Chapter 5: MULTI-ATTRIBUTE PORTFOLIO SELECTION

"The wind and waves are always on the side of the ablest navigators."[1)]

5.1 INTRODUCTION

The 'Multi-Attribute Approach to Portfolio Selection' consists of two stages:

(i) the formulation of a multi-attribute representation of securities;
(ii) the selection of an optimal portfolio.

The multi-attribute representation of securities distinguishes between two categories of attributes. The direct return related attributes comprise (explicit) expected return and risk measures. The risk measures come from a multi-factor representation of security returns. The key assumption in the multi-factor representation is that the returns of the securities in the opportunity set are influenced or generated by a series of economic forces or 'factors'. As the variability of returns is linked to the variability in various (identifiable economic) variables, investment risk becomes a multi-dimensional concept. In the context of this risk concept, the investor is assumed to have some idea of the securities' sensitivities for factor movements, which serve as risk measures. In addition to these explicit risk/return attributes, we leave room for other attributes that may be incorporated at the investor's discretion. These other attributes may be considered of general relevance in practice, but may also be relevant because of idiosyncrasies in the investor's personal decision context. These other attributes may in fact be proxies for (components of) expected return or risk. As a whole, this stage comprises a detailed security analysis.

The multi-attribute representation of securities thus assumes that an investor can demarcate a set of security attributes that he considers relevant. For the investor, a financial security then represents a basket of, say, k attributes and can fully be characterized by a k-tuple with scores or values that the attributes take on. In this view, when buying a security, an investor is actually buying an exposure to various attributes. Likewise, when composing a portfolio, an investor is actually composing an exposure to the various attributes.

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1) Edward Gibbon, 1787, **History of the Decline and Fall of the Roman Empire**, Bury, London UK

The selection of a portfolio is the copestone of the investment decision process. The issue of multi-attribute portfolio selection is to balance the attributes of the individual securities on the portfolio level. That is, given the security attributes and the investor's profile (personal context), the attributes of his portfolio must be fashioned in a way that suits his particular circumstances and preferences best.

As the attributes are evaluated in a portfolio context, the first issue in portfolio analysis, then, is aggregating the security attribute scores to the portfolio level. Given the non-linear nature of some attributes, this is not at all a trivial step. This is discussed in section 5.2.

The second issue is to confront the multi-attribute representation of securities with the preference structure of the investor. The problem we face is to construct a portfolio which best meets a given investor's preferences and desires. Because of the weak behavioral assumptions we make and the great flexibility we allow on the field of investor preferences, the choice of an adequate optimization approach is of paramount importance. This is the subject of section 5.3.

A distinguishing feature of the multi-attribute approach to portfolio selection is that it depends on the investor's specific decision context, as summarized by his goals, tastes and preferences, and restrictions. For this reason, we can only provide general areas of potential applications. This is done in section 5.4. These applications range from mere portfolio analysis to the actual selection of an optimal portfolio, of which we provide an illustrative example, and from defensive strategies to aggressive strategies.

Section 5.5 summarizes and concludes the chapter.

5.2 AGGREGATING SECURITY ATTRIBUTES TO THE PORTFOLIO LEVEL

A portfolio is simply an aggregate of the individual securities. Given the scores of any security i on any attribute j, summarized in the set $\{a_{ij}\}_{i,j}$, a portfolio p will be evaluated according to the scores on its attributes $\{a_{pj}\}_j$. Portfolio scores on attributes are obtained from aggregating security scores on these attributes, according to their investment fractions. Aggregating attribute scores to the portfolio level can cause problems, however, when it is not precisely known how the scores must be aggregated or when there is no simple linear relationship between security scores and portfolio scores. In other words: do portfolio characteristics equal the weighted average of security characteristics? In general terms, the answer is negative, of course, since the interrelationships between the securities constitute the prime motivation for composing portfolios. Considering, more specifically, the scores $\{a_{ij}\}_{i,j}$ on the selected attributes and the

portfolio weights $\{x_i\}_{i\epsilon p}$, we ask whether

$$(5.1) \quad a_{pj} \overset{?}{=} \sum_{i\epsilon p} x_i a_{ij} \qquad j = 1,\ldots,k$$

Only for linear attributes, the attribute score of a portfolio equals the weighted average of the securities' attribute scores. So, in order to keep the portfolio selection problem tractable, we restrict ourselves to linear attributes. Actually, most of the potential attributes considered so far are linear.

When some attributes prove to be non-linear, in the sense that eq.(5.1) is not satisfied, there are two possibilities. In the one case, the portfolio attribute score can still be obtained from the securities' attribute scores, but now in a non-linear (quadratic, e.g.) fashion. From the perspective of portfolio selection, this problem can be solved either by trying to find some linearizing transformation of the attribute scores, or by employing non-linear optimization techniques. The degree of complexity of the relationship between portfolio and security atributes scores then determines the feasibility of the latter non-linear approach. As an extension of the linear approach, to be presented in section 5.3, we let this issue rest here. In the other case, the portfolio attribute score simply is no longer a direct function of the securities' attribute scores, however complex this function may be. We provide an example of this case right below.

In the light of the above problem, we discuss in this section the linearity of various classes of attributes.

Direct return related attributes

Starting with the direct return related attributes, the expected return is a linear attribute, provided that percentage returns are used (cf. Appendix 4.B to chapter four).

Factor sensitivities are also linear attributes, provided that they are estimated as ordinary least squares coefficients. Note, however, that factor sensitivities can actually lose their linearity property when the estimators are adjusted for violations of the assumption underlying the classical regression model.[2)] One example is a correction for residual autocorrelation, for example according to the Cochrane-Orcutt or Prais-Winsten procedure (cf. Maddala [1977, p.278]). According to this procedure, we have the following transformations on the security returns and the changes in the k factors:

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2) Note that we here do not refer to the 'linearity' of an estimator in the common statistical interpretation that it is linear in the observations. We here refer to the linearity of the sensitivities in the sense that the sensitivity of a linearly weighted average of returns equals the linearly weighted average of the sensitivities.

$$\underline{r}_{it} \rightarrow \underline{r}_{it} - rho_i \cdot \underline{r}_{it-1} \quad \text{for} \quad i=1,\ldots,N$$
$$\underline{\delta}_{jt} \rightarrow \underline{\delta}_{jt} - rho_i \cdot \underline{\delta}_{jt-1} \quad \text{for} \quad j=1,\ldots,k$$

where rho_i is the first order serial correlation coefficient of security i's factor model residual. For simplicity, but without loss of generality of the conclusions, we limit ourselves to the univariate case (j=1). The adjusted estimator of the return sensitivities for the factor changes is simply the ordinary least squares estimator on the basis of the transformed variables:

$$(5.2) \quad b'_i = Cov(\underline{r}_{it} - rho_i \cdot \underline{r}_{it-1},\ \underline{\delta}_t - rho_i \cdot \underline{\delta}_{t-1}) \ / \ Var(\underline{\delta}_t - rho_i \cdot \underline{\delta}_{t-1})$$

Considering a portfolio p, characterized by the investment weights $\{x_i\}_{i \in p}$, its return is given by $\underline{r}_{pt} = \sum_i x_i \underline{r}_{it}$ and its adjusted sensitivity to the factor is:

$$(5.3) \quad b'_p = Cov(\underline{r}_{pt} - rho_p \cdot \underline{r}_{pt-1},\ \underline{\delta}_t - rho_p \cdot \underline{\delta}_{t-1}) \ / \ Var(\underline{\delta}_t - rho_p \cdot \underline{\delta}_{t-1})$$

Linear aggregation of the adjusted factor sensitivities of the individual securities, instead, gives:

$$(5.4) \quad b''_p = \sum_i x_i b'_i$$

$$= \sum_i x_i \frac{Cov(\underline{r}_{it} - rho_i \cdot \underline{r}_{it-1},\ \underline{\delta}_t - rho_i \cdot \underline{\delta}_{t-1})}{Var(\underline{\delta}_t - rho_i \cdot \underline{\delta}_{t-1})}$$

$$\neq b'_p$$

The reason for the inequality is that the securities' first order residual serial correlation coefficients rho_i show up in both the numerator and the denominator of b'_i.[3)]

Following the same line of reasoning, the same argument applies to some situations where ordinary least squares estimators are adjusted for the presence of residual heteroskedasticity. The argument holds *a fortiori* for intrinsic non-linear estimators (non-linear in the sense that they are no linear function of the observations), like robust estimators such as the least absolute deviation estimator.[4)]

[3)] Aside from this, we have $\sum_i x_i \cdot rho_i \neq rho_p$. Expanding (5.4) reveals that these problems are not removed by assuming that factor changes are uncorrelated over time and that there exist only contemporaneous relationships between the returns and the factor changes.

[4)] The least absolute deviation estimator minimizes the mean absolute deviation of the residuals. The analogy of this case with the heteroskedasticity correction case can easily be seen by considering the simple iterative weighted least squares procedure for obtaining least absolute deviation estimators (cf. Maddala [1977, p.311]). Discomforting in this respect is also Lord's Paradox (cf. Hildebrand [1984]): under certain conditions, the mean absolute deviation measure of dispersion does not increase when an additional

Speaking in general terms, although adjusted factor sensitivities may be more appropriate estimators for future ('true') factor sensitivities both on the securities' level and on the portfolio level, the weighted average of the adjusted sensitivities will very likely be different from the adjusted sensitivity of the weighted average (the portfolio). In fact, it is unclear what a weighted average of adjusted sensitivities **does** represent. This renders the use of adjusted factor sensitivities in a portfolio context dubious, to say the least. The problems that arise in aggregating adjusted response coefficients seems to be overlooked altogether in the context of market model betas.[5)]

Indirect return related attributes

'Price ratios' form an important class of indirect return related attributes. In price ratios, some quantity is related to the current price of a security. One example is the expected dividend yield that relates dividends, expected to be received over some horizon, to current stock price. In essence, the expected dividend yield is part of the expected total return and should therefore be classified as a direct return related attribute. Separate information about a portfolio's dividends can be important to an investor for several reasons, so it makes sense to distinguish between the two components of expected total return. The dividend yield indicates the fraction of the return expected to be received in cash. Aside from (but related to) this liquidity aspect, dividend income is taxed in a different way from capital income. Furthermore, there are indications that dividend yields are related to

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independent source of random variation is added.

