ECONOMETRIC ANALYSIS OF THE MARKET SHARE ATTRACTION MODEL

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ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2001-25-MKT
Publication	May 2001
Number of pages	31
Email address corresponding author	dfok@few.eur.nl
Address	Erasmus Research Institute of Management (ERIM) Faculty of Economics Erasmus Universiteit Rotterdam P.O. Box 1738 3000 DR Rotterdam, The Netherlands Phone: +31 10 408 1182 Fax: +31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

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BIBLIOGRAPHIC DATA AND CLASSIFICATIONS		
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Library of Congress Classification (LCC)	5001-6182	Business
	5410-5417.5	Marketing
	HF 5415.2+	Marketing research
Journal of Economic Literature (JEL)	M	Business Administration and Business Economics
	M 31 C 44	Marketing Statistical Decision Theory
	M 31 C 52	Marketing Model Evaluation and Testing
European Business Schools Library Group (EBSLG)	85 A	Business General
	280 G 255 A	Managing the marketing function Decision theory (general)
	280 K	Marketing Science (quantitative analysis)
Gemeenschappelijke Onderwerpsontsluiting (GOO)		
Classification GOO	85.00	Bedrijfskunde, Organisatiekunde: algemeen
	85.40 85.03	Marketing Methoden en technieken, operations research
	85.03 85.40	Methoden en technieken, operations research Marketing
Keywords GOO	Bedrijfskunde / Bedrijfseconomie	
	Marketing / Besliskunde	
	Marktaandeel, Forecasting	
Free keywords	Market share attraction model; model selection; estimation; diagnostics; forecasting	

Econometric Analysis of the Market Share Attraction Model

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Abstract

Market share attraction models are useful tools for analyzing competitive structures. The models can be used to infer cross-effects of marketing-mix variables, but also the own effects can be adequately estimated while conditioning on competitive reactions. Important features of attraction models are that they incorporate that market shares sum to unity and that the market shares of individual brands are in between 0 and 1. Next to analyzing competitive structures, attraction models are also often considered for forecasting market shares.

The econometric analysis of the market share attraction model has not received much attention. Topics as specification, diagnostics, estimation and forecasting have not been thoroughly discussed in the academic marketing literature. In this chapter we go through a range of these topics, and, along the lines, we indicate that there are ample opportunities to improve upon present-day practice.

Key words: market share attraction models; model selection; estimation; diagnostics; forecasting

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1 Introduction

The implementation of econometric models has become increasingly fashionable in marketing research. The main reason for this is that nowadays marketing research can involve the analysis of large amounts of data on revealed preferences, such as sales, market shares, brand choices and interpurchase times, and stated preferences such as opinions, attitudes and purchase intentions. Many firms collect data on these performance measures for their current and their prospective customers, and they usually try to relate these measures with individual-specific characteristics and marketing-mix efforts, see Franses and Paap (2001a) for a recent survey on quantitative models for revealed preference data. The main reason for considering econometric models is that in many cases the number of data points and the number of variables is rather large, and hence simply performing a range of bivariate analyses seems impractical.

The econometric analysis of a certain model for the above mentioned measures usually involves a range of steps. The first step amounts to specifying a model given the available data, the relevant explanatory variables, and the marketing problem at hand. Once the model has been specified, one needs to estimate the parameters and their associated confidence regions. Third, one usually considers the empirical validity of the model by performing diagnostic tests on its adequacy, where one typically focuses on the properties of the unexplained part of the model. Given the potential availability of two or more adequate rival models, one seeks to compare these models either on within-sample fit or on out-of-sample forecasting performance. Finally, one can use the ultimately obtained model for forecasting or for policy analysis. It should be noted that the focus in econometric textbooks tends to be on parameter estimation, but it is by no means the single most important issue. Indeed, in practice it is often difficult to specify the model and to compare it with alternatives.

In this chapter we will consider the econometric analysis of a popular model in marketing research, which is the market share attraction model. This model is typically considered for data on market shares, where the data have been collected at a weekly or monthly interval. Market share attraction models are seen as useful tools for analyzing competitive structures, see Cooper and Nakanishi (1988) and Cooper (1993), among various others. The models can be used to infer cross-effects of marketing-mix variables, but one can also learn about the effects of own efforts while conditioning on competitive reactions. Important features of attraction models are that they rightfully assume that market shares sum to unity and that the market shares of individual brands are in between 0 and 1. This complicates the econometric analysis, as we will see below. Typically, an attraction model can be written as a system of equations concerning all market shares, and the parameters can be estimated using standard methods, see for example Cooper (1993) and Bronnenberg *et al.* (2000).

Interestingly, a casual glance at the relevant marketing literature on market share attraction models indicates that there seem to have been little attention to how to specify the attraction model, how to estimate its parameters, how to analyse it virtues in the sense that the models capture the salient data characteristics, and about how to use the models for forecasting. In sum, it seems that an (empirical) econometric view in these models is lacking. Therefore, in this chapter we aim to contribute to this view by addressing these issues concerning attraction models when they are to be used for describing and forecasting market shares. The first issue concerns the specification of the models. A literature check immediately indicates that many studies simply assume one version of an attraction model to be relevant and start from there. In this chapter we first start with a fairly general and comprehensive attraction model, and we show how various often applied models fit into this general framework. We also indicate how one can arrive from the general model at the more specific models, thereby immediately suggesting a general-to-simple testing strategy. Second, we discuss the estimation of the model parameters. We show that a commonly advocated method is unnecessarily complicated and that a much simpler methods yields equivalent estimates. Along the lines, we also propose a few diagnostic measures, which to our knowledge have rarely been used, but which really should come in handy. Finally, we address the issue of generating forecasts for market shares. As the market share attraction model ultimately gets analyzed as a system of equations for (natural) log transformed shares, generating unbiased forecasts is far from trivial. We discuss a simulation-based method which yields unbiased forecasts.

The outline of this chapter is as follows. In Section 2 we first discuss the basics of the attraction model by reviewing various specifications of the model. We discuss the interpretation of the model in Section 3, and we discuss parameter estimation of the model in Section 4. We discuss diagnostic measures in Section 5 and forecasting in Section 6. Finally we touch upon the topic of model selection in Section 7. We conclude in Section 8 with suggestions for further research.

2 Representation

In this section we start off with discussing a general market share attraction model and we deal with various of its nested versions which currently appear in the academic marketing literature. We first start with the so-called fully extended attraction model in Section 2.1. This model has a flexible structure as it includes many variables. Naturally this increases the empirical uncertainty about the relevant parameters. Therefore, in practice one may want to consider restricted versions of this general model. In Section 2.2, we discuss some of the restricted versions, where we particularly focus on those models which are often applied in practice.

2.1 A general market share attraction model

Let $A_{i,t}$ be the attraction of brand i at time t, $t = 1, \ldots, T$, given by

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}} \qquad \text{for } i = 1, \ldots, I, \tag{1}$$

where $x_{k,j,t}$ denotes the k-th explanatory variable (such as price level, distribution, advertising spending) for brand j at time t and where $\beta_{k,j,i}$ is the corresponding coefficient for brand i. The parameter μ_i is a brand-specific constant. Let the error term $(\varepsilon_{1,t}, \ldots, \varepsilon_{I,t})'$ be normally distributed with zero mean and Σ as a possibly non-diagonal covariance matrix, see Cooper and Nakanishi (1988). As we want the attraction to be non-negative,

$x_{k,j,t}$ has to be non-negative, and hence rates of changes are usually not allowed. The variable $x_{k,j,t}$ may be a 0/1 dummy variable to indicate promotional activities for brand j at time t. Note that for this dummy variable, one should transform $x_{k,j,t}$ to $\exp(x_{k,j,t})$ to avoid that $A_{i,t}$ becomes zero in case of no promotional activity.

