UNPREDICTABLE AFTER ALL?

A SHORT NOTE ON EXCHANGE RATE PREDICTABILITY

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Unpredictable after all?

A short note on exchange rate predictability

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Abstract:
Earlier research has shown that it is very hard to outperform the random walk model with respect to forecasting exchange rates. In this paper we propose an extension to the regular regime-switching model in order to capture the exchange rate dynamics. The model is extended by including macro-economic variables, like inflation and interest rates, into both regimes. The regimes not only have different means and volatility's, but also different sensitivities to the macro-economic variables. We will show that this approach doesn't work over the whole sample, although previous research work might indicate otherwise. Furthermore we will elaborate on sub samples, in which the model showed a better performance than the random walk model, and show that this is rather coincidence.

Keywords:
Forecasting, exchange rates, regime-switching model, econometrics, economic variables

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Unpredictable after all?
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Introduction

The modeling and forecasting of exchange rates has been an important topic for many years. Meese & Rogoff (1983) tested several structural models, like the flexible-price and the sticky-price monetary models. They found that none of these models were able to give better out-of-sample forecasts than the random walk model at one to twelve months horizons for several dollar exchange rates. Meese & Rogoff (1988) regress log real exchange rates on real interest-rate differentials. In 32 out of 36 experiments the forecasts of this model gives a lower root-mean-square error (RMSE) than the random walk forecasts. However, these forecasts are not significantly better than the forecasts of the random walk model.

Mark (1995) presents evidence that long-horizon changes in the logarithm of the real exchange rate are predictable. He determines a "fundamental value" for the exchange rate based on the log relative real incomes and log relative money stocks. The deviation of the exchange rate from this fundamental value is used as an explanatory variable for the exchange rate to be predicted. This results in a good in-sample fit and better forecasts than the random walk model. This approach works well for larger horizons, like 12 or 16 quarters. Mark (1995) uses quarterly exchange rate data of the Canadian dollar, German mark, Swiss franc and Japanese yen against the U.S. dollar for the period 1973:II to 1991:IV. When this methodology is used for forecasting exchange rates in the nineties, we found that this approach does not work anymore[2]. Probably the results found by Mark (1995) depend on the specific sample chosen, however, it provides some evidence that economic variables might have some explanatory power.

The articles mentioned above all use linear models in the log exchange rate. Engel & Hamilton (1990) use a non-linear approach in order to find better forecasts for the log exchange rate. They mainly found better out-of-sample forecasts for the short horizon of one quarter in comparison with the random walk with drift (the Root Mean Squared Error of the forecasts made by their model were 8 to 17 percent lower than the forecasts of the random walk with drift model). Engel (1994) tests a Markov regime-switching model for different exchange rates. The model fits well in sample,

[2] These results are available from the author upon request.

but does not give superior forecasts over the random walk. Bollen, Gray and Whaley (1998) use a regime-switching model with independent shifts in mean and variance. They do not, however, use it to forecast the exchange rate, but they forecast the variance of the exchange rate and use this for currency option valuation.

All of these models weren't able to show consistent outperformance in comparison with the random walk model. It looks like the exchange rate dynamics can neither be captured by economically orientated models (like Mark(1995)) nor by regime-switching models (see Engel and Hamilton (1990)). Strangly, there is not much research that combines these two methods. Hence, we present a model that is based on the Engel and Hamilton (1990) model extended with macro-economic variables of the countries involved. In this paper we want to test the forecasting performance of this model and show that the random walk is very hard to best with respect to out-of-sample forecasting.

The paper is organized as follows. In section II the econometric model will be presented followed by a section about forecasts generated from the model. Section IV discusses the data and section V presents the results. We end up with the conclusions in section VI.

II Econometric specification

This section discusses the Markov regime-switching model that is going to be used for creating forecasts. This kind of model assumes the existence of two different regimes, which could correspond to an episode of a rising or falling exchange rate. The regime at any point in time is unobservable and controlled by a Markov chain. The model is estimated using an EM-algorithm, as described in Hamilton (1990 and 1994, pp.688-689). I will explain the principles of the model in this paper and I will refer to Hamilton (1994) for further details.

