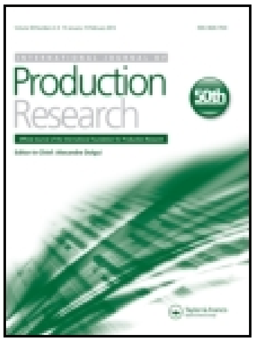

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Impact of required storage space on storage policy performance in a unit-load warehouse

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The performance of a storage policy in a warehouse is usually evaluated on the basis of the average one-way travel distance/time needed to store/retrieve a load. Dividing the storage space into zones based on item turnover frequency can reduce the travel distance. However, for a given number of stored items, a larger number of storage zones also requires more storage space, because of reduced space sharing between the items, which increases travel time. This study considers the required space consumption by storage zoning in comparing the performance of random, full turnover-based and class-based storage policies for a unit-load warehouse operated by a forklift in single-command mode. A generalised travel distance model that considers the required space consumption is developed to compare the performance of these policies. Results show that the one-way travel distance of a random policy decreases with the increase in skewness of the demand curve. By considering the required space consumption, a class-based storage policy performs generally better than a full turnover-based policy. In addition, the optimal warehouse shape factor (ratio of warehouse width to depth) appears to decrease with the skewness of the demand curve. Warehouse managers are advised to adopt a wide-shallow warehouse layout when the item demands are approximately equal, whereas a narrow-deep layout is preferred when the demand curves are steep.

Keywords: warehousing systems; logistics; class-based storage; required storage space; unit-load warehouse

1. Introduction

Warehouses play an important role in order fulfillment. They are the main point of contact with customers, responsible for meeting customer expectations set in the sales process. According to Handfield et al. (2013), warehouses are responsible for about 15% of the total logistics cost in developed countries such as Germany (other components such as packaging, inventory, value addition, managing flows and returns handling are activities that also typically take place in the warehouse). Mayer et al. (2009) estimate the warehousing and inventory carrying costs to be more than 40% of total logistics costs in Europe. Companies therefore focus on making warehouse operations efficient. Among these operations, retrieving items from their storage locations typically accounts for a high proportion of operational costs (Frazelle and Frazelle 2002; Tompkins et al. 2010), taking up to half of the total warehouse operating expense (De Koster et al. 2007).

Class-based storage, which divides the stored items of a warehouse into different classes according to the ABC demand curve (the ranked turnover of the items), is commonly applied in practice and is widely studied in the literature (Rosenblatt and Eynan 1989; Eynan and Rosenblatt 1994; Kouvelis and Papanicolaou 1995; Gu, Goetschalckx, and McGinnis 2007). Two special cases are random storage, with only one storage class exists, and full turnover-based storage, where the number of classes equals the number of items. That is, under a random storage policy, all items share a common class, whereas with full turnover-based storage, each item has its own, dedicated, class.

The required storage space (RSS) of items is important in warehouse management, particularly for warehouse design, storage policy selection and one-way travel distance evaluation. Existing research on class-based storage assumes that the RSS of an item equals its average inventory level (expressed in pallet quantities). Some well-known findings are based on this assumption. For example, the average one-way travel time for the storage and retrieval (S/R) machine to retrieve a unit-load in an automated storage and retrieval (AS/R) system with a normalised (to 1) square-in-time (SIT) rack, under a random storage policy equals 2/3 (Hausman, Schwarz, and Graves 1976). This value is not affected by the demand frequency curve of the stored items (i.e. the skewness of the demand curve; Hausman, Schwarz, and Graves 1976). Another well-known result is that the full turnover-based storage policy minimises the expected

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one-way travel time in a warehouse. This policy is widely used as a benchmark to evaluate the performance of class-based policies (Rosenblatt and Eynan 1989; Teunter et al. 2010).

However, as demonstrated in this study, the real RSS of an item is higher than its average inventory level, and it decreases with the number of items sharing the storage space in the same storage class (Bartholdi and Hackman 2011; Yu, De Koster, and Guo 2015). For example, when full turnover-based storage is applied, each item has a dedicated storage region, and its RSS equals its order quantity. However, if the number of items sharing a common storage zone is very large, items within the zone can share space much better and the RSS for each item is close to its average inventory level, which is only half of its order quantity (with deterministic demand). Thus, the possibility of sharing space among different items in the same storage class affects the RSS. As a result, by dividing the items into different classes according to their turnover, a class-based storage policy reduces the average one-way travel distance, and simultaneously increases the travel distance as the RSS grows. Consequently, a tradeoff exists between the space-sharing effect and the effect of turnover ranking according to item demands, when class-based storage is adopted. However, by making an (often implicit) assumption that '*the required storage space of an item equals its average inventory level*', conventional research only considers the retrieval-time reduction effect of ranking the items based on their turnover, but neglects the effect of space sharing among different items in the same class. The RSS measured in number of locations of an item is influenced by several factors, such as the number of items sharing the item class, its annual demand and the replenishment policy (Yu, De Koster, and Guo 2015). Different ABC demand curves therefore yield a different cumulative RSS of the items in the warehouse. The present paper aims to determine the size of the RSS, which depends on item demand and item space-sharing, as well as its effect on storage policy performance and on warehouse layout in a traditional warehouse.

A generalised travel distance model (TDM) is developed, based on the RSS measured in the number of locations, to find the optimal travel times for the random, full turnover-based and class-based storage policies. The travel distance performances are compared for different shapes of the ABC demand curve.

Computational results show that with the increase in skewness of the demand curve, the average one-way travel distance does not remain constant, but decreases for all three storage policies because the RSS decreases for a given total demand. Even for the random storage policy, a significant gap exists in travel time (almost 40%) between the 20%/20% and 20%/90% demand curves. For class-based storage, with the optimal number of classes and class boundaries, results show that less than five classes can yield the global minimum average one-way travel distance for a unit-load warehouse. This result reflects that a class-based policy outperforms the full turnover-based policy which is considered as a perfect benchmark in the literature. We also find that, under a class-based policy with optimal item classification, the optimal warehouse shape factor (the ratio of warehouse width to depth) decreases with the increase in skewness of the demand curve. This finding suggests that warehouse managers should adopt a *wide-shallow* warehouse layout with a large number of short storage aisles when items have similar demand volumes (i.e. a flat- or non-skewed ABC-demand curve), and a *narrow-deep* layout with a small number of long storage aisles when the ABC-demand curve is skewed.

