Nonparametric Efficiency Estimation in Stochastic Environments (II):

Noise-to-Signal Estimation, Finite Sample Performance and Hypothesis Testing

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NONPARAMETRIC EFFICIENCY ESTIMATION IN STOCHASTIC ENVIRONMENTS (II):

NOISE-TO-SIGNAL ESTIMATION, FINITE SAMPLE PERFORMANCE AND HYPOTHESIS TESTING.

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ABSTRACT

We consider the issues of noise-to-signal estimation, finite sample performance and hypothesis testing for the nonparametric efficiency estimation technique proposed in Cherchye, L., T. Kuosmanen and G. T. Post (2001) 'Nonparametric efficiency estimation in stochastic environments', forthcoming in *Operations Research*. In addition, we apply the technique for analyzing European banks.

KEY WORDS: nonparametric efficiency estimation, noise-to-signal estimation, finite sample performance, hypothesis testing, European banks.

1. INTRODUCTION

Empirical techniques for analyzing firm efficiency can be roughly categorized into parametric vs. non-parametric methods and deterministic vs. stochastic methods. The nonparametric techniques can model firm behavior without assuming a functional form for the production frontier or the statistical distribution of inefficient deviations from the frontier. This is an attractive feature, because production theory generally does not imply particular functional forms or statistical distributions, and reliable empirical specification tests are not available in many cases. The other side of the coin is a possible lack of power especially in small samples; the data can 'speak for themselves' only 'if there is a story to tell', i.e. if the sample includes many efficient observations for a wide range of production vectors. Deterministic techniques assume full accurate measurement. By contrast, stochastic techniques account for the possibility of errors-in-variables, e.g. due to debatable valuation and depreciation schemes for accounting data or due to uncontrollable external factors. The most popular techniques are (1) stochastic frontier analysis (SFA; see Kumbhakar and Lovell, 2000), which is parametric and stochastic, and (2) data envelopment analysis (DEA; see Cooper *et al.*, 2000), which is nonparametric and deterministic.

Cherchye, Kuosmanen and Post (CKP; 2001) introduced a novel nonparametric and stochastic technique. CKP demonstrated that the approach is asymptotically unbiased and has an asymptotic variance that is comparable to that of the estimators used in SFA (provided the latter use a correct specification). In addition, the CKP approach is computationally attractive, as the efficiency estimates can be computed using a simple enumeration algorithm. This is an important feature, because computational intensity is one of the key reasons why nonparametric regression methods have not yet infiltrated the applied literature to the degree that one might expect (see e.g. Yatchew, 1998).

CKP provided a structured research program for further analysing and developing this new technique. In this paper, we address three (strongly related) research topics of that program:

ESTIMATING THE NOISE-TO-SIGNAL RATIO

The CKP procedure requires an estimate for the noise-to-signal ratio prior to the analysis. CKP suggested different routes for obtaining such an estimate, including empirical estimation, cross-validation and sensitivity analysis. However, they left a detailed treatment of this issue for further research. This paper presents some important considerations about the relevance of good noise-to-signal ratio estimators.

ANALYZING FINITE SAMPLE PERFORMANCE

The attractive asymptotic properties of the CKP procedure relate to large sample performance only. Techniques that impose little a priori structure can involve substantial finite sample error, because they rely heavily on the data density near the frontier. Therefore, it is interesting to analyze the statistical goodness in finite samples. This paper presents the outcomes of a Monte-Carlo simulation study of the statistical goodness in finite samples of the CKP technique relative to DEA and SFA.

HYPOTHESIS TESTING

Like all techniques for efficiency analysis, the CKP estimates involve substantial error variance (even in large samples) if the noise-to-signal ratio is high. For this reason, it is important to develop hypothesis tests that can assess whether or not observed differences are statistically significant. This paper develops such tests. To remain consistent with the nonparametric orientation, we focus on tests that do not impose strong assumptions about the statistical distribution of the inefficiency terms.

CKP focused on cardinal measurement of inefficiency. However, in many cases, ordinal ranking is equally informative as cardinal measurement. First, ranks allow for hypothesis testing in the nonparametric fashion, i.e. without imposing strong assumptions about the statistical distribution of the inefficiency terms. Second, inefficiency estimates are frequently transformed in a non-linear way, i.e. to obtain the Debreu-Farrell measure for inefficiency in percentage terms (see also our application in Section 6). If non-linear transformations are used (e.g. for the Debreu-Farrell measure), then cardinal measures are no longer meaningful. By contrast, ranks remain meaningful (provided the transformations are monotone). Ondrich and Ruggiero (2001) used a similar argument in their assessment of parametric efficiency estimation techniques. Third, in many practical applications, efficiency analysis is used for attention direction, i.e. for identifying problem cases or for setting priorities for follow-up analysis. For such purposes, the ranks are equally informative as the true values. For these reasons, we focus on inefficiency in terms of ordinal ranking rather than cardinal measurement in our analysis. In addition, we use as a goodness measure the statistical association between the rank order of the estimated values of the inefficiency terms and the true values (e.g. measured by Spearman's rho or Kendall's tau) rather than Pearson product-moment correlation or linear correlation (as used in CKP).

The remainder of this paper unfolds as follows. Section 2 briefly recaptures the CKP method. Section 3 discusses the role of good noise-to-signal ratio estimators for the goodness of the CKP estimators in terms of the statistical association for ranks. Section 4 presents a Monte-Carlo simulation study of the finite sample performance of the CKP method (in terms of Spearman's rho) relative to DEA and SFA. Section 5 considers hypothesis testing; we demonstrate that two well-established nonparametric tests (the Wilcoxon rank sum test and the Kruskal-Wallis test) apply in large samples. Section 6 illustrates these tests using an empirical application for large European Union banks. Finally, Section 7 presents our conclusions.