5) Blume's [1971] adjustment for the beta's regression tendencies is linear, but Blume's [1975, p.790] theoretical correction for the 'order bias' (i.e. measurement error bias) is intrinsically non-linear. Blume's [1975, p.791] statement that "[i]t might be noted that these adjusted portfolio betas [as the average of adjusted security betas] could alternatively be obtained by adjusting the unadjusted portfolio betas directly" is only correct because he implements the procedure by estimating the variance in (true and estimated) individual betas in cross-section and by restricting these variances to be the same for any security.

Non-linearity is also implied by various robust estimation techniques, like the beforementioned least absolute deviation estimation (Sharpe [1971a]) or other methods (Chan & Lakonishok [1992], e.g.), as well as by some of the techniques that correct for the intervalling and thin trading effect biases on beta (like Cohen, Hawawini, Maier, Schwartz & Whitcomb's [1983] asymptotic beta estimator).

Sharpe [1971a, p.B-12,fn.10] only mentions the aggregation issue in a footnote; neither of the other studies mention the non-linearity of the estimators in a portfolio context. An implication of this is that portfolio risk, when measured by beta or betas, can be monitored, but cannot be designed or revised on the basis of individual security estimates.

future expected returns.[6] Considering the variety of these aspects, we decide to categorize dividend yield as an indirect return related attribute. Like total expected return, the (percentage) expected dividend yield is an intrinsically linear attribute.

Ratios like the price/earnings ratio, the cash flow to price ratio and the book to price ratio, in contrast, look like non-linear expressions. Even if they would be linear and could be aggregated to the portfolio level, one could take the view that the resulting average figure would not make sense since only the individual variables tell us something about the corresponding individual firm. However, it **does** make sense to incorporate these attributes when it is believed that they are in some indirect way related to (future) return. The question then is in what form to express the price ratios. In the 'anomaly studies' as cited in chapter one, section 1.4, the (ratio) attributes are moulded in a form that is believed to yield an adequate cross-sectional relationship with expected (average) returns.[7] For portfolio analysis, however, we are looking for a form which enables simple aggregation of the individual figures. The problem, then, is **how** to aggregate them to the portfolio level.

Let us first consider the price/earnings ratio P/E. For some portfolio p, the 'standard' way is to compute the portfolio price/earnings ratio $(P/E)_p$ as the weighted average of the securities' P/E's. This can be contrasted with the way in which the price/earnings ratio of a value weighted stock market index is commonly computed, i.e. the market capitalization of the incorporated stocks divided by the aggregated (expected) earnings. The latter method seems to be justified, but appears at the same time to imply some non-linear transformation of the individual P/E-ratios. We will show below that this intuition is correct, but can easily be overcome.

Define n_i as the number of securities i incorporated in portfolio p. Denoting the price of security i by P_i, the portfolio's price P_p is:

$$(5.5) \qquad \sum_{i \in p} n_i P_i = P_p$$

As before, x_i is the (relative) portfolio weight of security i in portfolio p:

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6) Cf. chapter one, section 1.4.

7) Fama & French [1992, p.444] use the logs of leverage and book/price variables "because preliminary tests indicated that logs are a good functional form for capturing leverage effects in average returns. Using logs also leads to a simple interpretation of the relation between the roles of leverage and book-to-market equity [i.e. book/price ratio] in average returns." Using log ratios in portfolio analysis, however, implies aggregating individual ratios in geometric form to portfolio ratios.

$$(5.6) \quad x_i \equiv \frac{n_i P_i}{P_p}$$

By analogy to eq.(5.5), we have for the (expected) earnings per share i, E_i, and the (expected) earnings on the portfolio level, E_p:

$$(5.7) \quad \sum_{i \in p} n_i E_i = E_p$$

Solving (5.6) for n_i and plugging this into (5.7) yields:

$$(5.8) \quad 1 = \sum_i n_i \frac{P_i}{P_p} = \sum_i x_i \frac{P_p}{P_i} \cdot \frac{E_i}{E_p}$$

Hence,

$$(5.9) \quad P_p/E_p = \left[\sum_i x_i \cdot E_i/P_i \right]^{-1}$$

This suggests that price/earnings ratios $(P/E)_i$ should be aggregated in **harmonic form** into a portfolio price/earnings ratio $(P/E)_p$.[8)]

We have thus shown that price/earnings ratios should be converted into to earnings/price ratios before they can serve as attributes in a portfolio context.[9)] This also removes some other undesirable property of the price/earnings ratio: when (projected) earnings approach to zero, the P/E-ratio approaches (plus or minus) infinity, yielding an awkward asymptote at zero.

So it is more natural to consider the reciprocal of P/E and scale earnings by current price. The attribute E/P is then expressed in the same form as expected total return $E\{\underline{\Delta P}+\underline{D}\}/P$. By the same argument, it follows that cash flow to price ratios and book to price ratios are already expressed in the right form and can serve as linear attributes in a portfolio context. There are several forms in which a financial leverage variable can be expressed.[10)] By analogy to the former cases, we suggest using the ratio of either asset or debt value to market equity value.

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8) When not all securities have the same price/earnings ratio and when the portfolio weights are positive, the harmonic mean of the P/E's will be lower than the arithmetic mean $\sum_i x_i \cdot P_i/E_i$.

9) Note, however, that now the weights of the prices and earnings depend on the composition of the particular portfolio, and not on the market capitalization of the corresponding stocks.

10) Bhandari [1988, p.509], for example, uses the ratio of book value of debt to market value of equity as the leverage variable. Fama & French [1992, p.441] use the (log of the) ratio book value of assets to market value of equity as one leverage variable.

Another possibility to incorporate information that is enclosed in attributes like earnings/price ratios, book/price ratios, cash-flow/price ratios and size, is to form mimicking portfolios for these attributes. By regressing security returns on the returns on these mimicking portfolios, sensitivity coefficients are obtained. Then the securities' return sensitivities to these portfolios are used as a substitute for their scores on the initial attributes.[11)]

The form in which attributes in a pricing context are incorporated in cross-section regressions, is motivated by their hypothesized linear relationship with average returns. In a time series context, the relationship between returns and factors can be non-linear, for example because of a discounting effect. This raises the question how non-linear multi-factor models for securities may be aggregated to a portfolio model. There is no simple answer to this question, but one practical solution is to aggregate the securities' sensitivities in a linear way whereafter non-linearity can be restored in an approximate way by using the portfolio sensitivity in a non-linear formulation as suggested in Appendix 3.D to chapter three. Another route is to distinguish between sensitivities for factor increases and decreases and to aggregate these partial sensitivities separately in a linear way. (Note that ordinary least squares partial sensitivities are linear.)

5.3 CHOOSING AN OPTIMAL PORTFOLIO

We assume that the outputs of the security analysis (for example as described in chapter one, section 1.4, and chapter four), are expressed in an adequate form so that they can be aggregated to a portfolio level and serve as inputs for the portfolio analysis. The evaluation of feasible portfolios and the selection of an optimal portfolio entail the combination of investor preferences and portfolio attribute exposures.

Section 5.3.1 discusses tastes and preferences about securities, expressed in scores on attributes. Section 5.3.2 discusses standard approaches that are employed in the portfolio selection decision. Finally, we present an alternative approach that does justice to the flexibility allowed in the security analysis and preference analysis.

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11) This procedure is suggested by Fama & French [1993, pp.4-5], who use mimicking portfolios for size and book/price. See chapter one, section 1.4.

5.3.1 Choosing between attribute exposures

A multi-attribute representation of securities entails a detailed security analysis. Preference information is used to demarcate the set of k attributes that an investor considers important. As a result, each of the N securities in the opportunity set can be adequately characterized by the values that the respective k attributes take on. That is, for any security i, the set of its relevant characteristics can be adequately summarized by the k-tuple:

$$\{ a_{i1} , a_{i2} , \ldots , a_{ij} , \ldots , a_{ik} \} \qquad i \in N$$

where a_{ij} is the value that the attribute j takes for security i. In other words: it is security i's **score** on that attribute as a measure for its **exposure** to the attribute. Considering the discussion in section 5.2, it is important to express security attributes in a form that can (linearly) be aggregated to the portfolio level.[12] Some or all of these attributes may be sensitivities to factor surprises, as set out in chapter four.[13]

In the foregoing, we implicitly and silently made the step from securities to their representation in terms of attribute scores. In a (E,σ^2)-expected utility context, portfolios can be fully characterized by mean and variance because the expected utility preference functional is a function of only mean and variance. Aside from appropriate distribution assumptions, this can be achieved by assuming a single dimensional utility function, quadratic in return. For a multi-attribute approach to portfolio selection, we must clearly escape from a uni-dimensional return context. We can justify a multi-attribute representation by referring to consumer theory, where 'characteristics models' have been developed for describing consumer behavior. In this respect we especially note Lancaster [1966, p.133], whose contribution is "breaking away from the traditional approach that goods are the direct objects of utility and, instead, supposing that it is the properties or characteristics of the goods from which utility is

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12) As put forward in chapter 3, section 3.4.1, all security attributes are linear when evaluated in an optimal portfolio. For example, when extending the (E,σ^2)-approach to multi-moment analysis, the relevant security attributes are their marginal contributions to the portfolio moments, and these are linear (see section 3.3.3). This, of course, is only **local** linearity, because changing the portfolio weights switches the portfolio status from optimal to non-optimal, destroying linearity.