The remainder of this paper is organised as follows. After reviewing the related literature in the next section, we describe the studied unit-load warehouse and provide the related notations in Section 3. In Section 4, the RSS and TDM are developed for a unit-load warehouse with n storage classes. Optimal travel solutions are then derived for the studied storage policies. A solution methodology (SM) based on dynamic programming is provided in Section 5. Section 6 presents the numerical results. Finally, Section 7 concludes this paper by summarising the findings and providing future research directions.

2. Literature review

The effect of storage policies on retrieval time is widely studied in scientific research papers (Hausman, Schwarz, and Graves 1976; Graves, Hausman, and Schwarz 1977; Eynan and Rosenblatt 1994; Kouvelis and Papanicolaou 1995; Thonemann and Brandeau 1998; De Koster et al. 2007; Gu, Goetschalckx, and McGinnis 2007; Roodbergen and Vis 2009), and some well-known operations management textbooks (e.g. Adams et al. 1996; Tompkins et al. 2010). A common assumption in this stream of literature is that the RSS of an item equals its average inventory level. Hausman, Schwarz, and Graves (1976) seem to be the first to study the effect of random storage, full turnover-based and class-based storage on travel time in an AS/RS with a SIT storage rack. The optimal class boundaries for the two- and three-class-based storage policies are obtained through numerical studies, and the optimal retrieval travel time is calculated. Results show that under a random storage policy and for a normalised rack (i.e. the size equals 1), the average travel time is 2/3, for any ABC demand curve. Furthermore, full turnover-based policy performs best, i.e. it yields the minimum expected travel time. Rosenblatt and Eynan (1989) derive a recursive function for the travel time to determine the optimal-class boundaries for any number of classes. Thereafter, numerous researchers studied class-based storage for AS/RSs (e.g. Goetschalckx and Ratliff 1990; Van den Berg 1996; Ashayeri et al. 2002; Koh, Kim, and Kim 2002;

Sari, Saygin, and Ghouali 2005; Lerher 2006; Lerher et al. 2006; Park 2006; Yu and De Koster 2009; Lerher, Šraml, and Potrč 2011; Bessenouci, Sari, and Ghomri 2012; Cardin et al. 2012; Zaerpour, De Koster, and Yu 2013).

Storage policies have also been studied in traditional, aisle-based warehouses, which have yielded similar results for the comparison of random, full-turnover- and class-based storage policies (Jarvis and McDowell 1991; Petersen 1999; Petersen and Aase 2004; Petersen, Aase, and Heiser 2004; De Koster, Le-Duc, and Zaerpour 2012; Roodbergen 2012). Le-Duc and De Koster (2005) optimise the shape of the storage classes based on a closed-form travel time estimation for a 2-block discrete warehouse. Petersen, Aase, and Heiser (2004) compare order-picking performance between class-based storage and random storage. Their numerical results show that class-based storage allows savings in picker travel over random storage. Rao and Adil (2013) study the travel distance for two- to four-class-based low-level order picking systems, for a traditional warehouse with a 2-block layout. Petersen and Schmenner (1999) compare three variations of the class-based storage policy: diagonal storage, within-aisle storage and across-aisle storage, in an order-picking operation. Petersen (1999) discusses the impact on order-picking warehouse efficiency of the diagonal and within-aisle storage policies. Glock and Grosse (2012) study the storage policies and order picking strategies in a U-shaped order picking system, and propose different storage location assignment policies. Based on a systematic literature review, Grosse et al. (2015) propose a conceptual framework for integrating human factors into planning models of order picking. Such factors may have a considerable impact on performance. Roodbergen, Vis, and Taylor (2015) integrate the warehouse layout and control policies in a model and show through simulation how this can help to improve warehouse performance. Çelk and Süral (2014) investigate retrieval strategies under random, full-turnover-based storage policies in a fishbone-aisle-layout warehouse, storing unit loads. They find that a fishbone design can obtain a 20 to 30% improvement over parallel-aisle based warehouses. Previous literature only considers storage assignment or travel distance estimation, but Dekker et al. (2004) and Battini et al. (2015) propose a joint model for optimising product allocation and travel distance reduction.

Yu, De Koster, and Guo (2015) was the only paper which calculates retrieval travel times while considering the RSS for an AS/RS warehouse with a finite number of items. They explicitly consider the space-sharing effect. They normalise the RSS by the total average inventory level of the system. Thus, the effect of RSS on retrieval time for traditional unit load warehouses has not been investigated. In order to fill this research gap, the present paper examines the performance of various storage policies while considering a realistic RSS in a unit-load warehouse operated by a forklift in single-command mode with a given annual demand.

3. Problem description and related notations

This section first describes the system studied in this paper. The research problem is then introduced. We consider a unit-load warehouse operated by a unit-capacity forklift to store/retrieve items operating in single-command mode. Storage racks are arranged in a parallel-aisle layout, with one front aisle in which the forklift can switch between aisles, and with one depot located at the middle of the front aisle. All items leave and enter the warehouse via the depot. The details are shown in Figure 1.

All storage locations in the warehouse are assumed to have the same unit-load size and each unit-load location stores only one item (De Koster, Le-Duc, and Roodbergen 2007). Furthermore, because every retrieval command needs a constant travel distance to cross the front aisle, we neglect it in the model (i.e. the width of the front aisle is assumed to be 0).

