demand in period t is $D(t) = \sum_{s=1}^t p_d(s)X(t-s) = e^{b_0}\sum_{s=1}^t e^{b_2s}X(t-s)$. The past sales information captured by installed base is not rich enough to retrieve this expected demand, as it only stores the sum-total of past sales and not the specific distribution of sales over the various periods. An approximation of expected demand is obtained by replacing the age-specific weights (e^{b_2s}) by a single weighting factor evaluated at the weighted age of the products, that is, $e^{b_2\mathrm{AGE}(t)}$ where $\mathrm{AGE}(t)$ is the mean age of the installed base in period t. This approximation gives $D(t) = e^{b_0}e^{b_2\mathrm{AGE}(t)}\sum_{s=1}^t X(t-s) = e^{b_0}e^{b_2\mathrm{AGE}(t)}$ IB(t), or

$$\ln(D(t)) = b_0 + \ln(\mathrm{IB}(t)) + b_2 \times \mathrm{AGE}(t). \tag{4}$$

The coefficient 1 of logarithmic installed base follows from the assumptions that no products are disused and that every break-down of the product results in demand for repair. We replace this coefficient by an arbitrary one to account for the fact that some products will be disused and only a portion of all break-downs will be repaired, as the owner can also decide to buy a new product. The resulting relation is

$$\ln(D(t)) = b_0 + b_1 \times \ln(\mathrm{IB}(t)) + b_2 \times \mathrm{AGE}(t). \tag{5}$$

This model is, of course, still a simplification. For example, wear-out of spare parts and end-ofuse of products may not have constant hazard rates, which results in more complicated relations. Demand models involving more flexible functions of installed base and product age can be used, provided that sufficient data are available. We mention one example. In our case study, we do not know the original purchase date of products that lead to spare part demand, that is, we do not know the individual ages. If this information were available, model (5) could be replaced by the disaggregated model (3), with t the age of the product and IB(t) the installed base remaining from the sales t weeks before the current date.

4.2 Estimation and model selection

Our modelling approach is rather pragmatic. The amount of relevant available demand data is generally limited, so we do not assume to know all details of the data generating process. For

example, we do not know the purchase dates of products that lead to spare part demand. Our ambition is therefore limited to produce reasonably accurate forecasts and we choose for rather simple models that are evaluated in terms of their out-of-sample forecast performance. For our case study, we present forecast results that are all based on model (5). We also considered richer specifications, for example, including squared age, but the data available for estimation were in general not rich enough to improve forecasting power. Model (5) corresponds to a regression model if we add an unobserved error $\varepsilon(t)$ term to account for the (unknown) approximation errors of actual demand behavior. Further, to allow for zero values of demand and installed base, the estimated model is specified as follows.

$$\ln(1+D(t)) = b_0 + b_1 \times \ln(1+\text{IB}(t)) + b_2 \times \text{AGE}(t) + \varepsilon(t).$$
 (6)

The error term is assumed to follow an autoregressive (AR) process, so that the unknown coefficients b_0 , b_1 , and b_2 in (6) can be estimated by ordinary least squares. As the actual weekly demand data of the case study spare parts are quite erratic, these demand data are smoothed by taking an exponentially weighted moving average (EWMA) with smoothing factor 0.06. This means that the demand of the current week gets weight 0.06, whereas the combined weight of previous weeks is 0.94. We compared the results for this choice of left-hand variable in (6) with two other options: without weighting, that is, using actual weekly demand, or with eight-week averaging, as the company needs an average lead time of eight weeks to cover unanticipated demand. We found that models estimated from EWMA smoothed demand, denoted by $D_s(t)$, work best in forecasting, not only to forecast EWMA demand, but also to forecast actual demand and its eight-week average. We did not try to optimize the EWMA smoothing factor and simply used the value that has become popular since RiskMetrics of J. P. Morgan (1996).

Black box forecasts are obtained from pure AR models, that is, with $b_1 = b_2 = 0$ in (6), and with b_0 estimated from the demand data of the initial and mature product phases. The error term is modeled as $\varepsilon(t) = c_1 \times \varepsilon(t-1) + \ldots + c_p \times \varepsilon(t-p) + \omega(t)$, where $\omega(t)$ is a white noise process and where the AR order p is determined by forward selection, that is, by increasing this order until the extra lag term becomes insignificant (at 5% level). These orders range from one to three for the spare parts of the case study, and the corresponding AR models provide in general a very good in-sample fit for demand during the initial and mature phases (R-squared values are typically larger than 0.99). We tried out some extensions of this simple black box model by allowing for linear or quadratic time trends and by smoothing, but as these extensions did not improve the forecasts we will only consider pure AR models.

The AR order of the black box model is also used in all installed base models. The corresponding model (6) is estimated for the initial and mature phase of the product for each of the four considered installed base types, that is, IBL, IBW, IBE, and IBM. The mean age is defined in terms of the corresponding installed base type, for example, using only products under warranty in the model with IBW. The installed base term $\ln(1+\text{IB}(t))$ is removed from the model if it has a negative coefficient ($b_1 < 0$), as it is a natural requirement that demand is positively related to

installed base (for given mean age). Although the above analysis for fixed hazard rates suggests that the coefficient of the mean age term AGE(t) should be negative ($b_2 < 0$), we do not impose this condition. The reason is that other hazard rates provide other age effects, and it is not illogical that spare part demand could increase for higher mean age of the installed base. Insignificant terms are not removed from the model, because insignificance may be due to a short estimation period for the model.