The market shares for the I brands follow from the, what is called, Market Share Theorem, see Bell *et al.* (1975). This theorem states that the market share of brand i is equal to its attraction relative to the sum of all attractions, that is,

$$M_{i,t} = \frac{A_{i,t}}{\sum_{j=1}^{I} A_{j,t}} \qquad \text{for } i = 1, \ldots, I \tag{2}$$

The model in (1) with (2) is usually called the market share attraction model. Notice that the definition of the market share of brand i at time t given in (2) implies that the attraction of the product category is the sum of the attractions of all brands and that $A_{i,t} = A_{l,t}$ results in $M_{i,t} = M_{l,t}$.

The interesting aspect of the attraction model is that the $A_{i,t}$ in (1) is unobserved. As we will see below, this implies that neither μ_i nor $\varepsilon_{i,t}$ is identified. Another consequence is that the market researcher should make a decision on the specification of $A_{i,t}$ prior to empirical analysis. As we will indicate, there are many possible specifications. For example, to describe potential dependencies in market shares over time, which describe purchase reinforcement effects, one may include lagged attractions $A_{i,t}$ in (1). For example, one may consider

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) A_{i,t-1}^{\gamma_i} \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}}. \tag{3}$$

However, due to the fact that we do not observe $A_{i,t}$, it turns out only possible to estimate the parameters in this model if the lag parameter γ_i is assumed to be the same across brands, see Chen *et al.* (1994). As this may be viewed as too restrictive, an alternative strategy to account for dynamics is to include lagged values of the observed variables $M_{j,t}$ and $x_{k,j,t}$ in (1). The most general autoregressive structure follows from the inclusion of lagged market shares and lagged explanatory variables of all brands. In that case, the attraction specification with a P-th order autoregressive structure becomes

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \left(\prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}} \prod_{p=1}^{P} \left(M_{j,t-p}^{\alpha_{p,j,i}} \prod_{k=1}^{K} x_{k,j,t-p}^{\beta_{p,k,j,i}} \right) \right), \tag{4}$$

where the $\alpha_{p,j,i}$ parameters represent the effect of lagged market shares on attraction and where the $\beta_{p,k,j,i}$ parameters represent the effect of lagged explanatory variables. To illustrate, this model allows that the market share for brand 1 at $t-1$ has an effect on that of brand 2 at t, and also that there is a relationship between brand 2's market share and the price of brand 1 at $t-1$. The flexibility of this general specification is reflected by the potentially large number of parameters. For example with $I = 4$ brands, $K = 3$ explanatory variables and $P = 2$ lags, there are over 150 parameters to estimate (although they are not all identified, see below).

The model that consists of equations (4) and (2) is sometimes called the fully extended multiplicative competitive interaction [FE-MCI] model, see Cooper (1993). To enable parameter estimation, one can linearize this model in two steps. First, one can take one brand as the benchmark brand. Choosing brand I as the base brand leads to

$$\frac{M_{i,t}}{M_{I,t}} = \frac{\exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \left(\prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}} \prod_{p=1}^{P} \left(M_{j,t-p}^{\alpha_{p,j,i}} \prod_{k=1}^{K} x_{k,j,t-p}^{\beta_{p,k,j,i}} \right) \right)}{\exp(\mu_I + \varepsilon_{I,t}) \prod_{j=1}^{I} \left(\prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,I}} \prod_{p=1}^{P} \left(M_{j,t-p}^{\alpha_{p,j,I}} \prod_{k=1}^{K} x_{k,j,t-p}^{\beta_{p,k,j,I}} \right) \right)}. \tag{5}$$

Below we will discuss another approach to linearizing the model, but we will show that both transformations lead to same parameter estimates, while the estimation procedure based on (5) is much simpler. Next, one can take the natural logarithm (denoted by log) of both sides of (5). Together, this results in the $(I-1)$-dimensional set of equations given by

$$\begin{aligned} \log M_{i,t} - \log M_{I,t} = (\mu_i - \mu_I) + \sum_{j=1}^{I} \sum_{k=1}^{K} (\beta_{k,j,i} - \beta_{k,j,I}) \log x_{k,j,t} + \\ \sum_{j=1}^{I} \sum_{p=1}^{P} \left((\alpha_{p,j,i} - \alpha_{p,j,I}) \log M_{j,t-p} + \sum_{k=1}^{K} (\beta_{p,k,j,i} - \beta_{p,k,j,I}) \log x_{k,j,t-p} \right) + \eta_{i,t}. \end{aligned} \tag{6}$$

for $i = 1, \ldots, I-1$. Note that not all μ_i parameters $(i = 1, \ldots, I)$ are identified. Also for each k and p, one of the $\beta_{k,j,i}$ and $\beta_{p,k,j,i}$ parameters is not identified. In fact, only the parameters $\tilde{\mu}_i = \mu_i - \mu_I$, $\tilde{\beta}_{k,j,i} = \beta_{k,j,i} - \beta_{k,j,I}$ $\tilde{\beta}_{p,k,j,i} = \beta_{p,k,j,i} - \beta_{p,k,j,I}$ are identified. This is however sufficient to completely identify elasticities, see Section 3 below and Cooper and Nakanishi (1988, p. 145). Finally, one can only estimate the parameters $\tilde{\alpha}_{p,j,i} = \alpha_{p,j,i} - \alpha_{p,j,I}$.

The error variables in (6) are $\eta_{i,t} = \varepsilon_{i,t} - \varepsilon_{I,t}$, $i = 1, \ldots, I-1$. Hence, given the earlier assumptions on $\varepsilon_{i,t}$, $(\eta_{1,t}, \ldots, \eta_{I-1,t})'$ is normally distributed with mean zero and $((I-1) \times (I-1))$ covariance matrix $\tilde{\Sigma} = L\Sigma L'$, where $L = (\mathbf{I}_{I-1} \vdots \mathbf{i}_{I-1})$ with $\mathbf{I}_{I-1}$ an $(I-1)$-dimensional identity matrix and where $\mathbf{i}_{I-1}$ is an $(I-1)$-dimensional unity vector. Note that therefore only $\frac{1}{2}I(I-1)$ parameters of the covariance matrix Σ can be identified.

In sum, the general attraction model can be written as a $(I-1)$-dimensional P-th order vector autoregression with exogenous variables [sometimes abbreviated as VARX(P)], given by

$$\log M_{i,t} - \log M_{I,t} = \tilde{\mu}_i + \sum_{j=1}^{I}\sum_{k=1}^{K} \tilde{\beta}_{k,j,i} \log x_{k,j,t} + \sum_{j=1}^{I}\sum_{p=1}^{P} \left(\tilde{\alpha}_{p,j,i} \log M_{j,t-p} + \sum_{k=1}^{K} \tilde{\beta}_{p,k,j,i} \log x_{k,j,t-p} \right) + \eta_{i,t}, \quad (7)$$

$i = 1, \ldots, I-1$, where the covariance matrix of the error variables $(\eta_{1,t}, \ldots, \eta_{I-1,t})'$ is $\tilde{\Sigma}$. For further reference, we will consider (7) as the general attraction specification. We will take it as a starting point in our within-sample model selection strategy, which follows the general-to-specific principle, see Section 7 below.

2.2 Various restricted models

As can be understood from (7), the general attraction model contains many parameters and in practice this will absorb many degrees of freedom. Therefore, one usually assumes a simplified version of this general model. Obviously, the general model can be simplified in various directions, and, interestingly, the academic marketing literature indicates that in many cases one simply assumes some form without much further discussion. This may be a non-trivial exercise, as there are many possible simpler models. One can for example impose restrictions on the β coefficients, on the covariance structure Σ, and on the autoregressive parameters α. In this section we will discuss a few of these potentially empirically relevant restrictions on the attraction specification in (4).