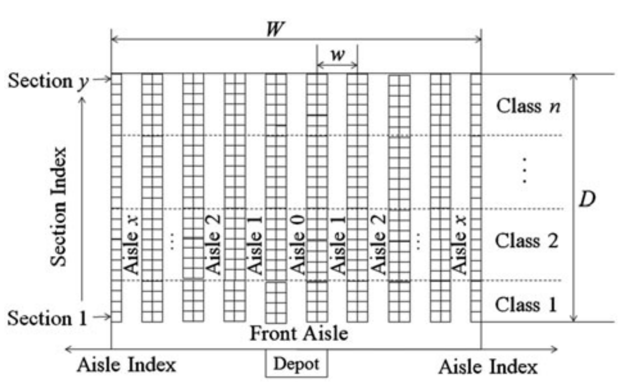

Figure 1. The warehouse system and class-based storage.

To facilitate the presentation of the problem at hand, the storage aisle opposite the depot is indexed as Aisle 0, and the aisle index number increases in both right and left directions. We therefore assume an odd number of aisles (as shown in Figure 1). The width of a storage aisle (or the centre to centre distance between two adjacent aisles) is denoted by w. As a result and as shown in Figure 1, the warehouse has x aisles on both sides of Aisle 0 and y sections of equal length along the aisles. Consequently, the travel distance for a command to visit location (a, b) takes aw in the horizontal direction to reach the target aisle, and b distance units in the depth direction to reach the target location.

Suppose that N items are stored in the warehouse and the turnover frequency (demand per unit-time) of each item is constant and known beforehand, as described by an ABC-demand curve. We assume that the classic economic-order quantity (EOQ) model is used as replenishment policy for the items (Hausman, Schwarz, and Graves 1976; Yu, De Koster, and Guo 2015).

The aim of this paper is to evaluate the performance of random, class-based and full turnover-based storage policies. Hence, the problem is to find the average one-way travel distance for a command under the three policies. A generalised TDM is developed to solve this problem. The model is based on an n-class based storage system, from which the solutions for the random and full turnover-based policies can be obtained, when $n = 1$ and $n = N$, respectively. In addition, an across-aisle storage policy (Petersen 2002) is adopted to describe the class boundaries for class-based and full turnover-based policies (as shown in Figure 1). The layout sketched in Figure 1 in combination with an across-aisle storage strategy is quite common in many industries. Examples include retail warehouses supplying stores (food and non-food), and wholesale warehouses (e.g. appliances, installation equipment). De Koster and Neuteboom (2001) and Matusiak et al. (2014) provide several examples.

The notations related to the problem include:

$i \in \{1, 2, \ldots, N\}$	Index of the ith item. The smaller the index is, the larger turnover the item has.
$b \in N^+$	Index of the bth section, numbered from the depot (front aisle).
i_k	Item index with the lowest turnover in class k (classes are ordered from fast to slow moving). It therefore also represents the total number of items in the first k classes, $k = \{1, 2, \ldots, n\}$.
b_k	Section index farthest from the depot in class k, the farthest boundary of class k.
t_k	Average one-way travel distance for storing/retrieving a unit load of class k.
$D(i)$	Annual demand for item i.
$\Lambda(k)$	Total turnover frequency of class k, in number of unit loads per unit-time period of all items stored in class k.
T_n	Average one-way travel distance of a unit load for an n-class-based warehouse.

With the notations given so far and in accordance with previous studies (e.g. Hausman, Schwarz, and Graves 1976; Rosenblatt and Eynan 1989) the average travel distance for an n-class-based storage system can be written as,

$$T_n = \sum_{k=1}^{n} t_k \left(\frac{\Lambda(k)}{\sum_{k=1}^{n} \Lambda(k)} \right), \quad k = 1, 2, \ldots, n, \tag{1}$$

where $\Lambda(k) \big/ \sum_{k=1}^{n} \Lambda(k)$ is the weighted turnover frequency of class k in the warehouse. Now the problem is to find the relationship between the average one-way travel time in each class and the corresponding total turnover, considering the RSS. The next section provides the detailed model.

4. Travel distance model considering RSS

This section gives the detailed TDM for an n-class based system in the unit-load warehouse. An ABC-demand curve is a plot of ranked cumulative percentage of expected demand per unit time. According to the well-known ABC-demand curve used in the literature (Hausman, Schwarz, and Graves 1976), the ABC curve for discrete items can be expressed as follows (also see Yu, De Koster, and Guo 2015),

$$G(i) = (i/N)^s = \sum_{j=1}^{i} D(j) \Big/ \sum_{j=1}^{N} D(j) \quad \text{for} \quad 0 < s \le 1 \quad i = 1, 2, \ldots, N, \tag{2}$$

where N is the total number of items in the warehouse, $D(i)$ is the annual demand for item i, and s is the shape factor of the ABC-demand curve. Let A be the annual demand of all items stored in the warehouse, i.e. $A = \sum_{j=1}^{N} D(j)$. The demand for item i can be expressed as,

$$D(i) = A((i/N)^s - ((i-1)/N)^s), \quad i = 1, 2, \ldots, N. \tag{3}$$

On the basis of Equation (3), the weighted turnover of class k in the total turnover of all items in the warehouse, $\Lambda(k)/\sum_{k=1}^{n}\Lambda(k)$ can be obtained as,

$$\frac{\Lambda(k)}{\sum_{k=1}^{n}\Lambda(k)} = \frac{\sum_{j=i_{k-1}+1}^{i_k} D(j)}{\sum_{j=1}^{N} D(j)} = (i_k/N)^s - (i_{k-1}/N)^s, \quad k = 1, 2, \ldots, n, \tag{4}$$

where $i_0 = 0$.

Furthermore, based on Equation (3) and the EOQ replenishment policy, the order quantity (in unit loads) of item i can be obtained as,

$$Q(i) = \sqrt{2KD(i)} = \sqrt{2KA((i/N)^s - ((i-1)/N)^s)}, \tag{5}$$

where K is the ratio of reorder cost to holding cost, which is assumed to be the same for all items as in previous studies (e.g. Hausman, Schwarz, and Graves 1976; Yu, De Koster, and Guo 2015).

