Multi-Attribute Portfolio Selection:

A Conceptual Framework

Winfried G. Hallerbach

MULTI-ATTRIBUTE PORTFOLIO SELECTION:

A CONCEPTUAL FRAMEWORK

(Multi-Attribute Portefeuilleselectie:
Een Conceptueel Raamwerk)

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"The trouble with modern education is you never know how ignorant people are. With anyone over fifty you can be fairly confident what's been taught and what's been left out. But these young people have such an intelligent, knowledgeable surface, and then the crust suddenly breaks and you look down into depths of confusion you didn't know existed."

Evelyn Waugh, 1945, **Brideshead Revisited**, Chapman & Hall, London UK (first edition), p.171

PREFACE

At a conference, quite some time ago, a man suddenly started scribbling down notes. He was observed by Albert Einstein, who asked him what he was doing. The scribbler replied: "I always carry a note pad so that I can write down any good ideas that pop up in my mind", whereupon Einstein soberly remarked, "I had only one or two good ideas in my whole life".

I feel a lot like that man at the conference. At the Department of Finance, I too did a lot of scribbling. More importantly, I learned that consolidating collected notes was not quite equivalent to writing a dissertation. I was as much hindered by my German roots (from which I obviously could not escape) as by the restricted space of my study, which looked like an exploded confetti factory. I hereby thank many vintages of student assistants (who copied most of the cited papers for me, and many, many more) to make this possible.

I'm deeply indebted to my supervisor Jaap Spronk and I thank him for his encouragement, support, guidance, never ceasing trust and – of course – for his notorious and contagious good moods. I look forward to our cooperation in the future.

I thank my colleagues at the Department of Finance (and my roommate Nico van der Sar in particular) for providing a stimulating research ambiance. Especially, I wish to thank my companion in distress Marc Goedhart. When time ran out and Occam's razor was sharpened, our conversations made me realize that there exists at least one exception to 'a trouble shared is a trouble doubled'.

Finally, I wish to thank my mother (who shows to be a real stayer) for never-ending support, and Jaap Jan for his long-long-time understanding and consideration. Words would be inadequate, but a promise for a long-long vacation is certainly in place.

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Winfried Hallerbach
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CONTENTS

Chapter 1: INTRODUCTION 1

1.1 The investment decision problem 1

1.1.1 Portfolio theory 1

1.1.2 Context 2

1.1.3 Deriving decision criteria 5

1.2 The representation of investment alternatives 11

1.2.1 The mean-variance framework 11

1.2.2 Some alternative approaches 16

1.3 The investment decision: a closer look 19

1.4 Refining the representation of investment alternatives 22

1.4.1 Direct return related attributes 22

1.4.2 Indirect return related attributes 24

1.5 Of market pricing and Procrustes 33

1.6 Risk management in an efficient market 39

1.7 Goal and outline of the study 43

Chapter 2: THE MARKET MODEL, INDEX MODELS AND FACTOR MODELS 49

2.1 Introduction 49

2.2 The Market Model (MM) 50

2.2.1 Definition 50

2.2.2 The market model as a theoretical concept 52

2.2.3 The market model as an empirical concept 63

2.3 The Single Index Model (SIM) and the Single Factor Model (SFM) 73

2.3.1 Definition 74

2.3.2 The SIM and SFM as theoretical concepts 78

2.3.3 The SIM and SFM as empirical concepts 81

2.4 Multi-Index Models (MIM) and Multi-Factor Models (MFM) 85

2.4.1 Definition 85

2.4.2 The MIM and MFM as theoretical concepts 86

2.5 MIMs and MFMs as empirical concepts 95

2.5.1 MIMs as empirical concepts 95

2.5.2 MFMs as empirical concepts 100

2.6 Summary and conclusions 104

Chapter 3: FACTOR MODELS AS STATISTICAL AND ECONOMIC CONCEPTS 107

3.1 Introduction 107

3.2 Conditioning security prices/returns on state variables 112

3.2.1 A general multi-factor representation 113

3.2.2 Linear and approximately linear multi-factor representations 115

3.2.3 Conditioning and least squares 119
3.2.4 Taylor series approximation versus least squares approximation 121
3.3 Factor models as simplifying distribution assumptions 127
3.3.1 Distributions of securities and portfolios 127
3.3.2 Conditioning and the reduction of complexity 129
3.3.3 (Multi-) factor models and multi-moment portfolio analysis 140
3.4 Sensitivity coefficients (beta & betas) as risk measures 149
3.4.1 The portfolio approach 150
3.4.2 The conditioning approach 155
3.4.3 Resume 157
3.5 Present value models as a basis for deriving factor models 158
3.5.1 Present value models for financial securities 158
3.5.2 From present value model to factor model: bonds 167
3.5.3 From present value model to factor model: stocks 173
3.5.4 The empirical validity of present value models 180
3.6 Summary and conclusions 181