Restricted Covariance Matrix [RCM]

If the covariance matrix of the error variables $\varepsilon_{i,t}$ in (4) is a diagonal matrix, where each $\varepsilon_{i,t}$ has its own variance σ_i^2, that is, $\Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_I^2)$, then the covariance matrix for the $(I-1)$-dimensional vector $\eta_{i,t}$ in (7) becomes

$$\text{diag}(\sigma_1^2, \ldots, \sigma_{I-1}^2) + \sigma_I^2 \mathbf{i}_{I-1}\mathbf{i}_{I-1}', \tag{8}$$

where $\mathbf{i}_{I-1}$ denotes a $(I-1)$-dimensional unity vector. In Section 7 we discuss how one can examine the validity of (8), which is not that trivial, even though this assumption has been regularly made. If this restriction holds, the errors in the attraction specifications are independent, implying that the unexplained components of the attraction equations are uncorrelated.

Restricted Competition [RC]

One can also assume that the attraction of brand i only depends on its own explanatory variables. This amounts to the assumption that marketing effects of competitive brands do not have an attraction effect, see for example Kumar (1994) among others. For (4), this corresponds to the restriction $\beta_{k,j,i} = 0$ (and $\beta_{p,k,j,i} = 0$) for $j \neq i$. More precisely, this RC restriction implies that (4) reduces to

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{k=1}^{K} x_{k,i,t}^{\beta_{k,i}} \prod_{j=1}^{I}\prod_{p=1}^{P}\left(M_{j,t-p}^{\alpha_{p,j,i}} \prod_{k=1}^{K} x_{k,i,t-p}^{\beta_{p,k,i}} \right), \quad \text{for } i = 1, \ldots, I, \tag{9}$$

where we write $\beta_{k,i}$ for $\beta_{k,i,i}$ and $\beta_{p,k,i}$ for $\beta_{p,k,i,i}$. Consequently, the linearized multiple equation model in (7) becomes

$$\log M_{i,t} - \log M_{I,t} = \tilde{\mu}_i + \sum_{k=1}^{K} \beta_{k,i} \log x_{k,i,t} - \sum_{k=1}^{K} \beta_{k,I} \log x_{k,I,t} + \\ \sum_{j=1}^{I}\sum_{p=1}^{P}\left(\tilde{\alpha}_{p,j,i} \log M_{j,t-p} + \sum_{k=1}^{K} \beta_{p,k,i} \log x_{k,i,t-p} - \sum_{k=1}^{K} \beta_{p,k,I} \log x_{k,I,t-p} \right) + \eta_{i,t}, \tag{10}$$

for $i = 1, \ldots, I-1$. Notice that this means that the coefficients $\beta_{k,I}$ are equal across the $(I-1)$ equations and that these restrictions should be taken into account when estimating the parameters. The RC assumption in (9) imposes $K(P+1)I(I-2)$ restrictions on the parameters in the general model in (7), which amounts to a substantial increase in the degrees of freedom. In Section 7 we will discuss how this restriction can be tested.

Restricted Effects [RE]

An even further simplified model arises if we assume, additional to RC, that the β parameters are the same for each brand, that is, $\beta_{k,i} = \beta_k$ (and $\beta_{p,k,i} = \beta_{p,k}$), see Danaher (1994) for an implementation of this combined restrictive model. This model assumes that marketing efforts for brand i only have an effect on the market share of brand i, and also that these effects are the same across brands. In other words, price effects, for example, are the same for all brands. It should be noted here that these similarities do not hold for *elasticities*, however, as will become apparent in Section 3. One may coin this model as an attraction model with restricted effects. Based on (4), the attraction for brand i at time t then further simplifies to

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{k=1}^{K} x_{k,i,t}^{\beta_k} \prod_{j=1}^{I} \prod_{p=1}^{P} \left(M_{j,t-p}^{\alpha_{p,j,i}} \prod_{k=1}^{K} x_{k,i,t-p}^{\beta_{p,k}} \right). \quad \text{for } i = 1, \ldots, I, \tag{11}$$

and the linearized multiple equation model (7) simplifies to

$$\log M_{i,t} - \log M_{I,t} = \tilde{\mu}_i + \sum_{k=1}^{K} \beta_k (\log x_{k,i,t} - \log x_{k,I,t}) + \sum_{j=1}^{I} \left(\sum_{p=1}^{P} \tilde{\alpha}_{p,j,i} \log M_{j,t-p} + \sum_{k=1}^{K} \beta_{p,k} (\log x_{k,i,t-p} - \log x_{k,I,t-p}) \right) + \eta_{i,t}, \tag{12}$$

for $i = 1, \ldots, I-1$. This RE assumption imposes an additional $K(P+1)(I-1)$ parameter restrictions on the β coefficients of (7). Of course, it may occur that the restrictions only hold for a few and not for all $\beta_{k,j,i}$ parameters, that is, for only a few marketing variables. In that case, less parameter restrictions should be imposed.

Restricted Dynamics [RD]

Finally, one may want to impose restrictions on the autoregressive structure in (4), implying that the purchase reinforcement effects are the same across brands. For example, the restriction that the attraction of brand i at time t only depends on its own lagged market shares $M_{i,t}$ corresponds with the restriction $\alpha_{p,j,i} = 0$ for $j \neq i$ in (4). The corresponding multivariate model then becomes

$$\log M_{i,t} - \log M_{I,t} = \tilde{\mu}_i + \sum_{j=1}^{I}\sum_{k=1}^{K} \tilde{\beta}_{k,j,i} \log x_{k,j,t} + \sum_{j=1}^{I}\sum_{p=1}^{P} \left(\alpha_{p,i} \log M_{j,t-p} - \alpha_{p,I} \log M_{I,t-p} + \sum_{k=1}^{K} \tilde{\beta}_{p,k,j,i} \log x_{k,j,t-p} \right) + \eta_{i,t}, \quad (13)$$

for $i = 1, \ldots, I-1$, where we again save on notation by using $\alpha_{p,i}$ instead of $\alpha_{p,i,i}$. Note that now the $\alpha_{p,I}$ parameters are the same across the $(I-1)$ equations and hence that these restrictions should be imposed when estimating the model parameters. To illustrate, Chen *et al.* (1994) additionally impose that $P = 1$ and $\alpha_{1,i} = \gamma$, which yields the estimable version of the attraction model in (3) which assumes that the purchase reinforcement effects are the same across brands. For further reference, we will call this last restriction the Common Dynamics [CD] restriction.

The above discussion shows that various attraction models, which are considered in the relevant literature and in practice for modeling and forecasting market shares, are nested within the general attraction model in (4). The fact that these models are nested automatically suggests that an empirical model selection strategy can be based on a general-to-simple strategy, see Franses and Paap (2001b).

3 Interpretation

As the market shares get modeled trough the attraction specification, and as this implies a reduced form of the model where parameters represent the impact of marketing efforts on the logarithm of relative market shares, the parameter estimates themselves are not easy to interpret. To facilitate an easier interpretation, one usually resorts to elasticities.

In fact, it turns out that the reduced-form parameters are sufficient to identify these (cross-)elasticities.

For model (4), the instantaneous elasticity of the k-th marketing instrument of brand j on the market share of brand i is given by

$$\frac{\partial M_{i,t}}{\partial x_{k,j,t}} \frac{x_{k,j,t}}{M_{i,t}} = \beta_{k,i,j} - \sum_{r=1}^{I} M_{r,t} \beta_{k,r,j}, \tag{14}$$

see Cooper (1993). To show that these elasticities are identified, one can rewrite them such that they only depend on the reduced-form parameters, that is,

$$\frac{\partial M_{i,t}}{\partial x_{k,j,t}} \frac{x_{k,j,t}}{M_{i,t}} = (\beta_{k,j,i} - \beta_{k,j,I})(1 - M_{i,t}) - \sum_{r=1 \wedge r \neq i}^{I-1} M_{r,t}(\beta_{k,j,r} - \beta_{k,j,I}), \tag{15}$$

see (6). Under Restricted Competition, these elasticities simplify to

$$\frac{\partial M_{i,t}}{\partial x_{k,j,t}} \frac{x_{k,j,t}}{M_{i,t}} = (\delta_{i=j} - M_{j,t})\beta_{k,j}, \tag{16}$$

where $\delta_{i=j}$ is the Kronecker δ which has a value of 1 if i equals j and 0 otherwise. Under Restricted Effects, we simply have

$$\frac{\partial M_{i,t}}{\partial x_{k,j,t}} \frac{x_{k,j,t}}{M_{i,t}} = (\delta_{i=j} - M_{j,t})\beta_{k}. \tag{17}$$

It is easy to see that the elasticities converge to zero if a market share goes to 1. From a marketing perspective, this seems rather plausible. If a brand controls almost the total market, its marketing efforts will have little if any effect on its market share. Secondly, in case the market share is an increasing function of instrument X, then if X goes to infinity the elasticity will go to 0. These two properties may seem straightforward, but among the best known market share models, the attraction model is the only model satisfying these properties, see also Cooper (1993).

