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Abstract	VMI (Vendor Managed Inventory) is a widely used cooperative inventory policy in supply chains in which each enterprise has its autonomy in pricing. This paper discusses a leader-follower Stackelberg game in a VMI supply chain where the manufacturer, as a leader, produces a single product with a limited production capacity and delivers it at a wholesale price to multiple different retailers, as the followers, who then sell the product in dispersed and independent markets at retail prices. An algorithm is then developed to determine the equilibrium of the Stackelberg game. Finally, a numerical study is conducted to understand the influence of the Stackelberg equilibrium and market related parameters on the profits of the manufacturer and its retailers. Through the numerical example, our research demonstrates that: (a) the market related parameters have significant influence on the manufacturer' and its retailers' profits; (b) a retailer's profit may not be necessarily lowered when it is charged with a higher inventory cost by the manufacturer; (c) the equilibrium of the Stackelberg equilibrium benefits the manufacturer.
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# Leader-follower Game in VMI System with Limited Production Capacity Considering Wholesale and Retail Prices

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VMI (Vendor Managed Inventory) is a widely used cooperative inventory policy in supply chains in which each enterprise has its autonomy in pricing. This paper discusses a leader-follower Stackelberg game in a VMI supply chain where the manufacturer, as a leader, produces a single product with a limited production capacity and delivers it at a wholesale price to multiple different retailers, as the followers, who then sell the product in dispersed and independent markets at retail prices. An algorithm is then developed to determine the equilibrium of the Stackelberg game. Finally, a numerical study is conducted to understand the influence of the Stackelberg equilibrium and market related parameters on the profits of the manufacturer and its retailers. Through the numerical example, our research demonstrates that: (a) the market related parameters have significant influence on the manufacturer' and its retailers' profits; (b) a retailer's profit may not be necessarily lowered when it is charged with a higher inventory cost by the manufacturer; (c) the equilibrium of the Stackelberg equilibrium benefits the manufacturer.

*Keywords:* Supply chain; Vendor managed inventory; Stackelberg game

## 1 Introduction

Enterprises can be non-cooperative and cooperative in a supply chain since they individually try to maximize their own profits. For a VMI (Vendor Managed Inventory) -type supply chain, the manufacturer and its retailers cooperate with each on their inventory control. The manufacturer decides on the appropriate inventory levels of each of the products for all enterprises, and the appropriate inventory policies to maintain these levels (Simchi-Livi et al. 2000). A VMI system has been widely adopted by many industries for years. The classical success story for VMI system is found in the partnership between Wal-Mart and Procter & Gamble (P&G). In 1985, the partnership had dramatically improved P&G's on-time deliveries and Wal-Mart's sales, and both of their inventory turns also increased (Buzzell and Ortmeyer 1995). Besides retailing industries, VMI is adopted by leading chemical companies to increase supply chain efficiency and to enhance customer and supplier relationships (Challener 2000). High-tech industries such as Dell, HP and ST Microelectronics also operate efficient supply chains through VMI to reduce inventory levels and costs (Shah 2002, Tyan and Wee 2003). In this system there are manufacturers ( P&G, Dell, HP etc.) and retailers, such as Wal-Mart, and all kinds of products sold, for example, HP selling printers, computers and scanners etc, P&G manufacturing cosmetics, household cleaners and paper products etc.

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However, it should be noted that VMI represents partial cooperation in the sense that their cooperation focuses only on inventory control and there still exist some degrees of autonomies for the individual enterprises/players to respond to their changing environments. For example, individual enterprises still enjoy the rights of determining retail prices for retailers and wholesale price for the manufacturer. Thus the enterprises face a question: how does every individual enterprise give optimal decision with its autonomies in order to maximize its own profit in VMI system? At this point the non-cooperation then occurs under a VMI system when every individual enterprise maximizes its own profit. The question motivates us to research this kind of semi-cooperation supply chain.

In the semi-cooperation setting, the competition among the different enterprises become inevitable, and the characteristic of the VMI setting will determine the game type among the enterprises. The characteristic of the VMI strategy is that the vendor generates the point of order decision and holds the ultimate inventory ownership. In order to make the strategy be implemented, under the VMI scheme, the vendor must have the capability to perform demand forecasting, and inventory management (Tyan and Wee 2003). Thus it is easy for the manufacturer to know the information of its retailers, and then anticipate the reaction of its retailers. At this point it is reasonable to assume that the manufacturer knows the information and reaction of its retailers, but the retailers do not know that of the manufacturer. The game approach addressing the kind of information asymmetry can be modeled as a Stackelberg game (see Chen et al. 2001a, Viswanathan and Wang 2003, etc.), where the enterprise who dominates the information of the system is considered as a leader, while the other enterprises are the followers. The followers' reaction process is known to the leader. In the VMI system, the manufacturer dominating the information motivates us to research how the manufacture can take advantage of the information to improve its profit with Stackelberg game. The manufacturer is the leader who knows and can participates the reaction of each retailer to maximizing its own profit by determine the inventory strategy of the VMI system and its wholesale price. Each retailer, as a follower, does not know the reaction of the manufacturer, but takes the manufacturer's decision results as input parameters to maximize its own profit by determining its retail price.

This paper is concerned with such a VMI supply chain that consists of one manufacturer (vendor) and multiple different retailers with a single product. The manufacturer produces a single product with a limited production capacity and distributes it to its retailers. Each retailer buys the product from the manufacturer at a wholesale price, and then sells it to the consumer market at a retail price. Retailers' markets are assumed to be dispersed and independent each other. In the supply chain, the manufacturer, as a leader, determines the wholesale price, inventory policy for the supply chain for maximizing its own profit, and each retailer, as a follower, in turn takes the manufacturer's decision results as given inputs to determine the optimal retail prices for maximizing its own profits.

The paper is organized as follows. In Section 2, the relevant literature is briefly reviewed. Section 3 gives the problem description and notations to be used. Section 4 develops the game model while Section 5 analyses the Stackelberg equilibrium for the developed model. Section 6 derives a method for solving the Stackelberg equilibrium. Section 7 presents a numerical study and corresponding sensitive analysis for some selected parameters.

## **2 Literature review**

This paper discusses the Stackelberg game in the supply chain where a manufacturer and multiple retailers maximize their own profits by determining their price strategies individually while inventory policies is cooperated with VMI strategy . So the research literature related to this paper can be divided into those on

integrated supply chain inventory model, those on integrating marketing policies into inventory decisions, and those on Stackelberg games in supply chains.