2. THE CKP TECHNIQUE

For each firm in the data set $J \equiv \{1,...,n\}$, we consider observations on a single output $y_j \in \mathbb{R}_+$ and multiple inputs $x_j \equiv (x_1...x_m) \in S$, where S is a convex subset of $\mathbb{R}_+^m$. Theoretically, the efficiency of the firms is defined relative to the efficient production frontier $f : S \to \mathbb{R}_+$.[1] No assumptions on the form of the frontier are imposed, apart from *smoothness*, i.e. $\lim_{e \to 0} |f(x+e) - f(x)| = 0$ for all x in the interior of S. Observed output can deviate from the frontier, because of an inefficiency term $u_j \in \mathbb{R}_+$ and a disturbance term $v_j \in \mathbb{R}$, i.e.:

(1) $$y_j = f(x_j) - u_j + v_j \quad j \in J\,.$$

The inputs, inefficiency terms and disturbances are treated as independent, continuous random variables that are homoskedastic across firms. The inefficiency terms have a mean m and standard deviation $s_u \in \langle 0, \infty \rangle$, and the disturbances have a zero mean and standard deviation $s_v \in \langle 0, \infty \rangle$.

Since reliable estimates of inefficiency in absolute terms can not be obtained without strong a priori assumptions about the statistical distribution of the inefficiency terms and the disturbance terms, CKP focused on estimating inefficiency relative to the mean rather than in absolute terms, i.e.:

(2) $$w_j \equiv u_j - m \quad j \in J\,.$$

[1] As discussed in CKP, the technique can account for multiple outputs in the context of estimating cost, revenue or profit functions. For example, the application in Section 6 estimates a cost function rather than a single-output production frontier.

As an estimator for relative efficiency for firm $j \in J$, CKP used the difference between the output of the evaluated firm and a reference output constructed as a weighted average of the outputs of all firms in the sample, i.e.

$$(3) \qquad \hat{w}_{CKP,j}\left(\hat{s}_v/\hat{s}_u\right) \equiv \sum_{i\in J} l_{ij}\left(\hat{s}_v/\hat{s}_u\right) y_i - y_j ,$$

where the weights $l_j\left(\hat{s}_v/\hat{s}_u\right) \equiv \left(l_{1j}\left(\hat{s}_v/\hat{s}_u\right) \cdots l_{nj}\left(\hat{s}_v/\hat{s}_u\right)\right)$ are selected by minimizing a penalized estimate for the (standardized) error variance, i.e.:[2]

$$(4) \qquad l_j\left(\hat{s}_v/\hat{s}_u\right) \equiv \arg\min\left\{\sum_{i\in J} l_{ij}^2 + \left(\sum_{i\in J} l_{ij}^2 - 2l_j + 1\right)\left(\hat{s}_v^2/\hat{s}_u^2\right) + \sum_{i\in J} l_{ij} x(x_i, x_j) \,\middle|\, \sum_{i\in J} l_{ij} = 1;\ l_j \in \square^n_+ \right\}.$$

In this expression $\left(\hat{s}_v/\hat{s}_u\right) \in \langle 0, \infty\rangle$ denotes the a priori estimate for the noise-to-signal ratio $\left(s_v/s_u\right)$. Further, $x(x_i, x_j)$ measures the multidimensional distance of the input vector of firm $i \in J$ from the input vector of firm $j \in J$. The selection of the appropriate distance function asymptotically does not affect the efficiency estimates. Still, the distance function may be relevant for small sample performance. As discussed in CKP, cross-validation and sensitivity analysis can guide in the selection of the appropriate measure. We leave this issue for further research and simply use the squared Mahalanobis (1930) distance in the simulations (Section 4) and application (Section 6). This measure standardizes each variable to zero mean and unit variance by using the covariance matrix in the distance calculation:

$$(5) \qquad x(x_i, x_j) = (x_i - x_j)\hat{\Sigma}^{-1}(x_i - x_j)^T ,$$

where $\hat{\Sigma}^{-1}$ is the inverted sample covariance matrix.

As discussed in CKP, the noise-to-signal ratio can be estimated using parametric estimation techniques. In addition, cross-validation and sensitivity analysis can guide in the selection of the appropriate estimator. Finally, one can use prior opinions concerning the precision with which the data have been measured. Those opinions could reflect knowledge about the industry under evaluation or the methods used for collecting the data. For example, if firms have substantial flexibility in allocating costs across different periods and different activities, a relatively high noise level seems appropriate. Similarly, a high noise level seems appropriate in case the data set includes survey data to quantify qualitative variables like service quality or customer

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[2] The original model formulation also included a smoothing parameter that represents the trade-off between the variance term and the distance term. For simplicity, we ignore this term, because any change in the smoothing parameter can equivalently be expressed as a change in the distance measure. In addition, we have divided the original objective function by $\hat{s}_u^2 > 0$. This does not affect the optimal weights. However, it does allow for a transparent formulation in terms of the noise-to-signal ratio.

satisfaction. Of course, prior opinions in most cases can not give an accurate estimate, and hence it is relevant to analyze the impact of inaccuracies.