The model postulates an unobservable variable s_t that denotes which "regime" or "state" governs an exchange rate. The probabilities of moving from one state to another are governed by a Markov transition matrix:

$$P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \qquad (1)$$

$$p_{ij} = p\left(s_t = j \middle| s_{t-1} = i\right)$$

In both regimes, the exchange rate is assumed to follow a linear time series with some specific explanatory variables, denoted by X, like the growth in domestic product or the interest rate, and *l* is the number of lags in the economic variables.

$$y_t = \begin{cases} a_1 + b_1 X_{t-l} + e_{1t} & if \quad s_t = 1 \\ a_2 + b_2 X_{t-l} + e_{2t} & if \quad s_t = 2 \end{cases} \qquad e_{it} \sim N\left(0, s_i^2\right) \tag{2}$$

The motivation for this specification is twofold. Firstly, earlier research did not find a linear relation between exchange rates and macro-economic variables, however, the results of Bansal (1996) show that there exists a non-linear relationship between exchange rates and fundamentals (in his case UIP). Furthermore, Engel & Hamilton (1990) showed that the regime-switching model is able to capture the long swings in exchange rates. Another possible specification is to make the transition probabilities time-dependent and include macro-economic variables in that relation (see for example Martinez Peria (1999) and Klaassen (1999-2)).

The parameters of the model are estimated by an EM-algorithm. This iterative process increases the loglikelihood of the model with every step, until the maximum likelihood solution is found. The EM-algorithm is explained in Appendix 1 or can be found in Hamilton (1994).

III Forecasts

As mentioned earlier, our purpose is to construct a model that is able to generate better forecasts than the random walk model. Hence, it is important to know how these forecasts are generated using the model. Note that the macro-economic variables are lagged in the model, since the future values of these variables should be known in order to be able to make a forecast.

$$\begin{aligned} \hat{y}_{t+1} &= p(s_{t+1} = 1) \cdot \left\{\hat{a}_1 + \hat{b}_1 \cdot X_{t-l+1}\right\} + p(s_{t+1} = 2) \cdot \left\{\hat{a}_2 + \hat{b}_2 \cdot X_{t-l+1}\right\} \\ p(s_{t+1} = 1) &= p_{11} \cdot p(s_t = 1) + p_{21} \cdot p(s_t = 2) \\ p(s_{t+1} = 2) &= 1 - p(s_{t+1} = 1) \end{aligned} \tag{3}$$

Such a forecast can also be calculated for the logarithmic difference in the exchange rate of k periods ahead. The probabilities are multiplied by the Markov transition matrix for every step.

$$\hat{y}_{t+k} = p(s_{t+k} = 1) \cdot \left\{ \hat{a}_1 + \hat{b}_1 \cdot X_{t-l+k} \right\} + p(s_{t+k} = 2) \cdot \left\{ \hat{a}_2 + \hat{b}_2 \cdot X_{t-l+k} \right\} \qquad l \geq k$$
$$\left(p(s_{t+k} = 1) \quad p(s_{t+k} = 2) \right) = \left(p(s_t = 1) \quad p(s_t = 2) \right) \cdot P^k \tag{4}$$

The lag of the explanatory variables should be at least as large as the forecast horizon, since only values up to time t are publicly known at that time t. The total logdifference for k-period forecast is the summation of the logdifferences of 1 to k:

$$\hat{e}_{t+k} = \hat{y}_{t+1} + \hat{y}_{t+2} + \ldots.. + \hat{y}_{t+k} \tag{5}$$

The forecasts are compared with the actual logdifferences in the exchange rate and are evaluated by the Root Mean Squared Error (RMSE) criterion:

$$RMSE = \frac{\sum_{t=1}^{T-k} \left(\hat{e}_{t+k} - e_{t+k} \right)^2}{T-k} \tag{6}$$

Dacco & Satchell (1999) discuss that the RMSE might be inappropriate for evaluating the out-of-sample performance of a regime-switching model. Hence, a second criterion for testing the out-of-sample performance of the models is used: the Mean Absolute Deviation (MAD).

$$MAD = \frac{\sum_{t=1}^{T-k} \left| \hat{e}_{t+k} - e_{t+k} \right|}{T-k} \tag{7}$$

IV Data

This section describes the data used for testing the out-of-sample performance of the several models. The data was downloaded from the OECD *Main Economic Indicators*. The sample consists of quarterly observations, from 1973:IV to 1998:IV and includes data of the United States, Canada, Germany, Japan and the United Kingdom.