Early researches based on simple supply chain inventory involved a single vendor and single retailer, as used by Goyal (1977) for studying a joint economic lot size (JELS) model to minimize the total relevant costs. Banerjee (1986) generalized Goyal's model (Goyal 1977) by incorporating a finite production rate for the vendor to obtain the optimal joint production or order quantity. Goyal (1988) extended Banerjee's model (Banerjee 1986) again by relaxing the lot-for-lot production assumption and argued that the economic production quantity is an integer multiple of the buyer's purchase quantity and showed that its model provides a lower or equal joint total relevant cost. Kohli and Park (1994) investigated joint ordering policies as a method to reduce transaction costs between a single vendor and a homogeneous group of retailers. They presented expressions for optimal joint order quantities assuming all products are ordered in each joint order. Lu (1995) considered a one-vendor multi-buyer integrated inventory model and gave a heuristic approach for joint replenishment policy. Banerjee and Banerjee (1992) considered a VMI system in which the vendor makes all replenishment decisions for its buyers to improve the joint inventory cost. Woo et al. (2001) and Yu and Liang (2004a) extended their discussions to three level supply chain in which only one raw material is considered. Recently, VMI is widely studied by other researchers, such as Achabal et al. (2000), Dong and Xu (2002), Disney and Towill (2003), Toni and Zamolo (2005), and Rusdiansyah and Tsao (2005) etc. How to give an optimal inventory policy for maximizing the joint inventory cost is the main objective for these literatures. Note that in the above VMI setting (Woo et al. 2001, Yu and Liang 2004a, etc.), in order to streamline the supply chain, vendors are expected to synchronize its production cycles with buyers' ordering cycles, such as common replenishment cycles/epochs adopted (Viswanathan and Piplani 2001, Woo et al. 2001, Mishra 2004, Yu and Liang 2004a, etc.), so that the total inventory cost for the entire chain can be reduced (Woo et al. 2001).

Many researchers have considered how to integrate marketing policies into inventory control decisions. For example, Kotler (1971) incorporated marketing policies into inventory decisions and discussed the relationship between economic ordering quantity and price decisions for infinite time horizon. Ladany and Sternleib (1974) studied the effect of price variations on demand and consequently on EOQ (economic order quantity). Roslow et al. (1993) and Yu and Liang (2004b) studied co-op advertisement or pricing. Chen and Chen (2005) and Anjos et al. (2005) established a pricing and inventory policy that maximizes the revenue from selling a given inventory of items with continuous decay. These papers mainly discuss how the end market policies influence system wide profit to show the importance of market parameters.

Stackelberg games for analyzing the game in supply chains are studied by quite a lot of researchers. In recent years, Weng (1995) studied the supply chain with one manufacture and multiple identical retailers, shows that Stackelberg game is used to guarantee perfect coordination considering quantity discounts and franchise fees. In the setting studied by Weng (1995), Chen et al. (2001a) showed that when the retailers are not identical, such a scheme is not guaranteed to perfectly coordinate the channel. They consider two Stackelberg games with the supplier as the leader and the retailers as followers. In one, the supplier sets a constant wholesale price, and in the other, the supplier offers an order-quantity discount scheme with one breakpoint. Viswanathan and Wang (2003) studied a similar setting with one manufacturer and one retailer by Stackelberg game with three price discount schemes, namely, (1) volume discounts, (2) quantity discounts, and (3) quantity discounts and simultaneous offer of volume discounts when demand is constant but price-sensitive. It is shown that quantity discount schemes help the supplier achieve economies in order processing and inventory costs by encouraging buyers to increase the size of each of lot. However, quantity discounts tend to raise the cycle inventory of the supply chain. With demand that is price-sensitive, Qin et al. (2006) considered volume discounts and franchise fees as coordination mechanisms in a system consisting of a supplier and a buyer. The problem is analysed as a Stackelberg game. The competition/coordination

mechanism in supply chains with Stackelberg game was also discussed by Huang and Li (2001), Viswanathan and Rajesh (2001), Chen et al. (2001b), Sarmah et al. (2005) , and Parlar and Weng (2006) etc. A critical assumption made throughout this literature, though, is that the supplier has full information, and can design the quantity discount/franchise scheme without giving a reasonable reason. Most of these researchers took quantity discounts or/and franchise fee with a contract as incentive schemes to influence buyers' ordering behaviour, thus reducing the supplier's (and the total supply chain's) costs. In the above literature, none of them take the wholesale price and retail price as a decision variables in the setting with one vendor and multiple different retailers. The game in VMI system is few concerned.

### 3 Problem description and notations

#### 3.1 Problem description

In this paper, we consider one manufacturer and multiple retailers in a VMI setting in which:

- (1) The manufacturer produces one type of product with a limited production capacity, and supplies it to its multiple retailers.
- (2) The retailers are geographically dispersed to serve the markets in their own regions. The demand function for every retailer is the decreasing and convex function with respect to its retail price.
- (3) The manufacturer is a leader in the supply chain. The retailers' response is available to the manufacturer (vendor) who determines the inventory replenishment plan and wholesale price. The manufacturer produces the product with fixed product rate and its production capacity is limited.
- (4) The retailers are assumed to be followers in the supply chain. However, they have the right to make decisions on their own retail prices.
- (5) The manufacturer is responsible for the chain-wide inventory control with the policy of VMI in which the manufacturer provides the product to its multiple retailers with a common replenishment cycle, and thus incurs all inventory related costs in order to eliminate the influence of the variations of the common replenishment cycle and backorder rate of every retailer on its retailers. Each retailer bears some inventory cost by repaying it to the manufacturer, and the cost for each retailer is in direct proportion to its demand rate. The inventory cost per unit is a constant that is previously negotiated by the manufacturer and its retailers.