3. THE NOISE-TO-SIGNAL RATIO ESTIMATE

CKP demonstrated that the a priori estimate for the noise-to-signal ratio is an important determinant for statistical goodness in terms of linear correlation. Specifically, the Pearson product-moment correlation coefficient is given by

$$(6) \qquad r_P\left(\hat{s}_v / \hat{s}_u\right) = \sqrt{1 - \frac{s_v^2}{s_u^2}\left[1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right]^2 - \left[\frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right]^2},$$

and achieves a minimum if the noise-to-signal ratio is estimated correctly, i.e. $\left(\hat{s}_v / \hat{s}_u\right) = \left(s_v / s_u\right)$.

However, the noise-to-signal ratio is less important if we adopt as a goodness measure the statistical association between the ranks of the estimated values of the inefficiency terms and the true values. Define the rank order of $\hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right) \quad j \in J$ as:

$$(7) \qquad \hat{r}_j\left(\hat{s}_v / \hat{s}_u\right) \equiv \text{card}\left\{i \in J : \hat{w}_{CKP,i}\left(\hat{s}_v / \hat{s}_u\right) \leq \hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right)\right\},$$

and define the rank order of the true inefficiency term $w_j \quad j \in J$ as:

$$(8) \qquad r_j \equiv \text{card}\left\{i \in J : w_i \leq w_j\right\}.$$

The distribution of the inefficiency estimates is continuous and hence no ties will occur. However, due to measurement problems or rounding, ties may occur. A method to correct for ties is to assign to each tied value the average of the ranks that would have been assigned to the value if no ties were present (see e.g. Daniel, 1978).

To analyse how the noise-to-signal ratio estimate affects the association in terms of ranks, we first give the following corollary to the asymptotic results derived in CKP:

COROLLARY 1 For all $j \in J$, $\lim_{n \to \infty} \hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right) = \left(w_j - v_j\right)\left(1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right)$.

PROOF For each evaluated firm $j \in J$, the reference output is computed as a weighed average with 'very small weights' for 'a very large number' of observations 'very close to' the evaluated firm, i.e.:

(i) $$\begin{cases} \lim_{n\to\infty} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) = 0 \quad x(x_i, x_j) \geq d \\ \dfrac{\hat{s}_v^{\,2}}{\hat{s}_v^{\,2} + \hat{s}_u^{\,2}} \leq \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right) \leq \dfrac{\hat{s}_v^{\,2}}{\hat{s}_v^{\,2} + \hat{s}_u^{\,2}} + e \\ \lim_{n\to\infty} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) \leq e \quad x(x_i, x_j) \leq d, i \neq j \end{cases}$$

with e and d for non-Archimedean infinitesimal small positive values (see CKP, equation (18) and Proof B). This equation, $\sum_{i\in J} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) = 1$ and smoothness, i.e. $\lim_{e\to 0}\left| f(x+e) - f(x)\right| = 0$, imply:

(ii) $$\lim_{n\to\infty} \sum_{i\in J\setminus j} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) f(x_i) = \left(1 - \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)\right) f(x_j)\,.$$

Since u_i and v_i $i \in J$ are independently and identically distributed random variables with mean m and 0 respectively, we find

(iii) $$\lim_{n\to\infty} \sum_{i\in J\setminus j} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) u_i = \left(1 - \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)\right) m\,;$$

and

(iv) $$\lim_{n\to\infty} \sum_{i\in J\setminus j} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) v_i = 0\,.$$

Combining (ii), (iii), and (iv), we find

(v) $$\begin{aligned} \lim_{n\to\infty} \hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right) &= \lim_{n\to\infty} \sum_{i\in J} l_{ij}\left(\hat{s}_v / \hat{s}_u\right) y_i - y_j \\ &= \lim_{n\to\infty} \sum_{i\in J} l_{ij}\left(\hat{s}_v / \hat{s}_u\right)\left(f(x_j) - u_j + v_j\right) - y_j \\ &= \lim_{n\to\infty} \sum_{i\in J\setminus j} l_{ij}\left(\hat{s}_v / \hat{s}_u\right)\left(f(x_j) - u_j + v_j\right) - \left(1 - \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)\right) y_j \\ &= \left(1 - \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)\right)\left(f(x_j) - m - y_j\right) \\ &= \left(1 - \lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)\right)\left(w_j - v_j\right). \end{aligned}$$

Substituting $\lim_{n\to\infty} l_{jj}\left(\hat{s}_v / \hat{s}_u\right)$ with $\dfrac{\hat{s}_v^{\,2}}{\hat{s}_v^{\,2} + \hat{s}_u^{\,2}}$ gives

(vi) $$\lim_{n\to\infty} \hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right) = \left(w_j - v_j\right)\left(1 - \frac{\hat{s}_v^{\,2}}{\hat{s}_v^{\,2} + \hat{s}_u^{\,2}}\right).$$

This corollary has important implications for the goodness of the noise-to-signal estimates:

THEOREM 1 The goodness of the noise-to-signal ratio asymptotically does not affect the rank order of the CKP inefficiency estimates, i.e.
$\hat{r}_j\left(\hat{s}_v / \hat{s}_u\right) = \hat{r}_j\left(g\hat{s}_v / \hat{s}_u\right) \quad \forall g > 0\,.$

PROOF Corollary 1 implies that multiplication of the estimated noise-to-signal ratio with a positive constant asymptotically yields a multiplication of the estimator with a positive constant, i.e.

(i) $$\lim_{n\to\infty} \hat{w}_j\left(g\hat{s}_v / \hat{s}_u\right) = b \lim_{n\to\infty} \hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right),$$

with $b \equiv \left(1 - \frac{g\hat{s}_v^2}{g\hat{s}_v^2 + \hat{s}_u^2}\right)\left(1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right)^{-1} > 0$ for all $g > 0$.

The ranking is not affected by scalar multiplication i.e.