We summarize the steps needed to estimate the candidate models for spare part demand during the initial and mature product phases. First, determine the following characteristics of the product: average lifetime, sales for each period during the initial and mature phases. From the sales data, determine the numerical values of installed base types IBL, IBW, IBE, and IBM, for each period over the full life cycle (initial, mature, and EOL phases). This requires information on warranty period (for IBW), cost of the spare part (for IBE), and consumer segmentation (for IBM). For each installed base, compute the associated values of mean age for each period of the full life cycle. Next, determine the spare part demand data for each period during the initial and mature phases, and compute smoothed values by EWMA. For these smoothed demand data, estimate an AR model and select the AR order *p*. Estimate four types of installed base models, each with the same AR order, and keep the installed base variable only if it has a positive coefficient.

4.3 Forecast evaluation

The procedure described in the previous section provides a set of five models (AR, IBL, IBW, IBE, and IBM) that can be used to forecast spare part demand over the EOL phase. These forecasts use only information that is available at the end of the mature phase, as no EOL information is of course available at the start of EOL. Note, however, that the values of installed base and mean age are completely determined by product sales before EOL, so that these variables can be extrapolated perfectly for the full EOL phase. The forecasts are determined iteratively, as the AR structure implies that forecasts of previous periods affect the forecast for the current period. These forecasts for earlier periods during EOL are not replaced by realized demands, because this information is not available at the start of EOL. The forecasts concern the variable $\ln(1+D_s(t))$, which are easily translated into forecasts of $D_s(t)$. For periods where the installed base is zero, the forecasted demand is manually set equal to zero. This is a logical condition, provided that the installed base type is correct, because no spare parts can be demanded if the installed base has disappeared. The resulting forecasts are for smoothed demand, but in practice one is interested in actual demand. To prevent erratic behavior, the model forecasts are not "unsmoothed', and the forecasts of $D_s(t)$ are directly compared with actual demand D(t).

The forecast performance of the various models can be compared graphically with actual demand by means of a joint time plot for the EOL phase. The forecasts are also numerically compared by means of the following three criteria. The summed error is the difference between

the summed forecasts and the summed demand over the EOL phase. Suppose that actual demand data D(t) are available for periods $t_1 \le t \le t_2$ of the EOL phase, and let F(t) be the forecasts for these periods; then

$$SUM = \sum_{t=t_1}^{t_2} (F(t) - D(t)) / \sum_{t=t_1}^{t_2} D(t).$$
 (7)

Positive values correspond to over-estimation of the total need for spare parts during EOL, and negative values correspond to under-estimation. This is our main criterion to compare forecast methods, as it is measures the global quality of forecasting the spare part need during EOL. We also consider two other criteria that measure the local, week-by-week forecast quality, that is, the mean absolute prediction error (MAPE) and the root mean squared prediction error (RMSPE).

$$MAPE = \frac{\sum_{t=t_1}^{t_2} |F(t) - D(t)|}{\sum_{t=t_1}^{t_2} D(t)} , RMSPE = \frac{\sqrt{\frac{1}{t_2 - t_1 + 1}} \sum_{t=t_1}^{t_2} (F(t) - D(t))^2}{\frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} D(t)} = \frac{\sqrt{\sum_{t=t_1}^{t_2} (F(t) - D(t))^2}}{\sum_{t=t_1}^{t_2} D(t) / \sqrt{t_2 - t_1 + 1}} .$$
(8)

5 Illustrative case: compressor of refrigerator

5.1 Product and demand characteristics

The spare parts in our case study are provided by the Western European warehouse of Samsung Electronics. Refrigerators are one of their products with a relatively stable sales pattern. In this section, we analyze the demand for the compressor of a specific type of refrigerator, which we call type 1 to distinguish it from another refrigerator type 2 that is studied in the next section. The compressor is indispensable, as the refrigerator does not function if the compressor is out of order. Compressors are very reliable, and malfunctioning is mainly caused by extreme operating conditions. The compressor is somewhat expensive, as it costs about 100 euro, which is about 18.3 percent of the price of around 550 euro of the refrigerator. The warranty period is 2 years, and the average lifetime of refrigerators is 13 years according to the National Association of Home Builders (2007).

In Section 3.2, we formulated a set of research hypotheses for the four installed base types. Consumers see refrigerators as a necessary product without much distinguishing characteristics, apart from size, price, and reliability. We therefore do not expect that consumers vary much in their subjective evaluation of remaining value of the refrigerator, so that a mixed economic installed base does not seem useful. The interim period for economic installed base is obtained from the formula in Section 3.1 for the (yearly) decay rate, $a = -\ln(p)/L = -\ln(550)/13 = -0.485$. The remaining value is 18.3 percent when $0.183 = \exp(-0.485 \times y)$, that is, after y = 3.5 years. The interim period of Figure 2, starting after the warranty period of 2 years, is therefore only 1.5 years. This is rather short for products with an average replacement cycle of 13 years, so that we do not expect large differences between economic and warranty installed base during initial periods. As refrigerators are such stable products, we expect that many consumers are willing to replace the

compressor at their own cost for the benefit of considerable extra lifetime of the refrigerator. Our hypothesis is therefore that IBL will provide the best forecasts, better than black box (AR) and alternative installed base types (IBW, IBE, and IBM).

Time plots of the various installed base types are shown in Figure 3. All installed base types are very similar in the initial phase, but start to get differentiated in the mature phase. The sales data run from week 12 in 2008, when sales of this refrigerator started, to week 29 of 2013, when sales ended. Weekly replacement data for the compressor are available from week 12 in 2008 to week 13 of 2014. Total sales of the refrigerator are more than half a million, and the total replacement demand for compressors is 5,678, of which 247 occur in the observed EOL phase. The six years of observations, with 279 weekly sales and 315 weekly demand data, cover less than half of the lifecycle of the product, and the observed EOL phase is only 36 weeks. The forecasting task is to predict demand for these 36 weeks, and there is a relatively long estimation period of 279 weeks for the initial and mature periods. As the case study has been conducted in April 2014, we are not able to evaluate forecasts beyond week 13 of 2014.