13) Care must be taken when considering multi-moment attributes in conjunction with factor sensitivities. As factor sensitivities specify a conditional return distribution, only moments of the **residual** return component (like residual variance, skewness &c.) should be regarded.

derived."[14] These implied characteristics models opened the way for a theory of multi-attribute choice. Transposed to the investment decision, we can assume that investors buy securities for the attributes they offer and that different securities are essentially different packages of attributes. This implies that investors choose between security attribute exposures instead of between the uni-dimensional securities or their returns. Hence, an investor's preference functional is directly specified in the multi-dimensional terms of relevant security attributes.[15] For an investor, a financial security then represents a basket of, say, k attributes and can fully be characterized by a k-tuple with scores or values that the attributes take on. In an investor's view, when buying a security, he is actually buying an exposure to various attributes. Likewise, when composing a portfolio, he is actually composing an appropriate exposure to the various attributes. Hence, we can specify a mapping of the securities in the space spanned by the attributes:

$$(5.10) \quad \text{security } i \rightarrow \{ a_{i1} , a_{i2} , \ldots , a_{ij} , \ldots , a_{ik} \} \quad , i \in N$$

Not surprisingly, by considering multiple attributes, the portfolio selection problem becomes multi-dimensional. As an investor will show different preferences and tastes for different attributes, the investment decision becomes a multi-criteria decision process. This is depicted in Figure 5.1. After performing the security analysis, the investment decision can be shaped by confronting the investor's preferences (expressed in attributes) with the securities' exposures to the attributes. In other words, the problem becomes how to construct a portfolio that is tailored to an investor's needs and preferences. This is further elaborated in the next sections.

5.3.2 Standard approaches

Given the perceived attributes of the securities, we can assume that an investor strives to attain various goals or criteria, related to a number of perceived attributes. The problem the investor faces is to construct a portfolio which best meets his preferences and desires. The

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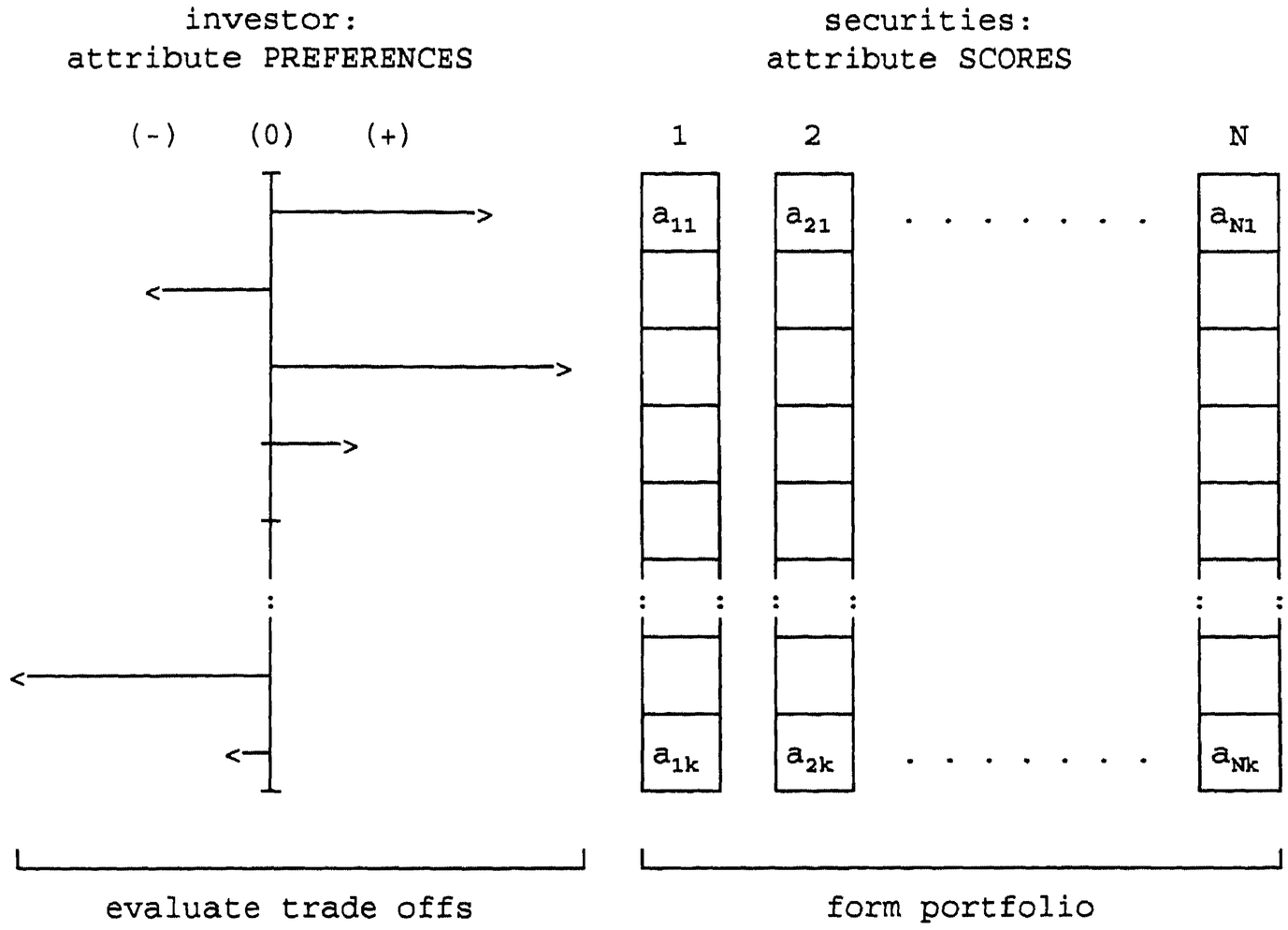
14) For a discussion and review of characteristics models, we refer to Ratchford [1975] and Deaton & Muellbauer [1980, pp.250-253].

15) Consumer theory considers decision making under certainty. Hence, characteristics models specify **utility** functions in terms of attributes. Because we are dealing with situations of uncertainty or risk, we express the **expected utility** function, or more general: the preference functional, in terms of multiple attributes. It must then be assumed that some subset of attributes captures the risk aspects, so that attitudes towards risk can be reflected in the preference functional. A preference functional specified in attributes is sometimes termed a 'Lancaster-type of utility function'.

Figure 5.1: The multi-criteria decision process: confronting investor preferences with attributes scores; k attributes and N securities in the opportunity set.



decision process now involves searching for the portfolio that is considered most desirable over all the attribute dimensions.

More concrete, the investor will strive for a specific constellation of portfolio attribute exposures. For a given portfolio, its exposure to a certain attribute can be calculated as a weighted average of the attribute exposures of the individual securities contained in this portfolio. The fractions invested in each of these securities can thus be treated as instrumental variables. Therefore, the attribute exposures can be seen as goal variables which are linear in the portfolio holdings. Often, the investor will try to either minimize or maximize each of these goal variables. Alternatively, the investor may strive to attain a target level or desired score on some attribute(s). Depending on the investor's insights and preferences, the relative importance of each of these goals may vary. Generally, the goal variables will be interrelated. This implies that no portfolio can be found for which each of the goal variables reaches its optimal value or for which all criteria are met. As a consequence, the investor has to evaluate the trade-offs between the various goal variables.

This section discusses various approaches that are suggested to cope with a multi-attribute investment decision. As the number of attributes is assumed to be much less than the number of securities in the opportunity set, the dimensionality of an initial choice problem in terms of securities is greatly reduced. The evaluation and representation of the trade-offs between the attributes, however, is now the main problem.

There are several routes leading to the selection of a portfolio, depending on the amount of information available on the investor's preference structure. On the one extreme, we have a minimal amount of preference information, allowing only ranking on an attribute or a lexicographic ordering over several attributes, for example. Note in this respect, that the investor himself indicates what attributes he considers important (see chapter one, section 1.3). This implies that in any case **some** preferences towards the selected attributes do exist, but they need not necessarily be expressed in some more or less detailed form. At the other extreme, a fully specified preference functional can be obtained. In that case, we can simply formulate an optimization problem and use techniques like linear programming, non-linear optimization or goal programming for solving it. The more preference information is available and the better this information is captured in an adequate formulation of the optimization problem, the stronger guarantee we have for an optimal solution to the portfolio decision. Conversely, when preference information is incorporated into the problem formulation in an incomplete or inadequate way, the optimality of the implied 'optimized' decision must seriously be doubted.

Below, we discuss these routes to portfolio selection in more detail.

Ranking methods

The most simple approach entails a simple ranking of the securities according to their attribute exposures. In Salomon Brothers' multi-attribute model (Sorensen, Mezrich & Thum [1989]), for example, ranking is used in order to screen an opportunity set for stock suggestions. In this way, the investment opportunity set is reduced, but no guidelines are provided for portfolio composition. Goldman, Sachs & Co.'s (Jones [1987]) use of the multi-attribute model also aggravates to ranking the stocks according to their scores, for each attribute separately. Here also, no guidelines are provided for portfolio composition. We can imagine the possibility of using intersections of the sorts on the various attributes and forming top and bottom quantile portfolios, but we doubt whether such a procedure would be satisfying.[16)]

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16) Martel, Khoury & Bergeron [1988] present a multi-criteria procedure for portfolio comparisons based on outranking relationships on various attributes. However, in this case it is assumed that the

Ranking involves only a very indirect and incomplete use of the multi-attribute information. More importantly, the trade-offs between the various attributes cannot be evaluated.

Multi-attribute utility theory

A more sophisticated possibility is to extend the traditional utility framework to a multi-dimensional context by casting a utility function in terms of multiple portfolio attributes:

$$(5.11) \quad U(\underline{r}_p) \rightarrow U(a_{p1},\ldots,a_{pk})$$

One could then, for example, extend the optimization procedure proposed by Sharpe [1987] to a more general context and replace the $Z(E_p,\sigma^2_p)$ preference functional by some other 'Lancaster [1966]-type' of function $Z(a_{p1},\ldots,a_{pk})$. The complexity of specifying a multi-attribute preference functional, however, is enormous and is not likely to be overcome in practice.