(ii) $\quad \text{card}\{i \in J : \hat{w}_{CKP,j}(\hat{s}_v/\hat{s}_u) \leq \hat{w}_{CKP,j}(\hat{s}_v/\hat{s}_u)\} =$

$\quad \text{card}\{i \in J : b\hat{w}_{CKP,i}(\hat{s}_v/\hat{s}_u) \leq b\hat{w}_{CKP,j}(\hat{s}_v/\hat{s}_u)\} \quad \forall b > 0$

and hence

(iii) $\quad \hat{r}_j(\hat{s}_v/\hat{s}_u) = \hat{r}_j(g\hat{s}_v/\hat{s}_u) \quad \forall g > 0$.

The result implies that CKP overestimated the importance of good a priori estimates for the noise-to-signal ratio; the estimated ratio asymptotically does not affect the goodness of the inefficiency estimates in terms of the statistical association between the ranking based on the estimators and the true ranking, e.g. Spearman's rho or Kendall's tau. Similarly, the estimated ratio asymptotically does not affect the outcome of hypothesis tests that are based on rank order, including the Wilcoxon rank sum test and Kruskal-Wallis test used in Section 5. Still, the goodness of the estimated noise-to-signal ratio may affect small sample performance, and therefore further research could focus on analysing the effects of estimation error in small samples. In the simulation below we will use the 'naïve' value of unity. The above theorem suggests that this 'naïve' value (in large samples) should not affect the goodness of the inefficiency estimates or the outcome of ranking based hypothesis tests.

4. SIMULATING FINITE SAMPLE PERFORMANCE

At least the following dimensions are relevant for analysing the statistical goodness of frontier efficiency techniques: the form of the frontier, the number of input variables, the statistical distribution of the inputs, the statistical distribution of inefficiency terms, the sample size, and the value of the noise-to-signal ratio. As discussed above, for the CKP approach, also the goodness of the estimated noise-to-signal ratio is relevant. We have little hope for analytically deriving the impact of these dimensions and therefore resort to Monte-Carlo simulations. Unfortunately, there currently is no generally accepted framework for performing Monte-Carlo studies for empirical efficiency analysis. It is not clear what simulation conditions best describe real-life problems. This is a cause for concern, because the simulation results can have little significance for research practice if the simulations do not represent a wide range of real-life research environments. Also, different studies are difficult to compare without a generally accepted framework. In this study we use simulation conditions tailored to our research objective: analyzing the effects of sample size and the noise-to-signal ratio.

The possibilities for varying the form of the frontier and the number of input variables are unlimited. This paper focuses on a single technology exclusively. One of the attractions of the CKP method is that it does not assume that the frontier is monotone increasing and hence can account for congestion. Brockett *et al.* (1998), Cooper *et al.* (2001) and Cherchye *et al.* (2001) used an example technology to discuss congestion in the context of DEA. We use that example technology in our simulations. The technology has the shape of a three dimensional polytope involving a single output and two-inputs. The domain of inputs is partitioned in 4 subdomains:

$$(9)\quad \begin{aligned} \Theta_1 &\equiv \left\{(x_1, x_2) \in \mathbb{R}^2_+ : x_2 \geq -x_1 + 6;\ x_2 \leq 5;\ x_1 \leq 5\right\} \\ \Theta_2 &\equiv \left\{(x_1, x_2) \in \mathbb{R}^2_+ : x_2 \geq 1.25 x_1 + 3.75;\ x_2 \geq 5;\ x_1 \leq 5\right\}. \\ \Theta_3 &\equiv \left\{(x_1, x_2) \in \mathbb{R}^2_+ : x_2 \leq -x_1 + 15;\ x_2 \geq 5;\ x_1 \geq 5\right\} \\ \Theta_4 &\equiv \left\{(x_1, x_2) \in \mathbb{R}^2_+ : x_2 \leq 0.8 x_1 - 3;\ x_2 \leq 5;\ x_1 \geq 5\right\} \end{aligned}$$

Frontier output corresponds to[3]

$$(10)\quad f(x_{1j}, x_{2j}) = \begin{cases} -2.5 + 2.25x_1 + 2.25x_2 & (x_1, x_2) \in \Theta_1 \\ 17.75 + 2.25x_1 - 1.8x_2 & (x_1, x_2) \in \Theta_2 \\ 38 - 1.8x_1 - 1.8x_2 & (x_1, x_2) \in \Theta_3 \\ 17.75 - 1.8x_1 + 2.25x_2 & (x_1, x_2) \in \Theta_4 \end{cases}$$

Figure 1 displays the technology (the coordinates $(x_1, x_2, f(x_1, x_2))$ of the extreme points are given within brackets).

