Multi-Attribute Portfolio Selection:

A Conceptual Framework

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MULTI-ATTRIBUTE PORTFOLIO SELECTION:
A CONCEPTUAL FRAMEWORK

(Multi-Attribute Portefeuilleselectie:
Een Conceptueel Raamwerk)

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"The trouble with modern education is you never know how ignorant people are. With anyone over fifty you can be fairly confident what's been taught and what's been left out. But these young people have such an intelligent, knowledgeable surface, and then the crust suddenly breaks and you look down into depths of confusion you didn't know existed."

PREFACE

At a conference, quite some time ago, a man suddenly started scribbling down notes. He was observed by Albert Einstein, who asked him what he was doing. The scribbler replied: "I always carry a note pad so that I can write down any good ideas that pop up in my mind". Whereupon Einstein soberly remarked, "I had only one or two good ideas in my whole life".

I feel a lot like that man at the conference. At the Department of Finance, I too did a lot of scribbling. More importantly, I learned that consolidating collected notes was not quite equivalent to writing a dissertation. I was as much hindered by my German roots (from which I obviously could not escape) as by the restricted space of my study, which looked like an exploded confetti factory. I hereby thank many vintages of student assistants (who copied most of the cited papers for me, and many, many more) to make this possible.

I’m deeply indebted to my supervisor Jaap Spronk and I thank him for his encouragement, support, guidance, never ceasing trust and – of course – for his notorious and contagious good moods. I look forward to our cooperation in the future.

I thank my colleagues at the Department of Finance (and my roommate Nico van der Sar in particular) for providing a stimulating research ambiance. Especially, I wish to thank my companion in distress Marc Goedhart. When time ran out and Occam’s razor was sharpened, our conversations made me realize that there exists at least one exception to ‘a trouble shared is a trouble doubled’.

Finally, I wish to thank my mother (who shows to be a real stayer) for never-ending support, and Jaap Jan for his long-long-time understanding and consideration. Words would be inadequate, but a promise for a long-long vacation is certainly in place.

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NOTATION

Although the meaning of symbols is explained in the text, we here provide some general remarks on notation.

Underscores denote random variables, for example $x_i$.

Boldface types denote matrices (upper case, for example $\Sigma$) and (column) vectors (lower case, $\mathbf{x}$). In this context, a prime (') denotes the transpose.

The braces \{\} indicate a set, like $\{x_i\}_{i=1}^n$, mostly abbreviated as $\{x_i\}_i$ or simply as $\{x_i\}$.

The square brackets [ ] indicate the elements of a (column) vector (like $\mathbf{x} = [x_1, \ldots, x_n]$) or a matrix (like $\mathbf{E} = [e_{ij}]_{i=1}^n$ or $[e_{ij}]_{i,j=1}^n$).

The summation operator $\sum_i$ is abbreviated as $\sum_{i=1}^n$ or simply as $\sum_i$.

Where it is clear from the context, a function $\Phi(\mathbf{A}, \ldots, \mathbf{A})$ with arguments $\{A_i\}_i$ is simply denoted as $\Phi(\cdot)$. Derivatives are then indicated by primes or parenthesized superscripts, for example $f'(\cdot)$, $f''(\cdot)$, $f^{(n)}(\cdot)$ and so on.

The expectations, variance and covariance operators are denoted by $E(\cdot)$, $\text{Var}(\cdot)$ and $\text{Cov}(\cdot, \cdot)$, respectively.

Quotation marks (" ") denote literal citation whereas inverted commas (' ') are used as ordinary quotation marks in our own text.

LHS and RHS is used to indicate the left-hand side and right-hand side of an equation.

Boldface plain text, finally, indicates our emphasis.