The top left diagram in Figure 4 shows a time plot of weekly demand as well as its EWMA smoothed version. The other diagrams show scatter plots, with EWMA smoothed demand on the vertical axis and installed base (IBL, IBW, and IBE) on the horizontal axis. The task is to determine a relation between installed base and demand from data for the initial and mature phases, and to use this relation to forecast demand during EOL. The scatter diagrams indicate that this is not an easy task, as this relation changes among the various phases of the product. If, for example, one would fit a straight line in the IBW scatter diagram for the initial and mature phases, then this would systematically over-estimate actual demand during the EOL phase. It seems very difficult to make a choice among alternative installed base types solely from the statistical data information in the initial and mature phases. In the foregoing, we used economic arguments on consumer behavior to motivate the choice for IBL.

5.2 Forecast results

The obtained black box model is the following AR(2) model: $\ln(1+D_s(t)) = 3.30 + \varepsilon(t)$, with $\varepsilon(t) = 1.13 \times \varepsilon(t-1) - 0.14 \times \varepsilon(t-2) + \omega(t)$. This model provides a very good fit for the smoothed demand data, with $R^2 = 0.997$. Each of the four installed base models has a negative coefficient for installed base, so that this variable is removed. The reduced models contain mean age and AR(2) error terms. The mean age variable has an insignificant negative coefficient in all cases, with the following (one-sided) p-values: 0.09 for IBL, 0.44 for IBW, 0.47 for IBE, and 0.40 for IBM. Even though the effects are weak, we try to exploit this installed base information in forecasting.

<< Insert Figure 5 about here. >>

Figure 5 shows time plots for the observed EOL phase of actual demand, its EWMA smoothed version, and the five alternative forecasts. IBL is clearly doing best and is rather successful in tracking the EWMA series that was used to estimate the model. The real interest lies in forecasting actual demand over the EOL phase, and IBL is also the best method in this respect. The models for IBW, IBE, and IBM provide forecasts that are nearly identical to that of the black box model, which is explained by the lack of significance of mean age in these models. The actual total demand over the observed EOL phase is 247, and the predicted totals are as follows: 551 (AR), 404 (IBL), 424 (IBW), 552 (IBE), and 545 (IBM). The relative total error, defined by SUM in (6), is 0.64 (IBL), 0.72 (IBW), 1.21 (IBM), 1.23 (IBE), and 1.23 (AR). IBL provides the best forecasts also in this respect, with IBW as second-best. This ranking is confirmed by the criteria MAPE (0.77 for IBL, 1.05 for IBW, 1.21 for IBE, 1.30 for IBM, 1.33 for AR) and RMSPE (0.87 for IBL, 1.21 for IBW, 1.44 for IBM, 1.46 for IBE, and 1.46 for AR).

Our conclusion is that lifetime installed base provides helpful information to forecast spare part demand for compressors for this type of refrigerator. As compared to black box forecasting, the error in total EOL demand is reduced by a factor of about two (from 304 to 157). These results confirm our first and second research hypotheses formulated in Section 3.2.

6 Results for three types of consumer products

6.1 Overview of eighteen spare parts

We apply the methodology of Section 4 to a set of eighteen spare parts related to six products, and we test the relevant research hypotheses of Section 3.2 for each case. The products fall in three categories, that is, refrigerators, flat panel televisions, and smartphones. Consumer sentiments and behavior differ among these three products. Sales data are available for two types of each product, and for three spare parts for each product. These spare parts differ in functionality and price. The products differ substantially in terms of both the available estimation data, that is, length of initial and mature phases, and the forecast challenge as measured by the length of the observed EOL phase. Figure 6 shows installed base of various types for the two types of each product, refrigerators (top row), televisions (middle row), and smartphones (bottom row). IBE and IBM depend on the (cost of the) spare part, and Figure 6 shows these two types of installed base for the most expensive spare part per product. The top left graph is identical to Figure 3.

Some characteristics of the products and spare parts are summarized in Table 1. The price of spare parts does not incorporate handling and labor costs. These additional costs are rather marginal for televisions and smartphones, because customers can easily bring these products to repair shops. The labor costs are more substantial for refrigerators, because their repair requires that a skilled technician visits the owner at home. Table 1 also shows our hypothesis on which installed base is expected to be most useful in forecasting EOL demand. The motivation for each

of these hypotheses is described below in Sections 6.2-6.4, together with the main forecast results for the eighteen spare parts. Further details on models and forecasts are available from the authors upon request.

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<< Insert Figure 6 about here. >> << Insert Table 1 about here. >>
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6.2 Refrigerator spare parts demand

Refrigerators have a long lifecycle, and Table 1 shows that the forecast evaluation period covers only a small part of the full EOL phase. The product warranty lasts for two years.

An essential and somewhat expensive spare part is the compressor (price 14-18% of that of the refrigerator). As was explained in Section 5.1, we expect IBL to perform best for this spare part, because repairing the compressor provides large gains in expected lifetime and refrigerators do not quickly get out of fashion. A less expensive and less essential part is the circuit board (relative price 6-8%). The refrigerator works well even if some functions of the circuit board have broken down. As the product has a much longer lifetime than the warranty period and keeps high utility until end of life, our third hypothesis in Section 3.2 suggests that consumers make economic decisions and replace this part if the remaining value of the refrigerator is higher than the replacement costs. That is, we expect that IBE will provide the best forecasts. A cheap and non-essential spare part is the door gasket (relative price about 3.5%). Even though this spare part is relatively cheap, replacement by the manufacturer is costly as a specialized repairer should visit the owner at home. We therefore expect consumers to demand this spare part only during the warranty period and to find other solutions after warranty has expired. That is, we expect IBW to provide the best forecasts.