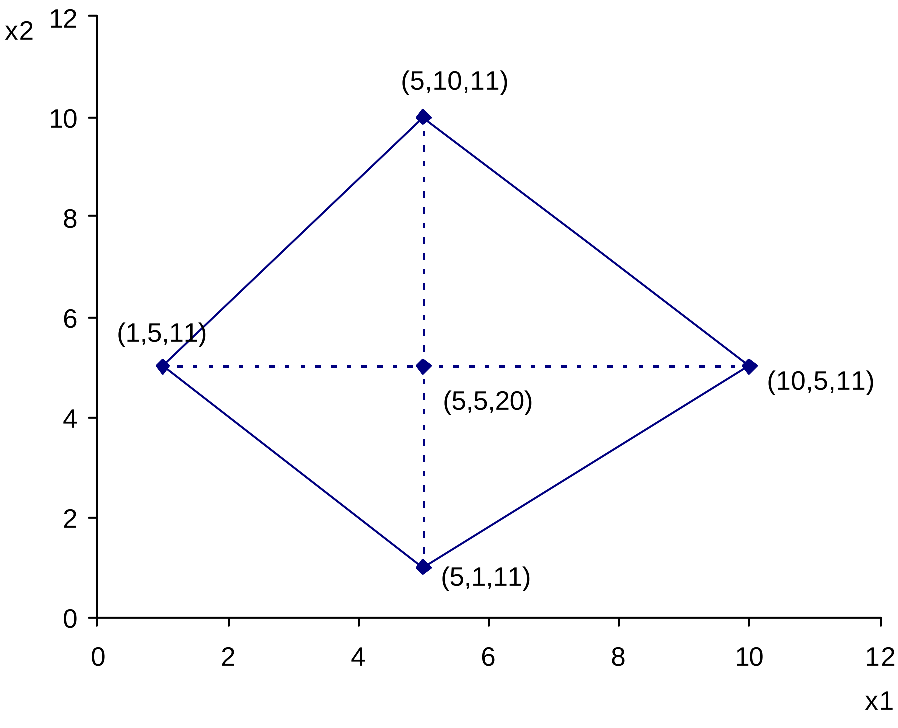

Figure 1 Source: Brockett, P.L., W.W. Cooper, H.C. Shin, and Y. Wang (1998): Inefficiency and Congestion in Chinese Production Before and After the 1978 Economic Reforms, *Socio-Economic Planning Sciences* 32, 1-20

[3] For convenience, we shifted up the Brockett *et al*. frontier with scalar 10, so as to exclude negative values for the outputs.

The following frontier model represents the data generating process:

(11) $$y_j = f(x_{1j}, x_{2j}) - u_j + v_j$$
$$(x_{1j}, x_{2j}) \sim U(\Theta)$$
$$u_j \sim |N(0,2)|$$
$$v_j \sim N(0, s_v)$$
$$j = 1,\ldots,n$$

Inputs are drawn from a bivariate uniform distribution on the union of the subdomains $\Theta \equiv \bigcup_{i \in \{1,2,3,4\}} \Theta_i$. Output is computed by first calculating the frontier output from the inputs and subsequently subtracting an efficiency term and adding a disturbance term. The inefficiencies are the absolute values of a variable with mean 0 and standard deviation 2; i.e. they follow a half normal distribution with mean 1.6 and standard deviation 1.2.[4] With these values, the simulated average of inefficiency as a percentage of output equals approximately 12.5%. The disturbances follow a normal distribution with mean 0 and standard deviation s_v. Standard deviations of 0.12, 0.3, 0.6 and 1.2 are considered (associated with noise-to-signal ratios of 0.1, 0.25, 0.5 and 1 respectively). The sample size *n* is set at 20, 50, 100 and 1000.

Each experiment consists of generating a set of artificial data from the above data-generating process, and employing these data to compute each of three different estimators (see below). For each of the combinations of sample size and noise-to-signal ratio, 10,000 experiments are done. Next, these estimates are used to gauge the finite sample performance of the competing estimators. Goodness is gauged by Spearman's rank order correlation coefficient or Spearman's rho:

(12) $$r_S = 1 - \frac{6\sum_{j \in J}\left(\hat{r}_j\left(\hat{s}_v / \hat{s}_u\right) - r_j\right)}{n(n^2 - 1)}.$$

As was first demonstrated by Kendall (1948), if there are no ties (or if ties are corrected using the method discussed in Section 3), then Spearman's rho is equivalent to the traditional Pearson product-moment correlation coefficient applied to the rank order of the estimated values of the inefficiency terms and the true values.

We consider three different estimators for the inefficiency term. First, we consider the CKP estimator $\hat{w}_{CKP,j}\left(\hat{s}_v / \hat{s}_u\right)$ with the estimated noise-to-signal ratio set at the 'naïve' value of unity. Second, we consider the following additive, congestion-adjusted DEA estimator:

(13) $$\hat{w}_{DEA,j} \equiv \max\left\{\sum_{i \in J} l_{ij} y_i - y_j \,\middle|\, \sum_{i \in J} l_{ij} x_{1i} = x_{1j}; \sum_{i \in J} l_{ij} x_{2i} = x_{2j}; \sum_{i \in J} l_{ij} = 1;\ l_j \in \square_+^n \right\}.$$

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[4] If a variable z is distributed half-normally, i.e. $z \sim |N(0, s)|$, then its mean is given by $E(z) = s\sqrt{2/p} \approx 0.8s$ and its standard deviation by $s_z = \sqrt{(p-2)/p}\, s \approx 0.6s$.

Third, we consider a SFA estimator based on a quadratic functional form. Ondrich and Ruggiero (2001) demonstrated that deterministic parametric techniques perform equally well as SFA (in terms of rank correlation), because the error decomposition does not affect the goodness of SFA inefficiency estimates. Since we compare the techniques in terms of rank correlation, ordinary least squares estimates can be expected to be equally good as SFA (see Ondrich and Ruggiero). Therefore, we use the following estimator:

(14) $$\hat{w}_{SFA,j} \equiv (a_0 + a_1 x_{1j} + a_2 x_{2j} + a_3 x_{1j}^2 + a_4 x_{2j}^2 + a_5 x_{1j} x_{2j}) - y_j ,$$

with the parameters $a_i \quad i = 1,...,5$ selected to minimize the sum of squares $\sum_{i \in J} \hat{w}_{SFA,i}^2$.