#### 3.2 Notations

Parameters	Definition
$m$	Total number of retailers
$i$	Index of retailers or markets, $i = 1, 2, \dots, m$
$b_i$	Fraction of backlogging per unit time for retailer $i$ , decision variables for the manufacturer
$C$	Common replenishment cycle time for the product, decision variable for the manufacturer (time)
$c_p$	Wholesale price of the product determined by the manufacturer, decision variable for the manufacturer (\$/unit)

$c_m$	Production cost per unit product (\$/unit)
$IR_p$	Inventory cost for the manufacturer managing all retailers' product inventory (\$/time)
$D_i(p_i)$	Demand rate in the market $i$ per unit time served by retailer $i$ , a decreasing function of $p_i$
$H_{b_i}$	Holding cost paid by the manufacturer to manage retailer $i$ 's inventory (\$/unit/time)
$H_p$	Holding cost at the manufacturer's side (\$/unit/time)
$K_i$	Constant in the demand function of retailer $i$
$L_{b_i}$	Backorder cost paid by the manufacturer to market $i$ (\$/unit/time)
$p_i$	Retail price charged by retailer $i$ (\$/unit), decision variable for retailer $i$
$P$	Production rate for the manufacturer, which is a known constant and $\sum_{i=1}^m D_i(p_i) \leq P$
$S_{b_i}$	Fixed cost paid by the manufacturer for managing retailer $i$ 's inventory (\$ per order)
$S_p$	Fixed order cost in a common replenishment cycle time for the manufacturer producing the product (\$ per setup)
$\phi_i$	Transportation cost shipped from the manufacturer to retailer $i$ (\$/unit)
$\zeta_i$	Inventory cost paid by retailer $i$ (\$/unit/time)
$x$	Binary variable to indicate whether the production capacity of the manufacturer is abundant, $x=1$ indicating that the capacity is surplus and vice versa
$TIC_p$	Total inventory cost for the product (\$/time)
$TIDC_p$	Total indirect cost for the product (\$/time)
$TDC_p$	Total direct cost for the product (\$/time)
$NP_{b_i}$	Net profit for retailer $i$ (\$/time)
$NP_m$	Net profit for the manufacturer (\$/time)

### 3.3 Game model

This section models the leader-follower relationship between the manufacturer and the retailers as a Stackelberg game with the manufacturer as the leader and retailers as its followers. In this game, the manufacturer maximizes its net profit by giving its optimal wholesale price and inventory control policy for the VMI system. All retailers decide their optimal retail prices to maximize their own net profits.

Here we consider a two-echelon supply chain with a supplier distributing a single product to  $m$  separated retailers who in turn sell the product on their own markets. The demand at each retailer is described by a general demand function of the retail price. The demand functions are, almost invariably, downward sloping and a convex function with respect to  $p_i$ . We have

$$\frac{\partial D_i(p_i)}{\partial p_i} < 0 \text{ and } \frac{\partial^2 D_i(p_i)}{\partial p_i^2} > 0. \quad (1)$$

This common demand function can go back to Samuelson (1947), see also Vives (1990) and often be described as Cobb-Douglas demand function as follows:

$$D_i(p_i) = K_i p_i^{-e_{p_i}} \quad i = 1, 2, \dots, m, \quad (2)$$

in which  $K_i$  and  $e_{p_i}$  represents the market scale of retailer  $i$  and the demand elasticity of retailer  $i$  with respect to its retail price respectively. Following the problem description that  $D_i(p_i)$  is a decreasing and convex function in the 2<sup>nd</sup> point in Subsection 3.1, we have  $e_{p_i} > 0$ .

It is assumed that the supply chain adopts a VMI strategy where the manufacturer is responsible for the chain-wide inventory control and each retailer pays its inventory cost again to the manufacturer in proportion to its demand rate  $D_i$  and thus the inventory cost per unit that is a constant  $\zeta_i$   $i=1,2,\dots,m$  negotiated by the manufacturer and its retailers. Retailer  $i$ 's inventory cost then is  $\zeta_i D_i(p_i)$ . Retailer  $i$ 's product procurement cost is  $c_p D_i(p_i)$  and the revenue is  $p_i D_i(p_i)$ . Therefore, the net profit for retailer  $i$  is given as Equation (3):

$$NP_{bi} = (p_i - c_p - \zeta_i) D_i(p_i). \quad (3)$$

Consider the transfer payment from a retailer to the manufacturer in our model. According to the 5<sup>th</sup> point of the problem description in Subsection 3.1, the payment consists of two components; a wholesale price  $c_p$  and an inventory charge  $\zeta_i$ . The reason is that a) any VMI system, of course including our VMI setting, is established under the setting that the manufacturer and its retailers have the long time cooperation of inventory control. In this case, it is not acceptable for a retailer to buy its product from the manufacturer at a higher price than that of the other retailers. The manufacturer must sell the same product at the same price to its retailers. So the first component, the wholesale price, as the manufacturer's decision variable, occurs. b) The second component  $\zeta_i$  occurs in our manuscript is also necessary since each retailer may have differences each other, such as inventory holding cost, ordering cost and demand rate, distance etc. So for the manufacturer, it is reasonable to charge different inventory costs from different retailers according to their inventory condition and demand rate.

After the net profit function for individual retailers is established as above, let us consider the net profit function for the manufacturer. Consider the components of the total inventory cost first. Figure 1 shows the inventory levels for all retailers and the manufacturer. As indicated in the 5<sup>th</sup> point of the problem description in Subsection 3.1, the inventory cost spent for all retailers consist of two parts, one is paid by all retailers and the other is paid by the manufacturer. According to the given VMI policy, retailer  $i$  pays inventory cost by demand rate only, that is  $\zeta_i D_i(p_i)$ , and the manufacturer pays the rest of the total inventory cost. So what the manufacturer spent is equal to abstracting the inventory cost paid by all retailers from that paid by the whole VMI system. To manage the product inventory in retailer  $i$ , VMI system spends  $S_{b_i} / C$  on fixed order cost,  $H_{b_i} (D_i(p_i)(1-b_i)C) / 2 = D_i(p_i)(1-b_i)^2 C^2 H_{b_i} / 2$  on holding cost, and  $L_{b_i} (D_i(p_i)b_i C) / 2 = D_i(p_i)b_i^2 C^2 L_{b_i} / 2$  on backorder cost per unit time according to Figure 1. Thus the inventory cost for the manufacturer managing all retailers' product inventory is given by Equation (4):

$$RI_p = \frac{1}{C} \left[ \sum_{i=1}^m S_{b_i} + \sum_{i=1}^m \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^m \frac{D_i(p_i)b_i^2 C^2}{2} L_{b_i} \right] - \sum_{i=1}^m \zeta_i D_i(p_i). \quad (4)$$

[Insert Figure 1 about here]