We applied the forecast methodology of Section 4 for each of the six spare parts, in the same way as was illustrated in Section 5 for compressors of the first type of refrigerator. The results are shown in Table 2. The outcomes for compressors support our hypotheses, as IBL provides more accurate EOL demand forecasts than the other installed base types. It also performs much better than the black box AR method. The results for the circuit board are mixed. Our hypothesis that IBE performs best is confirmed for the first type of refrigerator, but not for the second type where black box forecasts are much better. The cause of the bad forecast performance of IBE in the latter case is that the model has a significant positive coefficient for mean age, which increases steadily over EOL, whereas actual EOL demand is rather stable. Note that this bad forecast performance could not be foreseen at the start of the EOL phase, as the future trends in spare part demand have somehow to be extrapolated from the past. The results for the door gasket are mixed, as our hypothesis that IBW is best is confirmed for the second type of refrigerator, but not for the first type where IBM performs better. Overall, the outcomes provide support for the first and second hypotheses of Section 3.2, and partial support for the third hypothesis.

6.3 Television spare parts demand

High-tech products, like televisions, experience decreasing lifecycles because of fast innovations and quickly developing consumer trends. Consumers tend to base their buying decisions for these products less on economic value and more on the wish to own new product features. Therefore, the value of these products experienced by consumers often decays much faster than the economic lifecycle would suggest. We therefore expect that warranty installed base for these products is in general more informative than lifetime and economic installed base. The warranty period is two years.

Both types of flat screen televisions have very short sales periods of approximately one year. The estimation period for all models is too short to provide reliable forecasts. All methods, including black box, are far off the mark. We conclude that full EOL forecasting after such a short estimation period is too challenging, and instead we consider the more modest task to forecast remaining EOL demand one year after the end of product sales. The estimation period then becomes about two years, and the forecast evaluation period is about three years for television type 1 and about two years for type 2, as type 2 was introduced about one year after type 1.

An expensive and indispensable spare part of the television is the LCD panel. LCD panels are very expensive, with a price of about half that of a new television. As utility of televisions declines rather rapidly over time due to fast product innovations, we expect that IBW provides the best forecasts. A less expensive and non-essential part is the circuit board (relative price 8-10%). In many cases, the circuit board keeps its main functionalities even if some options are lost. We expect that customers demand this spare part during the warranty period, but not afterwards, as they may accept some function losses of relatively old televisions. Again, we expect IBW to perform best. A relatively cheap spare part is the cover of the television (relative price 3-5%), but repair is more expensive as it requires a service engineer. As this part is not essential, we expect that owners demand this part only during the warranty period, so that IBW is again expected to give the best forecasts.

Table 3 shows the forecast results for television spare parts. Most of the outcomes support our hypotheses, as IBW provides the best EOL demand forecasts in five out of six cases. The only exception is found for circuit boards for the first type of television, where IBE and IBL do slightly better. The differences between the four installed base types are small for this specific spare part, and each of these four methods is considerably better than the black box method. The outcomes therefore support our first and second hypotheses of Section 3.2.

<< Insert Table 3 about here. >>

6.4 Smartphone spare parts demand

The market for smartphones has expanded very rapidly in recent years. Consumers are fast in adopting new technologies, as product innovations expand the functionalities of new phones. This market is highly competitive, not only between brands but also between products of the same brand. The two smartphones in our case study are of the same brand. The first type is an early version, which is followed up within a year of its introduction by a second type that has much better functionalities. Both phones have a warranty period of two years, and the second type is introduced before the warranty period of the first type has expired. The far majority of phones is sold by telecom companies in combination with mandatory financial and maintenance contracts that last for one or two years. Consumers with such contracts are forced to use the product during this mandatory period, irrespective of their subjective evaluation of the remaining product value. We therefore expect that owners of the first type of phone will demand spare parts only during the warranty period, as later on they will wish to switch to the much improved new version. The second type of phone is still up-to-date and fashionable after the warranty period, and late adopters can continue using this phone whereas early adopters will move to newer products. We therefore expect that consumer decisions for the second phone will be varied. Summarizing, we expect in general that IBW will be the best method for the first phone, and IBM for the second one.

Phones of type 1 have a short sales period of slightly more than a year, which is too short for all forecast models, including black box, to provide reliable forecasts for the full EOL phase. For this phone, we make the same choice as for televisions in the previous section, that is, we forecast remaining EOL demand one year after the end of product sales. The sales period of phones of type 2 is nearly two years, which is sufficiently long to forecast the full observed EOL phase.

Expensive spare parts of smartphones are their circuit board (relative price 29-40%) and touch screen (20-26%). For the reasons stated above, we expect that IBW provides the best forecasts for phone 1 and IBM for phone 2. A very cheap spare part is the back cover (relative price 1-2%). As this part is so cheap and very easy to buy at shops or via internet, we expect that the demand for this spare part is best predicted by IBL.

Table 4 shows the forecast results for smartphone spare parts. Our hypotheses for the circuit board are confirmed, as IBW is clearly the best for phone 1 and IBM for phone 2. IBW is also best for touch screens, and this confirms our hypothesis for phone 1 but not for phone 2. The outcomes show that users of both phones replace the touch screen mostly during the warranty period. IBW is also best for back covers, contrary to our expectation that this cheap spare part would be replaced also after warranty has expired. Smartphone users seem not to wish repairing cosmetic parts after warranty has expired, even if the spare part is cheap. Overall, the outcomes provide support for the first hypothesis of Section 3.2, and partial support for the second and fourth hypothesis.