The quadratic functional form is a flexible functional form that can give a second-order Taylor series approximation for an arbitrary twice-differentiable frontier. Note however, that the frontier specification (9) consists of four different facets and is not continuously differentiable. A quadratic function is not flexible enough to approximate this frontier with full accuracy. Therefore, the SFA technique is not evaluated under most favourable conditions.

Table 1 gives the simulation results in terms of Spearman's rho.

Table 1 Simulation results*

		$\frac{s_v}{s_u}$=0.10	$\frac{s_v}{s_u}$=0.25	$\frac{s_v}{s_u}$=0.50	$\frac{s_v}{s_u}$=1.00
N=20	CKP	0.599	0.561	0.543	0.424
	DEA	0.533	0.458	0.436	0.291
	SFA	0.168	0.161	0.143	0.138
N=50	CKP	0.734	0.696	0.588	0.530
	DEA	0.659	0.608	0.561	0.380
	SFA	0.253	0.238	0.229	0.208
N=200	CKP	0.853	0.829	0.761	0.611
	DEA	0.834	0.811	0.707	0.520
	SFA	0.335	0.327	0.313	0.288
N=1000	CKP	0.928	0.893	0.827	0.650
	DEA	0.928	0.890	0.789	0.571
	SFA	0.384	0.379	0.367	0.353

*) Each cell (combination of noise-to-signal ratio and sample size) contains Spearman's rho (12) for the CKP, DEA and SFA estimators. The simulation is based on the data generating process (11). Random numbers were generated using the *GAUSS* software by *Aptech Systems*.

As is true for the outcomes of all statistical estimation techniques, the inefficiency estimates of all three techniques deteriorate for high noise-to-signal ratios and small samples. However, inefficiency estimation is more difficult than 'ordinary' estimation problems. Inefficiency estimation compares (1) frontier output with (2) firm output (see equations (3), (13) and (14)). Errors-in-variables and sampling error reduce the goodness of the estimated frontier. Still, the frontier can be approximated with high accuracy in large samples. By contrast, firm output cannot be measured accurately in case of errors, because the data set (in cross-section studies) contains just a single observation for the evaluated firm. Therefore, efficiency estimates are not statistically consistent; they involve an intrinsic variance that does not disappear in large samples. This is true for parametric techniques with a correct specification (see Waldman, 1984), and even more so for nonparametric techniques (or parametric techniques with an erroneous specification). This calls for caution when interpreting the results of efficiency estimation techniques. Still, the following considerations imply that the techniques can add value in many research situations:

- Large data sets are available for many industries of current interest. For example, applications in the area of financial services typically use data sets with thousands of observations (see Berger and Humphrey, 1997, for a survey).

- Using panel data can improve the noise-to-signal ratio.

- Discriminating power can be increased by introducing additional production information like disposability, convexity or returns-to-scale assumptions, and by using price data to aggregate variables into monetary aggregates like cost, revenue or profit.

- Even if the individual inefficiency estimates are poor, they can be useful in the context of testing hypotheses about the central tendency of inefficiency for groups of firms (see Section 5).

The SFA technique in all cases performs worse than the nonparametric estimators. Presumably, the polyhedral shape of the frontier is too complex for an accurate second-order Taylor series approximation. The SFA approach is not evaluated under most favourable conditions; to demonstrate the full potential of the parametric approach, we could have used a simpler functional form for the true frontier or a more flexible functional form for the estimation (e.g. a polynomial function of higher order or a Fourier flexible form). However, the poor performance of the SFA with quadratic functional form for this technology clearly indicates the value added of the nonparametric approach as a complement to the parametric approach especially if little prior information is available and if large data sets are available.

Ondrich and Ruggiero (2001) demonstrated that deterministic parametric techniques perform equally well (in terms of rank correlation) as the parametric stochastic SFA technique for a wide range of distributions for the disturbance terms (such as the normal, logistic or Laplace distributions). By contrast, our results suggest that a stochastic nonparametric approach can improve upon a deterministic nonparametric approach; the CKP approach outperforms the DEA approach in all cases. For the case with a large sample (*n*=1000) and a low noise-to-signal ratio ($\frac{s_v}{s_u}$ =0.10), both techniques involve a rank correlation of approximately 93 percent. Lowering the sample size and increasing the noise-to-signal ratio substantially reduces the goodness of both techniques. However, the CKP approach is much more robust, especially with respect to increases in the noise-to-signal ratio.

Finally, the DEA model is evaluated under relatively favorable conditions; we used a convex and congestion-adjusted model for a convex and congested technology. Specification error can seriously reduce the goodness of DEA estimators. For example, the standard DEA model (see e.g. Banker, Charnes and Cooper, 1984, and Banker, 1993) imposes both convexity and free disposability (i.e. no congestion), and would presumably perform worse in our simulations than the congestion-adjusted model. Unfortunately, technology properties like convexity and monotonicity can usually not be verified in practical situations (see the original CKP study for further discussion). Therefore, we believe that in practice the difference between the CKP technique (that does not require these technology assumptions) and the DEA technique will usually be even more pronounced than in the above simulations.

5. HYPOTHESIS TESTING

Hypothesis tests for firm-specific inefficiency are problematic because the variance level for individual efficiency estimates typically is too high to reach significant conclusions (even in large samples), especially in small samples and if the noise-to-signal ratio is high (see Section 4). Still, hypotheses can be tested about the differences between groups of firms in central tendency (e.g. mean, mode or median) of inefficiency. Many research questions can be phrased in terms of such hypotheses. For example, returns-to-scale can be tested by e.g. comparing the central tendency of firms in different size categories. In addition, comparing the central tendency of different ownership types can test the impact of ownership structure. Banker (1993) considered this type of hypotheses in the context of DEA.

