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ERIM REPORT SERIES RESEARCH IN MANAGEMENT	
ERIM Report Series reference number	ERS-2006-068-LIS
Publication	December 2006
Number of pages	31
Persistent paper URL	
Email address corresponding author	lkroon@rsm.nl
Address	Erasmus Research Institute of Management (ERIM)
	RSM Erasmus University / Erasmus School of Economics
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	P.O.Box 1738
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REPORT SERIES
RESEARCH IN MANAGEMENT

ABSTRACT AND KEYWORDS	
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Free Keywords	Passenger Railways, Shunting, Optimization
Availability	The ERIM Report Series is distributed through the following platforms: Academic Repository at Erasmus University (DEAR), DEAR ERIM Series Portal Social Science Research Network (SSRN), SSRN ERIM Series Webpage Research Papers in Economics (REPEC), REPEC ERIM Series Webpage
Classifications	The electronic versions of the papers in the ERIM report Series contain bibliographic metadata by the following classification systems: Library of Congress Classification, (LCC) LCC Webpage Journal of Economic Literature, (JEL), JEL Webpage ACM Computing Classification System CCS Webpage Inspec Classification scheme (ICS), ICS Webpage

Shunting of Passenger Train Units:

an Integrated Approach

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Abstract

In this paper, we describe a new model for the Train Unit Shunting Problem. This model is capable of solving the matching and parking subproblems in an integrated manner, usually requiring a reasonable amount of computation time for generating acceptable solutions. Furthermore, the model incorporates complicating details from practice, such as trains composed of several train units and tracks that can be approached from two sides. Computation times are reduced by introducing the concept of virtual shunt tracks. Computational results are presented for real-life cases of NS Reizigers, the main Dutch passenger railway operator.

1 Introduction

During the rush hours, the rolling stock of a passenger railway operator is typically operating the timetable or it is in maintenance. However, outside the rush hours, an operator usually has a surplus of rolling stock. In order to be able to fully exploit the main railway infrastructure, the idle rolling stock is parked at a shunt yard. Since only a few passenger night services exist in the Netherlands, most rolling stock has to be parked during the night.

In the Netherlands most train services are operated by train units, which are classified according to types and subtypes. Train units can move bi-directionally without the need for locomotives. Only train units of the same type can be combined to form longer train compositions, taking into account certain restrictions on the length of the resulting train. Subtypes belonging to the same type are discerned by their numbers of carriages per train unit. The different subtypes of train units have different characteristics such as seating capacity and length. The upper part of Figure 1 shows an example of a Dutch train unit with 3 carriages. This type of train unit (ICM) consists of subtypes with 3 or 4 carriages and is typically used for intercity services.

Figure 1: An ICM train unit with 3 carriages (ICM 3).

The process of parking train units at a shunt yard together with several related processes is called *shunting* and the corresponding planning problem is called the *Train Unit Shunting Problem* (TUSP). A major complicating issue is the fact that train units are strongly restricted in their movements by the railway infrastructure. Moreover, time is a restrictive resource for shunting. For example, a minimum headway time between two train movements on the same track is required. Moreover, railway passenger services are typically described by a timetable with planned times and exact compositions of the arriving and departing train services. Finally, it is common to solve TUSP station by station and for a 24-hour period.

During the night, the goal of shunting is to select the positions and compositions of the trains at the shunt yard in such a way that the operations in the next morning can start up as smoothly as possible. The shunt plans should be robust, since disruptions in real time are possible. In case of a disruption, the plans should need a minimum number of changes.

In general, train units of the same subtype can be used interchangeably. This flexibility implies that, given a timetable with times and exact compositions of the arriving and departing train services, a planner has to determine a matching of arriving and departing units at a station. A large part of this matching is already prescribed by the timetable. For example, the matching of arriving and departing train units of through train services, which continue passenger service after a short dwell time, is typically fixed.

Figure 2 shows the layout of station Zwolle, which is a station in the northeastern part of the Netherlands. The left side of the station is defined as the A-side, while the right side is defined as the B-side. The black areas in the figure represent the platforms, while around those tracks several shunt tracks are located.
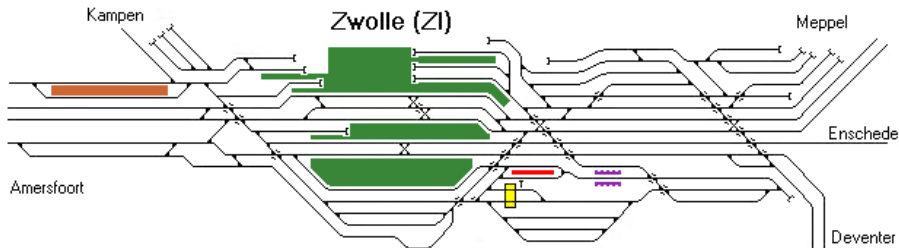

Figure 2: The layout of station Zwolle.

The choice to park a train unit at a particular shunt track has several implications. First, if train units of different subtypes are parked simultaneously at the same shunt track, then the order of the train units is important. In this case, it is important that no train unit is blocking the arrival or departure of another one. Second, many shunt tracks can be approached from both sides, leading to additional decisions to be made. Third, robustness of the resulting shunt plans

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against small disturbances is also relevant. Other aspects that play a role are the routing of train units between platforms and shunt tracks, preferences for shunt tracks, cleaning of train units that lay over, and the availability of crew to carry out the resulting shunt activities within certain time intervals. However, these aspects are considered outside the scope of this paper,

The main contribution of this paper is that we provide a model for solving this shunting problem for general shunt track configurations, where trains may consist of several units. In the model, the matching and parking subproblem are treated simultaneously. The model described in this paper is an extension of models described in earlier papers, e.g. Freling et al. [2005].

This paper continues with an in-depth problem description, containing an overview of relevant literature. Then, we discuss a model for restricted shunt track configurations. In addition, we investigate the influence of tracks with multiple subtypes of train units in this model and we extend the model to general track configurations. Computational results for several real-life cases precede the conclusions.

2 Problem Description

To start with, we introduce the term *arriving (departing) shunt units* for train units that need to be parked at (supplied from) the shunt tracks. We say that a *crossing* occurs at a shunt track whenever a train unit i obstructs the arrival or departure of a train unit j. This term was introduced by Gallo and Di Miele [2001] in the context of buses. Now, we formally define the *Train Unit Shunting Problem (TUSP)* as follows:

Definition 1. Given

- the infrastructure of a railway station and a nearby shunt yard, usually geographically separated from the station,

- a timetable, with for each train service the arrival and / or departure time and platform at the involved station, and its composition,

the *Train Unit Shunting Problem (TUSP)* consists of (*i*) matching the arriving and departing shunt units, and (*ii*) parking these shunt units at the shunt tracks, such that the total shunting costs are minimal and no crossings occur.

A solution to TUSP assigns arriving shunt units to departing ones and to a shunt track. Moreover, if such a track can be approached from both sides, the solution also describes the arrival and departure sides for each train unit parked at the track at any point in time. Typically, TUSP is solved for 24-hour periods.