In multi-attribute utility theory, this complexity is somewhat reduced by assuming (strong) separability of the preferences. When this assumption is satisfied, a series of uni-dimensional (i.e. single attribute) utility functions $U_j(a_{pj})$ can be assessed, whereafter these component functions are combined (in a linear, multiplicative or other fashion), using information about attribute trade-offs:

$$(5.12) \quad U(a_{p1},\ldots,a_{pk}) \rightarrow \Omega[U_1(a_{p1}),\ldots,U_k(a_{pk})]$$

In this way, the exposures are evaluated attribute by attribute and then combined to obtain an overall measure of desirability. Still, this places a heavy information burden on the investor. The problem here is to *ex ante* specify the uni-dimensional preferences for each of the attributes as well as the overall preference functional that incorporates the evaluation of a combination of attribute exposures and their trade-offs.[17)]

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portfolios to be compared already do exist.

17) When specifying **utility** functions in terms of attributes, it is silently assumed that no risk (i.e. probability distribution) is attached to the attribute scores. When instead the attribute scores are random variables, the **expected utility** function must be considered for evaluating the decision alternatives. In contrast to the univariate utility functions that appeared in chapter one, the concept of risk aversion is less well-defined for multivariate utility functions (cf. Richard [1975]). In this regard, one could assume that some subset of attributes captures the risk aspects and that the attribute scores themselves are deterministic variables (cf. footnote 15 above). For the case in which the attribute scores are random variables, Roberts & Durban [1988] present a multi-attribute (footnote continued on next page)

In the seminal multi-attribute specification of Smith [1974], he uses a intuitive methodology that could be termed an additive multi-attribute utility approach. On the basis of the securities' (asset classes') exposures to the attributes, he lets the investor assign 'suitability measures' to each attribute score; this can be interpreted as the transformation $a_j \rightarrow U_j(a_j)$ for each attribute j. Next, he lets the investor specify weights that indicate the relative importance of the various attributes; this can be interpreted as specifying the weight coefficients in the function $\Omega(\cdot)$ in eq.(5.12), that is assumed to be linear. The weighted average suitability measures for the various investment alternatives are then considered as an overall index of suitability. Properly normalized, the latter indices are finally used as investment fractions.

Linear programming and multiple-goal programming

For conditional-normative uses, a multi-attribute utility theoretic approach will prove to be too sophisticated because of its information requirements. Another route is then to cast the multi-dimensional preference functional in the form of a (linear) programming model. One can imagine to maximize the portfolio's exposure to one attribute (expected return, e.g.) subject to restrictions on the other attribute scores. Or, as in Sorensen & Thum's [1992] portfolio optimizer, to maximize expected portfolio return while equating its factor sensitivities with those of a reference index (see section 5.4.3). The problem with these specifications is that they are intrinsically uni-dimensional: only one attribute is optimized, while the other attributes only serve as constraints. One could then try to extend the linear programming formulation to a multi-dimensional context by incorporating various attributes in a weighted fashion in the objective function. A recent example is Arthur & Ghandforoush [1987]. They follow Smith [1974] —although they do not refer to him— in allowing for various attributes (both quantitative and qualitative in nature) and in assigning normalized 'factor measures' to attribute exposures and normalized 'factor weights' to attributes. Their weighted average is a 'security performance measure', and this is maximized (under constraints) on the portfolio level in a linear programming problem.

A linear programming formulation like this can only be employed when the trade-offs between the attributes can properly be specified. This is especially relevant in the case where the investor's goals with respect to the attribute exposures may be conflicting.

In formulating priorities and targets with respect to attributes and attribute exposures, goal programming offers more flexibility. The

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utility model under risk. For (conditional-) normative uses, however, this approach is far too complex and unworkable.

applicability of multiple goal programming to the portfolio problem was recognized in an early stage. In the (E,σ^2)-context, we have Lee [1972, pp.227-239], Lee & Lerro [1973], Kumar, Philipatos & Ezell [1978], Lee & Chesser [1980], Spronk [1981] and, more recently, O'Leary & O'Leary [1987]. Aside from expected return and risk, some indirect return related atributes (notably dividend yield) are specified, but a truly multi-attribute representation is not pursued. In all studies but the last[18], risk is accounted for by Sharpe's [1967] linear approximation of portfolio return variance on the basis of the single index model, or simply by a target for a market index model beta. Of course, the number of attributes could easily be extended towards multi-dimensional risk measures and various additional indirect return related attributes.

Multiple goal programming indeed has some attractive properties. It shows a close correspondence with decision making in practice, the goals are formulated as aspiration levels and there is always a solution for a well-defined problem (with a non-empty feasible region), even if some goals are conflicting. An important drawback of multiple goal programming, still, is its need for fairly detailed *a priori* information on the decision-maker's preferences.

Summarizing, an investor will try to find a balance (depending on his insights and preferences with respect to the feasible portfolios) between the portfolio's exposures to the attributes. The degree of sophistication of an optimization procedure to be applied to this decision, as measured in terms of preference information requirements, must be matched both with the investor's ability to explicitize and provide this information, and with the possibility to incorporate this information in an adequate way in the optimization problem formulation. In the case in which an investor can provide a fully specified preference functional, the optimization is more or less straightforward.

However, it is more realistic to consider the situation where this condition is not satisfied and, hence, an investor's portfolio selection problem cannot be solved by standard techniques as linear (or quadratic) programming and multiple goal programming. Although multiple attributes could be specified, a truly multi-attribute approach to portfolio selection would not live up to its promise. Because of the weak assumptions and flexibility allowed on the the field of investor preferences, a more flexible methodology is called for, that even in the most general context allows for an effective and efficient scan of feasible, non-dominated portfolios. This leads us to an interactive approach.

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[18] In their formulation of a goal for risk, O'Leary & O'Leary [1987, p.192] assume a return volatility measure that is additive and simply opt for the returns' standard deviations. As standard deviations are not additive, this is incorrect. Furthermore, since the correlation structure is ignored, a fundamental aspect of the portfolio problem is missed.

5.3.3 An alternative approach

In contrast with the methods described above, interactive methods neither require an explicit representation or specification of the decision-maker's preference function nor an explicit quantitative representation of the trade-offs among conflicting goals. By its nature, an interactive procedure progresses by seeking this information from the investor, removing the need for explicitizing the preference structure. For the investment problem as sketched in this study, we propose Interactive Multiple Goal Programming (henceforth IMGP), as developed by Spronk [1981].

In broad lines, IMGP works as follows. Given a set of (investment) alternatives and a set of goal variables (attribute exposures), IMGP starts formulating minimum requirements for each of the goal variables, leaving a set of alternatives meeting the requirements. (For the ease of exposition, we assume that all goal variables are to be maximized.) This vector of minimum goal values is presented to the investor, together with a set of indicators of the potential improvements of these minimum goal values, within the set of feasible portfolios. In the first iteration, very low minimum goal values are chosen (viewed by the investor as absolute minimum conditions or even worse) in order to be sure that no potentially acceptable portfolios are excluded. Next, the investor has to indicate whether or not the portfolios meeting the minimum requirements are satisfactory. If so, he can choose one of these portfolios. If not, he has to indicate which of the minimum goal values should be increased. The constraint on the value of the corresponding goal variable is then reformulated.

On basis of the resulting new vector of minimum goals values, a new set of indicators of the potential improvements of these values is calculated and presented to the investor. The investor has to indicate whether the shift in the indicated minimum goal value is outweighed by the shifts in the potentially attainable values of the other goal variables. If so, the investor has the opportunity to revise his earlier wishes with respect to the changed minimum goal value. If not, the change of the minimum goal value is accepted and the investor can continue to raise any of the other or even the same minimum goal value. Of course, by successively raising the minimum goal values from iteration to iteration, the set of feasible portfolios is gradually reduced by keeping only the portfolios that meet the higher standards. Each iteration produces indicators showing the 'price' of the higher standard, so the investor can evaluate the trade-offs between the goals (attributes).

The investor has several options. He can continue until the remaining set of feasible portfolios becomes very small. Another possibility is to select a suitable portfolio from the set of portfolios satisfying the minimum requirements. In this respect, IMGP produces at each iteration a set of non-dominated portfolios. Finally, a set of feasible portfolios

satisfying the minimum conditions on the goal values can be subjected to a second analysis by the investor. In his decision context, the investor may wish (or need) some elbow-room, thus requiring more than just one portfolio. The procedure then offers adequate flexibility to incorporate other, hard to quantify, criteria into the decision making process.

Summarizing, IMGP incorporates all the advantages of 'traditional' goal programming, while circumventing the unnecessary burden of obtaining a 'complete' picture of the investor's preference pattern. In our opinion, this approach offers the desired degree of flexibility to be fruitfully applied to the multi-attribute portfolio selection problem.

By tuning the attribute exposures, a specific portfolio profile can be obtained that matches the investor's profile. The last two stages of the portfolio selection process –portfolio analysis and portfolio selection– are no longer treated separately but are integrated. The interactive method then is no optimizer, but can better be characterized as a 'combinizer': it allows systematic scanning of the set of feasible portfolios and the selection of an optimal portfolio via an interactive process. In the interactive decision process, a **learning process** is embedded. By scanning the feasible portfolios, the investor first gets a feeling for the trade-offs that in the opportunity set exist between the exposures to the various attributes. Second, the investor can shape and adjust his preferences when confronted with the trade-offs between the attributes. It is in no way required that the investor performs the interactive process only once. He can explore the opportunity set in all the dimensions the attributes offer, and it is even adviced that he do so in order to get insight into the characteristics of the opportunity set at hand. Since the interactive procedure is path-independent, no desirable (feasible) alternatives can be missed, only insight can be gained.

5.3.4 An example

In this section, we illustrate the approach, set forth in section 5.3.3. We first pay attention to the data used and to the formulation of the model. Then we present the results for a general case.

The data

We recognize the fact that the identification of the (relevant) factors, along with the computation of the sensitivities of the securities for these factors, is the first and very important stage of the portfolio problem. Here we constrain ourselves to the second stage of the problem: the selection of a portfolio, the characteristics of which satisfy the goals and desires of the particular investor. Our main purpose is to

Table 5.1: The attribute exposures of the 36 stocks in the opportunity set. (Estimations over the period 1984:11 - 1994:2.)