Following Yu, De Koster, and Guo (2015), the required space for storing item i in class k, in which $N_k = i_k - i_{k-1}$ items share the same region, can be established as,

$$a_i(N_k) = 0.5\left(1 + N_k^{-\varepsilon}\right)Q(i), \quad 0 < \varepsilon \le 1. \tag{6}$$

ε is the space sharing factor, which may be influenced by the initial inventory levels of the items, the replenishment policy, the ABC-demand curve shape and the inventory cost. Fortunately, according to Yu, De Koster, and Guo (2015), the value of ε is quite insensitive to these parameters and appears to be between 0.17 and 0.25. As a result, we adopt $\varepsilon = 0.22$, for our numerical results in Section 6, which is the average value obtained through a large number of simulations by Yu, De Koster, and Guo (2015).

Therefore, under an ABC demand curve with shape factor s, the RSS for class k to store N_k items is,

$$R_k = \sum_{i=i_{k-1}+1}^{i_k} a_i(N_k) = \sqrt{0.5KA}\left(1 + N_k^{-\varepsilon}\right) \sum_{i=i_{k-1}+1}^{i_k} \sqrt{((i/N)^s - ((i-1)/N)^s)}. \tag{7}$$

Further, the cumulative RSS for the first k classes is the sum of the required space of each class equals,

$$L_k = \sum_{l=1}^{k} R_l = \sqrt{0.5KA} \sum_{l=1}^{k} \left(\left(1 + N_l^{-\varepsilon}\right) \sum_{i=i_{l-1}+1}^{i_l} \sqrt{((i/N)^s - ((i-1)/N)^s)} \right). \tag{8}$$

Since the warehouse applies an across-aisle policy for class-based storage, the number of sections needed by the first k classes, y_k (this is also the theoretical farthest boundary of class k in the continuous space model), can be obtained from Equation (8) and the number of aisles in the warehouse as follows,

$$y_k = L_k/(4x + 2). \tag{9}$$

where x is the maximum aisle index in the warehouse. According to the properties described in Section 3, the warehouse has a total of $2x + 1$ aisles, each with 2 storage racks.

Note also that in practice-class boundaries should be an integer, so we find the practical-class boundary for class k, b_k, as the minimum integer that is no less than y_k, i.e.

$$b_k = \lceil y_k \rceil = \lceil L_k/(4x + 2) \rceil. \tag{10}$$

Therefore, the storage locations in Section b_k may be allocated not only to class k, but also partly to class $k + 1$ ($k < n$), if y_k is not an integer. Here, we further assume that, within Section b_k, $(y_k + 1 - b_k)(4x + 2)$ storage locations are allocated to class k, and the other $(b_k - y_k)(4x + 2)$ are assigned to class $k + 1$.

Because $2x + 1$ storage aisles exist in the warehouse and the aisles farthest from the depot with index x are located on both the right and left sides of the depot, the average one-way travel distance from the depot to a location of Section b is,

$$d_b = b + \frac{x(x+1)w}{2x+1}. \tag{11}$$

The average one-way travel distance for storing/retrieving a unit-load in class k can be then obtained as follows,

$$t_k = \frac{(b_{k-1} - y_{k-1})b_{k-1} + \sum_{b=b_{k-1}+1}^{b_k} b - (b_k - y_k)b_k}{y_k - y_{k-1}} + \frac{x(x+1)w}{2x+1}$$

$$= \frac{2(y_k b_k - y_{k-1}b_{k-1}) - (b_k - b_{k-1})(b_k + b_{k-1} - 1)}{2(y_k - y_{k-1})} + \frac{x(x+1)w}{2x+1}, \tag{12}$$

where $b_0 = y_0 = 0$. The first part of Equation (12) is the average one-way travel distance in the depth direction and the second part is that in horizontal direction.

Consequently, according to Equations (1), (4), (8–10) and (12), the TDM of the n-class-based storage system can be obtained as follows,

Model TDM

$$\text{Min} \quad T_n = \frac{x(x+1)w}{2x+1} + \sum_{k=1}^{n} \frac{2(y_k b_k - y_{k-1}b_{k-1}) - (b_k - b_{k-1})(b_k + b_{k-1} - 1)}{2(y_k - y_{k-1})} \left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s \right), \tag{13}$$

$$\text{s.t.} \quad L_k = \sqrt{0.5KA} \sum_{l=1}^{k} \left(\left(1 + N_l^{-\varepsilon}\right) \sum_{i=i_{l-1}+1}^{i_l} \sqrt{\left((i/N)^s - ((i-1)/N)^s\right)} \right), \quad k = 1, 2, \ldots, n, \tag{14}$$

$$y_k = L_k/(4x+2), \text{and}\, b_k = \lceil y_k \rceil, \quad k = 1, 2, \ldots, n. \tag{15}$$

$$\text{d.v.} \quad x, i_k,\ k = 1, 2, \ldots, n-1,\ i_0 = 0,\ y_0 = 0 \text{ and } i_n = N \text{ are known.}$$

Note that y_k and b_k both depend on i_k, through Equations (8–10). For a given number of storage aisles per side of the depot, x, the average one-way travel distance for a random storage policy can be obtained by substituting $n = 1$, and for the full turnover-based policy by substituting $n = N$, i.e. with only one item in each class. For the general case, the SM to obtain the optimal number of classes and the corresponding class boundaries is given in Section 5. Based on the solutions for the three policies with different values of x, the optimal layout of the warehouse can be obtained and the performance of the policies can be compared.

5. Solution methodology for class-based storage policy

We have x as a decision variable in Model TDM. Because x can be any natural number and the total number of sections in a warehouse can be quite large, full enumeration is not an efficient method to find an optimal solution for Model TDM. In order to find an efficient method to obtain the optimal solution for the class-based policy, we first focus on the continuous model with a continuous section index (i.e. $b_k = y_k$ for all classes). Thereafter, we decide the optimal number of classes, optimal-class boundaries and minimum average travel time for the discrete case. The continuous TDM can be written as follows,

Model CTM: (Continuous Travel Model)

$$\text{Min} \quad T_n = \frac{x(x+1)w}{2x+1} + \sum_{k=1}^{n} \left(\frac{y_k + y_{k-1}}{2} \left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s \right) \right),$$

$$\text{s.t.} \quad \text{Equation (14) and } y_k = L_k/(4x+2) \text{ for } \quad k = 1, 2, \ldots, n,$$

where $i_0 = 0$, $y_0 = 0$.

