Fat Tails in Power Prices

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Abstract	Spot power prices exhibit extreme price jumps and the tendency to oscillate around a long-term mean. Despite these well-known characteristics, electricity price models used for Monte Carlo simulations, VaR related measures, or derivatives valuation, often assume normally distributed residuals. In this paper, we examine the distributional characteristics of model residuals and show that the hypothesis of normality is rejected due to significant tail fatness and skewness. We then examine the Student-t distribution as a candidate fit for residuals and as an alternative distribution for random innovations in Monte Carlo simulations. The resulting price patterns clearly show that simulations based on the Student-t distribution resemble more closely actual power price patters. We then discuss the implications of our results for risk management.

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Fat Tails in Power Prices

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Spot power prices exhibit extreme price jumps and the tendency to oscillate around a long-term mean. Despite these well-known characteristics, electricity price models used for Monte Carlo simulations, VaR related measures, or derivatives valuation, often assume normally distributed residuals. In this paper, we examine the distributional characteristics of model residuals and show that the hypothesis of normality is rejected due to significant tail fatness and skewness. We then examine the Student-t distribution as a candidate fit for residuals and as an alternative distribution for random innovations in Monte Carlo simulations. The resulting price patterns clearly show that simulations based on the Student-t distribution resemble more closely actual power price patters. We then discuss the implications of our results for risk management.

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1 Introduction

It is commonly known that electricity market prices for day-ahead delivery exhibit mean reversion, seasonality, non-constant volatility, and spikes. These stylised facts affect all market participants in their risk exposure to spot prices. Appropriate pricing and risk management models should incorporate these facts.

The traditional models[1] for electricity spot price dynamics focus mostly on mean reversion and seasonal patterns. Spot prices of electricity are modelled as the sum of a deterministic component that captures seasonality and a stochastic component that captures mean reversion and a noise term. This error term is mostly assumed to be normally distributed or at least to be IID. These models do not capture one of the most important characteristics of power prices being the frequent occurrence of spikes. These are extreme price movements due to shocks in supply and demand and the fact that electricity is a non-storable commodity. For example, the average Dutch price of electricity was about 30 euro per MWh in 2002 and a maximum price has occurred in 2002 of 701 euro per MWh between noon and 3 pm on August 21st, 2002. The maximum baseload price, average price for whole day delivery, in 2002 equalled around 220 euro per MW. The following graph presents the daily prices of baseload electricity (whole day delivery) for 2002.