<< Insert Table 4 about here. >>

6.5 Discussion

Table 5 provides a summary of our hypothesis for each of the eighteen considered spare parts, together with the outcomes discussed in preceding sections. The table also shows the outcomes of two tests to compare the forecast performance of two candidate methods. The first test is a comparison-of-means *t*-test for the forecast errors over the EOL phase, that is, the SUM criterion. The second test is a forecast comparison test proposed by Diebold and Mariano (1995), which is a comparison-of-means *t*-test for the absolute forecast errors over the EOL phase. Their test provides automatic correction for the extensive serial correlation that is present in the weekly series of forecast errors. If our hypothesis is confirmed, then we test whether this method is significantly better than the second-best method as measured by the SUM and MAPE criteria. If our hypothesis is denied, then we test whether the best method is significantly better than the method of the hypothesis. Small p-values indicate that one method is significantly better than the other. The final column in Table 5 shows whether our hypothesis is significantly confirmed or denied by the two tests. Confirmation is found in twelve out of eighteen cases, whereas denial occurs in six cases. Possible causes of these denials were discussed in previous sections.

We summarize our findings in terms of the four hypotheses of Section 3.2. The first hypothesis, that installed base forecasts are better than the considered black box AR methods, is confirmed for seventeen out of the eighteen spare parts in Table 5, and Tables 2-4 show that the improvements are substantial. The second hypothesis concerns the relative performance of lifetime and warranty installed base. Table 5 shows that one of these two installed bases is expected to be best for fourteen spare parts, and the hypothesis is confirmed for ten of these spare parts. The most notable exception occurs for back covers of smartphones, where the outcome that IBW is better than IBL is reverse to what was expected. The third hypothesis is on economic installed base, which is our hypothesis for one spare part of refrigerators. This hypothesis is confirmed for one type of refrigerator, but not for the other type. The fourth hypothesis is on mixed economic installed base, which is our hypothesis for two spare parts of smartphone type 2. This hypothesis is confirmed for one of these spare parts, but not for the other one.

7 Conclusions

When production of a product stops, the producer faces the important question as to how many spare parts will be needed during the end-of-life (EOL) phase of the product to cover all future demand. The producer then needs to know how many users will repair the product when it fails and which spare parts are needed for the repair. Their decision will depend on the price of the required spare part and on the evaluation by the user of the remaining value of the product. We propose the concept of installed base to forecast EOL spare part demand as decision support for the final production size. The specific type of installed base depends on characteristics of the

product, the spare part, and the consumer market. Warranty installed base is advised for relatively expensive spare parts, and lifetime base for essential spare parts for products with long average lifetimes. Economic installed base is useful for products with long average lifetimes that are out of warranty. If consumers differ in their adoption attitudes for product innovations, a mixed economic installed base can be useful. Our case study shows that installed base forecasts improve much upon black box autoregressive extrapolation.

Although the specific results will vary across products, the proposed methodology is technically viable to forecast EOL spare part demand. The required information for each product and spare part is the following. First, and most important, time series of product sales and of spare part demand until start of EOL. Further, the average lifetime of the product (for lifetime installed base), the warranty period (for warranty installed base), the cost of the spare part (for economic installed base), and consumer segments (for mixed economic base). We propose to smooth the demand data after screening them for extreme values that may occur, for example, in case of extraordinary failure rates briefly after introduction of a new product. The quality of the forecasts depends on the richness of the available data, ideally with a long estimation period during the production phase and with a limited EOL phase. In practice, the production period is often relatively short as compared to the EOL phase, and this also applies for the products in our case study. We advise to use simple models in such cases, in order to prevent forecast deterioration due to over-fitting. We do not advise a fully automated forecast procedure, because consumer behavior differs widely across products. It can be helpful to cluster products and spare parts in groups, depending on their characteristics and on expected consumer demand behavior. Within each cluster, EOL spare part demand can be forecasted by using the same type of installed base that applies for that cluster.