4 Parameter estimation

In this section we discuss two methods for parameter estimation, and we show that they are equivalent. The first method is rather easy, whereas the second (which seems to be commonly applied) is more difficult.

4.1 Using a base brand

To estimate the parameters in attraction models, we consider the $(I-1)$-dimensional set of linear equations which results from log-linearizing the attraction model given in (7). In general, these equations can be written in the following form

$$\begin{array}{rcllcl} y_{1,t} & = & w'_{1,t}b_1 & + z'_{1,t}a & + & \eta_{1,t} \\ y_{2,t} & = & w'_{2,t}b_2 & + z'_{2,t}a & + & \eta_{2,t} \\ \vdots & = & \vdots & + \quad \vdots & + & \vdots \\ y_{I-1,t} & = & w'_{I-1,t}b_{I-1} & + \; z'_{I-1,t}a & + & \eta_{I-1,t} \end{array} \tag{18}$$

where $y_{i,t} = \log M_{i,t} - \log M_{I,t}$, $\eta_t = (\eta_{1,t}, \ldots, \eta_{I-1,t})' \sim \text{NID}(\mathbf{0}, \tilde{\Sigma})$, and where $w_{i,t}$ are k_i-dimensional vectors of explanatory variables with regression coefficient vector b_i, which is different in each equation, and where $z_{i,t}$ are n-dimensional vectors of explanatory variables with regression coefficient vector a which is the same across the equations, $i = 1, \ldots, I-1$. Each (restricted) version of the general attraction model discussed in Section 2.2 can be written in this format, see Franses and Paap (2001b).

To discuss parameter estimation, it is convenient to write (18) in matrix notation. We define $y_i = (y_{i,1}, \ldots, y_{i,T})'$, $W_i = (w_{i,1}, \ldots, w_{i,T})'$, $Z_i = (z_{i,1}, \ldots, z_{i,T})'$ and $\eta_i = (\eta_{i,1}, \ldots, \eta_{i,T})'$ for $i = 1, \ldots, I-1$. In matrix notation, (18) then becomes

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{I-1} \end{pmatrix} = \begin{pmatrix} W_1 & \mathbf{0} & \ldots & \mathbf{0} & Z_1 \\ \mathbf{0} & W_2 & \ldots & \mathbf{0} & Z_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & W_{I-1} & Z_{I-1} \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_{I-1} \\ a \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_{I-1} \end{pmatrix} \tag{19}$$

or

$$y = Xb + \eta \tag{20}$$

with $\eta \sim \text{N}(\mathbf{0}, (\tilde{\Sigma} \otimes \mathbf{I}_T))$, where $\otimes$ denotes the familiar Kronecker product.

One method for parameter estimation of (20) is ordinary least squares [OLS]. Generally, however, this leads to consistent but inefficient estimates, where the inefficiency is due to the (possibly neglected) covariance structure of the disturbances. Only if the explanatory variables in each equation are the same, or in case Σ is a diagonal matrix, and provided that there are no restrictions on the regression parameters ($w_{i,t} = \mathbf{0}$ for all

i, t), OLS provides efficient estimates, see Judge *et al.* (1985, Chapter 12), among others. Therefore, one should better use generalized least squares [GLS] methods to estimate the model parameters. As the covariance matrix of the disturbances is usually unknown, one has to opt for a feasible GLS procedure, where we use the OLS estimator of the covariance matrix of the disturbances. This procedure is known as Zellner's (1962) seemingly unrelated regression [SUR] estimation method. An iterative SUR estimation method will lead to the maximum likelihood [ML] estimator of the model parameters, see Zellner (1962).

To estimate the parameters in attraction models, and to facilitate comparing various models, we favor ML estimation. The log of the likelihood function of (20) is given by

$$\ell(b, \tilde{\Sigma}) = -\frac{T(I-1)}{2}\log(2\pi) + \frac{T}{2}\log|\tilde{\Sigma}^{-1}| - \frac{1}{2}(y - Xb)'(\tilde{\Sigma}^{-1} \otimes \mathbf{I}_T)(y - Xb). \quad (21)$$

The parameter values which maximize this log likelihood function are consistent and efficient estimates of the model parameters.

For the FE-MCI model without any parameter restrictions in (7), the ML estimator corresponds with the OLS estimator, as the explanatory variables are the same across equations. In that case,

$$\hat{b}_{OLS} = (X'X)^{-1}X'y \quad (22)$$

and

$$\hat{\tilde{\Sigma}} = \frac{1}{T}\sum_{t=1}^{T} \hat{\eta}_t \hat{\eta}_t' \quad (23)$$

where $\hat{\eta}_t$ consists of stacked $\hat{\eta}_{i,t} = y_{i,t} - w_{i,t}'\hat{b}_{OLS,i} - z_{i,t}'\hat{a}_{OLS}$.

For the attraction models with restrictions on the regression parameters, that is, for the RC model in (10), the RE model in (12), and the RD model in (13), one can opt for the iterative SUR estimator which converges to the ML estimator. Starting with the OLS-based estimator for $\tilde{\Sigma}$ in (23), one constructs the feasible GLS estimator

$$\hat{b}_{SUR} = (X'(\hat{\tilde{\Sigma}}^{-1} \otimes \mathbf{I}_T)X)^{-1}X'(\hat{\tilde{\Sigma}}^{-1} \otimes \mathbf{I}_T)y, \quad (24)$$

that is the SUR estimator, see Zellner (1962). Next, we replace the estimate of the covariance matrix $\hat{\tilde{\Sigma}}$ by the new estimate of $\tilde{\Sigma}$, that is (23), where $\hat{\eta}_t$ now consists of stacked $\hat{\eta}_{i,t} = y_{i,t} - w'_{i,t}\hat{b}_{SUR,i} - z'_{i,t}\hat{a}_{SUR}$, to obtain a new SUR estimate of b. This routine is repeated until the estimates for b and $\tilde{\Sigma}$ have converged. These estimates are then the ML estimates of the model, that is, they maximize the log likelihood function (21).

A little more involved are the restrictions on the $\tilde{\Sigma}$ matrix. To estimate the attraction model under the restriction (8), one can either directly maximize the log likelihood function (21) with $\tilde{\Sigma} = \text{diag}(\sigma_1^2, \ldots, \sigma_{I-1}^2) + \sigma_I^2 \mathbf{i}_{I-1}\mathbf{i}'_{I-1}$ using a numerical optimization algorithm like Newton-Raphson or one can again use an iterative SUR procedure. In the latter approach, the new estimate of $\tilde{\Sigma}$ is obtained by maximizing

$$\ell(\tilde{\Sigma}) = -\frac{T(I-1)}{2}\log(2\pi) + \frac{T}{2}\log|\tilde{\Sigma}^{-1}| - \frac{1}{2}\hat{\eta}'(\tilde{\Sigma}^{-1} \otimes \mathbf{I}_T)\hat{\eta}, \quad (25)$$

where $\hat{\eta}$ are the residuals from the previous SUR regression. Again, we need a numerical optimization routine to maximize (25). Especially in cases where there are many brands, the optimization of (25) can become cumbersome. In can however be shown, see Appendix A, that the optimization can be reduced to numerically maximizing a concentrated likelihood over just σ_I^2 where one uses

$$\hat{\sigma_i}^2 = \frac{\hat{\eta}_i'\hat{\eta}_i}{T} - \hat{\sigma}_I^2 \qquad \text{for } i = 1, \ldots, I-1, \quad (26)$$

where $\hat{\eta}_i = (\hat{\eta}_{i,1}, \ldots, \hat{\eta}_{i,T})'$. Given an estimate of σ_I^2, this relationship can be used to obtain estimates of $\sigma_1^2, \ldots, \sigma_{I-1}^2$.