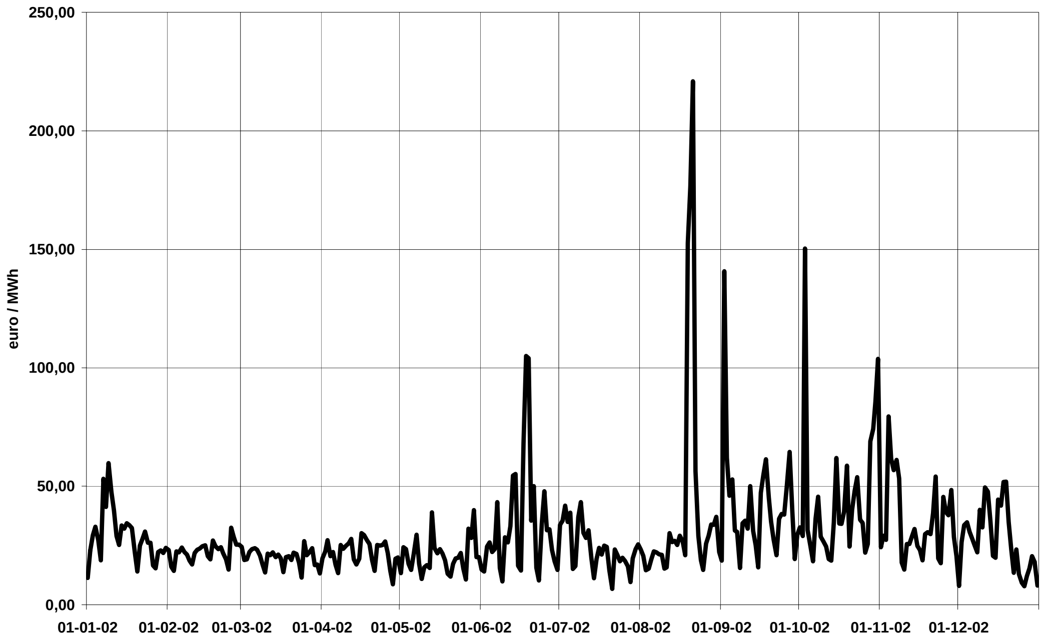

Figure 1. Baseload prices for Dutch day ahead delivery of electricity (APX) for 2002.

Monte Carlo simulations based on the traditional models do not resemble the actual price patterns of spot power. As practitioners use these simulations for risk management and valuation purposes, it is clear that researchers should improve upon the traditional models.

Motivated for reasons discussed above, models have been introduced recently that focus more closely on the spike characteristics. For example, jump processes and switching regime models have been introduced to model spikes in spot prices, thereby directly affecting the third and fourth moment of the noise term[2]. The advantage of these approaches is that they explicitly model the spikes and therefore allow for non-normal characteristics. Furthermore, switching regime models have the advantage of not affecting the estimates of the level of mean reversion as these models disentangle the mean-reversion from spikes from normal periods.

In this paper, we also focus on the tail characteristics of the noise term. We apply extreme value theory (EVT) to assess the level of tail fatness and we test whether the EVT can directly be applied to replace the normality assumption for a different distribution. We then demonstrate how to apply the Student-t distribution for Monte Carlo simulations. The advantage of the Student-t compared with the normal is that it captures tail fatness through its degrees of freedom parameter that can be calibrated by EVT. Our simulation results clearly improve upon the ones from the normal distribution, as the Student-t price patterns resemble more closely the true price pattern of power.

The paper is structured as follows. In section 2 we present the summary statistics of the data that we used. Section 3 discusses the price model used and outlines the parameters estimates. In section 4 we elaborate on the tail shape of the residual. In section 5 we show the impact of assuming normality of the residual process versus an alternative distribution that better describes the residual process by applying Monte Carlo simulations on the electricity spot prices. Section 6 concludes.

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1 Examples are discussed in Pilipovic (1998) and Lucia and Schwartz (2002) among others.

2 Examples include Deng (1999), Huisman and Mahieu (2001), and de Jong and Huisman (2002).

2 Data

The data used in this paper is derived from the Dutch Amsterdam Power Exchange (hereafter APX). We use baseload prices from January 1st 2001 through July 22nd, 2003 being 933 daily price observations[3]. Table 1 provides an overview of summary statistics of this data, both in price levels, log prices and log price changes. It clearly reveals the non-normal characteristics of the data being the positive skewness and excess kurtosis.

Table 1: Summary statistics of APX baseload day ahead prices.

	Price level	Log price	Log price changes
Mean	32.69	3.34	0.00
Median	26.92	3.29	-0.02
Standard deviation	25.03	0.50	0.48
Minimum	1.48	0.39	-2.37
Maximum	263.14	5.57	2.77
Skewness	4.67	0.64	0.69
Excess kurtosis	32.22	7.18	7.19

3 Modelling day ahead power prices

Following Lucia and Schwartz (2002), we model electricity prices by decomposing the log spot power price s(t) at time t (t = 1,2,…,T) in a deterministic component, f(t), and stochastic component, x(t), such that:

1) $s(t)= f(t) + x(t)$.

Introducing Δ as the differencing operator, we can write the daily changes in the log price of power as

2) $\Delta s(t)= \Delta f(t) + \Delta x(t)$.