The manufacturer's capacity is limited and produces the product with a fixed production rate. When the sum of all retailers' demand rate is less than the production rate, it means that capacity is redundant and the production process is not continuous. The setup cost occurs at every beginning of the common replenishment cycle, and  $x=1$ . Otherwise  $x=0$  and the production capacity is used up. The whole production process being continuous without production setup cost  $S_p$ . Thus, the manufacturer's total inventory cost at its own side for the product can be expressed by Equation (5):



$$TIC_p = \frac{1}{C} [x \cdot S_p + H_p \sum_{i=1}^m \frac{D_i(p_i)^2 C^2}{2P}]. \quad (5)$$

The total indirect cost of the manufacturer is rearranged in Equation (6):

$$TIDC_p = TIC_p + RI_p. \quad (6)$$

The direct cost per unit time which consists of manufacturing cost and transport cost is formulated in Equation (7):

$$TDC_p = \sum_{i=1}^m D_i(p_i)(c_m + \phi_i). \quad (7)$$

As the total revenue for the manufacturer is given as  $\sum_{i=1}^m D_i(p_i)c_p$ , its net profit can be determined by

$$NP_m(b_1, b_2, \dots, b_m, C, c_p, x) = \sum_{i=1}^m D_i(p_i)c_p - TDC_p - TIDC_p. \quad (8)$$

We have now obtained the net profit functions for the retailers and the manufacturer in Equations (3) and (8) respectively. Then the lead-follower relationship for the manufacturer and its retailers can be formulated as the Stackelberg game model below:

$$\max_{b_1, b_2, \dots, b_m, C, c_p, x} NP_m(b_1, b_2, \dots, b_m, C, c_p, x) = \sum_{i=1}^m D_i(p_i)c_p - TDC_p - TIDC_p \quad (9)$$

subject to

$$\sum_{i=1}^m D_i(p_i(c_p)) \leq P, \quad (10)$$

$$P - \sum_{i=1}^m D_i(p_i(c_p)) \leq x \cdot M, \quad (11)$$

$$0 \leq b_i \leq 1, \quad i=1, 2, \dots, m, \quad (12)$$

$$C \geq 0, c_p > 0 \quad x=0 \text{ or } 1; \quad (13)$$

$$\max_{p_i} NP_{b_i} = (p_i - c_p - \zeta_i)D_i(p_i) \quad i = 1, 2, \dots, m \quad (14)$$

subject to

$$p_i \geq c_p + \zeta_i, \quad i = 1, 2, \dots, m, \quad (15)$$

$$D_i(p_i) = K_i p_i^{-e_{p_i}}, \quad i = 1, 2, \dots, m. \quad (16)$$

Here Equations (10) and (11) indicate that the capacity for the manufacturer; when the production capacity is enough,  $P - \sum_{i=1}^m D_i(p_i(c_p)) > 0$ , and  $x=1$  is satisfied by Equation (11); and vice versa. That is,

when the capacity of is insufficient with  $P - \sum_{i=1}^m D_i(p_i(c_p)) = 0$ , then  $x=0$  can be satisfied by the optimal solution of the model since it will make  $TIDC_p$  decrease and  $NP_m$  increase. Equation (15) gives the least acceptable price for retailer  $i$  since  $p_i < c_p + \zeta_i$  will make  $NP_{b_i} < 0$ .

*The game mechanism:* the manufacturer is treated as the leader who first determines the common replenishment cycle, the backorder fraction and wholesale price, etc. The retailers are treated as the followers

who take the manufacturer's decision results as the given input parameters in determining the retail prices  $p_i$   $i = 1, 2, \dots, m$  in their markets when they maximize their net profits respectively. The results of the retailers then influence the net profits of the manufacturer. Then the manufacturer adjusts its optimal decisions in order to maximize its net profit. The process continues until the manufacturer can't increase its profit by changing its decision variables, and then the equilibrium, called *Stackelberg equilibrium*, obtained. That is to say, during this process, as the leader in the game, the manufacturer knows all retailers' reactions and therefore considers them when it maximizes its profit by working out the common replenishment cycle  $C$ , the backorder fraction  $b_i$   $i = 1, 2, \dots, m$ , wholesale price  $c_p$  and  $x$ . Every retailer takes the manufacturer's results as its input parameters in determining its retail price  $p_i$   $i = 1, 2, \dots, m$ .

#### 4 Analysis of the Stackelberg equilibrium

From the game mechanism in the preceding section, in order to determine the Stackelberg equilibrium, we first solve the reaction functions of all retailers in the lower level of the proposed Stackelberg game model, and then give the manufacturer's optimal decisions considering the reaction functions of its retailers.

##### 4.1 The retailers' reactions

Let us replace  $D_i(p_i)$  in Equation (14) with Equation (16), and we obtain Equation (17):

$$\max_{p_i} NP_{b_i} = (p_i - c_p - \zeta_i) K_i p_i^{-e_{p_i}} \quad i = 1, 2, \dots, m. \quad (17)$$

Taking the first derivative of Equation (17) with respect to  $p_i$  as follows:

$$\frac{\partial NP_{b_i}}{\partial p_i} = (1 - e_{p_i}) K_i p_i^{-e_{p_i}} + e_{p_i} (c_p + \zeta_i) K_i p_i^{-(e_{p_i} + 1)} \quad i = 1, 2, \dots, m. \quad (18)$$

If  $0 < e_{p_i} \leq 1$ ,  $\partial NP_{b_i} / \partial p_i > 0$ , the optimal solution is  $NP_{b_i} \rightarrow +\infty$  at  $p_i \rightarrow +\infty$ .

This is impractical. Therefore, we must ignore the situation where  $0 < e_{p_i} \leq 1$ . Let us focus on the situation where  $e_{p_i} > 1$  by setting Equation (18) to zero, we obtain Equation (19):

$$p_i^* = \frac{(c_p + \zeta_i) e_{p_i}}{e_{p_i} - 1} \quad i = 1, 2, \dots, m. \quad (19)$$

From Equation (19), it can be seen that retail  $i$ 's price is determined by its price elasticity  $e_{p_i}$ , wholesale price  $c_p$  and inventory cost per unit  $\zeta_i$ . Since  $e_{p_i} > 1$ ,

$$p_i^* = \frac{(c_p + \zeta_i) e_{p_i}}{e_{p_i} - 1} > (c_p + \zeta_i) \quad i = 1, 2, \dots, m. \quad (20)$$

This indicates that Equation (15) is satisfied naturally for a retailer to maximize its net profit. By substituting (19) into (17) and rearranging the result, we have

$$NP_{b_i} = K_i \left( \frac{(c_p + \zeta_i) e_{p_i}}{e_{p_i} - 1} - c_p - \zeta_i \right) \left( \frac{(c_p + \zeta_i) e_{p_i}}{e_{p_i} - 1} \right)^{-e_{p_i}} \quad i = 1, 2, \dots, m. \quad (21)$$

## 4.2 The manufacturer's decisions

The manufacturer determines the optimal common replenishment cycle  $C$  for the VMI system, wholesale price  $c_p$ , and the backorder fraction  $b_i$  etc. to maximize its own net profit subject to the constraints imposed by Equations (10)-(13), considering the retailers' reaction Equation (19). Hence, substituting Equation (19) to Equation (9) the manufacturer's model can be formulated as

$$\max_{b_1, b_2, \dots, b_m, C, c_p, x} NP_m = \sum_{i=1}^m D_i(p_i^*)c_p - TDC_p - TIDC_p. \quad (22)$$

subject to: (10)-(13) and (19).