Finally, in all the above cases the standard errors for the estimated regression parameters b are to be estimated by

$$\hat{V}(\hat{b}) = (X'(\hat{\tilde{\Sigma}}^{-1} \otimes \mathbf{I}_T)X)^{-1}, \quad (27)$$

where one should include the appropriate ML estimator for $\tilde{\Sigma}$. When taking the square roots of the diagonal elements of this matrix, one obtains the appropriate standard errors.

4.2 An alternative estimation method

The above estimation routine is based on the reduced-form model, which is obtained from reducing the system of equations using the base brand approach. An alternative method is the, what is called, log-centering method advocated by Cooper and Nakanishi (1988). We will now show that this method is equivalent to the above method, although a bit more complicated.

The log-centering approach is based on the following transformation. After taking the natural logs for the I model equations, the log of the geometric mean market share over the brands is subtracted from all equations. The reduced-form model is now specified relative to the geometric mean. So instead of reducing the system of equations by using a base brand, this methodology reduces the system by the "geometric average brand". Note that the reduced-form model in this case still contains I equations.

To demonstrate the equivalence of parameters obtained through the log-centering technique of Cooper and Nakanishi (1988) and those using the base brand approach, we show that there exists an exact relationship between these sets of parameters. The parameters for the base brand specification can uniquely be determined from the parameters for the log-centering specification and vice versa. Given the 1-to-1 relationship the likelihoods are the same, that is, the discussed FGLS estimator yields the same maximum value of the likelihood as we can use the invariance principle of maximum likelihood, see for example Greene (1993, page 115). All that needs to be shown is the 1-to-1 relationship between the parameters in the two specifications.

Consider a general attraction specification, that is

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}}, \tag{28}$$

where the market shares are defined by

$$M_{i,t} = \frac{A_{i,t}}{\sum_{j=1}^{I} A_{j,t}}. \tag{29}$$

Dynamics are not included in this specification to keep the notation relatively simple. The presented arguments however are invariant to the chosen specification. Written in a vector notation the model for the natural logarithm of attraction becomes

$$\log A_t := \begin{pmatrix} \log A_{1,t} \\ \vdots \\ \log A_{I,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_I \end{pmatrix} + \sum_k \begin{pmatrix} \beta_{k,1,1} & \beta_{k,2,1} & \dots & \beta_{k,I,1} \\ \beta_{k,1,2} & \beta_{k,2,2} & \dots & \beta_{k,I,2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k,1,I} & \beta_{k,2,I} & \dots & \beta_{k,I,I} \end{pmatrix} \begin{pmatrix} \log x_{k,1,t} \\ \vdots \\ \log x_{k,I,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{I,t} \end{pmatrix}$$

$$= \mu + \sum_k B_k \log x_{k,t} + \varepsilon_t. \tag{30}$$

The definition of market share in (29) implies that $\log M_{i,t} = \log A_{i,t} - \log \sum_{j=1}^{I} A_{j,t}$. In a vector notation this gives

$$\log M_t := \begin{pmatrix} \log M_{1,t} \\ \vdots \\ \log M_{I,t} \end{pmatrix} = \log A_t - \mathbf{i}_I \log \sum_{j=1}^{I} A_{j,t}, \tag{31}$$

where $\mathbf{i}_I$ denotes a $(I \times 1)$ unity vector.

As the model in (31) cannot be estimated directly due to the nonlinear dependence of $\log(\sum_{j=1}^{I} A_{j,t})$ on the model parameters, a reduced-form model should be considered. The log-centering method now subtracts the average of the log market shares from the equations to give a reduced-form specification. The dependent variable in this system of equations is now

$$\begin{pmatrix} \log M_{1,t} \\ \vdots \\ \log M_{I,t} \end{pmatrix} - \begin{pmatrix} \frac{1}{I}\sum_{j=1}^{I} \log M_{j,t} \\ \vdots \\ \frac{1}{I}\sum_{j=1}^{I} \log M_{j,t} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{I} & -\frac{1}{I} & \dots & -\frac{1}{I} \\ -\frac{1}{I} & 1-\frac{1}{I} & \dots & -\frac{1}{I} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{I} & -\frac{1}{I} & \dots & 1-\frac{1}{I} \end{pmatrix} \log M_t$$

$$= H_{lc} \log M_t, \tag{32}$$

where H_{lc}, with rank $I-1$, denotes the transformation matrix corresponding to the log-centering approach. The reduced-form model then becomes

$$H_{lc} \log M_t = H_{lc} \log A_t - H_{lc}\mathbf{i}_I \log \sum_{j=1}^{I} A_{j,t} \tag{33}$$

which equals

$$H_{lc} \log M_t = H_{lc}\mu + \sum_k H_{lc} B_k \log x_{kt} + H_{lc}\varepsilon_t, \tag{34}$$

as $H_{lc}\mathbf{i}_I = 0_{I\times I}$. Due to the reduced rank of H_{lc}, the system in (34) contains I equations, but it only has $I-1$ independent equations.

Alternatively, the base brand approach in Section 4.1 gives as the dependent variables in the reduced-form model

$$\begin{aligned}\begin{pmatrix}\log M_{1,t}\\ \vdots \\ \log M_{I-1,t}\end{pmatrix} - \begin{pmatrix}\log M_{I,t}\\ \vdots \\ \log M_{I,t}\end{pmatrix} &= \begin{pmatrix}1 & 0 & \dots & 0 & -1\\ 0 & 1 & \dots & 0 & -1\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \dots & 1 & -1\end{pmatrix}\log M_t \\ &= H_{bb}\log M_t,\end{aligned} \tag{35}$$

with H_{bb} as the relevant transformation matrix. As $H_{bb}\mathbf{i}_I = \mathbf{0}_{I-1\times I}$, the reduced-form model becomes

$$H_{bb}\log M_t = H_{bb}\log A_t = H_{bb}\mu + \sum_k H_{bb}B_k \log x_{kt} + H_{bb}\varepsilon_t, \tag{36}$$

which is to be compared with (34). We have seen that this system contains only $I-1$ equations.

The 1-to-1 relation between the parameters in the two approaches follows from the fact that the equation $CH_{lc} = H_{bb}$ yields a unique solution C, given by

$$C = \begin{pmatrix}1 & 0 & \dots & 0 & -1\\ 0 & 1 & \dots & 0 & -1\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \dots & 1 & -1\\ 1 & 1 & \dots & 1 & 1\end{pmatrix}. \tag{37}$$

Hence, the matrix C relates the "log-centered" parameters to the "base brand" parameters. The inverse transformation from the base brand specification to the log-centered specification follows from applying the Moore-Penrose inverse of C, denoted by C^+, that is,

$$C^+ = \begin{pmatrix}1-\frac{1}{I} & -\frac{1}{I} & \dots & -\frac{1}{I}\\ -\frac{1}{I} & 1-\frac{1}{I} & \dots & -\frac{1}{I}\\ \vdots & \vdots & \ddots & \vdots\\ -\frac{1}{I} & -\frac{1}{I} & \dots & 1-\frac{1}{I}\\ -\frac{1}{I} & -\frac{1}{I} & \dots & -\frac{1}{I}\end{pmatrix}. \tag{38}$$

Note that the matrix C^+ satisfies $H_{lc} = C^+ H_{bb}$.