Several important economic variables were downloaded. First of all, the exchange rates are U.S. dollar prices for the foreign currency, thus four different exchange rates were examined.

Furthermore, three different economic variables were tested, these are the growth domestic product, the consumer price index and the interest rate. These can also be found in the OECD Statistical Compendium, *Main Economic Indicators*. I used the 3-months interest rate for all countries except for the United Kingdom, because the 3-months interest rate was only available after 1978. Hence, I used the London clearing banks' rate, which is available after 1971. The first published interest rate of Japan in the OECD Statistical Compendium is from 1979:III. Hence, the dataset used for the exchange rate between Japan and the U.S. starts at this date.

In the econometric model the logdiffences of the exchange rate is used, which can be calculated by the following formula:

$$y_t = 100 * \left(\log(e_t) - \log(e_{t-1})\right) \qquad (8)$$

where e_t is the exchange rate at time t and y_t is the logdifference of the exchange rate. The same methodology is used for the economic variables (except for the 3-month interest rate), because we are mainly interested in the relative growth of the economic variables, which is defined by:

$$x_t = \log\left(\frac{x_t^{US}}{x_t^{G}}\right) - \log\left(\frac{x_{t-1}^{US}}{x_{t-1}^{G}}\right) \qquad (9)$$

where x_t^G is the value of an economic variable at time t for country G.

Another methodology is used for the interest rate. Following Meese & Rogoff (1988) the actual differences in the interest rate is used to compute the importance of this parameter on the exchange rate forecasts. A fourth economic variable, which we called Purchasing Power (PP), is defined as the ratio between the GDP and the CPI of a certain country. In the research the logdifferences of this PP is used and calculated following (9).

The parameters are estimated using the logdifferences of 1974:I till 1986:IV, which means that the estimates are based on 52 observations. In this way we have approximately half of the data set left to test the out-of-sample performance of the model. The first prediction is thus made for time $t=T_1$ + horizon, where T_1=1986:IV. After the predictions are made, the regression window is moved forward by one quarter, the parameters are re-estimated and another prediction is made. This whole procedure is done for several horizons. The lag in the explanatory variables in the model is set equal to the forecast horizon, such that all (at the time of the forecast) known information is used in the forecast. Applying formula (4) results in:

$$
\begin{aligned}
&for \quad j = T_1 + horizon, \ldots\ldots, T \\
&\qquad \hat{y}_j = p(s_j = 1) \cdot \left\{ \hat{a}_1^{\,j-horizon} + \hat{b}_1^{\,j-horizon} \cdot X_{j-horizon} \right\} + \\
&\qquad\qquad p(s_j = 2) \cdot \left\{ \hat{a}_2^{\,j-horizon} + \hat{b}_2^{\,j-horizon} \cdot X_{j-horizon} \right\}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&with: \\
&\qquad \hat{a}_i^{\,t} = \hat{a}_i \quad found \quad with \quad data \quad until \quad time \quad t \\
&\qquad \hat{b}_i^{\,t} = \hat{b}_i \quad found \quad with \quad data \quad until \quad time \quad t \\
&\qquad T = 1998 : IV
\end{aligned}
$$

The number of forecasts is equal to T-(T_1+horizon)+1, which comes down to 48, 45, 41 and 37 forecasts for the 1, 4, 8 and 12 quarter horizons respectively.

As mentioned earlier, the data set for the Japanese Yen/U.S. Dollar exchange rate was smaller, because the interest rate of the Japan is only known after 1979:III. In order to get the same number of forecasts, the estimation window of this exchange rate is somewhat smaller and the first forecast is based on only 30 observations.