Since  $x$  is a binary variable, so the Stackelberg game model can be discussed with  $x=1$  and  $x=0$  separately. Firstly we analyse the model with  $x=1$  from Equations (23) to (29) below.

When  $x=1$ , because the second derivative of (22) with respect to  $b_i$   $i=1, 2, \dots, m$  is negative, we have

$$\frac{\partial^2 NP_m(b_1, b_2, \dots, b_m, C, c_p)}{\partial b_i^2} = -CD_i(p_i^*)(H_{b_i} + L_{b_i}) < 0. \quad (23)$$

Thus,  $NP_m(b_1, b_2, \dots, b_m, C, c_p)$  is a concave function of  $b_i$  for any other given  $b_1, b_2, \dots, b_{i-1}, b_{i+1}, \dots, b_m, C$  and  $c_p$ .

Set the first derivative of (22) with respect to  $b_i$  equal to zero, then  $b_i$  can be obtained as

$$b_i^* = \frac{H_{b_i}}{H_{b_i} + L_{b_i}} \quad i = 1, 2, \dots, m. \quad (24)$$

It can be seen that  $0 < b_i^* < 1$   $i=1, 2, \dots, m$  is in the feasible area of the manufacturer's model. And  $b_i^*$   $i=1, 2, \dots, m$  is the optimal solution of the manufacturer.

By substituting (24) into (22) and rearranging the result, we obtain

$$NP_m(C, c_p) = \sum_{i=1}^m D_i(p_i^*)(c_p + \zeta_i - c_m - \phi_i) - \frac{1}{C}[S_p + \sum_{i=1}^m S_{b_i}] - \frac{C}{2}[H_p \sum_{i=1}^m \frac{D_i(p_i^*)^2}{P} + \sum_{i=1}^m \frac{D_i(p_i^*)L_{b_i}H_{b_i}}{L_{b_i} + H_{b_i}}]. \quad (25)$$

The second derivative of (25) with respect to  $C$  is

$$\frac{\partial^2 NP_m(C, c_p)}{\partial C^2} = -\frac{2}{C^3}[S_p + \sum_{i=1}^m S_{b_i}] < 0. \quad (26)$$

$NP_m(C, c_p)$  is a concave function of  $C$  for any given  $c_p$ .

Thus, from  $\partial NP_m(C, c_p) / \partial C = 0$ , the optimal value of  $C^*$  is obtained as

$$C^* = \sqrt{\frac{2(S_p + \sum_{i=1}^m S_{b_i})}{H_p \sum_{i=1}^m \frac{D_i(p_i^*)^2}{P} + \sum_{i=1}^m \frac{D_i(p_i^*)L_{b_i}H_{b_i}}{L_{b_i} + H_{b_i}}}}. \quad (27)$$

In the radical sign of Equation (27), the denominator,  $H_p (\sum_{i=1}^m D_i(p_i^*)^2) / P + \sum_{i=1}^m D_i(p_i^*) L_{b_i} H_{b_i} / (L_{b_i} + H_{b_i})$  is composed of the items  $D_i(p_i^*)$ ,  $L_{b_i}$  and  $H_{b_i}$  etc. and the numerator is composed of setup costs  $S_p$  and  $S_{b_i}$ . This is similar to that of the EOQ model in terms of overall demand rate and relevant cost components; The optimal  $C^*$  increases when  $D_i(p_i^*)$ ,  $L_{b_i}$  and  $H_{b_i}$  go up, whereas decreases with the increase of  $S_p$  and  $S_{b_i}$ . Substituting (27) into (25), the manufacturer's net profit becomes

$$NP_m(c_p) = \sum_{i=1}^m D_i(p_i^*)(c_p + \zeta_i - c_m - \phi_i) - \sqrt{2H_p \left( \sum_{i=1}^m \frac{D_i(p_i^*)^2}{P} + \sum_{i=1}^m \frac{D_i(p_i^*) L_{b_i} H_{b_i}}{L_{b_i} + H_{b_i}} \right) (S_p + \sum_{i=1}^m S_{b_i})}. \quad (28)$$

Equation (28) is a continuous function of variable  $c_p$ . Since the capacity of the manufacturer is enough ( $x = 1$ ) and the maximal of (28) does exist, the optimal  $c_p$  to maximize (28) is to satisfy

$$\frac{\partial NP_m(c_p)}{\partial c_p} = 0. \quad (29)$$

Let us denote the solution of Equation (29) that maximizes Equation (28) as  $c_p^*$ .

From the analysis from Equation (23) to (29), all optimal variables are determined with  $x = 1$ . Now we discuss the optimal results with  $x = 0$ .

When  $x = 0$ , Equations (10) and (11) are equivalent to  $P = \sum_{i=1}^m D_i(p_i^*(c_p))$  considering the react function of Equation (19). There is only one variable solution  $c_p$  in  $P = \sum_{i=1}^m D_i(p_i^*(c_p))$  since

$$\frac{d \sum_{i=1}^m D_i(p_i^*(c_p))}{dc_p} = \frac{dD_i(p_i^*)}{dp_i} \frac{dp_i^*(c_p)}{dc_p} < 0. \quad (30)$$

The only solution is denoted by  $c_p^{\min}$  here.