We partition the sample *J* into *h* exclusive and exhaustive subsamples J_i of size n_i, $i \in H \equiv \{1, \cdots, h\}$, and we consider hypotheses about differences across subsamples in median inefficiency. For that purpose, we propose to use the Wilcoxon rank sum test (or Mann-Whitney U test) and the Kruskal-Wallis test (see e.g. Lehman, 1975). These tests are of nonparametric nature and do not require strong assumptions about the statistical distribution of the estimators. In addition, the tests are formulated in terms of ranks, and hence the goodness of the a priori estimate for the noise-to-signal ratio does not affect the test results for large samples (see Theorem 1). Precondition for these tests is that the estimators are identically and independently distributed. The CKP estimators asymptotically satisfy this requirement:

THEOREM 2 The CKP estimators asymptotically are independent and have an identical distribution with zero mean and variance $(s_u^2 + s_v^2)\left[1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right]^2$.

PROOF From Corollary 1,

(i) $$\lim_{n\to\infty} \hat{w}_{CKP,j}\left(\hat{s}_v/\hat{s}_u\right) = \left(w_j - v_j\right)\left(1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right)$$

$$= \left(u_j - m - v_j\right)\left(1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right).$$

As discussed in Section 2, u_j and v_j are independently and identically distributed random variables with mean m and zero respectively and variance s_u^2 and s_v^2 respectively. Therefore, $\lim_{n\to\infty} \hat{w}_{CKP,j}\left(\hat{s}_v/\hat{s}_u\right)$ are independent and have zero mean and variance $(s_u^2 + s_v^2)\left[1 - \frac{\hat{s}_v^2}{\hat{s}_v^2 + \hat{s}_u^2}\right]^2$. ¦

We first consider the hypothesis of equal median inefficiency for two subsamples, say J_1 and J_2, where J_1 represents the subset of smallest size. The Wilcoxon rank sum test is an appropriate test for this purpose. This test uses as a test statistic the sum of ranks (computed relative to the combined sample) of estimates in the smaller subsample:

(15) $$W\left(\hat{s}_v/\hat{s}_u\right) \equiv \sum_{j\in J_1} \hat{r}_j\left(\hat{s}_v/\hat{s}_u\right)$$

In many cases, a normal approximation to the distribution of $y\left(\hat{s}_v/\hat{s}_u\right)$ can be used. In particular, the normalized statistic $W^*\left(\hat{s}_v/\hat{s}_u\right) \equiv \dfrac{W\left(\hat{s}_v/\hat{s}_u\right) - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}}$

approximately obeys the standard normal distribution if $n_1 \geq 5$ and $n_2 \geq 10$. Hence, using $\Phi(\cdot)$ to denote the cumulative standard normal distribution function, the hypothesis of equal median inefficiency is rejected at level of significance of $a \in [0,1]$ if $W^*\left(\hat{s}_v/\hat{s}_u\right) < \Phi^{-1}(a/2)$ or $W^*\left(\hat{s}_v/\hat{s}_u\right) > \Phi^{-1}(1-a/2)$.

In some cases, it is interesting to test the hypothesis that all subsamples J_i $i = 1,\cdots,h$ have the same median inefficiency. For this purpose, the Kruskal-Wallis test can be used. This test uses as a test statistic the weighted squared deviation of the average rank (relative to the full sample) of the subsamples from the average of the full sample:

(16) $$KW\left(\hat{s}_v/\hat{s}_u\right) \equiv \sum_{i\in H} n_i \left(\sum_{j\in J_i} \hat{r}\left(\hat{s}_v/\hat{s}_u\right)/n_i - (n+1)/2\right)^2$$

The standardized statistic $KW^{*}\left(\hat{s}_{v}/\hat{s}_{u}\right) \equiv \left[\frac{12}{n(n+1)}\right] KW\left(\hat{s}_{v}/\hat{s}_{u}\right)$ approximately obeys a chi-squared distribution with h-1 degrees of freedom if each sample size n_i is 5 or more and if there are more than 3 samples (i.e. $h > 3$). Hence, the hypothesis of equal median inefficiency is rejected at level of significance of $a \in [0,1]$ if $KW^{*}\left(\hat{s}_{v}/\hat{s}_{u}\right)$ exceeds the critical value of the chi-squared distribution function $c^{2}_{h-1,1-a}$. If the null hypothesis of equal medians is not rejected, then there is not convincing evidence that the medians are different. However, if there is evidence that the medians are in fact different, then it is interesting to know specifically which medians differ from the others. In order to determine this, one could use Bonferroni's method here. In this case, we would perform a Wilcoxon rank sum test for each pair of medians we wish to compare. If we make q comparisons, then we use a significance level a/$2q$ for each pair, thus guaranteeing an overall level of significance of no more than a.

6. EMPIRICAL APPLICATION

Efficient frontier analysis has seen extensive application for studying the financial industry (see Berger and Humphrey, 1997, for an elaborate survey). To illustrate the CKP technique and the associated hypothesis tests, we perform an empirical application in this area. Specifically, we use a data set with 1997 financial statement data of 838 large banks in the European Union[5].

Our analysis uses a simplified representation of the bank technology that includes a small number of variables (using aggregation whenever possible), so as to reduce the curse of dimensionality associated with nonparametric analysis. Specifically, we use a single input: total personnel and interest cost. This input uses input prices to aggregate a multitude of different input variables into a single economically meaningful measure. In addition, we use two aggregated outputs: total earning assets (TEA) and total off-balance-sheet (OBS) items. Table 2 lists some descriptive statistics for the data set.