	ERETURN	DIVYLD	RNL	USDLR	EFFXR	RISKPR	RESMF[a)]	R^2
ACF	10.38	3.13	-10.19	0.122	1.37	1.56	1.23	0.420
BSW	12.78	3.79	-5.79	0.067	0.72	0.86	0.75	0.235
CSM	22.12	2.76	-6.55	-0.085	1.37	0.60	0.86	0.433
GIS	11.77	3.40	-11.01	0.086	3.19	0.88	1.29	0.477
HEI	15.04	2.00	-7.71	-0.151	2.38	0.71	1.00	0.458
NUT	26.17	2.39	-4.59	0.483	0.57	-0.11	1.00	0.275
UNI	18.42	3.47	-8.97	0.378	0.64	0.43	0.90	0.573
FOK	13.85	1.78	-3.58	0.125	2.85	1.76	1.27	0.237
OCE	12.78	2.72	-1.43	-0.244	4.13	1.63	1.25	0.469
PHI	10.61	2.96	-4.88	0.128	2.42	1.39	1.18	0.321
STO	22.40	2.46	-5.22	-0.148	3.55	1.59	1.05	0.345
AKZ	16.65	5.11	-6.11	0.129	2.64	1.28	1.14	0.531
HOE	22.55	4.03	-7.42	0.129	0.90	-0.08	0.74	0.149
HO	17.97	2.54	-2.81	-0.196	6.62	1.52	1.67	0.311
HDG	21.82	2.49	-8.15	0.298	2.27	1.82	1.65	0.429
KKB	27.81	3.26	-9.51	-0.055	1.85	1.72	1.55	0.405
BAM	31.36	4.21	-9.04	0.347	0.18	0.52	1.01	0.229
HBG	19.15	4.74	-4.37	0.329	1.41	1.03	0.59	0.223
KLM	12.22	2.46	-3.49	0.330	1.58	1.87	1.09	0.316
NED	22.30	2.80	-1.51	0.304	3.18	2.31	1.49	0.364
PAK	21.21	4.58	-4.35	0.277	1.58	0.25	0.93	0.240
VO	15.50	5.53	-4.15	0.179	0.12	0.80	0.99	0.276
TEL	19.14	3.69	-4.91	0.056	0.73	0.71	0.85	0.349
VNU	23.63	3.41	-8.49	0.379	0.30	1.18	1.39	0.531
ELS	25.19	2.13	-9.68	0.125	0.79	0.72	1.19	0.474
AH	19.83	2.08	-6.19	0.409	1.68	1.19	0.96	0.495
BW	26.09	4.02	-5.02	0.021	1.36	1.99	0.79	0.221
HGM	23.34	3.63	-2.89	-0.197	3.37	1.12	1.03	0.307
IM	16.32	2.54	-12.74	0.113	1.29	0.32	1.17	0.314
KBB	18.36	2.11	-6.40	-0.123	0.36	0.10	0.73	0.115
AAB	14.34	5.57	-11.67	0.183	0.40	0.37	0.87	0.548
AGN	18.69	4.29	-7.78	0.274	0.74	0.46	1.09	0.551
AMV	13.39	4.80	-12.83	0.313	-0.51	0.66	0.92	0.489
ING	15.55	4.73	-11.48	0.328	0.55	0.79	1.12	0.673
SR	11.76	3.25	-6.02	0.476	-0.18	0.18	0.63	0.240
RD	17.46	5.64	-2.95	0.395	0.92	1.01	0.33	0.371
[a)] Auxiliary regression (RESMF):								
CBSGXR			-7.64	0.230	1.14	0.76		0.349

illustrate the proposed approach.

Our point of departure is the opportunity set of the 36 individual stocks, as described in Appendix 4.A to chapter four. We assume that the investor is interested in seven attributes. The first two attributes are expected return and dividend yield. We estimated these attributes as the historical averages of the (discrete) returns and dividend yields over the period 1984:11 - 1994:2. This is the same

subperiod as we used in chapter four, section 4.4. Over this same period, we estimated for these stocks the sensitivities of their returns for unanticipated changes in four economic factors, considered before, and a residual-market-index factor. The economic factors are the Dutch long term interest rate, the US dollar / guilder exchange rate, the non-dollar effective exchange rate, and the risk premium (proxied by the adjusted return on US long term low grade industrial bonds). The residual-market-index factor, discussed in chapter two, section 2.4.2, is the residual of a regression of the (actual) CBS total return index —Royal Dutch excluded— on the four economic factors. The average proportion of stock return variance explained by the five factors is 37%. The sensitivities from these factor models serve as the five other stock attributes.

The expected return, dividend yield, and the sensitivities of the stocks for the factors, are depicted in Table 5.1. In the bottom of that table, we find the auxiliary regression information of the residual-market-index factor.

The model

In our example, the investor is assumed to be interested in the values of the attributes or goal variables, as set out above. We assume that the investor wants the expected return and dividend yield maximized. The interest rate risk is to minimized, so that the negative sensitivities must be maximized. The non-dollar exchange rate risks are to be minimized also. As sensitivity of any feasible portfolio for non-dollar currency surprises will be positive, we can safely minimize this sensitivity. With two exceptions, the sensitivities for the risk premium are positive. We assume that our investor wants to minimize these sensitivities in the sense that he wants them to be as close to zero as possible. The dollar sensitivities can attain various positive and negative values. Since our investor desires to minimize dollar risk, he chooses a zero target value for the dollar sensitivity. Therefore, we have chosen for the formulation that the absolute values of these sensitivities are to be minimized. In addition, he wants the same sensitivity for omitted factors as the market index. Therefore, he specifies a target value of unity for the residual-market-index factor. In this way, the absolute value of the deviation from a unity residual-market-index beta is minimized.

The attribute exposures of a portfolio are the weighted average of the securities' exposures. Hence, the goal variables relating to the portfolio's different attributes can be computed as linear combinations of the securities' goal variables.

The set of feasible portfolios is defined by the budget constraint $\sum_i x_i = 1$, together with the maximum investment constraints $x_i \leq 0.07$ for all stocks, except Royal Dutch. For this (very large capitalization)

Table 5.2: Portfolios resulting when different goal variables are optimized, first iteration (entries show the percentage fractions of the budget, invested in each security).

	ERETURN	DIVYLD	DRNL	US$	EFFXR	RISKPR	RESMF
ACF	-	-	-	7.00	-	-	7.00
BSW	-	7.00	-	1.75	7.00	-	-
CSM	7.00	-	-	7.00	-	7.00	7.00
GIS	-	-	-	-	-	-	-
HEI	-	-	-	7.00	-	7.00	-
NUT	7.00	-	7.00	-	7.00	7.00	7.00
UNI	-	-	-	-	7.00	7.00	7.00
FOK	-	-	7.00	-	-	-	-
OCE	-	-	7.00	7.00	-	-	-
PHI	-	-	7.00	-	-	-	-
STO	7.00	-	-	7.00	-	-	-
AKZ	-	7.00	-	-	-	-	-
HOE	7.00	7.00	-	-	2.00	7.00	-
HO	-	-	7.00	7.00	-	-	7.00
HDG	7.00	-	-	-	-	-	1.23
KKB	7.00	-	-	7.00	-	-	-
BAM	7.00	7.00	-	0.25	7.00	7.00	7.00
HBG	2.00	7.00	7.00	-	-	-	-
KLM	-	-	7.00	7.00	-	-	7.00
NED	7.00	-	7.00	-	-	-	-
PAK	7.00	7.00	7.00	-	-	7.00	-
VO	-	7.00	7.00	-	7.00	-	-
TEL	-	1.00	7.00	7.00	7.00	7.00	7.00
VNU	7.00	-	-	-	7.00	-	7.00
ELS	7.00	-	-	-	7.00	2.00	-
AH	7.00	-	-	-	-	-	7.00
BW	7.00	7.00	1.00	7.00	-	-	-
HGM	7.00	-	7.00	7.00	-	-	-
IM	-	-	-	-	-	7.00	-
KBB	-	-	-	7.00	7.00	7.00	-
AAB	-	7.00	-	-	7.00	7.00	-
AGN	-	7.00	-	-	7.00	7.00	-
AMV	-	7.00	-	7.00	7.00	7.00	7.00
ING	-	7.00	-	7.00	7.00	-	7.00
SR	-	-	-	-	7.00	7.00	7.00
RD	-	15.00	15.00	-	-	-	7.77

stock, a portfolio weight of maximum 15% is allowed. In this way, any portfolio comprises at least 14 stocks. Furthermore, we have the usual non-negativity constraints $x_i \geq 0$ for all stocks.

Results

We show how an arbitrary investor might arrive at a single most preferred portfolio or at a set of portfolios, satisfying a series of

requirements formulated by this decison maker. As set out before, different investors will generally end up with other portfolios, depending on their insights and their preferences. However, the first iteration is the same for all investors (assuming they have formulated exactly the same set of goal variables). That is, in the first iteration, each of the goal variables is optimized **separately**. In the following iterations, each decision-maker formulates his own requirements with respect to the goal values.

Selected results of the various iterations are shown in Tables 5.2 to 5.4. In Table 5.2, the portfolios resulting from each of the separate optimizations in the first iteration are presented. It can be noted that the portfolios which are optimal in the first iteration are mutually significantly different: the 'unconstrained' optimizations lead to a great diversity in portfolio composition. Only one stock does not appear in either of the portfolios. No stock, however, appears in all portfolios.

In Table 5.3, the goal values associated with the first iteration portfolios are given, together with the best goal values which can be attained within the set of feasible portfolios. Also shown are the worst goal values resulting from each of the separate optimizations of the first iteration. It is not necessary to accept a goal value that is lower than this value.