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Figure 1

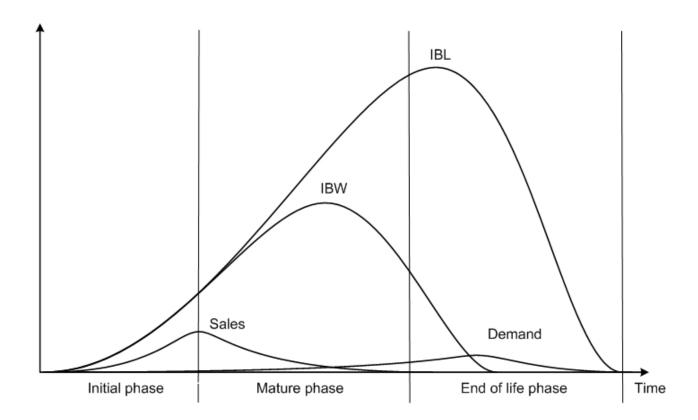


Figure 2

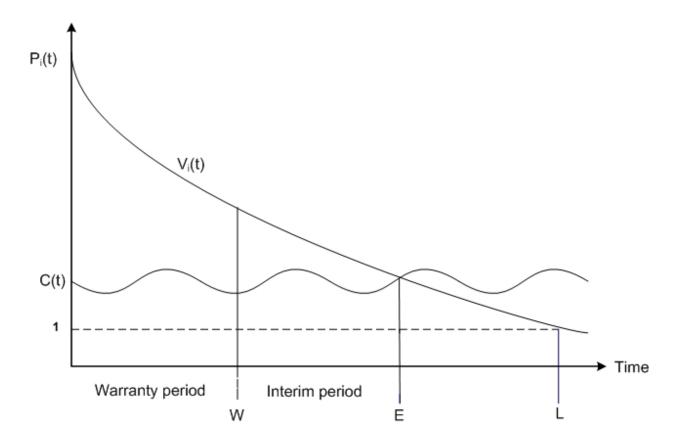


Figure 3

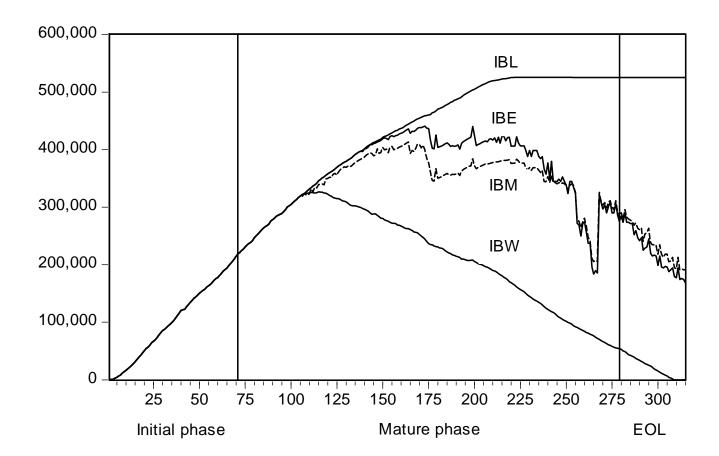


Figure 4

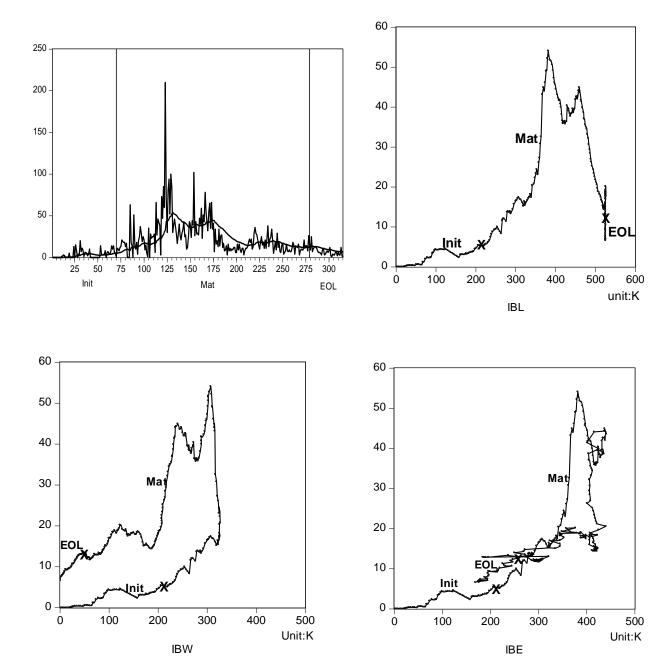


Figure 5

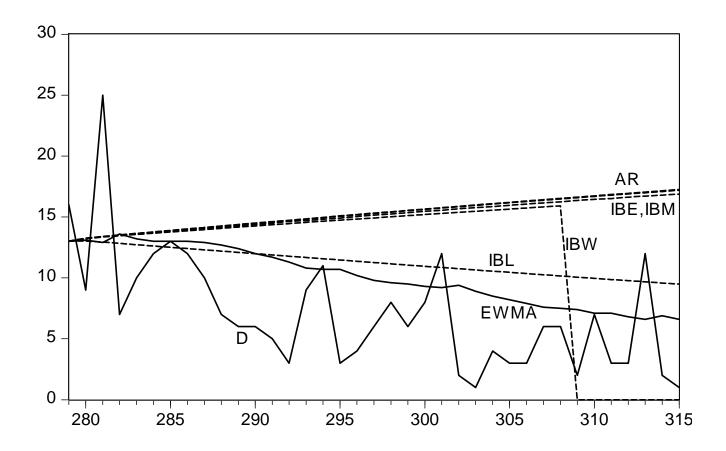


Figure 6

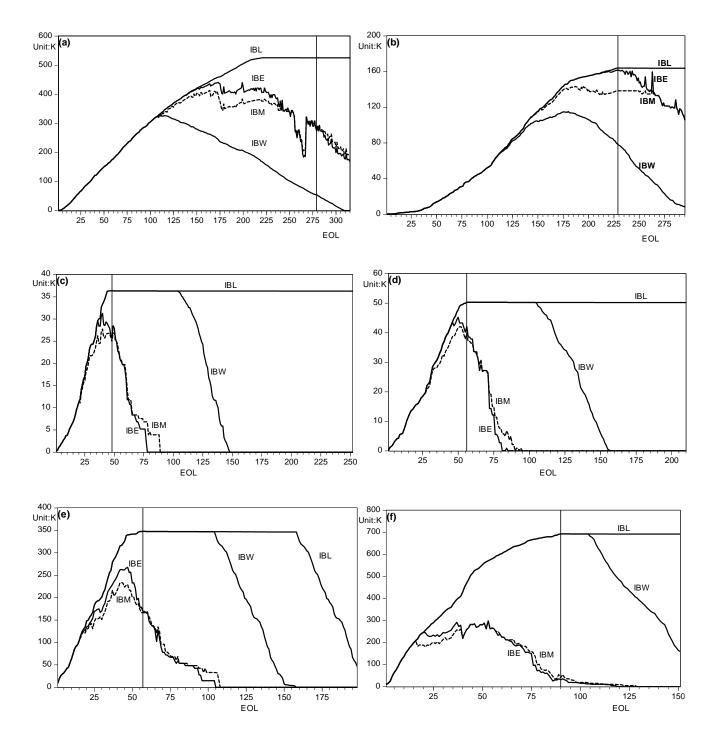


Table 1: Overview of six products and eighteen spare parts

Product	Consumer sentiments		Sales	Period	Estim.	Forec.	Lifecycle
	Life cycle	Tech trendy					
Refrigerator 1	Long	Low	538,386	08.12 - 13.29	279	36	676
Refrigerator 2	Long	Low	166,782	08.32 - 12.51	229	66	676
Television 1	Intermediate	Intermediate	36,766	09.23 - 10.17	100	152	360
Television 2	Intermediate	Intermediate	50,986	10.12 - 11.15	108	102	360
Smartphone 1	Short	High	348,153	10.24 - 11.28	109	89	160
Smartphone 2	Short	High	694,816	11.19 - 13.04	90	61	160

Spare part	Essential	Expensive	Den	Demand		EOL %		e %	Hypothesis
			1	2	1	2	1	2	
For refrigerator									
Compressor	Yes	Yes	5,678	6,090	4.4	13.6	46.7	39.8	L
Circuit board	No	No	9,596	3,518	17.7	36.5	7.6	5.6	E
Door gasket	No	No	4,581	698	17.7	10.0	3.5	3.4	W
For television									
LCD panel	Yes	Yes	868	889	24.9	20.6	47.0	39.4	W
Circuit board	No	No	562	774	37.7	39.1	9.6	8.2	W
Cover	No	No	152	230	9.2	16.1	3.2	4.5	W
For smartphone									
Touch screen	Yes	Yes	21,499	58,413	16.5	23.8	19.8	25.8	1W, 2M
Circuit board	No	Yes	6,325	14,492	14.2	37.6	28.6	40.0	1W, 2M
Back cover	No	No	5,259	11,033	42.4	45.8	1.6	1.2	L

^{*} Consumer sentiments describe aspects that affect consumer attidudes towards products and spare parts.

^{*} Sales is total product sales during sales period, indicated in format year.week (e.g., 08.12 is week 12 of 2008).

^{*} Estim. and Forec. show number of weeks of data available respectively for estimation and for forecast analysis.

^{*} Lifecycle is average lifetime in weeks, obtained for refrigerators from National Association of Home Builders (2007), for televisions from NPD DisplaySearch (2012), and for smartphones from Recon Analytis (2011).