V Empirical Results:

The results found for the out-of-sample performance of the regime-switching models for every considered economic variable and for several forecast horizons (from 1 quarter up to 3 years) are presented in this section. The models are compared to the random walk model and the regime-switching model without any economic variables. Thus, the influence of the economic variable on the out-of-sample forecasts can be seen directly from the performance, which is measured by the RMSE-criterion. The MAD-criterion is not stated, because it led to similar results.

Tables 1 to 4 show the RMSEs of all models tested for four exchange rates. The first thing that can be noticed from these numbers is that the (simple) random walk model always has the lowest RMSE, except for five cases. All occur with the U.S. dollar/Canadian dollar exchange rate. The regime-switching model with the GDP, the interest rate or PP as explanatory variable at some specific horizon has a lower RMSE than the random walk model. This result is in line with a lot of other research, like Meese and Rogoff (1983) and Engel (1994), who also found that it is hard to beat when computing out-of-sample performance.

It is also very interesting to examine the differences between the Hamilton regime-switching model with and without economic variables. The tables show that in most cases the addition of an economic variable does not give better predictions (measured by the RMSE-criterion). This shows that all economic information is incorporated in the exchange rate very quickly.

Engel & Hamilton (1990) showed that the in-sample performance of a Markov regime-switching model is very good. Our results are in line with this, because the R^2 of most regressions is very high (ranging from 20% up to 60%). Furthermore, they showed that the Markov regime-switching model outperforms the random walk model (with drift) in out-of-sample forecasts up to 17% lower RMSEs. Especially at a shorter horizon (1 or 2 quarters) their model has a better performance. When the results of the Hamilton model are compared to the random walk model with drift in this research, the conclusions of Engel & Hamilton (1990) are not supported. The RMSEs for all horizons and all exchange rates are very close to each other, but in 13 out of 16 cases the random walk with drift has a marginally better performance than the Hamilton model, which contradicts the statements of Engel & Hamilton (1990).

Mark (1995) presented a model with economic variables that outperformed the random walk in out-of-sample forecasts. His results were the best for long horizons like 3 or 4 years. Looking at tables 1 to 4 we find that his conclusion is also not supported. The longer the forecast horizon is, the larger is the (average) difference between the random walk model and any other model.

Since we expected that a regime-switching model with economic indicators as explanatory variables would have a better out-of-sample performance than any other model, a closer look was taken at the results found. In some periods the random walk model was better, but in other periods the regime-switching model seemed to work very well. This might suggest that a better model can be found when time-varying parameters are used. We leave this for further research.

Figure 1 shows the absolute forecast error for all forecasts of two different models. In the first four years the random walk model has better predictions than the regime-switching model. From October 1991 until January 1994 it is the other way around. It also strikes that the forecast errors of the regime-switching model resemble the errors of the random walk. The reason for this might be that the state information is averaged out and hence the forecasts are very close to the random walk forecasts.

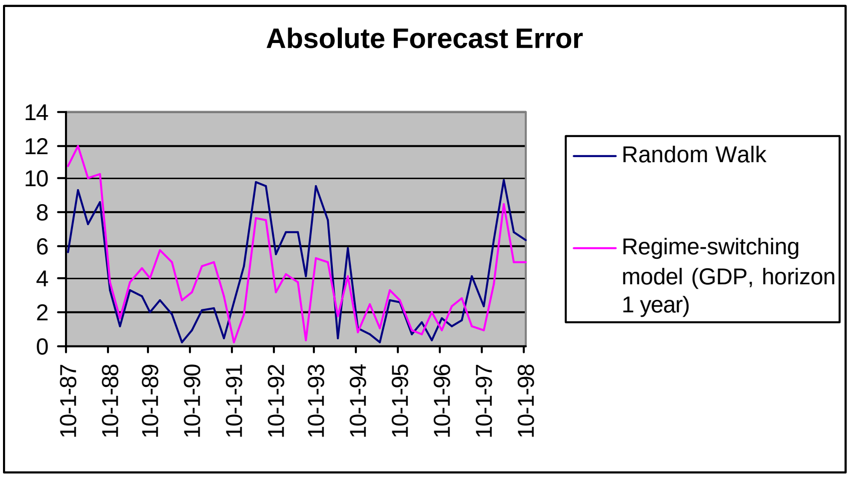

Figure 1: Absolute Forecast Error for two different models of the Canadian dollar/U.S. dollar exchange rate, with the GDP as economic variable and a forecast horizon of one year. The errors are expressed in logdifferences of the exchange rate.