Table 5.3: Goal values, associated with the first iteration portfolios, that result when the goal variables are optimized **separately**. The best attainable goal values are along the diagonal. The worst values (row-wise) are marked with an asterisk: it is not necessary to accept a value of the goal variable that is lower than this value.

	separately optimized goal variable						
	ERETURN	DIVYLD	DRNL	US$	EFFXR	RISKPR	RESMF
ERETURN	**23.89**	18.72	17.88*	18.26	18.95	19.32	18.56
DIVYLD	3.19	**4.76**	3.67	3.18*	3.82	3.52	3.47
DRNL	-6.29	-6.79	**-3.50**	-6.45	-7.97*	-7.96	-7.02
USDLR	0.17	0.24	0.17	**0.00**	0.25	0.19	0.27*
EFFXR	1.63	0.85	2.15*	2.02	**0.40**	0.75	1.14
RISKPR	1.06	0.78	1.17	1.18*	0.55	**0.37**	0.88
RESMF	1.11	0.82*	0.99	1.08	0.96	0.91	**1.00**

Table 5.4 shows how the final solution is reached. The first entry summarizes Table 5.3 and gives (in the form of a 'potency matrix') the best and worst goal values from the separate optimizations of the first iteration. First, we require an expected portfolio return of at least 20%. We see interest rate risk and non-dollar exchange rate risks increase only little. Next, the dividend yield is restricted to a

Table 5.4: Potential goal values during the interactive process. Shown first are the ideal or best goal values; required values appear in the row below. An asterisk denotes the goal value that is chosen to be improved in the next iteration. The arrows indicate the new required values.

iteration	goal variables						
	ERETURN	DIVYLD	DRNL	US$	EFFXR	RISKPR	RESMF
1	**23.89***	**4.76**	**-3.50**	**0.00**	**0.40**	**0.37**	**1.00**
	17.88	3.18	-7.97	0.27	2.15	1.18	0.82
2	**23.89**	**4.70***	**-3.76**	**0.00**	**0.44**	**0.38**	**1.00**
	20.00 <	3.18	-7.86	0.24	2.15	1.16	1.12
3	**22.77**	**4.70**	**-3.88**	**0.04**	**0.47**	**0.46***	**1.00**
	20.00	4.00 <	-7.50	0.24	2.15	1.13	0.88
4	**22.49**	**4.63**	**-4.58***	**0.08**	**0.47**	**0.46**	**1.00**
	20.00	4.00	-7.50	0.24	1.48	0.76 <	0.88
5	**22.40**	**4.56**	**-4.58**	**0.08**	**0.68**	**0.53**	**1.00***
	20.00	4.00	-6.00 <	0.24	1.48	0.76	0.88
6	**21.78**	**4.27**	**-5.40**	**0.10**	**1.07***	**0.66**	**1.00**
	20.00	4.00	-6.00	0.24	1.48	0.76	1.00 <
7	**20.94**	**4.03**	**-5.95**	**0.22***	**1.07**	**0.74**	**1.00**
	20.00	4.00	-6.00	0.24	1.10 <	0.76	1.00
8	**20.64***	**4.01**	**-5.99**	**0.22**	**1.09**	**0.75**	**1.00**
	20.00	4.00	-6.00	0.22 <	1.10	0.76	1.00
9	**20.64**	**4.00**	**-6.00**	**0.22**	**1.10**	**0.76**	**1.00**
	20.64 <	4.00	-6.00	0.22	1.10	0.76	1.00

minimal value of 4%. Now again the interest rate risks and exchange rate risks increase. We then focus on the risk variables. In the third iteration, the risk premium exposure is restricted to the general market index' value of .76. The increasing interest rate risk worries us and we we constrain the interest rate sensitivity to -6. Next, we require a unit sensitivity to the residual-market-index. The difference between the required and optimal goal values starts to narrow. We limit the non-dollar exchange rate sensitivity to 1.1 and observe that as a result the dollar sensitivity doubles. Thus, exposure to non-dollar currency exposure is traded with dollar exposure. We therefore restrain the dollar sensitivity to .22. The difference between the required and optimal values of expected return and dividend yield is now very small. In the last iteration, the differences are eliminated by requiring an expected return of 20.64.

This final iteration leaves us with seven solutions. As there is only negligable discrepancy between the worst and best goal values (Table 5.4, entry #9) indicates that there is practically no more elbow-room left in forming portfolios that satisfy our constraints. The set of portfolio holdings that belong to each of these solutions is presented in Table 5.5. Indeed, the solutions are only very slightly different. It is the set of portfolios that are tailored to the needs and preferences

Table 5.5: Portfolios resulting when different goal variables are optimized, final iteration (entries show the percentage fractions of the budget, invested in each security)

	ERETURN	DIVYLD	DRNL	US$	EFFXR	RISKPR	RESMF
ACF	-	-	-	-	-	-	-
BSW	3.76	3.80	3.80	3.81	3.80	3.79	3.83
CSM	-	-	-	-	-	-	-
GIS	-	-	-	-	-	-	-
HEI	-	-	-	-	-	-	-
NUT	7.00	7.00	7.00	7.00	7.00	7.00	7.00
UNI	-	-	-	-	-	-	-
FOK	-	-	-	-	-	-	-
OCE	-	-	-	-	-	-	-
PHI	-	-	-	-	-	-	-
STO	-	-	-	-	-	-	-
AKZ	7.00	7.00	7.00	7.00	7.00	7.00	6.94
HOE	7.00	7.00	7.00	7.00	7.00	7.00	7.00
HO	-	-	-	-	-	-	-
HDG	-	-	-	-	-	-	-
KKB	5.12	5.10	5.10	5.10	5.11	5.10	5.11
BAM	7.00	7.00	7.00	7.00	7.00	7.00	7.00
HBG	-	-	-	-	-	-	-
KLM	-	-	-	-	-	-	-
NED	7.00	7.00	7.00	7.00	7.00	7.00	7.00
PAK	7.00	7.00	7.00	7.00	7.00	7.00	7.00
VO	7.00	7.00	7.00	7.00	7.00	7.00	7.00
TEL	7.00	7.00	7.00	7.00	7.00	7.00	7.00
VNU	7.00	7.00	7.00	7.00	7.00	7.00	7.00
ELS	-	-	-	-	-	-	-
AH	-	-	-	-	-	-	-
BW	-	-	-	-	-	-	-
HGM	4.57	4.57	4.57	4.57	4.57	4.58	4.62
IM	-	-	-	-	-	-	-
KBB	2.16	2.14	2.14	2.14	2.14	2.14	2.08
AAB	3.97	3.97	3.97	3.97	3.97	3.97	3.99
AGN	7.00	7.00	7.00	7.00	7.00	7.00	7.00
AMV	-	-	-	-	-	-	-
ING	-	-	-	-	-	-	-
SR	5.55	5.56	5.56	5.55	5.56	5.56	5.58
RD	4.88	4.86	4.86	4.86	4.86	4.86	4.84

of the investor. Of course, we could have stopped the interactive process in an earlier stage. The difference in portfolio composition would have been larger then. This allows us to screen and select one of those portfolios on the basis of other criteria.

5.4 SOME APPLICATIONS

Section 5.3 centers around techniques; in this section we present some applications of the multi-attribute approach to portfolio selection.

We will mainly focus on factor sensitivities as relevant attributes. The multi-dimensional risk concept that emerges from multi-factor models provides opportunities for a sophisticated management of investment risks. Parallelling the discussion in chapter one, section 1.5, we can distinguish between an 'APT approach' and a 'multi-factor model approach'. The former is restricted to manipulating the exposure to priced risk factors, whereas the latter considers both priced and non-priced risks. In this respect, Roll & Ross [1984, p.25] clearly distinguish between the strategic level and the tactical level: "The strategic decisions determining the level of exposure to systematic economic factors influence the average return, but the tactical decisions can be made without any sacrifice of portfolio return, because they deal merely with idiosyncratic risk."[19] However, an investor will measure and evaluate the trade-off between return and risk exposure on the level of his own investment opportunity set, and not on the general market level. When both levels do not coincide, the term 'idiosyncratic risk' has different meanings and content, depending on the span of the opportunity set and the restrictions an investor faces. Roll & Ross [1984, p.23] stress that portfolios with the same level of market risk (CAPM-β) can be exposed to different types of factor risk and that there is hence no reason why investors should be indifferent with respect to equi-β portfolios. We can then extend this argument to the level of the factors themselves and allow investors to perceive different sets of relevant factors, as argued in section 1.5. This in turn implies that the difference between 'strategic' and 'tactical', as evaluated on the general market level, fades. These terms are then defined relative to an investor's specific decision context. Here, we do not intend to contrast 'strategic' with 'tactical'. In the following, a portfolio strategy entails purely the choice of an appropriate pattern of factor sensitivities.

Several active and passive portfolio strategies with respect to risk can be followed. We will start, however, describing a general application of the framework: scanning. Next we discuss tilting and hedging.

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19) Likewise, Berry, Burmeister & McElroy [1988b, p.79] remark: "Fortunately, in a multifactor world, you do not need to assume any equilibrium relationship between realized [sic] returns and the associated risk factors."

5.4.1 Scanning a portfolio

A general application of the framework is making a 'scan' of a portfolio's profile. A scan entails an inventarization of its exposures to the attributes considered relevant. By determining the sensitivities for various factors, the systematic risk profile of a given portfolio can be summarized in terms of its economic factor exposures. However, this does not yield a complete picture. First of all, a portfolio is not likely to be perfectly diversified with respect to the economic risk factors. Hence, the factor risk profile must be complemented with information about non-factor risks. This can be done by incorporating a residual market factor, which represents the influences of both omitted economic factors and 'market mood' (psychological) factors on a portfolio's return behavior. For large portfolios, most of the return can be attributed to the economic factors and the residual market factor. The residual return variability can expected to be small.

Second, also indirect return related attributes may be relevant, especially when they represent proxies for risk and/or return components (see chapter one, section 1.4). The scores on these attributes complete the portfolio profile.

Scanning implies a passive use of the framework. Next, an investor may either accept the uncovered portfolio profile for given or he may be induced to revise the portfolio. The latter case leads us either to the general approach to portfolio selection as presented above or to one of the portfolio strategies as described below.