The component f(t) is a deterministic function of time and models predictable regularities, such as periodic behaviour and trends. Let f(t) account for the fact that prices for electricity delivered on weekend days is lower than the price on an average working day. To do so we introduce

[3] This data can be retrieved from the APX website (www.apx.nl).

two dummy variables: $D_1(t)$ equals 1 on Saturdays and 0 on other days and $D_2(t)$ equals 1 on Sundays and 0 on other days. We therefore specify f(t) as follows:

3) $f(t) = \mu + \beta_1 D_1(t) + \beta_2 D_2(t),$

where parameter μ reflects the average log price level.

The stochastic component x(t) in equation 1 reflects the movement of the electricity price out of its deterministic behaviour at time t. One important characteristic is mean reversion. Following Pilipovic (1998), let α be the speed with which the spot price of power reverts back to it's long term mean. As the long-term mean μ is captured by the deterministic component in equation 3, we model mean reversion in the stochastic part as reversion due to a deviation from 0:

4) $\Delta x(t) = -\alpha\, x(t-1) + \sigma\, \varepsilon(t), \;\; \varepsilon(t) \sim IID\ (0, 1),$

where $\varepsilon(t)$ represents the noise term and σ the standard deviation of this noise term. After substituting equations 4 and 3 into equation 2, we come to the following model for daily log price changes of power:

5) $\Delta s(t) = \alpha\mu + \beta_1 \{D_1(t) + (\alpha - 1)\, D_1(t-1)\} + \beta_2 \{D_2(t) + (\alpha - 1)\, D_2(t-1)\} - \alpha\, s(t-1) + \sigma\, \varepsilon(t).$

As equation 5 is non-linear in its parameters, we apply non-linear least squares (NLS) to estimate the parameter value. These NLS estimates are listed in table 2.

Table 2: NLS parameter estimates and residual characteristics for model 5.

μ	3.482 (0.029)	σ	0.333
β_1	-0.293 (0.028)	Jarque-Bera	1099.2
β_2	-0.644 (0.028)	White	16.53
α	0.385 (0.025)		

Standard errors are in parentheses. The estimate for μ of 3.482 relates to a long-term average price of 32.52 euro per MWh. The adjusted R^2 equals 0.513.

From table 2, we observe a long-term average log price of 3.5, which is 0.29 lower on Saturdays and 0.64 lower on Sundays. The estimate for the speed of mean reversion equals 0.39 and is significant. The Jarque-Bera statistic is much higher than its critical value of 5.99. Therefore, we reject the hypothesis of normally distributed residuals. The White statistic is the outcome of a White test on homoskedasticity of the residuals (including cross-terms). The critical value is 15.51 and the higher value of 16.53 for White makes that we reject homoskedasticity for the residuals. In order to examine whether the residuals exhibit autocorrelation, we applied the Ljung-Box autocorrelation test. We find that autocorrelation is clearly apparent for the 7th lag, which can be explained as a weekly pattern in power prices.

As the residuals are not clean in this respect, we added an autoregressive term to the price model 5 resulting in:

6) $$\Delta s(t) = \alpha\mu + \beta_1 \{D_1(t) + (\alpha - 1) D_1(t-1)\} + \beta_2 \{D_2(t) + (\alpha - 1) D_2(t-1)\} - \alpha\, s(t-1) + \theta\Delta s(t-7) + \sigma\, \varepsilon(t).$$

Table 3 shows the parameter estimates for model 6.

Table 3: NLS parameter estimates and residual characteristics for model 6.

μ	3.479 (0.033)	θ	0.080 (0.032)
β_1	-0.303 (0.030)	σ	0.324
β_2	-0.646 (0.030)	Jarque-Bera	1034.3
α	0.360 (0.026)	White	9.48

Standard errors are in parentheses. The estimate for μ of 3.479 relates to an long-term average price of 32.43 euro per MWh. The adjusted R^2 equals 0.515.