If we would only consider the period from 1991 till 1994 , the regime-switching model would clearly be preferred. However, the model doesn't work much better for the total period of 1987:IV till 1998:IV. Maybe the model works well only for specific cases, possibly, there is a variable that triggers the performance of the regime-switching model. We examined two possible triggers: we tested the influence of the current state (at the time of forecasting) and the influence of the exchange rate volatility on the forecasts.

It followed that there is no clear relation between the current state an exchange rate is in and the difference between the RMSEs of the model and the random walk. This was tested using the following regression:

$$RMSEdiff_t = a + b \cdot state_t + e_t \qquad e_t \sim N\left(0, s^2\right) \tag{11}$$

$RMSEdiff_t$ stands for the difference in the RMSE-criterion between the random walk and the regime-switching model, $state_t$ is the probability that the exchange rate is in state 1 at the time when the forecast is made, thus lies between 0 and 1.
The following table presents the results of this regression:

α	β
0.864 (0.745)	-1.01 (0.863)

Table 5: results of the regression of formula (11)

The R-squared of this regression was only 2.9%, which shows that there is no relation between the current regime of the exchange rate and the effectiveness of the regime-switching model.

The relationship between the exchange rate volatility and the forecasts is also examined, using the following regression:

$$RMSEdiff_t = a + b \cdot vol_t + e_t \qquad e_t \sim N\left(0, s^2\right) \tag{12}$$

$RMSEdiff_t$ is defined as before and vol_t is calculated as the 90-day variance of the daily exchange rate volatility. These tests were performed for all horizons and the results of the one quarter horizon are stated in table 6:

α	β
-0.194 (0.607)	0.076 (0.186)

Table 6: results of the regression of formula (12)

and resulted in an R-squared of 3.69 % or lower. So, the exchange rate volatility also can not seem to explain why the regime-switching model performs better in some periods.

The overall conclusion that can be made, is that it is very hard to come up with a model that outperforms the random walk in out-of-sample forecasting. If a model predicts well, the scatterplot should show a positive relationship. Figure 2 shows the relation between the forecasted logdifferences and the actual logdifferences. This figure shows no relation at all between the forecast of the regime-switching model and the actual difference. Therefore, this strengthens the results found: a regime-switching model (with economic variables) is unable to outperform the random walk model.

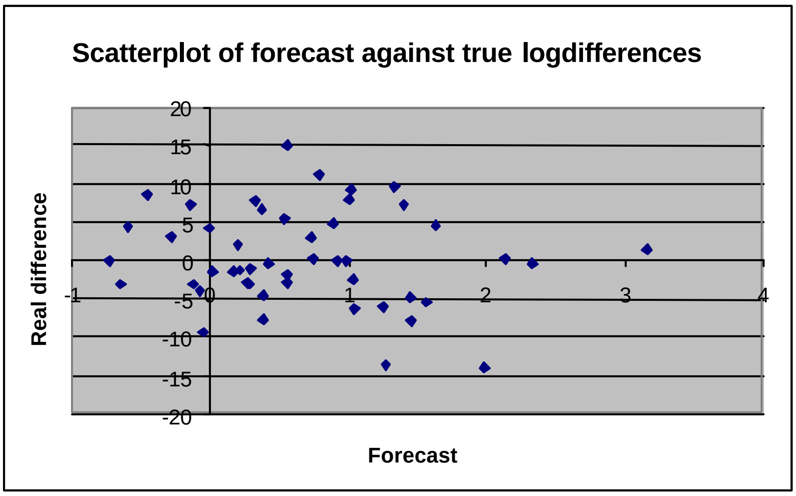

Figure 2: A scatterplot of the forecasted logdifferences against the actual differences. The model used the GDP as explanatory variable, lagged with one quarter. The underlying exchange rate is the German Mark / U.S. dollar rate

<u>VI Conclusions</u>

In this paper we propose an extension to the Engel & Hamilton (1990) model in order to capture the exchange rate dynamics. Engel & Hamilton (1990) introduced a regime-switching model, which captured the long-swings of an exchange rate. We extend this model by incorporating macro-economic variables, like inflation and interest rates, into both regimes. The inclusion of other variables has been covered before but then in the relation of the transition probabilities and not in the regime regressions. We used this new approach in order to investigate this issue.