The cost of a shunt plan is determined by the number of splits of train units resulting from the same train service to different shunt tracks and the number of tracks with multiple subtypes of train units parked at it. The number of splits is a proxy for the amount of resource consumption at the station, such as crew and railway infrastructure. Indeed, if two train units from the same train service are split, this results in two routes to shunt tracks, which requires two drivers and two reservations of parts of the infrastructure, compared to keeping the units together. The main characteristics of real-life instances of TUSP are the following:

- Arrivals and departures of train units are typically mixed in time. This implies that, within the planning horizon, the first departure takes place before the last arrival.

- Shunt units may belong to different types or subtypes (and thus have different lengths). The type of a unit may restrict the set of shunt tracks at which the unit can be parked. For example, electrical train units can only be parked at a track with catenary.

- Shunt tracks may have different types and lengths. The type of a track determines how a unit may approach the track. Some tracks may be approached from one side only. These tracks will be called *LIFO tracks*. Other tracks may be approached from both sides, which will be called *free tracks*. Note that for these track configurations, it is possible to arrive from

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one side and depart to the other, as well as arriving from and departing to the same side.

As opposed to similar versions of this problem already known in literature, see e.g. Blasum et al. [2000] and Winter [1999], subtype mismatches are not allowed in Dutch railway rolling stock deployment. An important reason for this restriction is the fact that previous planning processes already decided upon the exact configurations of the train services and local changes in these configurations are highly undesired.

Throughout this paper we assume that the shunt yard is empty at both the start of the planning period and the end of it. This also implies that no train unit of any subtype is permanently parked at a shunt yard. The presented models can also be applied in case of cyclic planning. However, for ease of description, we will disregard the cyclic case in the remainder of this paper.

2.1 Related literature

Cordeau et al. [1998] provide a recent overview of the use of Operations Research in railway systems focusing on train routing and scheduling problems. Moreover, Huisman et al. [2005] discuss how Operations Research supports the planning processes of a typical European passenger railway operator.

An elaborate introduction to TUSP, including a solution approach and computational results, can be found in Freling et al. [2005]. Here, the matching and parking subproblems are solved separately, nevertheless resulting in solutions with high quality. Moreover, Lentink [2006] discusses models and algorithms for other elements of shunt planning, including routing and cleaning of train units. Haijema et al. [2006] proposes a heuristic based on dynamic programming for solving TUSP. The first results of this approach are promising.

Some special cases of TUSP have been dealt with by Winter and Zimmermann [2000] and Blasum et al. [2000] for dispatching trams in a depot. Winter [1999] theoretically extends this approach with length restrictions and mixed arrivals and departures. Moreover, he also discusses an application to a bus depot, including computational results.

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Di Stefano and Kǒci [2003] study the computational complexity of several variants of subproblems of TUSP. Furthermore, they also present algorithms for solving some of these subproblems, including bounds on the objectives and on the complexity of the algorithms.

Furthermore, Gallo and Di Miele [2001] discuss an application for dispatching buses in a depot, with an extension of their models taking account mixed arrivals and departures. Another application of bus dispatching is described in Hamdouni et al. [2006]. Here, robust solutions are emphasized by having as little different types of buses as possible in one lane, and within one lane by grouping together the buses of the same type as much as possible. In subsequent work, Hamdouni et al. [2005] develop an alternative formulation, where type-mismatches between requested and supplied buses are allowed at some cost.

Tomii et al. [1999] and Tomii and Zhou [2000] propose a genetic algorithm that takes into account some related processes of TUSP. However, their parking problem is of a less complex nature, since in their context at most one train unit can be parked at a shunt track at the same time.

Lentink et al. [2006] introduce the routing subproblem of TUSP, accompanied by a solution methodology and computational results. Zwaneveld [1997] studied a routing problem for train units over railway infrastructure at a station, which is strongly related to TUSP. In this problem, one is looking for a set of routes and platforms for train services in a one hour period where arrival and departure times are fixed. This approach is applied to a number of railway stations in the Netherlands.

Lübbecke and Zimmermann [2005] discuss a related problem that arises at an in-plant private freight railroad. In this problem, one assigns transportation requests to certain regions of the in-plant railroad and selects cars of specific types from a shunt track in this region for servicing a specific request. However, the authors only discuss LIFO tracks and assume that there is no prescribed order of different types of cars in a train. In addition, it is assumed that there are no limitations for the temporary parking of cars, when these are not servicing a request.

Dahlhaus et al. [2000] discuss the related problem of rearranging carriages in a freight train to group them by destination. Their goal is to use a minimum number of tracks for this rearrangement and they show that this problem is $\mathcal{NP}$-hard. More information on similar topics in railway freight transportation can be found in Cordeau et al. [1998].

In addition, He et al. [2000] extend this problem by considering multiple trains. In their approach, the rearranging of carriages is a two-stage process where arriving carriages are firstly classified and secondly assembled before making up the departing train. Classification and assembly take place at different sets of parallel tracks. The occupation of shunt tracks by carriages is not taken into account. He et al. [2003] propose an integrated model and solution approach for the problem described in He et al. [2000], where the occupation of the tracks by the trains is also taken into account.

3   Basic Model for LIFO Tracks

To start with, we discuss a model for TUSP in the special case that all tracks are LIFO tracks. Before discussing this model, we introduce some notation.

The set S contains all the shunt tracks at which units can be parked. For each $s \in S$, c_s gives the length of track s. Each station has an A-side and a B-side. Furthermore, we define the A-side of a track as the side which is closest to the A-side of the station, and similarly the B-side of a track. A shunt track s can be open at the A-side, the B-side or at both sides. This uniformly defines the side of a shunt track. In this section we assume that the tracks are only open at the A-side.

We define $T = \{1, \dots, n\}$ as the set of train units that arrive or depart. For each train unit $t \in T$, we know whether it is arriving or departing. Therefore, we can partition the set T in a subset of arriving train units, denoted by T_+, and a subset of departing train units, denoted by T_-, with $T = T_+ \cup T_-$ and $T_+ \cap T_- = \oslash$. Moreover, we introduce Y as the set of subtypes of train units and the mapping τ_t, which maps a train unit $t \in T$ to its subtype $\tau_t \in Y$. We

assume that the arrivals and departures of each subtype are balanced. Also, the mapping l_t maps a unit $t \in T$ to its length. For each $t \in T$, we know the train service i_t in which it arrives at or departs from the station.

As mentioned before, the timetable consists of a sequence of arriving and departing train services. For each service, we know the planned time of arrival or departure as well as the exact composition of the train operating this service. Similar to the A-side of a track, we introduce the A-side of a train as the side of the train which is closest to the A-side of the station, whenever the train is within the boundaries of the station.