Through the similar analysis we can obtain optimal  $b_i^*$  which is the same as Equation (24) and

$$C^* = \sqrt{\frac{2(\sum_{i=1}^m S_{b_i})}{H_p \sum_{i=1}^m \frac{D_i(p_i^*(c_p^{\min}))^2}{P} + \sum_{i=1}^m \frac{D_i(p_i^*(c_p^{\min})) L_{b_i} H_{b_i}}{L_{b_i} + H_{b_i}}}}, \quad (31)$$

which is similar to Equation (27) with  $p_i^* = p_i^*(c_p^{\min})$ . Then the manufacturer's net profit is obtained as

$$NP_m(c_p^{\min}) = \sum_{i=1}^m D_i(p_i^*(c_p^{\min}))(c_p^{\min} + \zeta_i - c_m - \phi_i) - \sqrt{2H_p \left( \sum_{i=1}^m \frac{D_i(p_i^*(c_p^{\min}))^2}{P} + \sum_{i=1}^m \frac{D_i(p_i^*(c_p^{\min})) L_{b_i} H_{b_i}}{L_{b_i} + H_{b_i}} \right) (\sum_{i=1}^m S_{b_i})}. \quad (32)$$

From the above analysis, we can obtain the algorithm steps to calculate the equilibrium of the Stackelberg game as the following section.

## 5 Algorithm steps for Stackelberg equilibrium

With the above analysis, this section presents an algorithm for solving the Stackelberg game model with the following steps:

*Step 1:* Calculate the minimal  $c_p^{\min}$  by  $P = \sum_{i=1}^m D_i(p_i^*(c_p))$  and substitute  $c_p^{\min}$  into Equation (32) to calculate the corresponding  $NP_m(c_p^{\min})$  with  $x=0$ . In this case the manufacturer's production capacity is used up.

*Step 2:* Calculate the optimal  $c_p^*$  by Equation (29). If  $c_p^* > c_p^{\min}$ , substitute  $c_p^*$  into Equation (28) to calculate the corresponding  $NP_m(c_p^*)$  with  $x=1$ . In this case the manufacturer's production capacity is redundant and we go to Step 3. Otherwise we go to Step 4.

*Step 3:* Compare  $NP_m(c_p^{\min})$  with  $NP_m(c_p^*)$ . If  $NP_m(c_p^{\min})$  is larger than  $NP_m(c_p^*)$ , then go to Step 4, otherwise go to Step 5.

*Step 4:* The optimal  $x$  and  $c_p$  are 0 and  $c_p^{\min}$  respectively. Substitute  $c_p^{\min}$  into (24), (31), (19) and (21) to calculate optimal  $b_i, c_p, p_i$  and  $NP_{b_i}$  respectively.

*Step 5:* The optimal  $x$  and  $c_p$  are 1 and  $c_p^*$  respectively. Substitute  $c_p^*$  into (24), (27), (19) and (21) to calculate optimal  $b_i, c_p, p_i$  and  $NP_{b_i}$  respectively.

Step 1 and Step 2 are to calculate two candidate results of the manufacturer's maximum net profit. In Step 2 the manufacturer lacks of capacity with  $c_p^* \leq c_p^{\min}$  and the optimal  $c_p$  will be  $c_p^{\min}$  in Step 1 with  $x=0$  since there is no  $S_p$  in that case. In Step 3, the larger  $NP_m$  is selected as the final optimal results by the manufacturer maximizing its profit since the manufacturer is the leader and prefers a larger profit. Step 4 and Step 5 calculate the other optimal results of all retailers and the manufacturer under two kinds of settings;  $x=0$  and  $x=1$  respectively.

## 6 Numerical example

This section presents a numerical example to the proposed game model. Here one manufacturer and three retailers are selected. The related input parameters for the base example are  $e_{p_1}=1.4$ ,  $e_{p_2}=1.3$ ,  $e_{p_3}=1.5$ ,  $H_{b_1}=6$ ,  $H_{b_2}=6$ ,  $H_{b_3}=6$ ,  $K_1=2 \times 10^6$ ,  $K_2=2.5 \times 10^6$ ,  $K_3=1.5 \times 10^6$ ,  $L_{b_1}=300$ ,  $L_{b_2}=300$ ,  $L_{b_3}=300$ ,  $S_{b_1}=50$ ,  $S_{b_2}=50$ ,  $S_{b_3}=50$ ,  $\phi_1=5$ ,  $\phi_2=5$ ,  $\phi_3=5$ ,  $\zeta_1=7$ ,  $\zeta_2=7$ ,  $\zeta_3=7$ ,  $c_m=150$ ,  $H_p=3$ ,  $S_p=150$ ,  $P=200$ . The base example shows that retailer 1's market is worse than retailer 2', but better than retailer 3'. By applying the above solution procedure in Section 6, the corresponding results for the base example and sensitivity analysis with some selected parameters are shown in Table 1. The following are only a few interesting observations derived from Table 1 although many more can be obtained.

[Insert Table 1 about here]

1. The market related parameters have *significant influence* on the manufacturer' and all retailers' profits. For example, when  $e_{p_1}$  increases from 1.4 in the base example to 1.5, the manufacturer's and retailer 1's profits go down from 68255 to 60090 and 66808 to 31728, decreased by 11.96% and 52.51% respectively. However

retailer 2's and 3's profits remain relatively stable, going up slightly from 181391 to 182806 and 23490 to 23796, increased by 0.78% and 1.3% respectively. It can be seen that retailer 1's related parameters have a significant influence on its own profit, and then on its vendor's/the manufacturer's profit. The other retailers' profit is not much influenced.

2. Retailer 1's parameters not only have impacts its *own* retail price and the manufacturer's wholesale price, but also on the *other* dispersed retailer's retail prices via the change of the manufacturer's decisions. For example, if  $K_1$  increases from  $2 \times 10^6$  in the base example to  $3 \times 10^6$ , retailer 1 changes its retail price from original 2114 to 1924, and the manufacturer decreases its wholesale price from 597.11 to 542.63, deduced by 9.12%, and this reduced wholesale price then makes retailer 2 and retailer 3 have the opportunity to decrease their retail prices, from 2618 to 2382 and 1812 to 1649 respectively. The improvement of the retailer 1's market makes the manufacturer change its market strategy from the strategy of high price with low demand rate to that of low price with high demand rate. The change of the strategy not only lets retailer 1 and the manufacturer enjoy the increased profits, but also lets the other retailers benefit from added profits from the decreased wholesale price from 597 to 543.