APPENDIX 3.A: A multi-factor model in continuous time 187
APPENDIX 3.B: Taylor series approximation versus least squares approximation 190
APPENDIX 3.C: A simple flexible growth model 197
APPENDIX 3.D: Modelling non-linearities with first order elasticities 203

Chapter 4: SOME EMPIRICAL RESULTS FOR THE DUTCH STOCK MARKET 209
4.1 Introduction 209
4.2 The stock market data 215
4.2.1 Stocks and indices 215
4.2.2 Choosing a return definition 217
4.3 The relationship between common stock returns and interest rates 218
4.3.1 Introduction and summary 218
4.3.2 Estimating 'equity duration': some empirical studies 219
4.3.3 The interest rate series 220
4.3.4 The interest rate sensitivities 229
4.3.5 Non-linearities: equity convexity 239
4.3.6 Confronting theoretical and empirical durations 243
4.4 Towards a multi-factor model 247
4.5 Summary and conclusions 251

APPENDIX 4.A: The stock market data 253
APPENDIX 4.B: Return definitions and biases 257
APPENDIX 4.C: A correction for spurious serial correlation from averaging 262

Chapter 5: MULTI-ATTRIBUTE PORTFOLIO SELECTION 265
5.1 Introduction 265
5.2 Aggregating security attributes to the portfolio level 266
5.3 Choosing an optimal portfolio 272
5.3.1 Choosing between attribute exposures 273
5.3.2 Standard approaches 274
5.3.3 An alternative approach 280
5.3.4 An example 281
5.4 Some applications 288
5.4.1 Scanning a portfolio 289
5.4.2 Tilting a reference portfolio 291
5.4.3 Hedging economic risks 292
5.5 Summary and conclusions 294

Chapter 6: SUMMARY AND EVALUATION 299

REFERENCES 307

SAMENVATTING (Summary in Dutch) 337

Curriculum Vitae 347

NOTATION

Although the meaning of symbols is explained in the text, we here provide some general remarks on notation.

Underscores denote random variables, for example $\underline{r}_{it}$.

Boldface types denote matrices (upper case, for example $\mathbf{\Sigma}$) and (colum) vectors (lower case, $\mathbf{x}$). In this context, a prime (') denotes the transpose.

The braces { } indicate a set, like $\{\underline{r}_i\}_{i=1,..,N}$, mostly abbreviated as $\{\underline{r}_i\}_{i\in N}$ or simply as $\{\underline{r}_i\}_i$.

The square brackets [ ] indicate the elements of a (colum) vector (like $\mathbf{x} = [x_i]_{i=1,..,N} = [x_i]_i$) or a matrix (like $\mathbf{\Sigma} = [\sigma_{ij}]_{i,j=1,..,N} = [\sigma_{ij}]_{i,j}$).

The summation operator $\sum_{i=1}^{N}$ is abbreviated as $\sum_{i\in N}$ or simply as $\sum_i$.

Where it is clear from the context, a function $\Phi(\underline{\delta}_1,...,\underline{\delta}_k)$ with arguments $\{\underline{\delta}_j\}_j$ is simply denoted as $\Phi(\cdot)$. Derivatives are then indicated by primes or parenthesized superscripts, for example $f'(\cdot)$, $f''(\cdot)$, $f^{(3)}(\cdot)$ and so on.

The expectations, variance and covariance operators are denoted by $E(\cdot)$, $Var(\cdot)$ and $Cov(\cdot,\cdot)$, respectively.

Quotation marks (" ") denote literal citation whereas inverted commas (' ') are used as ordinary quotation marks in our own text.

LHS and RHS is used to indicate the left-hand side and right-hand side of an equation.

Boldface plain text, finally, indicates our emphasis.