A graphical representation of different sides of a station, a track and a train is presented in Figure 3. Note that it is very likely that shunt tracks open at the A-side are located at the B-side of the station and vice versa.
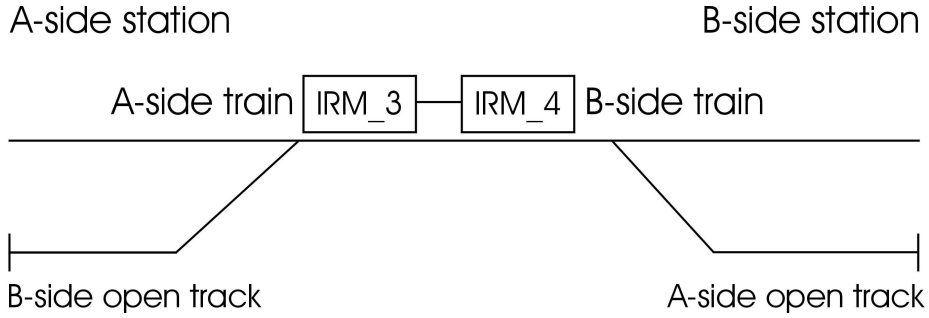

Figure 3: The A-side and the B-side of a station, a train and two LIFO shunt tracks.

The set T is sorted according to the partial ordering $<_A$ on the train units. By definition $X_i <_A X_j$ if and only if one of the following conditions is satisfied:

1. Unit X_i arrives or departs in a train with an earlier planned time than the train to which unit X_j belongs.

2. Arriving units X_i and X_j arrive in the same train and X_j is closer to the A-side of the train than X_i.

3. Departing units X_i and X_j depart in the same train and X_i is closer to the A-side of the train than X_j.

Consider an arriving train composition $X_1 \cdots X_k$, where X_1 is closest to the A-side of the train and X_k is farthest from it. Case 2 states that this composition is ordered as $X_k <_A \ldots <_A X_1$ in T. Note that this is the order in which the units arrive at a shunt track open at the A-side. Since the units of a departing service $X_1 \cdots X_k$ leave the shunt track via the A-side in the order $X_1 <_A \ldots <_A X_k$, Case 3 states that a departing service is ordered in this manner in T. Finally, the relation $X_i \leq_A X_j$ holds if and only if $X_i <_A X_j$ or $X_i = X_j$.

Figure 4 gives an example of an arriving and a departing train service at the platforms 5a\b and 7a\b. For clarity, we numbered the train units ①,②,③,④ which results in $T = \{①, ②, ③, ④\}$ with order ①$<_A$②$<_A$③$<_A$④ for shunt tracks that are open at the A-side.
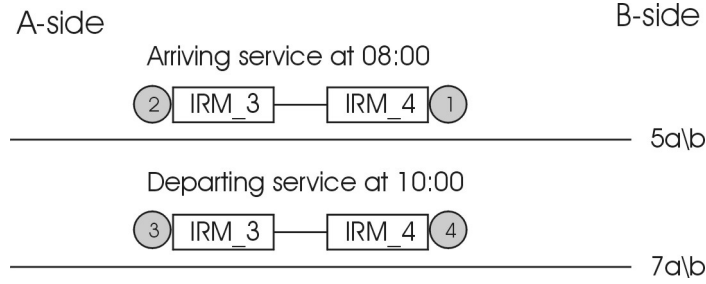

Figure 4: An example of the ordering of train units.

We define I as the set of pairs of train units $(t, t+1) \in T^2$ that arrive or depart in the same train service, i.e. $i_t = i_{t+1}$. Finally, we define L as the set of pairs of train units that can be matched:

$$L := \{(t, u) \mid t \in T_+, u \in T_-, t <_A u, \tau_t = \tau_u\}. \tag{1}$$

We assume that the conditions in the definition of the set L are sufficient for a feasible matching of two train units, although this is not necessarily the case. Indeed, even if the conditions in (1) are met, a train unit t cannot be matched to a train unit $u >_A t$ if no train unit of subtype τ_t is present at some point in time between the arrival time of unit t and the departure time of unit u. The set X of potential crossings at a LIFO track is defined as:

$$X := \{\{(t, u), (v, w)\} \mid ((t, u), (v, w)) \in L^2, t <_A v <_A u <_A w\}$$

Given these definitions, we introduce the following decision variables:

$$z_{t,s} = \begin{cases} 1 & \text{if train unit } t \text{ is parked at or retrieved from track } s; \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{t,u,s} = \begin{cases} 1 & \text{if arriving train unit } t \text{ is matched to departing unit } u \\ & \quad \text{and parked at track } s; \\ 0 & \text{otherwise.} \end{cases}$$

$$b_{t,s} = \begin{cases} \text{the length of the train units at track } s \text{ immediately after the} \\ \text{arrival or departure of unit } t. \end{cases}$$

$$d_t = \begin{cases} 1 & \text{if units } t \text{ and } t+1 \text{ are related to the same train service} \\ & \quad \text{and are parked at or retrieved from different tracks;} \\ 0 & \text{otherwise.} \end{cases}$$

$$m_{\tau,s} = \begin{cases} 1 & \text{if at least one unit of subtype } \tau \text{ is parked at track } s; \\ 0 & \text{otherwise.} \end{cases}$$

$$n_s = \text{the number of subtypes } \tau \text{ in excess of 1 parked at track } s.$$

The penalty N on the variables n_s models a preference for solutions with less different subtypes parked at a track. This adds to a more robust solution in practice. Moreover, a penalty D is incurred for train units from the same train, which are parked at different shunt tracks. The initial model reads:

$$\text{minimize} \quad D \sum_{t \in T} d_t + N \sum_{s \in S} n_s \tag{2}$$

$$\text{subject to} \quad \sum_{s \in S} z_{t,s} = 1 \qquad \forall t \in T \tag{3}$$

$$\sum_{u:(t,u) \in L} x_{t,u,s} = z_{t,s} \qquad \forall t \in T_+, s \in S \tag{4}$$

$$\sum_{u:(u,t) \in L} x_{u,t,s} = z_{t,s} \qquad \forall t \in T_-, s \in S \tag{5}$$

$$x_{t,u,s} + x_{v,w,s} \leq 1 \qquad \forall s \in S, \{(t,u),(v,w)\} \in X \tag{6}$$

$$b_{t-1,s} + l_t z_{t,s} = b_{t,s} \qquad \forall t \in T_+, s \in S \tag{7}$$

$$b_{t-1,s} - l_t z_{t,s} = b_{t,s} \qquad \forall t \in T_-, s \in S \tag{8}$$

$$b_{t,s} \leq c_s \qquad \forall t \in T_+, s \in S \tag{9}$$

$$z_{t,s} - z_{t+1,s} \leq d_t \qquad \forall s \in S, (t,t+1) \in I \tag{10}$$

$$z_{t,s} \leq m_{\tau_t,s} \quad \forall s \in S, t \in T \tag{11}$$

$$\sum_{\tau \in Y} m_{\tau,s} \leq n_s + 1 \quad \forall s \in S \tag{12}$$

$$z_{t,s} \in \{0,1\} \quad \forall t \in T, s \in S \tag{13}$$

$$x_{t,u,s} \in \{0,1\} \quad \forall (t,u) \in L, s \in S \tag{14}$$

We call (2)-(14) Model 1. In this model, restrictions (3) state that each train unit needs to be parked at a track. Restrictions (4) state that each arriving train unit is parked at a track and matched to a departing unit. This also holds for restrictions (5) for departing train units. In addition, restrictions (6) prohibit crossings. Restrictions (7) are used for administrating the length of the units parked at a track at arrival of a train unit. Restrictions (8) are similar, but for departing train units. Restrictions (9) ensure that the total length of the units parked at a track never exceeds the capacity of the track. Moreover, restrictions (10), (11), and (12) are used for determining the right values of the decision variables in the objective function (2). Finally, restrictions (13) and (14) are integrality restrictions on the z and x decision variables.