From table 3 we observe the significance of the lagged term. The Ljung-Box test for autocorrelation does reject the hypothesis that the residuals exhibit autocorrelation and from the White test we reject the hypothesis that the residuals are heteroskedastic. From this point, we conclude that the residuals are IID, but we cannot conclude that they are normally distributed from the Jarque-Bera test result. In the following section, we shall focus on the residuals from the model 6.

4 Dealing with fat tails

From the previous section we conclude that the residuals of model 6 are IID but not normal. This is in contrast with many proposed models that are being used especially for Monte Carlo simulations. In these cases, one assumes a model for the spot price development over time (such as model 5 or 6) and that the daily innovations $\varepsilon(t)$ are drawn from a particular distribution. In many cases, the normal distribution is chosen for reasons of convenience. But the drawback of this convenience can be enormous as one has an erroneous assessment of the true risks faced because of the fact that he or she neglects the non-normal properties of the innovations. Especially for electricity prices, these non-normal properties are pronoun. Figure 2 shows the histogram of the residuals and the fitted normal distribution function. The figure shows clearly that the normal distribution provides a poor fit to the histogram of the residuals. Not only in terms of tail fatness, but also for the probability mass in the middle as we see that the frequencies in the middle are much higher for the actual data than what the normal distribution would imply.

This observation is not unique for power but it is quite common for many financial assets[4]. Huisman, Koedijk, Kool, and Palm (2002) propose to use the Student-t distribution as an alternative to the normal distribution as its shape is taller in the middle and as it is capable of incorporating fat tails. Huisman, Koedijk, and Pownall (1998) suggest using the Student-t distribution in order to obtain better Value at Risk estimates for different assets. Following Huisman, Koedijk, Kool, and Palm (2002), we examine in to what extent the Student-t distribution would provide a better fit than the normal distribution function.

[4] See Huisman, Koedijk, and Pownall (1998) for bond and stock portfolios, Boothe and Glassman (1987), and Huisman, Koedijk, Kool, and Palm (2002) for exchange rates, and Campbell and Huisman (2002) for credit spreads.

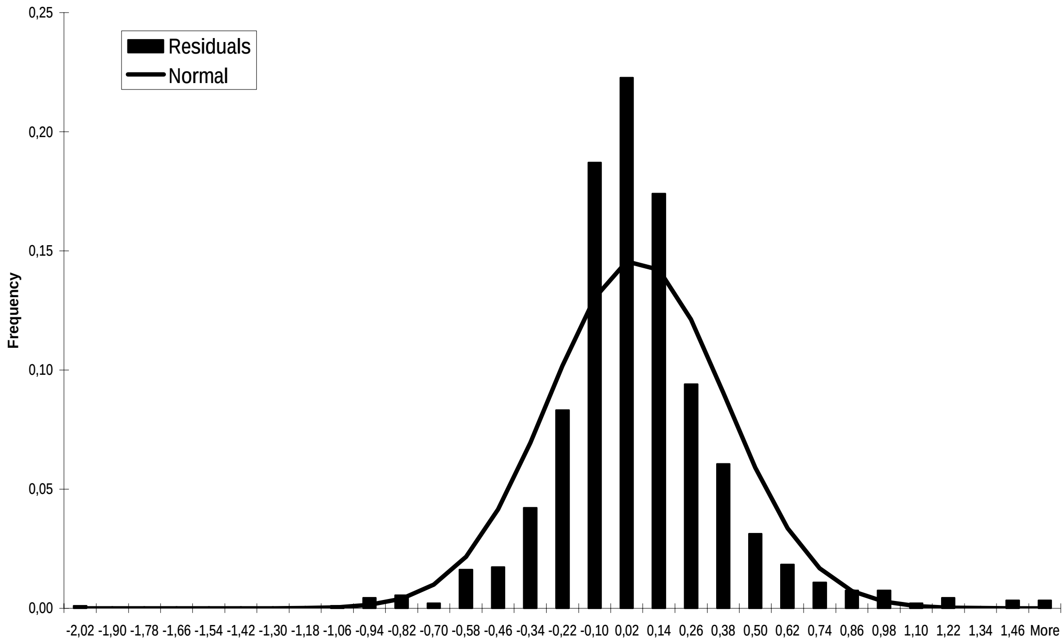

Figure 2. Histogram of the residuals from model 6 and the fitted normal distribution.