The above shows that the transformations yield equivalent parameters. For example, assume that the log-centered form of the model is estimated, giving estimates of $H_{lc}\mu$, $H_{lc}B_k$ and $H_{lc}\Sigma H'_{lc}$. By multiplying the estimated system of equations by C we get $CH_{lc}\mu$, $CH_{lc}B_k$ and $CH_{lc}\Sigma H'_{lc}C'$ as model coefficients. Using the invariance principle of maximum likelihood and the relation $CH_{lc} = H_{bb}$, these coefficients are the maximum likelihood estimates of $H_{bb}\mu$, $H_{bb}B_k$ and $H_{bb}\Sigma H'_{bb}$. These coefficients are exactly the same as the coefficients used in the base brand specification, see (36). Using the inverse of C, the procedure can be used the other way around. We can also obtain estimates of the coefficients in a log-centered specification from the estimates in a base brand specification by multiplying them with C^+.

In our opinion, the main reason to prefer taking a base brand to reduce the model is that the statistical analysis of the resulting model is more straightforward as compared to the log-centering technique. Recall that the log-centered reduced-form model contains I equations whereas the base brand reduced-form model only has $I-1$ equations. One of the equations in the log-centered specification is however redundant. This redundancy leads to some difficulties in the estimation and interpretation, as estimation usually requires the (inverse) covariance matrix of the residuals. In the log-centering case the residuals are linearly dependent, and the covariance matrix is therefore non-invertible. Further, direct interpretation of the coefficients obtained from the base brand approach is easier as each coefficient only concerns two brands, while a coefficient in the log-centering approach always involves all brands.

5 Diagnostics

In this section we present some basic diagnostics for the market share attraction model. First of all we present a test on the normality assumption in the attraction specification. Next, we discuss tests for outliers and tests for structural breaks.

5.1 Normality

One can test the normality of each of $\hat{\eta}_1, \ldots, \hat{\eta}_{I-1}$ separately using the familiar normality test by Bowman and Shenton (1975) which is based on the skewness, denoted by $\sqrt{b_1}$, and the kurtosis, denoted by b_2, of the residuals for every brand. However, Doornik and Hansen (1994) argue that this test is unsuitable except in very large samples. Instead, they propose to use the sum of squared transformed skewness and kurtosis measures, where the transformation involved is as in D'Agostino (1970). The resultant test statistic equals

$$E_p = z_1(b_1)^2 + z_2(b_1, b_2)^2, \tag{39}$$

where $z_1(\cdot)$ and $z(\cdot, \cdot)$ are the relevant transformation functions. Under the hypothesis of normally distributed η_i, $i = 1, \ldots, I-1$, the test statistic is approximately $\chi^2(2)$ distributed. Note that the normality of η_j depends on the normality of both ε_j and ε_I. It is not however straightforward to test the normality of one of the random attraction factors. Therefore, it is easier to use a joint test on the normality of all disturbances. Doornik and Hansen (1994) show that a joint test statistic for multivariate normality can easily be obtained by summing the individual test statistics. The resulting statistic has a $\chi^2(2(I-1))$ distribution under the null hypothesis of joint normality.

5.2 Outliers

Testing for outliers in market shares is not straightforward. A sudden event in the market share of one brand is by definition accompanied by an opposite effect in the remainder of the market. Outliers in market shares can therefore not be attributed to a single brand. It is then easier to test for an outlier in attractions. To test for this in the attraction of brand j at time T_b, we simply include $\exp(D_t)$ in the attraction specification of brand j. The dummy variable D_t is defined as

$$D_t = \begin{cases} 1 & \text{if } t = T_b \\ 0 & \text{elsewhere.} \end{cases} \tag{40}$$

Note that due to the multiplicative specification of attraction we need the exponential transformation to ensure that the new variable does not affect the attraction if $t \neq T_b$. For the specification of the reduced-form model it matters whether the brand with the aberrant observation is the base brand or not. In case $j < I$, so that brand j is not the base brand, we just add the variable D_t to the reduced-form equation for $\log M_{j,t} - \log M_{I,t}$. In case the brand with the aberrant observation happens to be the base brand the variable $-D_t$ is added to the equations for $\log M_{i,t} - \log M_{I,t},\ i = 1, \ldots, I-1$, where the corresponding coefficients are restricted to be equal across the equations.

Whether the observation at T_b actually corresponds with an outlier in the attraction of brand j can now easily be tested by testing the significance of D_t in the reduced-form model. In case the observation does turn out to be an outlier, one can opt to remove the observations at T_b from the data set to prevent the outlier from influencing the estimation results. One can also choose to include the above introduced variable into the model and base the interpretation of the model on the resulting specification. In fact, the inclusion of D_t "removes" the influence of the market share at time T_b of brand j.

5.3 Structural breaks

Testing for a structural break is much like testing for outliers. To test for a structural break in the attraction of brand j starting from time T_b, one can just add the variable $\exp(D_t^*)$ to the attraction specification of brand j, with

$$D_t^* = \begin{cases} 1 & \text{if } t \geq T_b \\ 0 & \text{elsewhere.} \end{cases} \tag{41}$$

Using the same reasoning as above, the reduced-form specifications can be obtained. The significance of D_t^* in the reduced-form model indicates whether there has been a break at time T_b.

The above methodology only considers a break in the level of the attraction. The structural break can also be in the effect of one of the marketing instruments. For example, due to a repositioning of brand j, the price elasticity of this brand may change. To test for this, one can add the variable $\exp[D_t' \log(P_{j,t})]$ to the attraction specification of brand

j, and correspondingly to the reduced-form equations.

6 Forecasting

There has been considerable research on forecasting market shares using the market share attraction model. Most studies discuss the effect of the estimation technique used in combination with the parametric model specification on the forecasts, see for example Leeflang and Reuyl (1984), Brodie and de Kluyver (1984) and Ghosh *et al.* (1984), among others. More recent interest has been on the optimal model specification under different conditions, see, for example, Kumar (1994) and Brodie and Bonfrer (1994). The available literature, however, is not specific as to how forecasts of market shares should be generated. In this section we show that forecasting market shares turns out not to be a trivial exercise and that in order to obtain unbiased forecasts one has to use simulation methods.

Furthermore, in empirical applications it should be recognized that parameter values are obtained through estimation. The true parameter values are usually unknown, and parameter values are at best obtained through unbiased estimators of the true values. In a linear model this parameter uncertainty can be ignored when constructing unbiased forecasts. However, in nonlinear models this may not be true, see for example Hsu and Wilcox (2000).

6.1 Forecasting market shares

To provide some intuition why forecasting in a market share attraction model is not a trivial exercise, consider the following. The attraction model ensures logical consistency, that is, market shares lie between 0 and 1 and they sum to 1. These restrictions imply that the model parameters can be estimated from a multivariate reduced-form model with $I-1$ equations. The dependent variable in each of the $I-1$ equations is the natural logarithm of a relative market share. More formally, it is $\log m_{i,t} \equiv \log \frac{M_{i,t}}{M_{I,t}}$, for $i = 1, 2, \ldots, I-1$. The base brand I can be chosen arbitrarily.

Of course, one is usually interested in predicting $M_{i,t}$ and not in the logs of the relative market shares. It is then important to recognize that, first of all, $\exp(\mathrm{E}[\log m_{i,t}])$ is not equal to $\mathrm{E}[m_{i,t}]$ and that, secondly, $\mathrm{E}[M_{i,t}/M_{I,t}]$ is not equal to $\mathrm{E}[M_{i,t}]/\mathrm{E}[M_{I,t}]$, where E denotes the expectation operator. Therefore, unbiased market share forecasts cannot be obtained by routinized data transformations, see also Fok and Franses (2001b) for similar statements.