3.1 Improvements on the basic model

The bottleneck for this model is the large number of restrictions (6), namely one for each potential crossing at each shunt track. A first step to reduce the number of crossing restrictions is to aggregate them, which also strengthens the formulation. In the second step, we improve these aggregated restrictions to *cliques*.

For a convenient discussion of the aggregation of crossing restrictions, we introduce the set Z as the set of pairs $(v,u) \in T^2$ such that there exist $(t,u), (v,w) \in L$ with $t <_A v <_A u <_A w$. Each pair in Z is an arrival, followed by a departure, which might be involved in a crossing. Given Z, we can replace restrictions (6) with:

$$\sum_{t <_A v:(t,u) \in L} x_{t,u,s} + \sum_{w >_A u:(v,w) \in L} x_{v,w,s} \leq 1 \quad \forall (v,u) \in Z, s \in S \tag{15}$$

These restrictions sharpen the restrictions (6) and are far less in number. Given this replacement, we define Model 2 by (2)-(5),(15), (7)-(14). In the remainder of this section, we try to reduce the number of restrictions even further.

In fact, the restrictions (6) and (15) are special cases of the well-known *clique* inequalities. Before discussing these inequalities, we define the graph $G = (L, E)$. In this graph, L is defined in (1) and $E := \{\{(t, u), (v, w)\} \mid ((t, u), (v, w)) \in L^2, t \leq_A v <_A u \leq_A w\}$, which is a slight extension of X. In this paper, a clique is a subset K of L, such that the elements in K are pairwise adjacent in G. Now, we can improve the crossing restrictions (15) to the following ones:

$$\sum_{(t,u)\in K} x_{t,u,s} \leq 1 \quad \forall \text{ cliques } K, s \in S \tag{16}$$

These restrictions (16) are at least as strong as restrictions (15). Indeed, for each $(v, u) \in Z$, the set

$$\{(t, u) \in L \mid t <_A v\} \cup \{(v, w) \in L \mid w >_A u\}$$

is a clique, and therefore appears in (16). However, real-life applications require a huge number of maximal cliques, which invalidates explicit enumeration of these restrictions in a solution approach. In addition, we can also find an implicit manner to describe all clique inequalities. Therefore, we define $T' := \{i \in T_+ \mid i + 1 \in T_-\}$ as the set of pairs of an arriving train unit directly followed by a departing train unit. Given $i \in T'$, we define $L_i := \{(t, u) \in L \mid t \leq_A i <_A i + 1 \leq_A u\}$, which is the set of possible matchings arriving no later than i, and not departing before $i + 1$. In the following discussion we will need the well-known notion of *comparability graphs*. A graph H is a comparability graph if a partial order $\preceq$ on the vertices of H exists, such that for each pair of vertices $(u, v) \in H$:

$$u \text{ and } v \text{ are adjacent in } H \Leftrightarrow u \preceq v \text{ or } v \preceq u. \tag{17}$$

Now, we can state the following two lemmas:

Lemma 1. *For each clique K there exists an $i \in T'$ such that $K \subset L_i$.*

Proof. Determine the largest v for which $(v, w) \in K$ for some w and the smallest u for which $(t, u) \in K$ for some t, according to the ordering $<_A$. Because $t \leq_A v <_A u \leq_A w$, we know that $v <_A u$. Moreover, $v \in T_+$ and $u \in T_-$. Therefore, an $i \in T'$ with $v \leq_A i <_A i + 1 \leq_A u$ exists. $\blacksquare$

Lemma 2. *The subgraph G_i of G induced by L_i is a comparability graph for each $i \in T'$.*

Proof. Define the following partial order on L_i:

$$(t, u) \preceq_A (v, w) \Leftrightarrow t \leq_A v \text{ and } u \leq_A w,$$

where the order $\leq_A$ represents the order defined at the start of Section 3. This order satisfies requirement (17). $\blacksquare$

The relation $(t, u) \prec_A (v, w)$ holds if and only if $(t, u) \preceq_A (v, w)$ and $(t \neq v$ or $u \neq w)$. If $(t, u) \prec_A (v, w)$, then a crossing occurs if both (t, u) and (v, w) are assigned to the same LIFO track, which is clarified in Figure 5. Since we consider shunt tracks only open at the A-side in this section, the unit v is closest to the A-side after arrivals t and v. However, unit u, which is matched to unit t, needs to depart before unit w, which is matched to unit v. Therefore, the arriving unit v is obstructing the departure of unit u, resulting in a crossing.
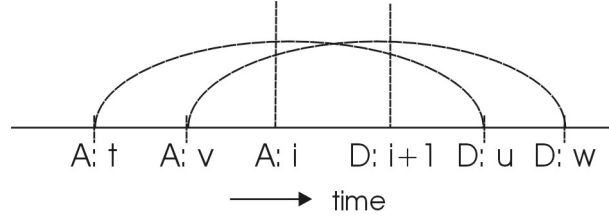

Figure 5: An example of a crossing at a specific LIFO track.

In order to implicitly describe the clique inequalities in the subgraph G_i in an efficient way, we introduce the variables $y_{v,w,s,i}$ for each $s \in S, i \in T'$, and

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$(v, w) \in L_i$:

$$y_{v,w,s,i} = \begin{cases} 1 & \text{if a conflicting matching } (t, u) \in L_i, (t, u) \prec_A (v, w), \\ & \text{is parked at track } s; \\ 0 & \text{otherwise.} \end{cases}$$

For each $s \in S$, $i \in T'$, and $(t, u), (v, w) \in L_i$, the following inequalities ensure that the $y_{v,w,s,i}$ variables are set to the appropriate values:

$$y_{v,w,s,i} \geq y_{t,u,s,i} + x_{t,u,s} \quad \text{if } (t, u) \prec_A (v, w) \tag{18}$$

$$y_{v,w,s,i} + x_{v,w,s} \leq 1 \tag{19}$$

Note that it suffices to restrict restrictions (18) to (v, w) directly subsequent to (t, u) according to $\prec_A$. Moreover, restrictions (19) are required only for pairs (t, u) without successor. Note that for each $i \in T'$ multiple cliques might exist and therefore, according to $\prec_A$, one (t, u) can have multiple pairs (v, w) as direct successors.