The standardized Student-t distribution is a symmetric distribution and it's being shaped by one parameter: the degrees of freedom. In the following, we let α be the degrees of freedom. The standardized Student-t distribution has zero mean and its variance equals $\alpha / (\alpha-2)$. An important property of the Student-t distribution is that it converges to the normal distribution for $\alpha \rightarrow \infty$. The degrees of freedom α equals the number of existing moments and α is directly related to the tails of the distribution function. For high values of α, the distribution has tails that corresponds closely with the normal distribution, but the lower α gets, the fatter the tails become. Huisman, Koedijk, Kool, and Palm (2002) show that α is between 3 and 8 for different exchange rates.

In order to examine the appropriateness of the Student-t distribution, we have to estimate the degrees of freedom parameter α. We apply the estimator proposed by Huisman, Koedijk, Kool, and Palm (2001) that is based on extreme value theory. This procedure estimates the tail index from an empirical distribution. The tail index is a measure for the amount of tail fatness of the distribution under investigation and may also be looked upon as an indicator for the pace with which the tail moves to zero. The fatter the tail the slower the speed and the lower the tail index given. The tail index has the attractive feature that it is equal to the number of existing moments of the distribution and thus can be used to parameterise the Student-t distribution. Table 4 shows the tail index parameter estimates for the residuals from model 6.

Table 4: Tail index estimates for the residuals of model 6.

Both tails	3.142 (0.005)
Left tail only	3.182 (0.003)
Right tail only	3.331 (0.004)

Standard errors are in parentheses.

The tail index estimates equal 3.3 for the right tail and 3.2 for the left tail. This implies that the left tail of the residual distribution is fatter than the right. Interestingly α is smaller than 4, indicating that the fourth moment of the residuals distribution does not exist. Therefore, one needs to be careful with interpreting kurtosis estimates.

Now we have estimated the degrees of freedom parameter α, we investigate whether the Student-t distribution is able to provide a better fit to the residuals of model 6 than the normal distribution. As the variance of the standardized Student-t equals is not equal to one, we need to adjust the residuals data to make its variance consistent. Let x be the residual under consideration. To fit the student-t-distribution we assume that $y = \varphi(x - \rho)$ is Student-t distributed with α degrees of freedom. Here ρ is a location parameter that equals the residuals mean, which is zero. The scale parameter φ is a function of the variance and can be derived as follows.

7) $\text{variance}(y) = \alpha / (\alpha - 2) = \varphi^2 \, \text{variance}(x)$, thus

8) $$\varphi = \sqrt{\frac{\alpha}{(\alpha - 2)\text{var}(x)}}.$$

The following figure shows the histogram of the residuals and the Student-t fit for which we assumed α equal to 3.142 (taken from table 4) and φ equal to 5.113 obtained from equation 8.