Scanning can be supplemented by conducting **sensitivity analyses**. This leads us back to chapter three, where we discussed the mechanism of conditioning. In effect, sensitivity analysis is directly suggested by the roots of multi-factor models and forms the logical complement to conditioning. As set out in chapter three, section 3.3, a sensitivity analysis entails a 'what if' approach. Given a portfolio's risk profile, the exposure to some factor can be combined with a hypothesized change in that factor. This yields a 'ceteris paribus' portfolio return that can be attributed to the factor.[20)]

Sensitivity analyses can greatly be enhanced by using **scenarios**. The combination of the multi-factor framework with the use of economic scenarios suggests itself immediately. When conditioning is complete (i.e. when the full set of underlying factors is considered), each scenario describes a state of the world. In more general terms, a scenario entails a specific constellation of potential factor realizations; this is depicted in Figure 5.2. Using the portfolio's risk

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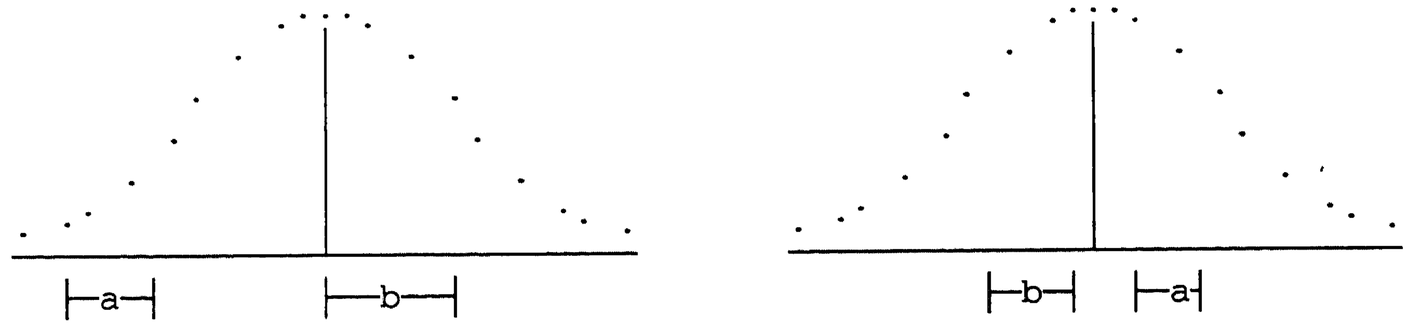
20) Another form of sensitivity analysis is to inspect to consequences of a change in a portfolio's composition on its profile. As the securities' exposures to the attributes represent their marginal contributions to the portfolio exposures, this analysis is straightforward. Implicitly, this analysis is performed in the process of constructing an optimal portfolio.

sensitivities, the set of factor changes corresponding to a specific scenario can be translated into a portfolio return. As the scenarios will cover various parts of the distributions of future factor movements, they can be interpreted as specific discrete approximations to these (perceived) distributions. By assigning probabilities to the occurence of each scenario and by combining this information with the portfolio's factor sensitivities, the (perceived) return distribution of the portfolio can be approximated. The use of scenarios in this way reminds of probabilistic modelling as discussed in chapter three, section 3.3. Markowitz & Perold [1981], for example, show the link between factor models and scenarios in estimating the variance-covariance matrix of security returns in a (E,σ^2)-context. Important is, however, that the use of economic scenarios enhances the economic transparancy of a portfolio's risk profile by linking the variability in portfolio return (beyond mean and variance) to changes in the economic environment as summarized by the scenarios.[21)]

Aside from being an analytical tool, scenarios can also be used to support portfolio strategies. This is especially relevant for the tilting strategies, discussed next.

Figure 5.2: Scenario specifications and factor distributions.



factor 1 factor 2

scenario a : a sharp decrease in factor 1 and a moderate incline in factor 2
scenario b : a positive change in factor 1 and a gradual decrease of factor 2

21) Some useful references on the specification and application of scenarios are Spiro [1976], Edesess & Hambrecht [1980] and Michaud [1985]. For the role of scenarios in the broader field of asset allocation, especially relating to the correlation between bond and stock returns, see Carman [1981] and Farrell [1989].

5.4.2 Tilting a reference portfolio

The portfolio strategy of tilting can be applied in the context of both risk attributes and other attributes.

Tilting risk exposures

Many portfolio strategies have a passive starting point in the sense that they pivot around a 'normal' portfolio. This portfolio may be the 'default' portfolio an investor holds —or a portfolio manager is expected to hold— because it is considered optimal over some longer period. Also, the normal portfolio may be a reference or target portfolio (for example a general stock market index) and the investor or portfolio manager in the first instance wishes to mimic the return behavior of the target. The default portfolio then is a mimicking or tracking portfolio with respect to the chosen target.[22] In investment practice, a normal portfolio often serves as an appropriate benchmark for performance measurement (Kritzman [1987]).

Given a normal portfolio, an investor may wish to deviate from this portfolio in a controlled way. This is done (implicitly or explicitly) by specifying an 'active' portfolio. This is a zero-investment portfolio that combined with the default portfolio yields the actual portfolio. So the active portfolio can be interpreted as a revision portfolio. In the case of tilting, the composition of the active portfolio depends on the investor's forecasts of future factor realizations. According to the degree in which the investor thinks that his ideas about the future movements of a factor differ from those of the market, he may increase or mitigate the portfolio's exposure to that factor.[23] For example, when the investor forecasts a decrease in some factor, he can decrease (in algebraic sense) the portfolio's sensitivity to that factor. By the same token, a high 'duration' portfolio can be formed by increasing the portfolio's interest rate sensitivity, while controlling its exposure to other factors. The factor exposures of the active portfolio reveal the degree in which the risk characteristics of the normal portfolio are altered. In specifying these 'factor tilts', 'factor plays' or 'factor bets', risk is considered as an opportunity and the investor intends to earn excess returns from this source. A crucial condition for adopting this active (and 'aggressive') strategy is that the investor has superior forecasting abilities, in order to predict factor movements that are unexpected to other investors. Not so much the actual size of the factor change is relevant, but its sign. The

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22) For a recent overview of and comparison between index tracking techniques (and empirical results for the Amsterdam Stock Exchange), we refer to Hallerbach [1994b].

23) In the case of factor-tilted index tracking, the tracking error is allowed to increase in turn for a more pronounced exposure to some factor.

application of the active strategy is pointed out by Berry, Burmeister & McElroy [1988a,b] and Sorensen, Mezrich & Thum [1989] and Sorensen & Thum [1992], among others.

Tilting other attributes

The tilting of a portfolio can be extended to the exposures to other attributes. For example, starting from a normal portfolio, an investor can tilt the dividend yield to a higher or lower level while controlling the other attributes. As set out in chapter one, section 1.4, there are indications that some attributes possess predictive abilities for future returns. While controlling risk exposures and other attributes, an investor could strive to tilt his portfolio to high dividend yield, low price/earnings ratio, low price/book ratio or small capitalization.[24)]

5.4.3 Hedging economic risks

Much of the idea underlying a multi-factor approach to investment management can already be recognized in Treynor [1972], who advocates separating security analysis from macro-economic forecasting. Treynor [1972, pp.150-151] remarks: "Used together, however, the two techniques [of hedging and diversification] represent powerful complementary instruments for focusing portfolio exposure on those risk factors a portfolio manager is most confident he can predict while minimizing the exposure to the others. To a remarkable degree, they transform the task of portfolio balancing from one of choosing among securities to one of choosing among causal factors. Securities are merely incidental vehicles for taking positions in underlying factors."

By tuning the sensitivity coefficients, a specific risk profile of the investment portfolio can be chosen. An active and aggressive strategy as described above depends on making forecasts of future factor realizations. We can confront the active strategy with passive strategies, that do not require factor forecasts. Here, risk is considered as a threat, not as an opportunity. These passive strategies can also be termed 'defensive', since they intend to shield a portfolio's return from undesired factor influences.

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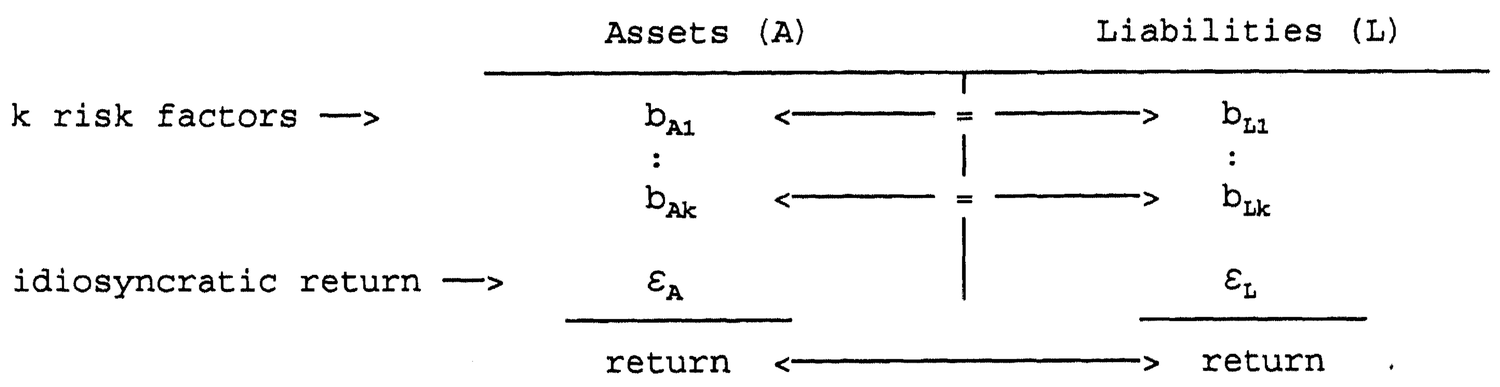
24) Sorensen & Thum [1992] stress the importance of monitoring factor risks when following a pure 'value strategy', i.e. picking the top quantile stocks according to rankings on a dividend discount model, price/earnings ratios, price/book ratios &c. They remark that "investment approaches associated with value rankings (...) may have reached their peak in popularity. The roster of value critics now seems to outnumber the list of believers." As described in section 1.4, however, the study by Fama & French [1992] gave new impetus and new content to 'value investing'.