Lemma 3. *Restrictions (16) are equivalent to the combination of restrictions (18) and (19).*

Proof. For both parts and without loss of generality, we start with a maximal clique K. According to Lemma 1, an $i \in T'$ exists such that $K \subset L_i$. In addition, we use the order $\prec_A$ on the elements of K:

$$(t_1, u_1) \prec_A (t_2, u_2) \prec_A \cdots \prec_A (t_k, u_k), \tag{20}$$

such that (t_{j+1}, u_{j+1}) is a successor of (t_j, u_j) and (t_k, u_k) has no successor. Now (18), (19) $\Rightarrow$ (16) follows directly from:

$$\sum_{(t,u) \in K} x_{t,u,s} = \sum_{j=1}^{k} x_{t_j,u_j,s} \leq \sum_{j=1}^{k-1} (y_{t_{j+1},u_{j+1},s,i} - y_{t_j,u_j,s,i}) + x_{t_k,u_k,s}$$
$$= y_{t_k,u_k,s,i} - y_{t_1,u_1,s,i} + x_{t_k,u_k,s} \leq y_{t_k,u_k,s,i} + x_{t_k,u_k,s} \leq 1.$$

Secondly, (16) $\Rightarrow$ (18), (19). For t, u, s, i let the maximal clique be attained by clique K. Let $K' := K \cup \{t, u\}$. Then

$$y_{v,w,s,i} \geq \sum_{\substack{(a,b) \in K' \\ (a,b) \prec_A (v,w)}} x_{a,b,s} = \sum_{\substack{(a,b) \in K \\ (a,b) \prec_A (v,w)}} x_{a,b,s} + x_{t,u,s} = y_{t,u,s,i} + x_{t,u,s}.$$

This completes the proof. ∎

This model requires far less constraints but far more variables than Model 2. Although it helps that the y variables are linear, this model requires too much computation time and is not considered in the remainder of this paper.

4   Restricting the Number of Heterogeneous Tracks

In our computational experiments it turned out that the models described so far often produce solutions in which a number of shunt tracks are occupied by train units of a single subtype only. In this section we describe how this structure of the solutions can be exploited to further reduce the numbers of decision variables and constraints of the models.

Suppose one would know *beforehand* that a certain track s is to be occupied by train units of a single type only. In this case, the matching variables $X_{t,u,s}$ as well as the associated aggregated crossing restrictions (15), or their equivalents (18)-(19), are superfluous for track s. Indeed, since train units of the same subtype can be used interchangeably, the detailed matching and the order of the train units at such a track is irrelevant.

An additional advantage of a track with only 1 type of train units parked at it is that it adds to the robustness of a solution. Indeed, the solution will be better able to handle changes in the operations. The latter occur frequently in practice.

However, we do not want to choose beforehand which tracks are so-called *heterogeneous tracks*, containing several subtypes, and which ones are not. This can be achieved by introducing *virtual tracks* and by assigning these virtual tracks to physical tracks. We let S represent the set of virtual tracks. Moreover, we introduce the set P of physical tracks, with $|S| = |P|$. A matching from S to P assigns the virtual tracks to the physical ones. Let S' be the set of heterogeneous virtual tracks and let $S'' = S \setminus S'$ be the set of homogeneous virtual tracks. Then it suffices to include the crossing restrictions (15) only for

the heterogeneous tracks in S' instead of for all virtual tracks in S:

$$\sum_{t <_A v : (t,u) \in L} x_{t,u,s} + \sum_{w >_A u : (v,w) \in L} x_{v,w,s} \leq 1 \quad \forall (v,u) \in Z, s \in S' \qquad (21)$$

Similar changes for restrictions (18) and (19) are required. Moreover, the $x_{t,u,s}$ decision variables are superfluous for homogeneous tracks. Of course, only units of one subtype can be parked at a track in S'', which results in this additional set of restrictions:

$$n_s = 0 \quad \forall s \in S'' \qquad (22)$$

The matching of virtual tracks to physical tracks can be described by the following decision variables:

$$r_{s,p} = \begin{cases} 1 & \text{if virtual track } s \in S \text{ is assigned to physical track } p \in P; \\ 0 & \text{otherwise.} \end{cases}$$

We replace the parameters c_s describing the length of a track $s \in S$ with c_p representing the length of the physical track $p \in P$. The following restrictions make the assignment of virtual tracks to physical tracks a matching:

$$\sum_{s \in S} r_{s,p} = 1 \qquad \forall p \in P \qquad (23)$$

$$\sum_{p \in P} r_{s,p} = 1 \qquad \forall s \in S \qquad (24)$$

$$r_{s,p} \in \{0,1\} \quad \forall s \in S, p \in P \qquad (25)$$

Given this matching, we rewrite the restrictions (9) on the track capacity as:

$$b_{t,s} \leq \sum_{p \in P} c_p r_{s,p} \quad \forall t \in T_+, p \in P \qquad (26)$$

This results in Model 3 consisting of (2)-(5),(18)-(19),(7),(8),(21)-(26),(10)-(14). Note that (2)-(5),(18)-(19),(7),(8),(10)-(14) remain unchanged, and are defined in terms of virtual tracks now.

Further reductions in the resulting models are possible by choosing a specific subtype of train unit to be assigned to a homogeneous shunt track. For subtypes of train units, which occur frequently at a station it is logical to assume that at

least one track exists with only this subtype of train units parked at it. This holds especially for subtypes without other subtypes in the same type and for long subtypes. For a homogeneous virtual track $s \in \mathcal{S}$ with a pre-assigned subtype τ of train unit, restrictions (12) can be omitted while restrictions (11) simplify to $z_{t,s} = 0$ if $\tau_t \neq \tau$. The latter implies that restrictions (4) and (5) are only relevant for virtual track s if $\tau_t = \tau$. Note that if no heterogeneous virtual tracks are allowed, we know that each type of train unit has at least one track which only consists of units of this type.

5   Models Extended to Free Tracks

Until now, we have assumed that the shunt tracks can only be approached from the A-side of the track. In this section, we extend our model in such a way that we can also consider shunt tracks which can be approached from the B-side, and even from both sides of the track.
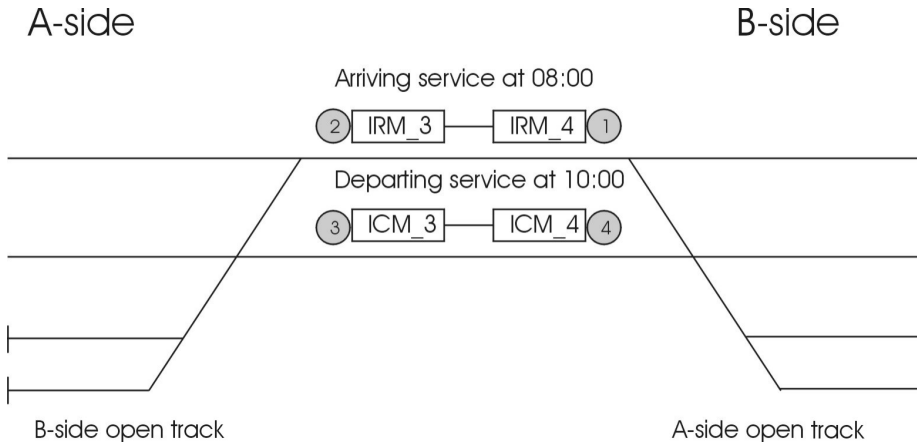

Figure 6: Tracks open at different sides require a different ordering of train units within a train.