The idea of combining the non-linear model (a Markov regime-switching model) with economic variables looked very promising, however, the results show that it is very hard to create a model that predicts the exchange rate better than the random walk model does. We found hardly any evidence that this model can outperform the random walk model: only in a few cases the RMSE (Root Mean

Squared Error) of the regime-switching model was lower than the RMSE of the random walk model. Most of the information seems to be incorporated in the exchange rates as soon as it is known and thus no predictability can be found based on this information.

What strikes us is that our model may perform very well in certain sub periods, but can not outperform the random walk model over the whole period. We examined two possible variables that might indicate during which period the regime-switching model is better or not. These are the current state an exchange rate is in and the volatility of the exchange rate. We didn’t find any relation between one of these variables and the performance of our model.

A possible extension to this research is using time-varying parameters to capture the dynamics of exchange rates better, because the characteristics of the regimes might change over time, especially when the time range reaches over 20 years. Another research direction is to make the transition probabilities dependent on the macro-economic variables instead of the mean and variance, as in Martinez Peria (1999) and Klaassen (1999-2). Applying this methodology might also give better predictions for the purpose of (out-of-sample) forecasting exchange rates.

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Appendix A

This appendix gives a detailed description of the EM-algorithm used to estimate the parameters of the regime-switching model. Rewrite the model stated in equation (2) as follows with this shorthand notation.

$$y_t = \begin{cases} \beta_1 X_{t-l} + \varepsilon_{1t} & if \quad s_t = 1 \\ \beta_2 X_{t-l} + \varepsilon_{2t} & if \quad s_t = 2 \end{cases} \qquad \varepsilon_{jt} \sim N\left(0, \sigma_j^2\right) \tag{A. 1}$$

where the state probabilities are governed by a Markov transition matrix:

$$\begin{gathered} P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \\ p_{jk} = p\left(s_t = k \middle| s_{t-1} = j\right) \end{gathered} \tag{A. 2}$$

These equations have six parameters in total: $\theta = \{\beta_1, \beta_2, \sigma_1, \sigma_2, p_{11}, p_{22}\}$. The EM algorithm finds the maximum likelihood parameters with an iterative procedure. Therefore, the density, based on the information set at time t-1 ($=\Omega_{t-1}$), should be calculated:

$$\begin{aligned} f\left(y_t \middle| \Omega_{t-1}; \theta\right) &= f\left(y_t, s_t = 1 \middle| \Omega_{t-1}; \theta\right) + f\left(y_t, s_t = 2 \middle| \Omega_{t-1}; \theta\right) \\ &= f\left(y_t \middle| s_t = 1, \Omega_{t-1}; \theta\right) \cdot P(s_t = 1) + f\left(y_t \middle| s_t = 2, \Omega_{t-1}; \theta\right) \cdot P(s_t = 2) \end{aligned} \tag{A. 3}$$

This is an easy expression as soon as the probabilities are known, since y_t given the parameters and the state follows a normal distribution and can be calculated with the following formula:

$$f\left(y_t \middle| s_t = j, \Omega_{t-1}; \theta\right) = \frac{1}{\sqrt{2\pi}\,\sigma_j} \exp\left\{ \frac{-\left(y_t - \beta_j X_{t-l}\right)^2}{2\sigma_j^2} \right\} \qquad j \in \{1,2\} \tag{A. 4}$$