For Model CTM, we offer the following theorem. The proof can be found in Appendix A.

Theorem 1: *Let* $M_n = \sum_{k=1}^{n} \left(\frac{L_k + L_{k-1}}{2} \left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right)$. *Then for any given number of classes and any ABC-demand curve,* $M_n < \sqrt{2NKA}$. *Consequently, the optimal value of* x *to minimise the travel distance is* $x^* = \sqrt{(2M_n - w)/(4w)} - 0.5$, *and* x^* *satisfies* $x^* < \sqrt{\sqrt{2NKA}/(2w)}$. *The optimal warehouse shape factor (the ratio of warehouse width, W, to warehouse depth, D) is* $r_n^* = 2(2M_n - w)/L_n$, *and satisfies* $r_n^* < 4$.

In this theorem, the optimal number of aisles is bounded. In practice, the bound is appears to be relatively small, e.g. smaller than 50 when $N = 1{,}000$, $A = 100{,}000$, $K = 2$ and $w = 4$. Furthermore, the total number of aisles is an odd integer. Hence, we can find all the optimal classifications of the warehouse for every possible value of x via the solution methodology (SM) provided blew.

We now turn to the practical (i.e. integer)-class boundaries and TDM for the discrete case. For a given value of x, Model TDM can be modified as,

Model SM

$$\text{Min} \quad T_n' = \sum\nolimits_{k=1}^{n} \frac{2(y_k b_k - y_{k-1}b_{k-1}) - (b_k - b_{k-1})(b_k + b_{k-1} - 1)}{2(y_k - y_{k-1})}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right), \tag{16}$$

$$\text{s.t.} \quad \text{Equations (14) and (15)},$$

$$\text{d.v.} \quad i_k,\ k = 1, 2, \ldots, n-1,\ i_0 = 0 \text{ and } i_n = N \text{ are known.}$$

For Model SM, the solution of an n-class-based storage system can be found through a dynamic programming algorithm described in the following and summarised in Figure 2.

We define k as the index of the stage corresponding to the kth class. The item with the lowest demand in class k, i_k, denotes the total number of items in the first k classes. i_k is bounded by k and $N - (n - k)$ since there is at least one item in each class. The number of items for class k, N_k, is the decision variable at stage k, and $1 \le N_k \le i_k - k + 1$ because at least $k - 1$ items have been assigned to the previous $k - 1$ classes. The state-transfer function of the model is $i_k = N_k + i_{k-1}$ and the evaluation function at stage k for given i_k is $f_k(i_k)$, where the recursive function can be written as follows, according to Equation (16),

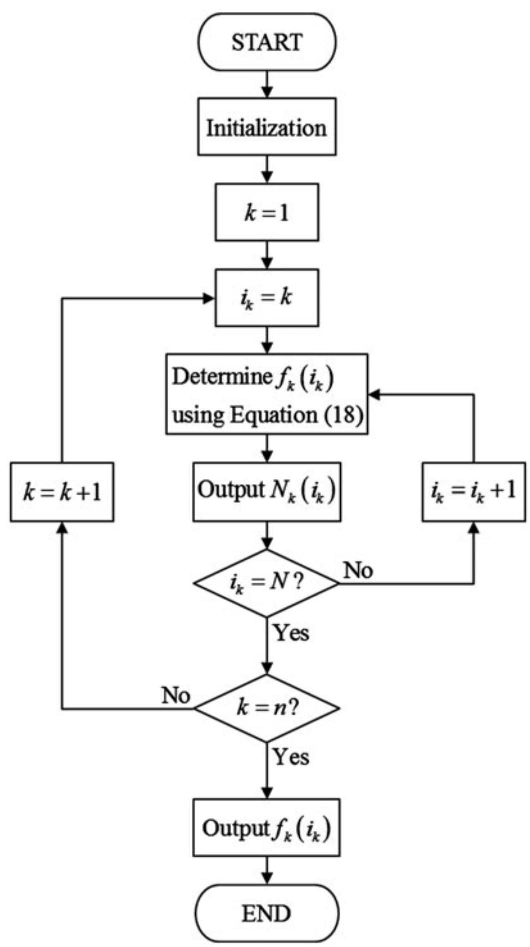

Figure 2. Flowchart of the dynamic programming algorithm.

$$f_k(i_k) = f_{k-1}(i_{k-1}) + \frac{2(y_k b_k - y_{k-1} b_{k-1}) - (b_k - b_{k-1})(b_k + b_{k-1} - 1)}{2(y_k - y_{k-1})}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right), \tag{17}$$

with initialisation $f_0(0) = i_0 = N_0 = 0$, and $f_0(i) = \infty$ for any $1 \le i \le N$.

Then the minimum objective value at stage $k > 1$ based on a known optimal solution at stage $k-1$ can be obtained by,

$$f_k^*(i_k) = \underset{1 \le N_k \le i_k - k + 1}{Min}\left\{f_{k-1}^*(i_k - N_k) + \frac{2(y_k b_k - y_{k-1} b_{k-1}) - (b_k - b_{k-1})(b_k + b_{k-1} - 1)}{2(y_k - y_{k-1})}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_k - N_k}{N}\right)^s\right)\right\}, \tag{18}$$

where y_k and b_k are determined according to Equations (14) and (15).

For any given n, $T_n' = f_n^*(i_n = N)$ gives the minimum objective value by optimising the number of items in each class (the optimal class boundaries can be found according to the state-transfer function $i_k = N_k + i_{k-1}$ and $i_0 = 0$). The optimal number of classes (and optimal-class boundaries) for a given value of x can be obtained from $n^*(x) = \arg\min_{1 \le n \le N}\{f_n^*(N)\}$. Finally, the optimal item classification and corresponding shortest travel distance for class-based storage policy can be identified according to Equation (13).