Before describing these extensions of our model, we need to take a closer look at the ordering $<_A$ of the train units. In Figure 6, both the IRM_3 and IRM_4 units as well as the ICM_3 and ICM_4 units arrive respectively depart in one train. Via the A-side, the partial ordering $<_A$ results in the order ①,②,③,④.

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However, for a track open at the B-side, the units arrive or depart in the order: ②,①,④,③. Therefore, the order $<_A$ is wrong for compositions with multiple train units parked via the B-side of a shunt track and needs to be reversed. This results in the partial ordering $<_B$ of the train units. The relation $X_i <_B X_j$ holds if and only if one of the following conditions is satisfied:

1. Unit X_i arrives or departs in a train with an earlier planned time than the service to which unit X_j belongs.

2. Arriving units X_i and X_j arrive in the same train and unit X_j is closer to the B-side of the train than arriving unit X_j.

3. Departing units X_j and X_i depart in the same train and unit X_i is closer to the B-side of the train than departing unit X_i.

Note that both orders $<_A$ and $<_B$ are the same for train units in different train compositions and only differ for train units in the same train composition.

We partition the set P of physical shunt tracks into the set P_A with tracks which can be approached from the A-side only, P_B with tracks which can be approached from the B-side only, and $P \setminus \{P_A \cup P_B\}$ with the tracks that can be approached from both sides.

As a straightforward extension of Models 2 and 3, one could add two indices to the decision variables $x_{t,u,s}$, resulting in $x_{t,u,s,d,e}$ variables, where d describes the arrival side at track s for unit t, and e describes the departure side at track s for unit u. Subsequently, crossing constraints, similar to restrictions (6), can be determined. Moreover, some restrictions on the sides of the tracks which can be used are required. However, since the number of decision variables roughly increases by a factor 4, this approach will not result in a model that is able to solve real-life instances. Therefore, we discuss a different approach in the remainder of this section.

We start with the introduction of one decision variable for each train unit, which indicates the side of a track via which the unit arrives or departs:

$$k_t = \begin{cases} 0 & \text{if train unit } t \text{ arrives or departs via the A-side of a track;} \\ 1 & \text{if train unit } t \text{ arrives or departs via the B-side of a track.} \end{cases}$$

Note that, compared to the straightforward extension, this results in a huge reduction in the number of decision variables, since the decision variables k_t make the additional indices d and e in the $x_{t,u,s,d,e}$ decision variables superfluous. One variable k_t is used for each train unit to denote the side of the shunt track where the unit is parked, instead of explicitly taking into account the sides of all shunt tracks.

In order to extend Model 2 to include free tracks, we need restrictions like $z_{t,s} - k_t \leq 0$ if track s is not accessible from the A-side, and $z_{t,s} + k_t \leq 1$ if track s is not accessible from the B-side. However, the model with virtual tracks, Model 3, requires the following restrictions:

$$z_{t,s} - k_t + \sum_{p \in P_B} r_{s,p} \leq 1 \quad \forall s \in S', t \in T \tag{27}$$

$$z_{t,s} + k_t + \sum_{p \in P_A} r_{s,p} \leq 2 \quad \forall s \in S', t \in T \tag{28}$$

Indeed, suppose that train unit t is parked at track s, resulting in $z_{t,s} = 1$ and virtual track s is assigned to a physical track $p \in P_B$, which can only be approached from the B-side. Then restrictions (27) imply $k_t = 1$, ensuring that unit t approaches track s from the B-side. Restrictions (28) can be explained similarly.
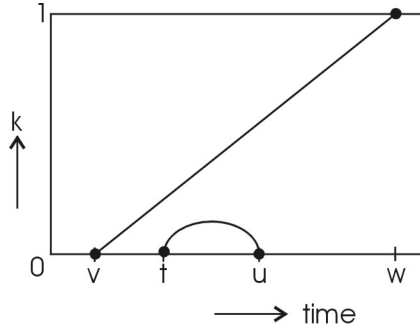

Figure 7: A case where (t, u) and (v, w) can be assigned to the same track.

Note that an arriving train unit t that is matched to a departing train unit u and is parked at shunt track s can be visualized by a continuous curve inside the area $\mathbb{R} \times [0, 1]$ from the point (t, k_t) to the point (u, k_u). Here the area $\mathbb{R} \times [0, 1]$ is

a so-called time-space diagram, where $\mathbb{R}$ represents time, and the interval $[0, 1]$ represents the shunt track. It is not difficult to see that the movements of several train units at a certain shunt track are feasible if and only if the corresponding curves do not cross each other. This representation holds both for LIFO tracks and for free tracks, but it is particularly useful for free tracks. Figure 5 shows an infeasible matching (crossing curves) at a LIFO track. Figure 7 shows a feasible matching at a free track. Note that this representation also holds for trains consisting of more than 1 train unit, provided that for train units in the same train that are arriving or departing from the A-side the ordering $<_A$ is used in the figure, and for train units in the same train that are arriving or departing from the B-side the ordering $<_B$ is used.

5.1 Trains with one unit

For ease of discussion, we start with the crossing restrictions in the special case that each train consists of exactly one train unit. Note, that in this case both orders $<_A$ and $<_B$ are the same. The resulting restrictions are generalized later on to trains with multiple units. For each $s \in S$, and $((t, u), (v, w)) \in L^2$, the conditions on k_t are:

$$\text{if } x_{t,u,s} = 1 \text{ and } x_{v,w,s} = 1 \text{ and } t <_A v <_A u <_A w, \text{ then } k_v \neq k_u \qquad (29)$$

$$\text{if } x_{t,u,s} = 1 \text{ and } x_{v,w,s} = 1 \text{ and } v <_A t <_A u <_A w, \text{ then } k_t = k_u \qquad (30)$$

Theorem 1. *If each train consists of exactly one unit, restrictions (29) and (30) are necessary and sufficient for describing the crossing constraints for $s \in S$, and $((t, u), (v, w)) \in L^2$.*

Proof. As was indicated above, the theorem is equivalent to showing that for each (t, u) and (v, w) in L with $\{t, u\} \cap \{v, w\} = \emptyset$ and $x_{t,u,s} = x_{v,w,s} = 1$, disjunct curves in $\mathbb{R} \times [0, 1]$ exist from (t, k_t) to (u, k_u) and from (v, k_v) to (w, k_w) if and only if restrictions (29) and (30) hold. Without loss of generality, we may assume $u <_A w$, and therefore $t <_A u <_A w$. Now, there are three possibilities for the position of v compared to $t, u,$ and w. If $u <_A v <_A w$ then such curves can be drawn, because the train units do not have a common

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time interval at track s. Moreover, if $t <_A v <_A u$, such curves exist only if $k_v \neq k_u$, conform Figure 5. In addition, if $v <_A t$, these curves exist only if $k_v = k_u$. Exactly these latter two restrictions on the k variables are described by restrictions (29) and (30). Note that if $v >_A w$ then v arrives later than w, which contradicts $(v, w) \in L$.