One passive strategy involves 'risk sterilization' (Berry, Burmeister & McElroy [1988a]). In that case, a portfolio's exposure to one or several factors is considered to be excessive. The portfolio composition is then shifted in order to mitigate or negate its exposures to selected factors.

The other passive strategy is 'hedging', the main point in Roll & Ross' [1984] approach to strategic portfolio planning. The choice of an appropriate pattern of risk exposures is explicitly linked to the uses to which the income generated by the portfolio is to be put. The risk exposure is structured in accordance with the economic profile of an investor, which encompasses the pattern of his expenditures, his other sources of income and the economic conditions he will face. This can include a sterilization strategy: the portfolio is immunized against a source of risk by driving its sensitivity for the corresponding factor to zero. In general terms, a hedging strategy takes the form of 'matching'. Given the investor's liabilities as specified by his economic profile, the risk exposure of the assets (i.e. investment portfolio) is manipulated to absorb the risks that are incurred on the liability side. By pairwise equating the factor sensitivities of assets and liabilities, one could strive to achieve a certain degree of hedging. The resulting 'factor matching' is depicted schematicaly in Figure 5.3. This form of risk management is especially relevant for institutional investors like pension funds.

Figure 5.3: The principle of factor matching.



The flexibility in structuring the risk exposures of a portfolio depends on the variety of risk exposures of the securities in the opportunity set. The variety of the securities' sensitivities is even more important for the potential success of hedging strategies. When insurance against some factor risk is seeked, then the securities' sensitivities must allow the composition of a portfolio that has a zero or near zero sensitivity for that factor. This implies either the existence of many securities with zero sensitivities, or a range in sensitivities that

stretches from negative values to positive values. Likewise for matching, the asset's sensitivities must allow the formation of a portfolio that exhibits (approximately) the same sensitivities as the liabilities. This implies that the assets' sensitivities must be 'right', not only in sign but also in size. In this context, the negative inflation sensitivity of many asset categories is troublesome for pension funds.[25] Aside from factor risks, there are residual risks attached to both assets and liabilities. The degree in which these risks are idiosyncratic determines the degree in which they can be managed.

Aside from practical complications, which are topics for futher study, the multi-factor context signifies as a conceptual framework for an integrated approach to risk management. Especially for hedging purposes, the analysis of the characteristics of liabilities and assets (investment portfolios) in one unified, consistent framework is indispensible (cf. Van Aalst [1995]).

5.5 SUMMARY AND CONCLUSIONS

In this chapter, the multi-attribute representation of securities is supplemented with the final stage of selecting an optimal portfolio. From the securities in the opportunity set, the investor wishes to compose a portfolio that suits his preferences and desires best.

In the light of this choice problem, we explicitized the step from choosing between securities to choosing between exposures on the attributes. A multi-attribute representation of securities, entailing a detailed security analysis, provides the attribute exposures. To this end, the investor himself demarcates a set of relevant security attributes, depending on his decision context. It is assumed that the investor buys securities for the attributes they offer. For him, a security is adequately characterized by its exposures to the set of delineated attributes. This implies that the investor chooses between security attribute exposures instead of between the uni-dimensional securities. Hence, an investor's preference structure is cast in the multi-dimensional terms of relevant attributes.

The evaluation of feasible portfolios and the selection of an optimal portfolio then entails the combination of investor preferences and portfolio attribute scores. As an investor will evaluate attribute exposures on a portfolio level, we indicated the importance of expressing security attributes in a form that can be (linearly) aggregated. We concretisized this for various attributes, discussed in chapter one, section 1.4.

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[25] Cf. the studies by Van Aalst et al. [1994a,b]. See also Van Aalst & Hallerbach [1994].

By considering multiple attributes, the portfolio selection problem becomes multi-dimensional. The decision process involves searching for the portfolio that is considered most desirable, as judged over all the attribute dimensions. As an investor will show different preferences and tastes for different attributes, the investment decision forms a complex multi-criteria decision problem. The attribute exposures can be seen as goal variables which are linear in the portfolio holdings. The investor will then try to either minimize or maximize each of these goal variables, or strive to attain target exposures to attributes. Depending on the investor's insights and preferences, the relative importance of each of these goals may vary.

Generally, the goal variables will be interrelated; several goal variables may even be conflicting. This implies that no portfolio can be found for which each of the goal variables reaches its optimal value or for which all criteria are met. As a consequence, the investor has to evaluate the trade-offs between the various attributes. Because of the weak behavioral assumptions we make and the great flexibility we allow on the field of investor preferences, the choice of an adequate optimization approach is of paramount importance.

We critically discussed some standard approaches that are suggested to tackle a multi-dimensional portfolio problem. Ranking, for example, involves only a very indirect and incomplete use of the multi-atribute information. In addition, the trade-offs between the various attributes cannot be evaluated. An alternative, sophisticated approach extends the traditional utility framework to a multi-dimensional context by casting a utility function in terms of multiple attributes. The crucial problem in this multi-attribute utility theory, however, is the enormous complexity of specifying a multi-attribute preference functional. Hence, for conditional-normative uses this approach is too sophisticated because of the heavy information burden placed on the investor.

Considering this problem, several other approaches are suggested; these can be interpreted as (simplified) operationalizations of multi-attribute utility theory. One route is to cast the multi-dimensional preference functional in the form of a (linear) programming model. In a simple approach along this line, the objective function is cast in terms of one attribute, whereas the remaining attributes only serve as constraints. One could then try to extend the linear programming formulation to a multi-dimensional context by incorporating various attributes in a weighted fashion in the objective function. Another route is to mould the problem in a multiple-goal programming formulation. Compared to linear programming optimization, multiple goal programming shows a closer correspondence with decision making in practice in that the goals are formulated as aspiration levels. Also, there is always a solution for a well-defined problem (with a non-empty feasible region), even if some goals are conflicting.

An important drawback of linear programming and multiple goal programming, however, is their need for fairly detailed *a priori*

information on the investor's preferences on the side of trade-offs between the attributes. It is realistic to assume that this information cannot readily be acquired from an investor and/or cannot adequately be incorporated in the problem formulation. In that case, the investor's portfolio selection problem cannot be solved by these standard techniques.

To do justice to the flexibility allowed in the security analysis and preference analysis, we propose an alternative approach: interactive multiple goal programming. Since the interactive procedure evolves by seeking information from the investor, this removes the need for explicitizing his preference structure. Neither an explicit representation or specification of a preference functional is required, nor an explicit quantitative representation of the trade-offs among conflicting goals. Even in the most general context, this allows an effective and efficient scan of feasible, non-dominated portfolios.

The procedure starts formulating minimum requirements for exposures to each of the attributes, leaving a set of portfolios meeting these requirements. In addition, a set of indicators of the potential improvements of these values is presented to the investor. To the discretion of the investor, the minimum requirements are increased interactively. Each iteration produces indicators showing the sacrifice of meeting the higher standard in terms of shifts in the potentially attainable values on the other attributes. In this way, the investor can evaluate the trade-offs between the various attributes. By successively narrowing the discrepancies between the required values and the potential values of the attribute exposures, the set of feasible portfolios is gradually reduced by keeping only the non-dominated portfolios. The investor can continue this process until only one feasible portfolio is left, or end the process when the set of feasible portfolios is sufficiently small. In the latter case, a suitable portfolio can be selected from the set of feasible portfolios on the basis of other considerations. In this way, the procedure offers adequate flexibility to incorporate other —qualitative— criteria into the decision process. We illustrated the potential of the approach by means of an example.

In our opinion, this approach offers the desired degree of flexibility to be fruitfully applied to the multi-attribute portfolio selection problem. It incorporates all the advantages of traditional sophisticated methodologies, while circumventing the unnecessary information burden of assessing the investor's detailed preference pattern. Note that the last two stages of the portfolio selection process —portfolio analysis and portfolio selection— are no longer treated separately in this approach but are integrated. The interactive method then is no optimizer, but a 'combinizer'. In the interactive decision process, a learning process is embedded. By scanning the feasible portfolios, the investor not only becomes acquainted with the trade-offs that in the opportunity set exist between the exposures to

the various attributes; also he can shape and adjust his preferences when confronted with these trade-offs. To explore the opportunity set in all the dimensions the attributes offer, the investor is adviced to perform the interactive process several times. Given the path-independency of the procedure, no desirable (feasible) portfolios can be missed, only insight can be gained.

After discussing the choice methodology, we indicated some derived applications. We mainly focussed on factor sensitivities as relevant attributes. The multi-dimensional risk concept, emerging from multi-factor models, provides ample opportunities for a sophisticated management of investment risks. In most of the cases, however, other attributes can be incorporated.

A general application of the framework is the scanning of a portfolio's profile by inventarizing its exposures to the attributes considered relevant. We indicated how this scanning can be supplemented by performing sensitivity analyses, and can be enhanced by using scenarios. Aside from being an analytical tool, scenarios can also be used to support portfolio strategies.

A more specific application is the tilting of a reference portfolio. Starting form a 'normal' or 'default' portfolio, an investor may wish to deviate from this portfolio in a controlled way. In the case of factor tilting, a portfolio's sensitivity to one or more factors is increase or mitigated, depending on the investor's forecasts of future factor realizations. These factor plays or factor bets form an active portfolio strategy. The tilting of a (normal) portfolio can be extended to the exposures to other attributes. For example, an investor can tilt the dividend yield to a higher or lower level while controlling the other attributes.

By tuning the sensitivity coefficients to the factors, a specific risk profile of the investment portfolio can be chosen. An active and aggressive strategy as tilting considers risk as an opportunity. This can be contrasted with a passive, defensive strategy in which a portfolio's composition is shifted in order to mitigate or negate its exposures to selected factors. In this sterilization strategy, risk is considered as a threat. Another passive strategy is hedging, where the choice of an appropriate pattern of portfolio risk exposures is explicitly linked to the risk exposures of liabilities in order to arrive at a pairwise match. The multi-factor context thus signifies as a valuable conceptual framework for an integrated approach to risk management.

We believe that the sketched methodology has a clear potential as a tool to support the investor in evaluating and selecting portfolios that meet his investment goals as close as possible.