To forecast the market share of brand i at time t, one needs to consider the relative market shares

$$m_{j,t} = M_{j,t}/M_{I,t}, \quad j = 1, 2 \ldots, I, \tag{42}$$

as $m_{1,t}, \ldots, m_{I-1,t}$ form the dependent variables (after log transformation) in the reduced-form model (7). As $M_{I,t} = 1 - \sum_{j=1}^{I-1} M_{j,t}$, we have that

$$\begin{aligned} M_{I,t} &= \frac{1}{1 + \sum_{j=1}^{I-1} m_{j,t}} \\ M_{i,t} &= M_{I,t} m_{i,t} = \frac{m_{i,t}}{1 + \sum_{j=1}^{I-1} m_{j,t}}, \text{ for } i = 1, 2, \ldots, I-1. \end{aligned} \tag{43}$$

Note that $m_{I,t} = M_{I,t}/M_{I,t} = 1$ and hence (43) can be summarized as

$$M_{i,t} = \frac{m_{i,t}}{\sum_{j=1}^{I} m_{j,t}}, \text{ for } i = 1, 2, \ldots, I. \tag{44}$$

As the relative market shares $m_{i,t},\ i = 1, \ldots, I-1$ are log-normally distributed by assumption, see (7), the probability distribution of the market shares involves the inverse of the sum of log-normally distributed variables. The exact distribution function of the market shares is therefore complicated. Moreover, correct forecasts should be based on the expected value of the market shares, and unfortunately, for this expectation there is no simple algebraic expression. Appropriate forecasts therefore cannot be obtained from the expectations directly.

If we ignore parameter uncertainty for the moment, we need to calculate the expectations of the market shares given in (44). This cannot be done analytically. However, we can calculate the expectations using simulations. The relevant procedure works as follows. We use model (7) to simulate relative market shares for various disturbances η randomly drawn from a multivariate normal distribution with mean 0 and covariance matrix $\tilde{\Sigma}$. In each run, we compute the market shares where parameter values and the realization of the disturbance process are assumed to be given. The market shares averaged over a number of replications now provide their unbiased forecasts. Notice that we only need the parameters of the reduced-form model in the simulations.

To be more precise about this simulation method, consider the following. The one-step ahead forecasts of the market shares are simulated using

draw

$$\eta_t^{(l)} \text{ from } N(0, \tilde{\Sigma}),$$

compute

$$m_{i,t}^{(l)} = \exp(\tilde{\mu}_i + \eta_{i,t}^{(l)}) \prod_{j=1}^{I} \left(\prod_{k=1}^{K} x_{k,j,t}^{\tilde{\beta}_{k,j,i}} \prod_{p=1}^{P} \left(M_{j,t-p}^{\tilde{\alpha}_{p,j,i}} \prod_{k=1}^{K} x_{k,j,t-p}^{\tilde{\beta}_{p,k,j,i}} \right) \right), \ i = 1, \ldots, I-1,$$

with

$$m_{I,t}^{(l)} = 1$$

and finally compute

$$M_{i,t}^{(l)} = \frac{m_{i,t}^{(l)}}{\sum_{j=1}^{I} m_{j,t}^{(l)}}, \ i = 1, \ldots, I, \tag{45}$$

where $l = 1, \ldots, L$ denotes the simulation iteration and where the FE-MCI specification is used, see (4). Every vector $(M_{1,t}^{(l)}, \ldots, M_{I,t}^{(l)})'$ generated this way amounts to a draw from the joint distribution of the market shares at time t. Using the average over a sufficiently large number of draws we calculate the expected value of the market shares. By the weak law of large numbers we have

$$\operatorname*{plim}_{L \to \infty} \frac{1}{L} \sum_{l=1}^{L} M_{i,t}^{(l)} = \mathrm{E}[M_{i,t}]. \tag{46}$$

For finite L the mean value of the generated market shares is an unbiased estimator of the market share. The estimate may differ from the expected market share, but this difference is only due to simulation error and this error will rapidly converge to zero if L gets large. Of course, the value of L can be set at a very large value, depending on available computing power.

The lagged market shares in (7) are of course only available for one-step ahead forecasting and not for multiple-step ahead forecasting. Hence, one has to account for the uncertainty in the lagged market share forecasts. One can now simply use simulated values for lagged market shares, thereby automatically taking into account the uncertainty in these lagged variables. Note that we do assume that the marketing efforts of all market players are known. It is possible to also model these efforts and use the estimated model to obtain forecasts that also account for that uncertainty.

6.2 Parameter uncertainty

The model parameters usually have to be estimated from the data. This implies that the parameter estimators are random variables. If estimated parameters are used for forecasting in combination with a nonlinear model, we should also take into account the uncertainty of these estimates. To take account of the stochastic nature of the estimator, we explicitly take the expectation of the market shares over the unknown parameters.

Unfortunately, the relevant distribution of the parameters is not known. To overcome this difficulty, we propose to use parametric bootstrapping by drawing parameter vectors from the distribution. The parameter vectors are sampled using the following scheme:

- Use the estimated parameter vector, the realizations of the exogenous variables and the first P observed realizations of y as starting values to generate artificial realizations of the market shares.
- Reestimate the model based on the artificial data.

The resulting parameter vectors can be seen as draws from the small sample distribution of $\hat{b}$. Based on L draws $\hat{b}^{(1)}, \ldots, \hat{b}^{(L)}$, in general we calculate an estimate $\hat{y}_t$ as $\frac{1}{L}\sum_{l=1}^{L} \mathrm{E}[y_t|\hat{b}^{(l)}]$.

In the market share attraction model the forecasting scheme becomes more complicated as the market shares do not depend linearly on the disturbances. From (7), (42) and (44) we have $M_{i,t} = g_i(X_t, \ldots, X_{t-P}, M_{t-1}, \ldots, M_{t-P}, \eta_t, \theta)$, where X_t contains all exogenous variables at time t, $M_t = (M_{1,t}, \ldots, M_{I,t})'$, θ a vector of all unknown parameters including the parameters of the unknown covariance matrix $\tilde{\Sigma}$ and $g_i()$ is a nonlinear function. As in this case $M_{i,t}$ also nonlinearly depends on the model disturbances $\mathrm{E}[M_{i,t}|\hat{\theta}] \neq g_i(X_t, \ldots, X_{t-P}, M_{t-1}, \ldots, M_{t-P}, \mathbf{0}, \hat{\theta})$. To obtain unbiased forecasts, we therefore have to take the expectation of $g_i()$ with respect to η_t **and** $\hat{\theta}$, that is

$$\widehat{M_{i,t}} = \int_{\hat{\theta}} \int_{\eta_t} g_i(X_t, \ldots, X_{t-P}, M_{t-1}, \ldots, M_{t-P}, \eta_t, \hat{\theta}) \phi(\eta_t|\, \hat{\theta}) f(\hat{\theta}) d\eta_t d\hat{\theta}, \qquad (47)$$

where $\phi(\eta_t|\hat{\theta})$ denotes the distribution function of the (normally) distributed disturbances given the parameter estimates and $f(\hat{\theta})$ denotes the distribution function of the parameter estimator conditional on the data. Again we choose to calculate the complex integral using simulation. The parameter vectors are simulated using the bootstrap methodology described above. For every realization of $\hat{\theta}^{(l)}$ we calculate $\mathrm{E}[M_{i,t}|\, \hat{\theta}^{(l)}]$, $i = 1, \ldots, I$ using the simulation technique in Section 6.1. The average of the forecasts over all generated parameter vectors constitutes unbiased forecasts of the market shares under uncertain parameters. It is not necessary to use many simulation rounds conditional on the parameters. Theoretically it suffices to use one round for every $\hat{b}_l$.

7 Model selection

Attraction models are often considered for forecasting market shares. It is usually assumed that, by imposing in-sample specification restrictions, the out-of-sample forecasting accuracy will improve. Exemplary studies are Brodie and Bonfrer (1994), Danaher (1994), Naert and Weverbergh (1981), Leeflang and Reuyl (1984) , Kumar (1994) and Chen *et al.* (1994), among others. A summary of the relevant studies is given in Brodie *et al.* (2000). A common characteristic of these studies, an exception being Chen *et al.* (1994), is that they tend to compare one or two specific forms of the attraction model with various more naive models. In this section we consider the question of obtaining the best (or a good) choice for the specification from the wide range of possible attraction specifications. We present a general-to-simple strategy for the model selection, following Hendry (1995). In Franses and Paap (2001b) it is shown that this strategy tends to work very well in empirical applications.