By applying the dynamic programming algorithm for other possible values of x, the optimal number of storage aisles can be found by comparing their corresponding minimum travel distances. From this, the optimal warehouse parameters can be obtained.

6. Performance evaluation

In this section, we numerically evaluate the performance for random, class-based and full turnover-based storage policies based on the proposed TDM. We use basic layout parameters (shown in Table 1) based on a wholesale warehouse in the Netherlands. Similar parameters were used by Matusiak et al. (2014) and Yu, De Koster, and Guo (2015).

Furthermore, eight cases with different ABC-demand curves are considered, where the first 20% items contribute 20, 30, 40, 50, 60, 70, 80 and 90% to the warehouse demand, respectively. Among the curves, the 20%/90% curve has the highest skewness and the 20%/20% curve has the lowest skewness, as all items have the same demand volume. The shape of the curves is shown in Figure 3.

Through the SM provided in Section 5, the optimal solutions for class-based policies can be obtained quickly. The optimal item classification under various demand curves is shown in Table 2. The one-way travel distances for the three polices (random, full turnover and class-based with optimum number of classes) under the eight demand curves are presented in Figure 4 as a function of different ABC demand curves, and the corresponding RSS is presented in Figure 5. Table 3 summarises the optimal warehouse parameters (optimal number of storage aisles horizontally and optimal number of sections in depth) under different storage policies and various demand structures, while the corresponding warehouse shape factors are shown in Figure 6. Finally, Table 4 shows the comparison of different warehouse shapes with different numbers of aisles based on two demand curves (the 20%/30% and 20%/90% curves).

Table 1. Basic parameters used in the numerical experiments.

Parameters	Values	Description
Unit-load deep	1.4 m-gross 1.2 m-net	From a wholesale warehouse in the Netherlands.
Unit-load wide	1.2 m-gross 1 m-net	
Basic aisle width (slot-to-slot distance)	3.6 m (3 unit-loads width)	
Number of items in the warehouse (N)	100	Yu, De Koster, and Guo (2015)
Total demand in an experiment period (A)	10,000	
The ratio of reorder cost to holding cost (K)	2	
The space sharing factor (ε)	0.22	

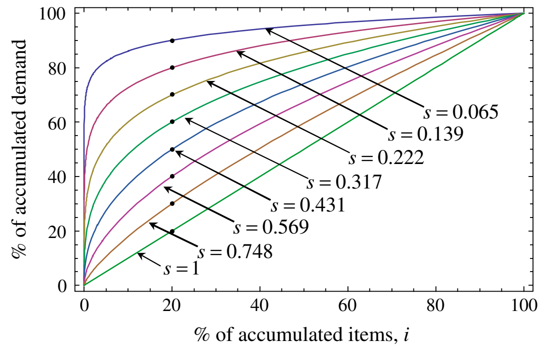

Figure 3. ABC-demand curves.

Table 2. Optimal item classification for class-based storage under different demand curves ($N = 100$).

Shape factor of demand curve s	Optimal number of classes	Optimal number of items in each class (also can be considered as class boundaries)					Total number of items
		Class 1	Class 2	Class 3	Class 4	Class 5	
1 (20%/20%)	1	100					100
0.748 (20%/30%)	2	73	27				100
0.569 (20%/40%)	3	26	68	6			100
0.431 (20%/50%)	4	9	40	50	1		100
0.317 (20%/60%)	4	4	26	51	19		100
0.222 (20%/70%)	4	2	19	45	34		100
0.139 (20%/80%)	5	1	11	38	36	14	100
0.065 (20%/90%)	5	1	9	27	33	30	100

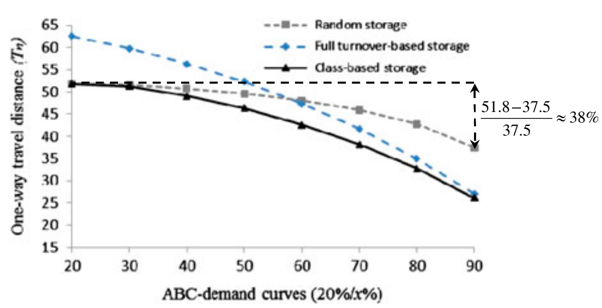

Figure 4. One-way travel distance as a function of demand curves under different storage policies with optimal item classification and optimal warehouse factor.

The results in Figure 4 show that the one-way travel distance decreases with the increase in skewness of the demand curve (or the decrease in shape factor s) under all three storage policies. An ABC-demand curve with high skewness, has a small portion of items contributing to a large part of demand. Therefore, the total RSS is smaller than the RSS for the case with less skewed demand (see also Figure 5). As a result, the average one-way travel distance becomes shorter when a larger portion of the demand is caused by a small percentage of the stored items. For the random policy, conventional research (e.g. Hausman, Schwarz, and Graves 1976; Rosenblatt and Eynan 1989; Eynan and Rosenblatt 1994) assumes that the average one-way travel time (or distance) is independent from the demand curve because the storage space of the warehouse is assumed to be equal to the total average inventory level. However, when we consider the realistic RSS, more space is needed, leading to a difference in travel times of about 38% for a 20%/90% curve compared with the 20%/20% curve.

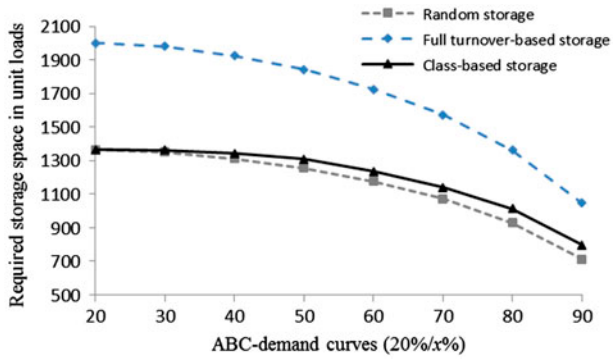

Figure 5. RSS as a function of demand curves under different storage policies with optimal item classification and optimal warehouse shape factor.