3. In VMI system, the inventory cost  $\zeta_i$   $i = 1, 2, \dots, m$  per unit paid by retailer  $i$  is decided by the negotiation between retailer  $i$  and the manufacturer. At the first glance, the manufacturer would get more profit with the increase of  $\zeta_i$   $i = 1, 2, \dots, m$ . Here the result from the example is opposite. When  $\zeta_1$  increases from 2 to 12, retailer 1 is willing to pay more inventory cost for per unit product, then enhances its retail price from 2102 to 2127, and then its net profit decreases from 66972 to 66644. In order to cope with this kind of change, the manufacturer takes the measure of reducing the wholesale price from 598.43 to 595.83, and its profit also decreases from 68263 to 68248. That is to say, an unreasonable value for  $\zeta_i$   $i = 1, 2, \dots, m$  in VMI system may cause both manufacturer's and retailer  $i$ 's profits to drop. This has the important managerial insight that the manufacturer ought to treat  $\zeta_i$   $i = 1, 2, \dots, m$  as market policy carefully to maximize their profits.

4. The mutual competition or promotion exists among retailers. For example, from the first conclusion, when  $e_{p_1}$  increases from 1.4 in the base example to 1.5, retailer 1's profit goes down from 66808 to 31728, reduced by 52.51%, but retailer 2's and 3's profits go up from 181391 to 182806 and 23490 to 23796, increased by 0.78% and 1.3% respectively. That is, the mutual *competition* among retailers occurs at this setting. However, with the improvement of  $K_1$  increasing from  $2 \times 10^6$  in the base example to  $3 \times 10^6$  the mutual *promotion* occurs; since three retailers get added profits, from 66808, 181391 and 23490 to 104073, 186608 and 24627, increased by 55.78%, 2.88% and 4.84% respectively.

5. From  $b_i^* = H_{b_i} / (H_{b_i} + L_{b_i})$   $i = 1, 2, \dots, m$ , it can be drawn that  $b_i^*$  is only influenced by  $H_{b_i}$  and  $L_{b_i}$ , which also can be seen from Table 2. When  $L_{b_i}$  decreases from 300 to 10, the backorder fraction increases significantly from 1.96% to 37.50%.

[Insert Figure 2 about here]

In the leader-follower game, the manufacturer maximizes its profit considering the retailers' maximizing their own profits. In order to illustrate the validity of the leader-follower game to the manufacturer, with the given parameters for the base example, we assume that the manufacturer only negotiates a wholesale price with its retailers at a fixed value. As an illustration, we fix the wholesale price at different levels between 200-1000, calculate the corresponding results, and compare them with that of the base example, as shown in Figure 2. The series denoted by  $NP_{m\_current}$  and  $NP_{m\_base}$  in Figure 2 represent the manufacturer's net profits corresponding to different fixed wholesale price, the base example respectively. "Decrease" represents the profit reduction and is measured by  $(NP_{m\_base} - NP_{m\_current}) / NP_{m\_base} \times 100\%$ . From Figure 2, we conclude that:

(a) The leader-follower game benefits the manufacture in VMI setting; the deviation of wholesale price from the optimal point 597.11 will bring a loss to the manufacturer. For example, when  $c_p = 200$ , the decrease of the manufacturer's profit is up to 85%.

(b) For a given deviation of the wholesale price from 597.11 (the Stackelberg equilibrium), the minus deviation has greater influence on the manufacturer's profit than that of the plus deviation. For example, setting the deviation=200, the manufacturer's profit decreases around 27% at  $c_p = 397.11$ , while it decreases only around 2% at  $c_p = 797.11$ .

## 7 Conclusion

This paper has discussed a VMI supply chain where a manufacturer and multiple retailers play a game with each other under the partial cooperation in the inventory control with VMI policies in order to determine mutually optimal product marketing (retail price and wholesale price) and inventory policies by maximizing their individual net profit. The retailers determine the optimal local retail prices and the manufacturer gives its wholesale price and the product's inventory replenishment in the supply chain level. This supply chain problem is modeled as a Stackelberg game model where the manufacturer is the leader and retailers are followers. An algorithm has been proposed to solve this game model. A numerical study is conducted to understand the Stackelberg equilibrium and significant influence of market related parameters on optimal policies and profits of the manufacturer and its retailers. The result of numerical example also shows that: (a) the competition or promotion still exists among the different retailer's markets via changed wholesale price even if they only sale the product in dispersed and independent product markets; (b) the Stackelberg equilibrium benefits the manufacturer; any deviation of the manufacturer from the equilibrium will bring a loss to the manufacturer.

However, this paper has the following limitations which may be extended in further research. The paper does not consider the horizontal competition among different retailers with one retailer's demand is the function of the other retailers' retail prices. Secondly, although the Stackelberg equilibrium benefits the manufacturer's profit, it can not guarantee that the system-wide profit is maximized. Thirdly, only a single product is assumed in the discussion. It is more realistic to include multiple product variants of a single product family. Finally, this paper has focused on a supply chain dominated by a manufacturer. Many supply chains may be dominated by retailers and the manufacturer may be just a follower. This is an interesting scenario for further research.