Hence, we need to calculate the regime probabilities given the history up till time t-1. In order to get the best estimates, three different kind of probabilities will be calculated, the so-called forecast, inference and smoothed inference. The forecast, denoted by $\xi_{t|t-1}$, is equal to the state in the previous period ($\xi_{t-1|t-1}$) times the transition probabilities of the underlying Markov process:

$$\hat{x}_{t|t-1} = P \cdot \hat{x}_{t-1|t-1} \tag{A. 5}$$

where $\xi_{i|j}$ is in vector notation. These forecasts are used to calculate the state probabilities, or inferences using:

$$\hat{x}_{t|t} = \frac{\hat{x}_{t|t-1} \otimes f_t}{i'\left(\hat{x}_{t|t-1} \otimes f_t\right)} \tag{A. 6}$$

where i is the unit vector, $\otimes$ represents element by element multiplication and f_t is a vector containing the conditional densities for both regimes at time t. Together with some starting values $\xi_{1|0}$ and values for all parameters (such that the densities are known) these two formulas are used to make forecasts and inferences for all state probabilities. Several starting values can be chosen, e.g. a fixed vector of constants which sum to unity. We used a starting vector that gave both possible scenarios the same weight, i.e. ½. Hamilton (1994, p.693-694) also gives other suggestions.

Since, the forecasts and inferences are known, the smoothed inferences of the regime probabilities can now be calculated. The following algorithm to calculate these smoothed inferences, denoted by $\xi_{t|n}$ was developed by Kim (1993):

$$\hat{x}_{t|n} = \hat{x}_{t|t} \otimes \left\{P'\left(\hat{x}_{t+1|n} \div \hat{x}_{t+1|t}\right)\right\} \tag{A. 7}$$

and $\div$ means element by element division. This algorithm runs backwards from t=n, n-1….1. As soon as the smoothed inferences are known, new parameter estimates can be made. Hamilton (1990) showed that the maximum likelihood estimates for the transition probabilities satisfy:

$$\hat{p}_{jk} = \frac{\sum_{t=2}^{n} P\left\{s_t = k, s_{t-1} = j \mid \Omega_n; \hat{q}\right\}}{\sum_{t=2}^{n} P\left\{s_{t-1} = j \mid \Omega_n; \hat{q}\right\}} \qquad j,k \in \{1,2\} \tag{A. 8}$$

where $\hat{q}$ denotes the maximum likelihood estimates of θ.

The maximum likelihood estimates for the other parameters are:

$$\hat{b}_j = \left[\sum_{t=1}^{n}[\tilde{x}_t(j)][\tilde{x}_t(j)]'\right]^{-1}\left[\sum_{t=1}^{n}[\tilde{x}_t(j)]\tilde{y}_t(j)\right] \qquad j \in \{1,2\} \tag{A. 9}$$

$$\hat{s}_j^{\,2} = \frac{\sum_{t=1}^{n}\left(y_t - x_t'\hat{b}_j\right)^2 \cdot P\left(s_t = j \mid \Omega_n; \hat{q}\right)}{\sum_{t=1}^{n} P\left(s_t = j \mid \Omega_n; \hat{q}\right)} \qquad j \in \{1,2\} \tag{A.10}$$

where

$$\begin{aligned} \tilde{y}_t(j) &= y_t \cdot \sqrt{P(s_t = j \mid \Omega_n; \hat{q})} \\ \tilde{x}_t(j) &= x_t \cdot \sqrt{P(s_t = j \mid \Omega_n; \hat{q})} \end{aligned} \qquad j \in \{1,2\} \tag{A.11}$$

Using these formulas an iterative procedure can be followed in order to estimate the parameters of the regime-switching model. Hamilton (1990) proved that this iterative procedure leads to the maximum likelihood estimates. Given a certain starting value θ_0, the smoothed inferences can be calculated using (A.5), (A.6) and (A.7). Combining these smoothed inferences with the old transition probabilities gives new values for the regime transition probabilities. The new transition probabilities in turn will be used with formulas (A.9) and (A.10) to find new estimates for β_j and σ_j, which complete θ_1. Iterating this procedure gives θ_2, θ_3 and so on. This procedure stops as soon as convergence occurs.

Table 1 The performance of several models on the U.S. dollar – Canadian Dollar exchange rate.

This table presents the performance of the Markov regime switching model with economic variables in comparison with the random walk model. The results of the random walk model with drift and Hamilton (1990) Markov regime switching model are also stated. The performance is measured by the Root Mean Squared Error (RMSE). Using the MAD (mean absolute deviation) as a performance measure delivers similar results. The true errors are only mentioned for the random walk, for all other models the ratio of the RMSE with the RMSE of the random walk is written, which makes the comparison easier.