∎

We can rewrite the restrictions (29) and (30) in linear form for each $s \in S$, and $((t, u), (v, w)) \in L^2$, resulting in:

$$x_{t,u,s} + x_{v,w,s} \leq 3 - k_u - k_v \quad \text{if } t <_A v <_A u <_A w \tag{31}$$

$$x_{t,u,s} + x_{v,w,s} \leq 1 + k_u + k_v \quad \text{if } t <_A v <_A u <_A w \tag{32}$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 - k_t + k_u \quad \text{if } v <_A t <_A u <_A w \tag{33}$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 + k_t - k_u \quad \text{if } v <_A t <_A u <_A w \tag{34}$$

For example, restrictions (34) prohibit the situation depicted in Figure 8, where there is no possibility to park (v, w) at the same track as (t, u) without introducing a crossing. Note that this is independent of the used sides for the units v and w.
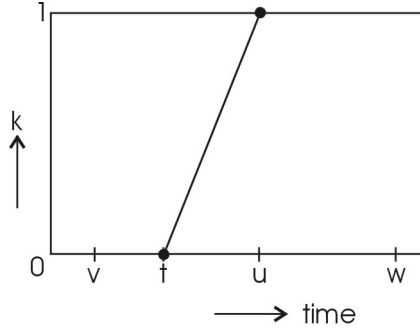

Figure 8: A graphical representation of restrictions (34).

5.2 Trains with multiple units

When trains might consist of several train units, the set of restrictions (31)-(34) generalizes to the following set, again for each $s \in S$, and $((t,u),(v,w)) \in L^2$:

$$x_{t,u,s} + x_{v,w,s} \leq 1 + k_u + k_v \qquad \text{if } t <_A v < u <_A w \qquad (35)$$

$$x_{t,u,s} + x_{v,w,s} \leq 3 - k_u - k_v \qquad \text{if } t <_B v < u <_B w \qquad (36)$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 - k_t + k_u \qquad \text{if } v <_B t, u <_A w \qquad (37)$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 + k_t - k_u \qquad \text{if } v <_A t, u <_B w \qquad (38)$$

$$x_{t,u,s} + x_{v,w,s} \leq 3 + k_t - k_u - k_v \qquad \text{if } t <_A v <_B t, u <_B w \qquad (39)$$

$$x_{t,u,s} + x_{v,w,s} \leq 3 + k_w - k_u - k_v \qquad \text{if } t <_B v, u <_A w <_B u \qquad (40)$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 - k_t + k_u + k_v \qquad \text{if } v <_A t <_B v, u <_A w \qquad (41)$$

$$x_{t,u,s} + x_{v,w,s} \leq 2 - k_w + k_u + k_v \qquad \text{if } t <_A v, w <_A u <_B w \qquad (42)$$

$$x_{t,u,s} + x_{v,w,s} \leq 3 + k_t - k_u - k_v + k_w \quad \text{if } t <_A v <_B t, u <_A w <_B u \quad (43)$$

We define Model 4 as the model consisting of (2)-(5),(7),(8),(10)-(14),(22)-(28), (35)-(43). Note that in restrictions (35)-(43) a relation like $u <_A w <_B u$ implies that the train units u and w belong to the same train. These restrictions are explained by similar arguments as the following one for restrictions (36).

Restrictions (36) are only restrictive if $k_u = 1$ and $k_v = 1$ since otherwise the righthand side is at least two, and the restriction is trivially fulfilled because $x_{t,u,s}$ and $x_{v,w,s}$ are binary. The situation at track s after both arrivals t and v is given in Figure 9, which is independent of the value of k_t. In this figure, the arrow from (t,u) represents the fact that this unit needs to depart from the B-side of shunt track s. This results in a crossing when unit u has to leave via the B-side before the departure of unit w, which explains this set of restrictions.

This formulation assumes that train units arriving or departing in the same train service may end up on the same shunt track in a different order than in the train service. This occurs when, e.g. one train unit is routed via the A-side and the other via the B-side of the same shunt track. In practice, this hardly ever happens because of the additional complexity in the local operations as well as the additional resource consumption. However, theoretical instances exist
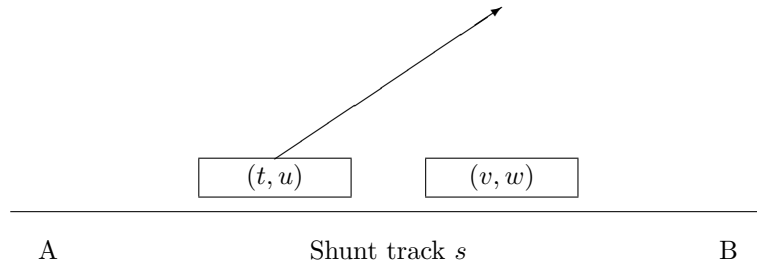
Figure 9: The situation at track s after both arrivals.

where only such impractical solutions are feasible.

6 Computational Results

We applied the presented models to the stations Zwolle (Zl) and Enschede (Es), for periods from 8:00 AM one morning until 8:00 AM in the next morning. Station Zwolle is considered a large station with 15 free tracks and 4 LIFO tracks. The number of train arrivals and departures ranges from 550 to 600 with 800 - 1,100 train units. These trains result in 50 - 125 train units that need to be parked, belonging to 12 subtypes. Parking the other units is not necessary since they are at the station for short periods. Station Enschede is a smaller station with 11 free tracks and 2 LIFO tracks. Here, between 140 and 160 arrivals and departures take place with 210 - 300 train units. These arrivals and departures result in approximately 30 - 45 train units that need to be parked, belonging to 3 subtypes.

The computations are performed on a PC with an Intel Pentium IV 2.8 Ghz computer with 512 Mb of RAM, and operating under Windows XP Professional. The models are solved using CPLEX 7.1, with some specific settings. We impose a branching order starting with d_t variables, followed by $m_{\tau,s}$ and $z_{t,s}$ variables. We start with branching up. Moreover, we perturb the objective function and allowed an aggressive probing strategy. In the objective, we penalize each train consisting of several units that is parked at different tracks with a penalty of 3 per split of the train. Each shunt track that is used for parking more than 1

train unit type is penalized with the number of train unit types exceeding 1.

We start the discussion of the computational results with the number of restrictions required for prohibiting crossings without homogenous tracks presented in Table 1. The relevant models are Models 1, 2, and 4. Note that the number of crossing restrictions for one track in Model 3 is equal to the similar number of Model 2. Moreover, the factors between the number of restrictions per track and in total represent the number of hetereoneous shunt tracks. The table shows that aggregation of the crossing restrictions is extremely successful, reducing the number of restrictions with a factor 20. By looking at the instance at Station Zwolle, it is also clear that Model 1 grows too large to solve real-life instances. Finally, we see that enabling the use of both sides of a free shunt track results in a significantly larger number of crossing restrictions.