The starting point of the model selection strategy is the most extended attraction model, that is, model (7) without any restrictions. Of course, in practice the size of the model is governed by data availability and sample size. The first step of a model selection strategy concerns fixing the proper lag order P of the model. It is well known that an inappropriate value of P leads to inconsistent and inefficient estimates. Lag order selection may be based on the BIC criterion of Schwarz (1978). Another strategy may be a sequential procedure, where one starts with a large value of P and tests for the significance of the $\tilde{\beta}_{P,k,j,i}$ and $\tilde{\alpha}_{P,j,i}$ parameters and imposes these restrictions when they turn out to be valid. These tests usually concern many parameter restrictions and may therefore have little power. Instead, one may therefore base the lag order determination on Lagrange Multiplier [LM] tests for serial correlation in the residuals, see Lütkepohl (1993) and Johansen (1995, p. 22). The advantage of these tests is that they concern less parameter restrictions and hence have more power. We would recommend to start with a model of order 1 and increase the order with 1 until the LM tests do not indicate the presence of any serial correlation.

Once P is fixed, we propose to test the validity of the various restrictions on (7) as proposed in Section 2.2. We test for the validity of restriction (8) on the covariance matrix $\tilde{\Sigma}$ [RCM] in model (7). Additionally, we test in model (7) for restricted dynamics [RD], common dynamics [CD], and, for each explanatory variable k, for restricted competition [RC], for restricted effects [RE] (12) and even for the absence of this variable. Finally, we propose to test for the significance of the lagged explanatory variables in the general model.

Next, we recommend to perform an overall test for all restrictions which were not rejected in the individual tests. If this joint test is not rejected, all restrictions are imposed, and this results in a final model that can be used for forecasting. However, if the joint test indicates rejection, one may want to decide to relax some restrictions, where the p-values of the individual tests can be used to decide which of these restrictions have to be relaxed.

To apply our general-to-simple model selection strategy, we have to test for restrictions on the covariance matrix $\tilde{\Sigma}$ and on the $b = (\beta, \beta_p, \alpha)$ parameters in model (7). To test these parameter restrictions, we opt for Likelihood Ratio [LR] tests, see for example Judge *et al.* (1985, p. 475). Denoting the ML estimates of the parameters under the null hypothesis by $(b_0, \tilde{\Sigma}_0)$ and the ML estimates under the alternative hypothesis by $(b_a, \tilde{\Sigma}_a)$, then

$$\text{LR} = -2(\ell(\hat{b}_a, \hat{\Sigma}_a) - \ell(\hat{b}_0, \hat{\Sigma}_0)) \underset{asy}{\sim} \chi^2(\nu), \tag{48}$$

where $\ell(\cdot)$ denotes the log-likelihood function as defined in Section 4 and where ν is the number of parameter restrictions.

8 Concluding remarks

In this chapter we have gone through part of the econometrics involved in analyzing market share attraction models. We believe that a systematic strategy enhances the possibility to compare various empirical findings and to understand deficiencies in case model forecasts turn out to be inaccurate.

There are a few more issues that need concern in future work. One of these involves the analysis of possibly differing short-run and long-run effects of marketing efforts, see Dekimpe and Hanssens (1995) and Paap and Franses (2000), among others. In Fok *et al.* (2001) we provide a first attempt in the context of a market share attraction model. Next, one may want to allow for the event of new brands entering the market or old brands leaving it. In Fok and Franses (2001a) we discuss techniques for doing so. Finally, one would want to allow for endogenous marketing efforts, like pricing strategies, which originate from attraction models.

A Estimation of restricted covariance matrix

Recall the log likelihood function (25)

$$\ell(\tilde{\Sigma}) = -\frac{T(I-1)}{2}\log(2\pi) + \frac{T}{2}\log|\tilde{\Sigma}^{-1}| - \frac{1}{2}\hat{\eta}'(\tilde{\Sigma}^{-1} \otimes \mathbf{I}_T)\hat{\eta}, \tag{49}$$

where $\tilde{\Sigma} = \text{diag}(\sigma_1^2, \ldots, \sigma_{I-1}^2) + \sigma_I^2 \mathbf{i}_{I-1}\mathbf{i}'_{I-1}$. For $i = 1, \ldots, I-1$ it holds that

$$\begin{aligned}
\frac{\partial \ell(\tilde{\Sigma})}{\partial \sigma_i} &= \left(\frac{\partial \ell(\tilde{\Sigma})}{\partial \text{vec}(\tilde{\Sigma}^{-1})}\right)' \frac{\partial \text{vec}(\tilde{\Sigma}^{-1})}{\partial \sigma_i} \\
\frac{\partial \ell(\tilde{\Sigma})}{\partial \text{vec}(\tilde{\Sigma}^{-1})} &= \frac{T}{2}\frac{\log|\tilde{\Sigma}^{-1}|}{\partial \text{vec}(\tilde{\Sigma}^{-1})} - \frac{1}{2}\frac{\partial \hat{\eta}'(\tilde{\Sigma}^{-1} \otimes \mathbf{I}_T)\hat{\eta}'}{\partial \text{vec}(\tilde{\Sigma}^{-1})} \\
&= \frac{T}{2}\text{vec}(\tilde{\Sigma}) - \frac{1}{2}\text{vec}(\begin{pmatrix} \hat{\eta}'_1 \\ \vdots \\ \hat{\eta}'_{I-1} \end{pmatrix}(\hat{\eta}_1, \ldots, \hat{\eta}_{I-1})) \\
&= \frac{1}{2}\text{vec}[T\tilde{\Sigma} - \begin{pmatrix} \hat{\eta}'_1 \\ \vdots \\ \hat{\eta}'_{I-1} \end{pmatrix}(\hat{\eta}_1, \ldots, \hat{\eta}_{I-1})] \qquad (50) \\
\frac{\partial \text{vec}(\tilde{\Sigma}^{-1})}{\partial \sigma_i} &= \text{vec}(-\tilde{\Sigma}^{-1}\frac{\partial \tilde{\Sigma}}{\partial \sigma_i}\tilde{\Sigma}^{-1}) = -(\tilde{\Sigma}^{-1})^2_{ii} e_{i,I-1} \\
\frac{\partial \ell(\tilde{\Sigma})}{\partial \sigma_i} &= \frac{1}{2}\text{tr}[-T\tilde{\Sigma}(\tilde{\Sigma}^{-1})^2_{ii} e_{i,I-1} + \begin{pmatrix} \hat{\eta}'_1 \\ \vdots \\ \hat{\eta}'_{I-1} \end{pmatrix}(\hat{\eta}_1, \ldots, \hat{\eta}_{I-1})(\tilde{\Sigma}^{-1})^2_{ii} e_{i,I-1}] \\
&= \frac{1}{2}[-T(\tilde{\Sigma})_{ii}(\tilde{\Sigma}^{-1})^2_{ii} + \hat{\eta}'_i\hat{\eta}_i(\tilde{\Sigma}^{-1})^2_{ii}] \\
&= \frac{1}{2}(\tilde{\Sigma}^{-1})^2_{ii}[\hat{\eta}'_i\hat{\eta}_i - T(\sigma_i^2 + \sigma_I^2)]
\end{aligned}$$

where $e_{i,k}$ is a zero vector of size $(k \times 1)$ with the i-th element equal to 1. Solving the last equation given $\hat{\sigma}_I^2$ yields

$$\hat{\sigma_i}^2 = \frac{\hat{\eta}'_i\hat{\eta}_i}{T} - \hat{\sigma}_I^2 \tag{51}$$

The concentrated likelihood is obtained by inserting (51) into the likelihood (49). The concentrated likelihood now has to be optimized over just one parameter, that is σ_I.

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