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Extending Bell's Theorem: Ruling out Paramater Independent Hidden Variable Theories

G J Leegwater

Faculty of Philosophy, Erasmus University Rotterdam, Burgemeester Oudlaan 50, 3062 PA Rotterdam, The Netherlands

E-mail: `leegwater@fwb.eur.nl`

Abstract. Bell's Theorem may well be the best known result in the foundations of quantum mechanics. Here, it is presented as stating that for any hidden variable theory the combination of the conditions Parameter Independence, Outcome Independence, Source Independence and Compatibility with Quantum Theory leads to a contradiction. Based on work by Roger Colbeck and Renato Renner, an extension of Bell's Theorem is considered. In this extension the theorem is strengthened by replacing Outcome Independence by a strictly weaker condition.

1. Introduction

Quantum mechanics provides limited predictions for measurement outcomes: generally, its predictions are probabilistic. It has been a long-standing question under what conditions quantum mechanics can be extended such that more precise predictions can be given. In such extensions, known as 'hidden variable theories', the quantum description of a system is supplemented with extra variables. A number of impossibility theorems have been proven, showing that large classes of hidden variable theories are incompatible with quantum mechanics. The best known example is Bell's Theorem [1, 2], which rules out hidden variable theories satisfying two locality conditions: Parameter Independence and Outcome Independence. Using recent work of Roger Colbeck and Renato Renner [3, 4], this result can be extended: Outcome Independence can be replaced by a strictly weaker condition, enlarging the class of hidden variable theories that is ruled out. In this extension, any hidden variable theory satisfying Parameter Independence is trivial, meaning that the probabilities of measurement outcomes are independent of the hidden variables, rendering such variables redundant.

2. Extending Bell's Theorem

2.1. Bell's Theorem

Bell's Theorem can be stated as the appearance of a contradiction when the following four conditions are combined:

- **ParInd: Parameter Independence**
  When considering measurements on subsystems of some larger system, ParInd guarantees that the outcome probabilities for a measurement on any subsystem are independent of what measurement is being performed on any other subsystem. Quantum mechanics itself satisfies ParInd. A violation of ParInd gives rise to tension with relativity theory.

- **OutInd: Outcome Independence**
  When considering measurements on subsystems of some larger system, OutInd guarantees that the outcome probabilities for a measurement on any subsystem are independent of the outcome of a measurement on any other subsystem. Quantum mechanics itself violates OutInd.
- **SrcInd: Source Independence**
  This condition guarantees that the distribution of the hidden variables is independent of what measurement is being performed. Since the choice of measurement can be based on anything an experimentator wishes, a violation of this condition is often seen as a 'conspiracy'.
- **CompQuant: Compatibility with Quantum Mechanics**
  According to this condition, when integrating out the hidden variables, the hidden variable theory gives the same probabilities for measurement outcomes as quantum mechanics.

2.2. Extension
Bell's Theorem can be strengthened by replacing OutInd by the following condition:

- **NonTriv: Nontriviality**
  Triviality means that for any measurement, outcome probabilities are independent of the hidden variables. This renders such variables redundant: with or without the hidden variables, the probabilities equal the probabilities given by quantum mechanics. Nontriviality is the opposite condition: there are measurements for which the outcome probabilities do depend on the hidden variables.

Note that, since quantum mechanics (without additional variables) violates OutInd, any hidden variable theory satisfying OutInd must predict outcome probabilities different from those of quantum mechanics. This means that the outcome probabilities depend on the hidden variables, redering such a theory nontrivial. So, OutInd implies NonTriv. Furthermore, NonTriv does not imply OutInd, since it is easy to construct a hidden variable theory which gives the same predictions as quantum mechanics for EPR experiments, thereby violating OutInd, while for some other measurement it gives nontrivial predictions, thereby satisfying NonTriv. This makes NonTriv a strictly weaker condition than OutInd.

3. Further reading
The above is explained in more detail in a forthcoming paper [5], where also the relation between this work and the work of Colbeck and Renner is elucidated.

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