Model	Zl: 19 tracks		Es: 13 tracks	
	Per track	In total	Per track	In total
1	21.150	401.850	2.294	29.822
2	1.023	19.437	129	1.677
4	18.833	357.827	2.452	31.876

Table 1: The number of crossing restrictions in TUSP.

In Table 2, we report computational results for different models at stations Zwolle and Enschede. For Models 3 and 4, we initially assumed 0 tracks with multiple types of train units parked at it. As mentioned before, this means that each train unit type has at least one virtual track, where only units of this type are parked. The table contains the numbers of arriving and departing trains with units at different tracks, as well as the sum of the n_s variables in the column 'Type'. Moreover, the computation time is reported in seconds. In case the optimal solution was not found in 3 hours, the indication (*) is added here. Note that for all instances a feasible solution was obtained. For the Models 3 and 4, we assumed 0 tracks with multiple types of train units parked at it. As mentioned before, this means that each train unit type has at least

one virtual track, where only units of this type are parked.

Model	Station Zwolle				Station Enschede			
	A	D	Type	Time	A	D	Type	Time
1	N/A	N/A	N/A	N/A	0	0	2	434.02
2	2	3	14	10800 (*)	0	0	2	286.34
3	3	1	0	23.81	4	4	0	1.20
4	3	1	0	9.14	4	4	0	0.39

Table 2: Computational results for different models.

From this table, we can conclude that the restriction of allowing only one type of train unit per shunt track is very strong. At station Zwolle, the first model cannot be solved within three hours of computation time. In this case, CPLEX requires all three hours for preprocessing the problem, and therefore is not even able to find a feasible solution in three hours. This justifies the extensions described in this paper. The results presented in this table represent two ends of a spectrum of possible models, where only Model 4 considers free tracks. On one hand, there are instances with much flexibility, but requiring large computation times. On the other hand, there are restrictive instances with low computation times. In order to support shunt planners in a practical way, a combination of these two extremes is required. Therefore, we continue with reporting computational results for Models 3 and 4 with one and two shunt tracks with multiple types of train units, and 0 to 3 shunt tracks with pre-assigned subtypes of train units in Tables 3 and 4. In these tables, the first two columns describe the model and the number of homogeneous virtual tracks that were demanded. Columns 3 to 10 are the same as columns 2 to 9 of Table 2.

By analyzing Table 3, we see that the instances at station Zwolle of Model 4 with only one heterogeneous shunt track can be solved, but are not solved to optimality within three hours of computation time. This is caused by the fact that the LP-relaxation is weaker for these instances than for the instances with LIFO tracks. At station Enschede, we also see that free track instances are more diffi-

Model	# pre-	Station Zwolle				Station Enschede			
	assigned	A	D	Type	Time	A	D	Type	Time
3	0	2	0	1	10800(*)	1	1	1	2.34
	1	2	0	1	101.48	1	1	1	1.39
	2	2	0	1	102.75	1	1	1	2.44
	3	2	0	1	91.33	1	1	1	1.30
4	0	2	0	1	10800(*)	1	1	1	12.19
	1	1	1	1	10800(*)	1	1	1	17.72
	2	2	0	1	10800(*)	2	0	1	13.19
	3	2	0	1	10800(*)	2	0	1	9.00

Table 3: Computational results with one track with multiple types of train units.

cult to solve. However, since these instances are significantly smaller, they can be solved to optimality within 20 seconds of computation time. Moreover, using one virtual shunt track with multiple types of train units is insufficient to prevent trains with units at different shunt tracks. Since such trains are considered more important than tracks with multiple types of train units, we continue the computational experiments by increasing the number of heterogeneous virtual tracks to 2 in Table 4.

For the instances at station Zwolle, we see that the instances with free tracks can be solved to optimality within somewhat more than 15 minutes. However, the instances with LIFO tracks cannot be solved to optimality within 3 hours of computation time. These results are opposite to the results reported in Table 3. Similar to the latter table, the negative results are caused by a weak LP-relaxation for the instances with LIFO tracks. We conclude that for the Zwolle instances the additional effort of modeling tracks as free tracks pays off. Observe that the instances with free tracks are able to avoid trains with units at different tracks in this case. This typically comes at the cost of one additional shunt track with multiple subtypes of train units. At station Enschede, the instances with LIFO tracks already provide sufficient flexibility for avoiding such

trains. Therefore, computational results for the instances with LIFO tracks are better at station Enschede since a solution with the same quality is found faster.

Model	# pre-assigned	Station Zwolle				Station Enschede			
		A	D	Type	Time	A	D	Type	Time
3	0	0	1	2	10800(*)	0	0	2	2.19
	1	0	1	2	10800(*)	0	0	2	1.59
	2	2	0	1	10800(*)	0	0	2	1.31
	3	0	1	2	10800(*)	0	0	2	0.84
4	0	0	0	2	1033.00	0	0	2	10.16
	1	0	0	2	598.03	0	0	2	19.18
	2	0	0	2	938.36	0	0	2	26.66
	3	0	0	2	213.14	0	0	2	49.66

Table 4: Computational results with two tracks with multiple types of train units.

7 Conclusions and Further Research

In this paper, we describe a new model for the Train Unit Shunting Problem, which was introduced in Freling et al. [2005]. This new model is capable of solving the matching and parking subproblems in an integrated manner, usually requiring a reasonable amount of computation time.

Compared to similar problems described in Winter [1999], Winter and Zimmermann [2000], Gallo and Di Miele [2001], Hamdouni et al. [2006], and Hamdouni et al. [2005], the model incorporates complicating details from practice. These details include trains composed of several train units and tracks that can be approached from two sides. Computational results are presented for real-life cases of NS Reizigers, the main Dutch passenger railway operator.

An initial model is refined further and further to incorporate these practical details and to improve the structure of the model, such that a good balance between the quality of the model and the computation time is found.

Computational results for stations Zwolle and Enschede indicate that the model is able to produce high quality solutions usually within reasonable amounts of computation time. Shunt planners have a large influence on solution times and quality by selecting the pre-assigned subtypes of train units. More of such pre-assigned subtypes restrict the solution space considerably, but might prohibit good solutions.

In future research, we intend to further research the structure of the problem and the models in order to reduce computation times. For example, shunt planners typically deploy rolling stock line by line, which reduces the matching options for a train unit. Moreover, future research regarding a combination of these models with a fixed matching are interesting since these will decrease computation times.

Regarding the scope of the presented models, we see possibilities to include estimates of routing costs. Finally, although timetabled train services have fixed arrival and departure times, shunt units have flexible arrival times at and departure times from the shunt tracks. For example, the departure time of an arriving shunt unit from a platform to a shunt track is flexible within a time interval starting at the arrival time of the unit at the platform and ending some time before the next arrival of another train service at the same platform. This flexibility presents an opportunity to further increase the quality of solutions.

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