Table 3. Optimal warehouse parameters for different storage policies under various demand curves.

Shape factor of demand curve s	Random policy				Full turnover policy				Class-based policy			
	No. of required locations	No. of aisles	No. of sections	Location utilisation (%)	No. of required locations	No. of aisles	No. of sections	Location utilisation (%)	No. of required locations	No. of aisles	No. of sections	Location utilisation (%)
1 (20%/20%)	1364	15	46	98.84	2000	19	53	99.30	1364	15	46	98.84
0.748 (20%/30%)	1350	15	45	100	1980	19	53	98.31	1362	15	46	98.70
0.569 (20%/40%)	1311	15	44	99.32	1923	17	57	99.23	1342	15	45	99.41
0.431 (20%/50%)	1256	15	42	99.68	1842	17	55	98.53	1308	15	44	99.09
0.317 (20%/60%)	1175	15	40	97.92	1724	15	58	99.08	1236	13	48	99.04
0.222 (20%/70%)	1073	15	36	99.35	1574	13	61	99.24	1141	11	52	99.74
0.139 (20%/80%)	929	13	36	99.25	1363	11	62	99.93	1014	11	47	98.07
0.065 (20%/90%)	713	11	33	98.21	1046	9	59	98.49	798	7	57	100

The results in Figure 4 also indicate that the full turnover-based policy performs worse than the optimal class-based policy for all eight demand curves, and even worse than random storage for the first four cases with a low demand curve skewness. That is to say, if the items in the warehouse have approximately similar demand volumes, warehouse managers should not apply the full turnover-based policy, because it is even outperformed by random storage. This result is caused by the tradeoff between the space-sharing effect among the items stored in an identical class and the turnover ranking according to item demand. A full turnover-based policy takes advantage of the turnover ranking: shortening the travel distance by increasing the frequency of retrievals close to the depot, but it loses the possibility of space sharing because each class contains only one item. Consequently, a class-based policy is preferred, because it balances this tradeoff and takes advantage of both the turnover ranking and the space sharing effect. The results in Table 2 show that a small number of classes are optimal for the class-based policy. Even for a 20%/90% curve with $s = 0.065$,

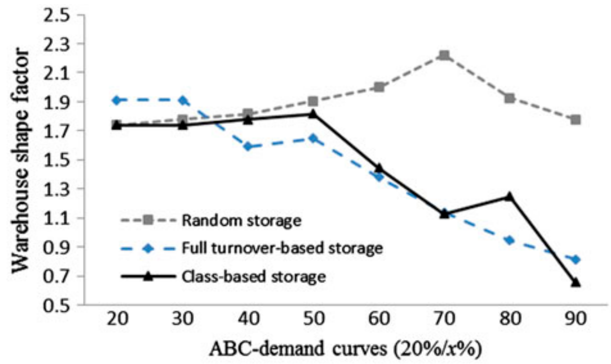

Figure 6. Optimal warehouse shape factor as a function of demand curves under different storage policies.

Table 4. Travel distance comparison for two ABC curves with different numbers of storage aisles.

	$x = 3$ (7 aisles)		$x = 5$ (11 aisles)		$x = 7$ (15 aisles)	
Shape factor of demand curve s	$T_n(x)$	$\frac{T_n(x)-T_n^*}{T_n^*} \times 100\%$	$T_n(x)$	$\frac{T_n(x)-T_n^*}{T_n^*} \times 100\%$	$T_n(x)$	$\frac{T_n(x)-T_n^*}{T_n^*} \times 100\%$
0.748 (20%/30%)	68.88	34.43%	54.53	6.4%	51.24	0%
0.065 (20%/90%)	26.14	0%	27.33	4.6%	31.30	19.74%

the optimal number of classes is only five. Thus, a small number of classes ($n^* \leq 5$) provides the minimum expected one-way travel distance for a unit-load warehouse with parallel aisle configuration and across-aisle storage policy. This result is in accordance with the findings in the literature (Yu, De Koster, and Guo 2015) and in practical operations (Petersen, Aase, and Heiser 2004; De Koster, Le-Duc, and Roodbergen 2007; Roodbergen and Vis 2009). Although more storage space is required for a warehouse using a class-based storage policy compared with the random policy, the space gap is quite small, as shown in Figure 5.

Figure 6 shows that the optimal warehouse shape factor ($r = W/D$) is influenced by the demand structure of the stored items, which seems to be a decreasing function of the skewness of the ABC curve under class-based and full turnover-based storage policies (the latter can be considered as a special case of class-based storage with $n = N$). The optimal warehouse shape depends on the size of the warehouse (see also Roodbergen 2001) and, for a given storage policy, the RSS decreases with the increase in skewness. For flat ABC-demand curves (i.e. with a large s) a *wide-shallow* warehouse layout with a large number of short storage aisles leads to short travel times. For skewed ABC-demand curves a *narrow-deep* warehouse layout with a small number of long storage aisles performs best. Table 3 illustrates that the optimal layout of the warehouse for the 20%/20% and 20%/30% demand curves is 15 aisles horizontally and 46 sections in depth whereas for the 20%/90% curve, the optimal layout is only 7 aisles horizontally but 57 sections in depth for each storage aisle. The required number of locations is obtained from Equation (8). Comparison of the one-way travel distance based on different warehouse layouts shown in Table 4 shows that a 34.43% efficiency loss occurs for the 20%/30% curve within a 7-aisle warehouse compared with that when 15 aisles are adopted. Similarly, a 19.74% efficiency loss occurs for the 20%/90% curve when a warehouse with 15 aisles is implemented, compared with the optimal layout with 7 aisles.

7. Conclusions and future research

This paper studies the travel time performance of different storage policies in a unit-load warehouse, with consideration of the required storage space. A generalised TDM for an n-class-based storage policy is developed, leading to some interesting findings. First, the average one-way travel distance for the random policy is not constant, but decreases with the increase in skewness of the ABC demand curve. In fact, a significant gap (nearly 40%) exists between the 20%/20% and 20%/90% curves. This result shows that the random storage policy performs better with more skewed demand curves, as the warehouse can become smaller.