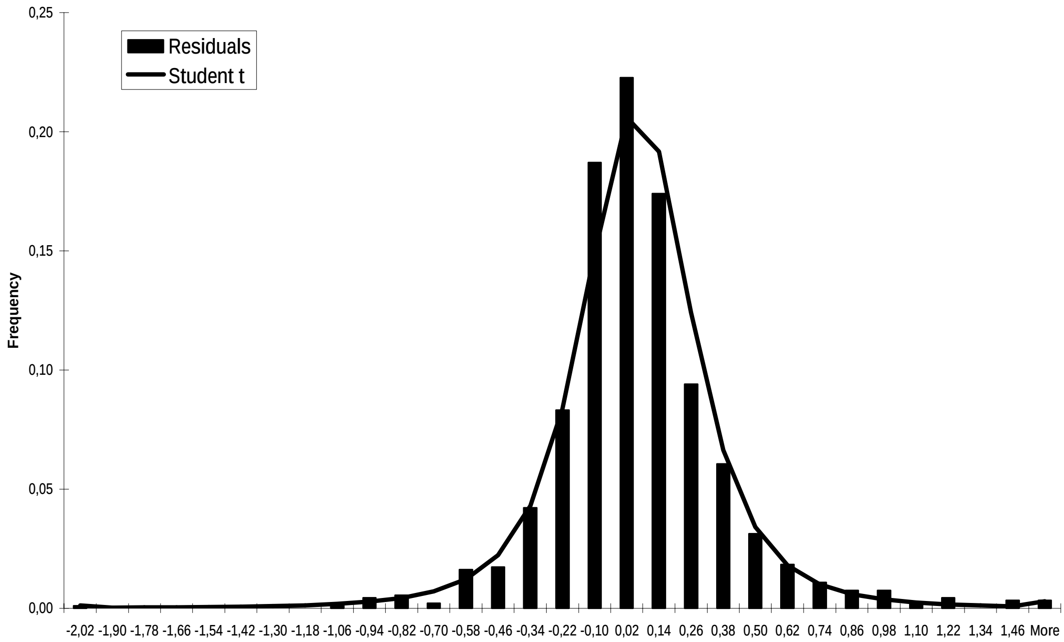

Figure 3. Histogram of the residuals from model 6 and the fitted Student-t distribution.

At first glance, the student-t distribution seems to graphically fit the complete empirical distribution of residuals very well for the residuals. Compared with the fit from the normal distribution shown in figure 2, we obtain a better fit for the tails and for the central part of the distribution. To formally test the hypothesis that the fitted Student-t distribution is a good approximation for the unconditional empirical distribution of the residuals, we apply the following goodness of fit-test similar to that used by Boothe and Glassman (1987) and Huisman, Koedijk, Kool, and Palm (2002). The goodness-of-fit test compares the observed and expected number of observations in c intervals over which the data is divided as follows:

9) $$G = \sum_{j=1}^{c} \frac{(o_j - e_j)^2}{e_j},$$

where o_j and e_j are the observed and expected number of observations in interval j. The test statistic G is Chi-squared distributed with (c-1) degrees of freedom. The intervals (except the first in the left tail and the last in the right tail that range to minus or plus infinitely respectively) are chosen such that they are of equal length. Note that each interval has at least five expected observations, which is to ensure that the Chi-squared approximation is fairly accurate.

Table 5 contains the goodness of fit results for the normal distribution and the Student-t distribution. According to the goodness of fit results in table 5, we cannot reject the hypothesis that the Student-t distribution provides a good fit to the residuals at the 99% confidence level, as the test statistics does not exceed the 99% critical value. For the normal distribution, we do reject the hypothesis of a good fit of the residuals. Based on these results we conclude that the Student-t provides a much better fit to the residuals of model 6 than the normal distribution function.

Table 5: Goodness of fit results for the residuals of model 6.

Normal distribution	194.16 (24.73)
Student-t distribution	26.02 (26.22)

NB: Chi-squared critical values for n-1 (number of intervals) at the 99% confidence level are in parentheses. Reject the null-hypothesis that the distribution is a good approximation for the residual distribution if the reported goodness-of-fit statistic exceeds the corresponding critical value.