^{* &}quot;Essential" describes whether spare part is essential for product, and "Expensive" indicates relative price of spare part compared to product price (excluding spare part repair costs if service engineer is needed).

^{*} Demand is total spare part demand until analysis date, i.e., week 13 of 2014, for products of type 1 and type 2.

^{*} EOL % shows demand during EOL phase as percentage of demand in initial and mature phases.

^{*} Price % is price of spart part as percentage of product price, for type 1 & for type 2.

^{*} Hypothesis shows installed base hypothesis for the spare part (L for IBL, W for IBW, E for IBE, M for IBM).

Table 2: Forecast results for refrigerator spare parts

Spare part		Installed base					
_	AR	L	W	Е	M		
1, Compressor							
SUM	1.23	0.64	0.72	1.23	1.21		
MAPE	1.33	0.77	1.05	1.21	1.30		
RMSPE	1.46	0.87	1.21	1.46	1.44		
2, Compressor							
SUM	0.81	0.05	0.32	0.17	0.12		
MAPE	0.98	0.49	0.62	0.53	0.52		
RMSPE	1.08	0.68	0.75	0.69	0.69		
1, Circuit board							
SUM	-0.10	0.03	-0.13	<u>-0.01</u>	0.03		
MAPE	0.27	0.25	0.39	0.25	0.25		
RMSPE	0.32	0.32	0.54	<u>0.31</u>	0.32		
2, Circuit board							
SUM	<u>-0.09</u>	3.04	-0.27	2.94	1.37		
MAPE	0.27	3.09	0.37	2.99	1.43		
RMSPE	<u>0.36</u>	4.07	0.48	3.93	1.68		
1, Door gasket							
SUM	-0.08	0.09	-0.15	0.07	<u>0.05</u>		
MAPE	<u>0.38</u>	0.47	0.43	0.45	0.44		
RMSPE	0.59	0.62	0.61	0.61	0.61		
2, Door gasket							
SUM	1.43	0.18	0.07	0.17	0.19		
MAPE	1.60	0.94	<u>0.85</u>	0.94	0.94		
RMSPE	1.79	<u>1.13</u>	<u>1.13</u>	<u>1.13</u>	<u>1.13</u>		

^{*} The two types of refrigerator are denoted by 1 and 2.

^{*} Installed base L denotes lifetime, W warranty, E economic, and M mixed economic.

^{*} SUM is the summed forecast error over observed EOL as fraction of EOL demand; a positive (negative) value corresponds to over-forecasting (under-forecasting) actual demand.

^{*} RMSPE and MAPE are respectively the root mean weekly squared prediction error and the mean weekly absolute prediction error over observed EOL, each as fraction of the mean weekly EOL demand.

^{*} The result for the best forecast method (per spare part and criterion) is underlined.