The top row of the table denotes the model used, and the left column gives the forecast horizon (which equals the lag of the economic variable in some models) in quarters per annum.

Horizon	Random Walk	Random Walk (with drift)	Hamilton	Regime-Switching GDP	Regime-Switching CPI	Regime-Switching Interest Rate	Regime-Switching PP
1	4.43	1.000	1.002	0.991	1.063	0.991	1.025
4	24.74	1.017	1.017	0.965	1.011	1.158	1.039
8	62.65	1.037	1.037	1.040	1.197	1.522	1.066
12	103.74	1.008	1.010	0.962	1.086	1.324	0.970

Table 2 The performance of several models on the U.S. dollar – German D-Mark exchange rate.

This table presents the performance of the Markov regime switching model with economic variables in comparison with the random walk model. The results of the random walk model with drift and Hamilton (1990) Markov regime switching model are also stated. The performance is measured by the Root Mean Squared Error (RMSE). Using the MAD (mean absolute deviation) as a performance measure delivers similar results. The true errors are only mentioned for the random walk, for all other models the ratio of the RMSE with the RMSE of the random walk is written, which makes the comparison easier.

The top row of the table denotes the model used, and the left column gives the forecast horizon (which equals the lag of the economic variable in some models) in quarters per annum.

Horizon	Random Walk	Random Walk (with drift)	Hamilton	Regime-Switching GDP	Regime-Switching CPI	Regime-Switching Interest Rate	Regime-Switching PP
1	38.02	1.013	1.029	1.050	1.007	1.083	1.049
4	108.3	1.070	1.073	1.076	1.092	1.665	1.202
8	149.75	1.260	1.292	1.291	1.370	2.395	1.567
12	175.86	1.198	1.262	1.574	1.083	2.734	2.083

Table 3 The performance of several models on the U.S. dollar – Japanese Yen exchange rate.

This table presents the performance of the Markov regime switching model with economic variables in comparison with the random walk model. The results of the random walk model with drift and Hamilton (1990) Markov regime switching model are also stated. The performance is measured by the Root Mean Squared Error (RMSE). Using the MAD (mean absolute deviation) as a performance measure delivers similar results. The true errors are only mentioned for the random walk, for all other models the ratio of the RMSE with the RMSE of the random walk is written, which makes the comparison easier.

The top row of the table denotes the model used, and the left column gives the forecast horizon (which equals the lag of the economic variable in some models) in quarters per annum.

Horizon	Random Walk	Random Walk (with drift)	Hamilton	Regime-Switching GDP	Regime-Switching CPI	Regime-Switching Interest Rate	Regime-Switching PP
1	44.84	1.012	1.015	1.084	1.017	1.046	1.069
4	139.02	1.126	1.165	1.197	1.080	1.088	1.181
8	315.81	1.434	1.461	1.823	1.557	1.863	1.658
12	508.19	2.040	2.237	2.719	2.502	2.031	3.066

Table 4 The performance of several models on the U.S. dollar – U.K. Pound Sterling exchange rate.

This table presents the performance of the Markov regime switching model with economic variables in comparison with the random walk model. The results of the random walk model with drift and Hamilton (1990) Markov regime switching model are also stated. The performance is measured by the Root Mean Squared Error (RMSE). Using the MAD (mean absolute deviation) as a performance measure delivers similar results. The true errors are only mentioned for the random walk, for all other models the ratio of the RMSE with the RMSE of the random walk is written, which makes the comparison easier.

The top row of the table denotes the model used, and the left column gives the forecast horizon (which equals the lag of the economic variable in some models) in quarters per annum.

Horizon	Random Walk	Random Walk (with drift)	Hamilton	Regime-Switching GDP	Regime-Switching CPI	Regime-Switching Interest Rate	Regime-Switching PP
1	30.36	1.026	1.053	1.050	1.069	1.119	1.057
4	80.85	1.124	1.122	1.160	1.273	1.254	1.248
8	97.71	1.147	1.171	1.263	1.274	2.459	1.263
12	112.12	1.238	1.295	1.295	1.670	1.800	1.571

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