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List of Tables and Figures

Table 1. The results for the manufacturer and its three retailers

Parameters	$b_1^*$	$b_2^*$	$b_3^*$	$c_p^*$	$x^*$	$C^*$	$p_1^*$	$p_2^*$	$p_3^*$	$NP_m^*$	$NP_{b_1}^*$	$NP_{b_2}^*$	$NP_{b_3}^*$
Base example	1.96	1.96	1.96	597.11	0	0.75	2114	2618	1812	68255	66808	181391	23490
$e_{p_1}=1.3$	1.96	1.96	1.96	642.47	0	0.73	2814	2814	1948	80769	141995	177493	22655
$e_{p_1}=1.5$	1.96	1.96	1.96	581.66	0	0.79	1766	2551	1766	60090	31728	182806	23796
$K_1=10^6$	1.96	1.96	1.96	608.21	0	0.82	2153	2666	1846	58386	33162	180403	23277
$K_1=3 \times 10^6$	1.96	1.96	1.96	542.63	1	0.46	1924	2382	1649	78271	104073	186608	24627
$\phi_1=1$	1.96	1.96	1.96	592.47	0	0.75	2098	2598	1798	68433	67014	181811	23581
$\phi_1=9$	1.96	1.96	1.96	601.75	0	0.76	2131	2638	1826	68079	66604	180975	23400
$\zeta_1=2$	1.96	1.96	1.96	598.43	0	0.75	2102	2624	1816	68263	66972	181272	23464
$\zeta_1=12$	1.96	1.96	1.96	595.83	0	0.75	2127	2612	1808	68248	66644	181506	23515
$P=150$	1.96	1.96	1.96	608.25	1	0.53	2153	2666	1846	68467	66322	180400	23276
$P=250$	1.96	1.96	1.96	596.69	0	0.76	2113	2616	1811	68267	66827	181429	23498
$H_{b_1}=2$	0.66	1.96	1.96	596.32	0	0.82	2112	2614	1810	68323	66843	181462	23505
$H_{b_1}=10$	3.23	1.96	1.96	597.83	0	0.70	2117	2621	1814	68195	66776	181326	23476
$L_{b_1}=10$	37.50	1.96	1.96	596.69	0	0.79	2113	2616	1811	68292	66827	181429	23498
$L_{b_1}=1000$	0.60	1.96	1.96	597.12	0	0.75	2114	2618	1812	68254	66807	181389	23490
$S_{b_1}=20$	1.96	1.96	1.96	596.52	0	0.71	2112	2615	1811	68296	66834	181444	23502
$S_{b_1}=80$	1.96	1.96	1.96	597.67	0	0.79	2116	2620	1814	68216	66783	181340	23479
$H_p=1$	1.96	1.96	1.96	595.67	0	0.79	2109	2612	1808	68296	66872	181521	23518
$H_p=5$	1.96	1.96	1.96	598.44	0	0.72	2119	2624	1816	68217	66749	181271	23464
$S_p=100$	1.96	1.96	1.96	596.10	0	0.69	2111	2613	1809	68325	66853	181482	23510
$S_p=200$	1.96	1.96	1.96	793.31	0	1.01	2801	3468	2401	61860	59700	166714	20409
$c_m=100$	1.96	1.96	1.96	490.52	1	0.46	1741	2156	1493	77847	72202	192268	25884
$c_m=200$	1.96	1.96	1.96	792.25	0	0.93	2797	3463	2398	61912	59731	166781	20422

#  $b_1^*$ ,  $b_2^*$ , and  $b_3^*$  is given in  $10^{-2}$ .

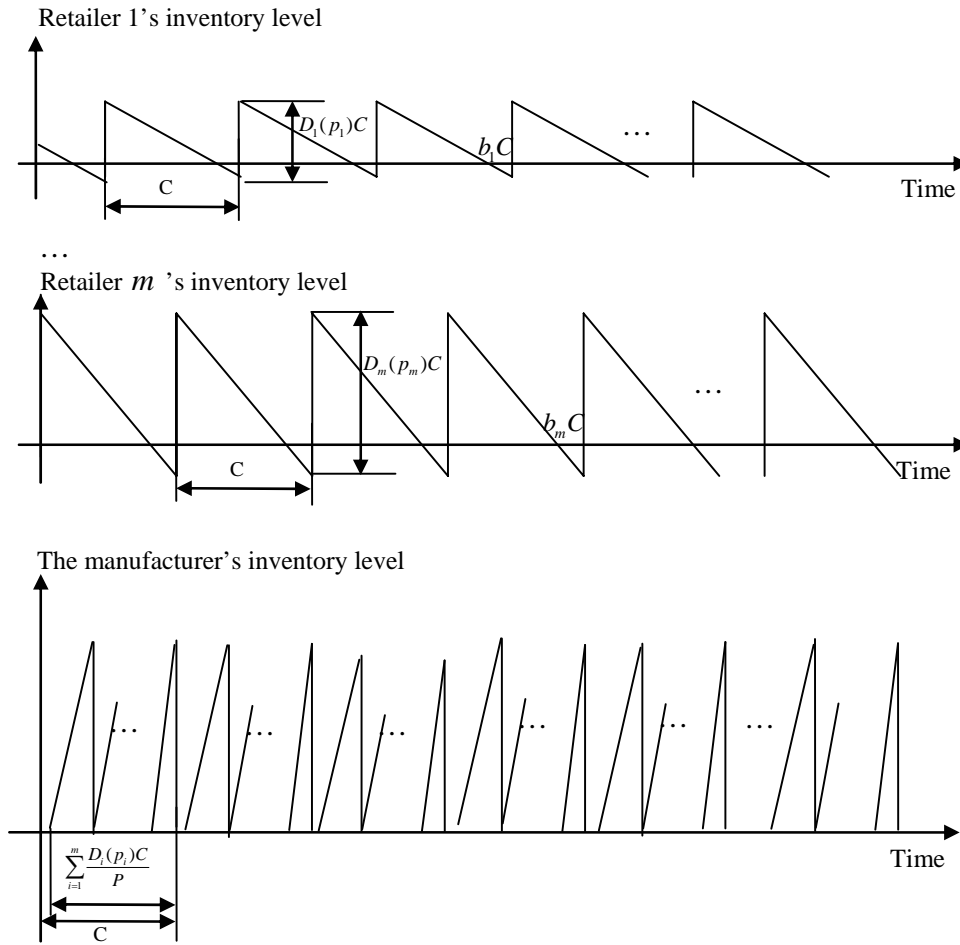


Figure 1. The inventory levels for the product in the VMI system

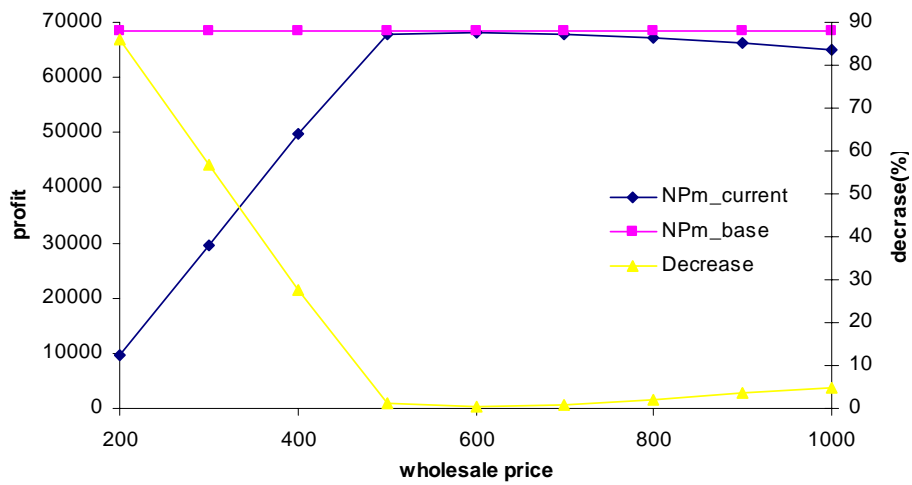


Figure 1. The inventory levels for the product in the VMI system

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