Table 3: Forecast results for television spare parts

Spare part			Installe	ed base	
	AR	L	W	Е	M
1, LCD panel					
SUM	4.60	22.03	<u>-0.22</u>	-1.00	-1.00
MAPE	4.64	22.03	0.89	0.99	1.00
RMSPE	4.79	24.42	<u>1.48</u>	1.76	1.76
2, LCD panel					
SUM	3.45	0.48	<u>0.46</u>	-1.00	-1.00
MAPE	3.46	<u>0.78</u>	0.95	1.00	1.00
RMSPE	3.73	<u>1.00</u>	1.34	1.39	1.39
1, Circuit board					
SUM	2.31	-0.16	-0.26	<u>-0.03</u>	0.15
MAPE	2.52	0.65	0.68	0.72	0.74
RMSPE	2.75	0.92	0.98	0.98	0.95
2, Circuit board					
SUM	6.26	7.55	<u>0.85</u>	5.77	8.03
MAPE	6.26	7.54	<u>1.48</u>	5.82	8.05
RMSPE	7.37	9.06	<u>1.95</u>	7.68	9.56
1, Cover					
SUM	13.86	5.86	<u>1.20</u>	3.58	3.62
MAPE	14.11	6.51	<u>2.17</u>	4.34	6.51
RMSPE	14.11	7.60	<u>4.34</u>	5.43	7.60
2, Cover					
SUM	4.45	3.53	0.25	2.40	2.01
MAPE	4.69	3.86	<u>1.38</u>	2.76	2.48
RMSPE	5.24	4.14	<u>2.76</u>	3.31	3.31

^{*} This table is similar to Table 2.

^{*} Spare part demand forecasts are not for the full EOL phase, but for the subperiod starting one year (52 weeks) after begin of EOL.

Table 4: Forecast results for smartphone spare parts

Spare part		Installed base				
	AR	L	W	Е	M	
1, Circuit board						
SUM	6.03	4.04	<u>-0.05</u>	-1.00	-1.00	
MAPE	6.05	4.08	<u>0.96</u>	1.00	1.00	
RMSPE	6.34	4.66	<u>1.76</u>	2.05	2.05	
2, Circuit board						
SUM	0.71	0.78	0.76	-0.24	0.03	
MAPE	0.85	0.91	0.89	0.73	0.69	
RMSPE	0.99	1.06	1.03	0.93	0.86	
1, Touch screen						
SUM	5.03	3.01	<u>-0.31</u>	0.97	2.20	
MAPE	5.18	3.16	<u>0.65</u>	1.37	2.44	
RMSPE	5.36	3.62	1.25	2.08	2.96	
2, Touch screen						
SUM	1.74	0.54	<u>0.36</u>	1.15	-0.45	
MAPE	1.77	<u>0.65</u>	0.67	1.26	0.88	
RMSPE	1.89	<u>0.80</u>	0.91	1.41	1.15	
1, Back cover						
SUM	3.53	4.46	0.83	5.74	5.69	
MAPE	3.84	4.76	<u>1.59</u>	6.03	5.99	
RMSPE	4.14	5.18	<u>2.07</u>	6.88	6.98	
2, Back cover						
SUM	0.42	1.88	<u>0.18</u>	0.70	1.13	
MAPE	0.86	2.17	<u>0.67</u>	1.08	1.49	
RMSPE	1.13	2.51	<u>0.99</u>	1.25	1.79	

^{*} This table is similar to Table 2.

^{*} Forecast evaluation period is full EOL phase for smartphone type 2, and subperiod starting one year (52 weeks) after begin of EOL for smartphone type 1.

Table 5: Evaluation of forecast strategies

Spare part	Hypothesis	Outcome		Test	Conclusion	
			Type	SUM	MAPE	
Refrigerator						
1, Compressor	L	L	L > W	0.000	0.000	Confirmed (2x)
2, Compressor	L	L	L > M	0.000	0.006	Confirmed (2x)
1, Circuit board	E	E	E > L	0.000	0.400	Confirmed (1x)
2, Circuit board	E	AR	AR > E	0.000	0.000	Denied (2x)
1, Door gasket	W	M	M > W	0.001	0.855	Denied (1x)
2, Door gasket	W	W	W > E	0.000	0.293	Confirmed (1x)
Television						
1, LCD panel	W	W	W > E	0.000	0.000	Confirmed (2x)
2, LCD panel	W	W	W > L	0.378	0.995	Weakly confirmed (1x)
1, Circuit board	W	E	E > W	0.000	0.802	Denied (1x)
2, Circuit board	W	W	W > E	0.000	0.000	Confirmed (2x)
1, Cover	W	W	W > E	0.000	0.000	Confirmed (2x)
2, Cover	W	W	W > M	0.000	0.000	Confirmed (2x)
Smartphone						
1, Circuit board	W	W	W > E	0.000	0.400	Confirmed (1x)
2, Circuit board	M	M	M > E	0.001	0.228	Confirmed (1x)
1, Touch screen	W	W	W > E	0.000	0.000	Confirmed (2x)
2, Touch screen	M	W	W > M	0.000	0.021	Denied (2x)
1, Back cover	L	W	W > L	0.000	0.000	Denied (2x)
2, Back cover	L	W	W > L	0.000	0.000	Denied (2x)

^{*} Hypothesis on installed base: lifetime L, warranty W, economic E, or mixed economic M.

^{*} Outcome shows the installed base that provides the best forecasts (taken from Tables 2-4); if best method varies across criteria, then method with best SUM is taken as outcome.

^{*} Test type A > B tests whether method A provides better forecasts than method B; if the outcome confirms the hypothesis, then A is the hypothesis and B is second-best method (with respect to SUM); if the outcome differs from the hypothesis, then A is the outcome and B is the hypothesis.

^{*} Test SUM is t-test for mean error, and MAPE is Diebold-Mariano test for absolute errors; the table shows the p-value for the one-sided alternative that base A is better than base B.

^{*} Conclusion "confirmed" denotes that the hypothesis base is significantly better (for 1 or 2 tests) than second-best; "weakly confirmed" means that hypothesis base is best, but not significantly better than the second-best base; "denied" means that the hypothesis base performs significantly worse than the best base; significance level is 5%.