Table 2 Descriptive statistics data set

	Cost (million €)	TEA (million €)	OBS (million €)
Mean	909.27	15120.08	3726.13
Minimum	12.15	570.61	1.00
Maximum	29141.99	509548.13	151917.32
Std. Dev.	2624.11	42196.20	13941.07
Skewness	6.64	6.28	6.86
Kurtosis	52.56	48.20	54.20

We stress that this application is for illustrative purposes only. A more realistic representation of the production technology would account for differences across

[5] We use BankScope data provided by Bureau van Dijk Nederland.

banks in e.g. quality of service, risk and capital structure, as well as uncontrollable environmental variables.

The data set includes banks from different EU regions and banks with different specialisation. Specifically, we distinguish between the regions South (France, Greece, Italy, Portugal, and Spain), NorthWest (Belgium, Denmark, Finland, Luxembourg, Netherlands, Sweden and United Kingdom), and Central (Germany and Austria). In addition, we distinguish between commercial banks, co-operative banks and savings banks. Table 3 gives the composition of the sample.

Table 3 Composition of the sample

	South	NorthWest	Central	Total
Commercial	139	151	57	347
Co-operative	113	7	68	188
Savings	109	8	186	303
Total	361	166	311	838

We test whether banks from different regions and banks with different specialization are equally cost efficient. We expect that efficiency in different regions has converged as a result of the common European market. In addition, we expect that co-operative banks are less efficient (from the perspective of cost minimization) than commercial and savings banks, because the objective of co-operatives generally is to maximize services to members rather than to minimize cost.

We first estimate the Debreu-Farrell cost efficiency of all banks relative to the full sample. For this purpose, we apply the CKP method (see Section 2) to the data converted to natural logarithms, and using unity as a 'naïve' estimate for the signal-to-noise ratio.[6] Next, the efficiency estimates are converted to ranks, with the highest rank assigned to the most efficient bank. Subsequently, we compute for each combination of country and specialization the average rank. Finally, we use the Wilcoxon rank sum test (see Section 5) to test whether the median inefficiency for each combination of region and specialization differs from the median for the other banks in the sample. Table 4 gives the results.

[6] Log variables are used for two reasons: (1) The CKP model assumes homoskedasticity for the inefficiency terms and the disturbance terms. Homoskedasticity for the log variables seems more appropriate than homoskedasticity for the original variables, especially given the substantial differences in bank size in our sample (see also the original CKP study). (2) If log variables are used, then Debreu-Farrell efficiency can be obtained as exp($-u_j$) i.e. using a monotone transformation, and monotone transformations preserve the ranking of the not-transformed inefficiency estimates, i.e. exp($-u_j$) involves the same ranking as $-u_j$ (see Ondrich and Ruggiero, 2001).

Table 4 Estimation results*

		South	NorthWest	Central	Total
Commercial	rank	428.22	430.27	432.74	429.86
	number	139	151	57	347
	p-value	0.68	0.73	0.67	0.85
Co-operative	rank	412.87	499.57	362.16	397.76
	number	113	7	68	188
	p-value	0.38	0.81	0.02	0.08
Savings	rank	402.63	382.88	435.87	422.51
	number	109	8	186	303
	p-value	0.22	0.33	0.85	0.61
Total	rank	415.69	430.91	419.18	419.5
	number	361	166	311	838
	p-value	0.35	0.75	0.49	

*) The table contains for each cell (combination of region and specialization) the average rank (rank), the number of observations (number), and the *p*-value for the hypothesis that the median inefficiency of the banks that cell equals the median inefficiency for the full sample (*p*-value).

The results suggest that commercial and savings banks in all three EU regions are are equally efficient; the average ranks do no significantly differ from the sample average. This finding is consistent with our prior expectation that efficiency in different regions has converged as a result of the common European market. In addition, we find evidence that co-operative banks are less efficient on average than commercial and savings banks. This finding is consistent with the assumption that co-operatives seek to maximize services to members rather than minimize cost. However, the 'inefficient' co-operatives are found predominantly in the Central region; co-operatives in the Southern and Northwestern regions are not significantly less efficient than commercial and savings banks. Further research could focus on rationalizing this surprising finding.

7. CONCLUSIONS

We have considered the issues of noise-to-signal estimation, finite sample performance and hypothesis testing for the CKP technique, and obtained the following insights:

1. The goodness of the estimated noise-to-signal ratio asymptotically does not affect the goodness of the CKP estimators in terms of the statistical association between ranks (which frequently is the appropriate goodness measure, as discussed in the Introduction). The estimated ratio does affect in a non-trivial way linear correlation, and it may also affect the rank association in small samples. Still, the asymptotic irrelevance for rank association implies that the estimation of the noise-to-signal ratio is not as important as was suggested in CKP.

2. The simulation results suggest that the CKP technique can substantially improve upon DEA, and can provide a valuable complement (and sometimes even a

substitute) for parametric techniques, especially if little prior production information is available and if large samples are available. We realize that the simulation results may be biased by our subjective choices regarding the shape of the technology and the statistical distribution of the inefficiency terms. Unfortunately, there currently is no generally accepted framework for performing Monte-Carlo studies for firm efficiency analysis. Therefore, we call for a critical discussion aimed at developing such a framework.

3. Finally, the CKP estimators allow for nonparametric statistical testing based on the Wilcoxon rank sum test and the Kruskal-Wallis test. The empirical application suggests that these tests can involve substantial power in practical applications. Still, we can not derive the power of the test from a single application, and a more rigorous analysis of power would be useful.

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