Second, results show that the full turnover-based policy (a special case of the class-based policy with only one item in each class) does not lead to the global minimum average one-way travel distance for the warehouse. This result contradicts conventional studies that consider the travel time with the full turnover-based policy as a lower bound for class-based policies (Hausman, Schwarz, and Graves 1976; Thonemann and Brandeau 1998; Yu and De Koster 2013). This result is caused by the fact that the full turnover-based policy needs nearly 50% more storage space than the random policy, since items cannot share space in the same class. Consequently, a tradeoff exists between the effects of item ranking and space sharing, depending on the number of classes and on the number of items in each class. In other words, ranking the items according to their turnover reduces the average travel distance by increasing the system efficiency, while also increasing the average travel distance because of the need for expanded storage space. The result shows that, as a result of balancing this tradeoff, a class-based policy with a small number of classes (no more than 5), is optimal. This finding is consistent with that of Yu, De Koster, and Guo (2015) on class-based storage for a square in time AS/RS with a finite number of items.

In addition, this paper offers the optimal warehouse shape factor (the ratio of warehouse width to depth) for different storage policies under various demand curves. The optimal shape factor depends on the size of the warehouse. For a given storage policy, the RSS decreases with the increase in the demand curve skewness. A *wide and shallow* layout (characterised by a large number of short storage aisles) is preferred if the stored items have similar demand volumes per unit time (i.e. with a flat ABC-demand curve and a large RSS). A *narrow and deep* layout (characterised by a small number of long storage aisles) is preferred if the stored items have significant different demand volumes (i.e. with a highly skewed ABC-demand curve and a small RSS). For instance, for the 20%/40% demand curve, the optimal warehouse shape factor is 1.78 with 15 storage aisles, whereas for the 20%/90% demand curve, the optimal shape factor is 0.65 with only 7 storage aisles.

This research can be extended in several directions. First, we only considered the across-aisle storage policy although within-aisle and diagonal policies are also widely applied in practice. Thus, studying the warehouse layout and optimal classification for these policies would also be useful. Second, research on the optimal warehouse layout based on flying-V or fishbone aisle configurations that consider space-sharing effect would be welcome. Third, this research focuses on a unit-load warehouse operating under single command, while similar research on dual-command operations or in an order-picking system could also be interesting. Finally, our results are based on the assumption of one depot, located at the middle of the front aisle. Relaxing this assumption may also be useful in practice.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix A: Proof of Theorem 1

Considering that $y_k = L_k/(4x+2)$ and $M_n = \sum_{k=1}^{n}\left(\frac{L_k+L_{k-1}}{2}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right)$, the objective function of T_n can be rewritten as,

$$\begin{aligned} T_n &= \frac{x(x+1)w}{2x+1} + \sum_{k=1}^{n}\left(\frac{y_k+y_{k-1}}{2}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right) \\ &= \frac{x(x+1)w}{2x+1} + \frac{1}{4x+2}\sum_{k=1}^{n}\left(\frac{L_k+L_{k-1}}{2}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right) \\ &= \frac{x(x+1)w}{2x+1} + \frac{M_n}{4x+2} \\ &= \frac{w(2x+1)}{4} + \frac{2M_n-w}{4(2x+1)}. \end{aligned}$$

Since $x \geq 0$ and $2M_n > w$ in practice, and $u^2+v^2 \geq 2uv, \quad \forall u, v$, we have $T_n \geq \sqrt{w(2M_n-w)}/2$, and $T_n = \sqrt{w(2M_n-w)}/2$ if and only if $x = \sqrt{(2M_n-w)/(4w)} - 0.5$.

Next, we give the proof of $M_n < \sqrt{2NKA}$ and $x^* < \sqrt{\sqrt{2NKA}/(2w)}$.

$$\begin{aligned} M_n &= \sum_{k=1}^{n}\left(\frac{L_k+L_{k-1}}{2}\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right) \\ &< \sum_{k=1}^{n}\left(L_k\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right) \\ &< \sum_{k=1}^{n}\left(L_n\left(\left(\frac{i_k}{N}\right)^s - \left(\frac{i_{k-1}}{N}\right)^s\right)\right) \\ &< L_n \end{aligned}$$

According to Equation (8), $M_n < L_n \leq L_N$, where L_N is the RSS for the warehouse with N classes each with only one item in the class and,

$$L_N = \sqrt{2KA}\sum_{i=1}^{N}\sqrt{(i/N)^s - ((i-1)/N)^s}.$$

Since $G(i) = (i/N)^s$ is a concave function of i, and $\sum_{i=1}^{N}\left((i/N)^s - ((i-1)/N)^s\right) = 1$, we find, $L_N \leq \sqrt{2KA}\sum_{i=1}^{N}\sqrt{i/N-(i-1)/N} = \sqrt{2NKA}$. That is to say, the maximum RSS is achieved only if the warehouse stores N items with identical demand (a 20%/20% ABC-demand curve). Therefore, $M_n < L_n \leq L_N \leq \sqrt{2NKA}$. Using $x^* = \sqrt{(2M_n-w)/(4w)} - 0.5$, we find $x^* < \sqrt{\sqrt{2NKA}/(2w)}$.

Finally, since L_n is the total RSS for an n-class-based storage warehouse with $2x+1$ storage aisles, the required storage sections in depth can be obtained as $L_n/(4x+2)$. Consequently, the optimal warehouse shape factor (the ratio of warehouse width to depth) can be obtained as $r_n^* = w(2x^*+1)/(L_n/(4x^*+2))$. Considering $x^* = \sqrt{(2M_n-w)/(4w)} - 0.5$, it follows $r_n^* = 2(2M_n-w)/L_n$. Furthermore, by considering $M_n < L_n$ for any number of classes under any ABC-demand curve, it follows $r_n^* < 4$.