5 Monte Carlo simulations of day ahead power prices

The result from the previous section has important implication for risk management and derivatives valuation. For both applications it hold that many use the normal distribution function to generate random numbers from in Monte Carlo simulations. As we have seen in the previous section, this is an erroneous assumption. As these Monte Carlo simulations are used to calculate portfolio Value at Risk type of risk statistics and to valuate options or forwards for which the underlying is the day ahead power price, one will underestimate the probability of very small and extreme price changes, and will overestimate the probability of medium price changes. This is clearly visible after comparing the histograms and the distributional fits in the figures 2 and 3. In order to show the difference for Monte Carlo simulations of day-ahead power prices, we show the outcomes of such a simulation in the following figure. We use model 6 as our data-generating model. We simulate two series for the APX time series: one where the residuals are normally distributed and the other where the residuals are Student-t with the number of degrees of freedom is set equal to 3.142. In both cases we assume the innovations to be IID and to have equal variance as indicated in table 3. We set the number of simulated observations equal to 933, being the number of APX prices that we have available.

Figure 4 shows graphs of the simulated time series. As we expected, it can be easily observed that the Student-t simulated prices resemble more closely the true price path of power prices than the normal simulated prices.

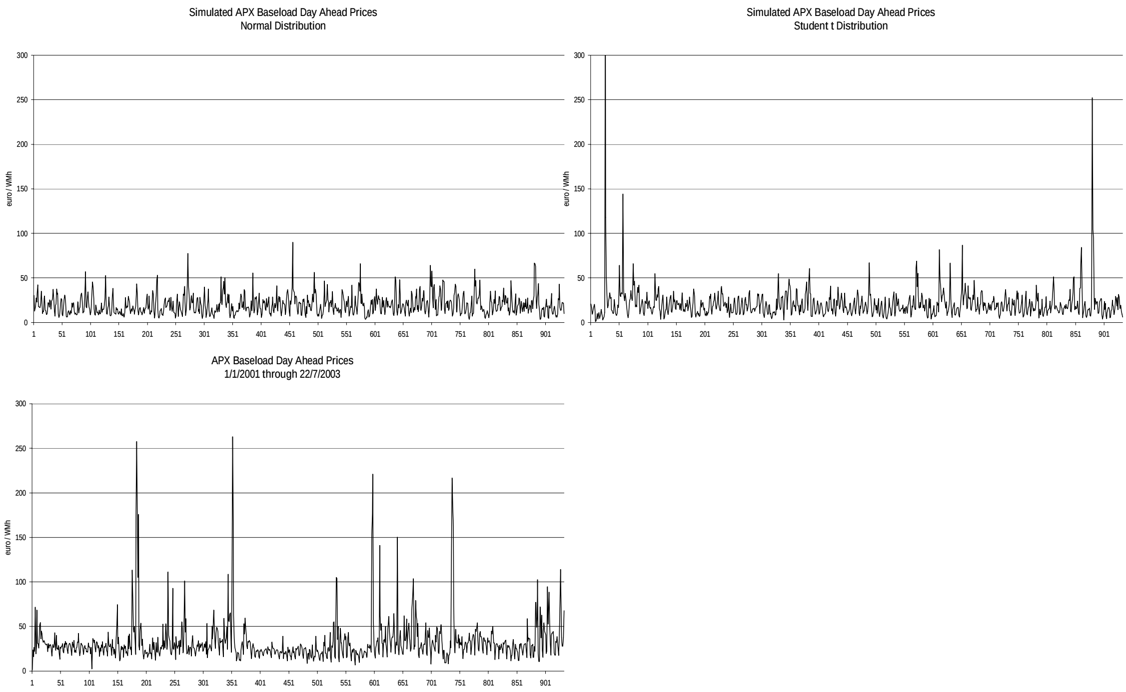

Figure 4. Simulated APX base load day-ahead prices based on model 6 with normal innovations (top left), Student t innovations (top right) and the actual APX prices (bottom left).

6 Concluding remarks

In this paper, we demonstrate that assuming normal innovations in Monte Carlo simulations for risk management and derivatives valuation purposes can have serious consequences for the true amount of risk faced. We propose the Student-t distribution as an alternative to the normal distribution as it is capable of capturing the fat tailed behaviour of electricity prices. We assess the amount of tail fatness using extreme value theory and use its results directly to parameterise the Student-t. It is then shown that the Student-t provides a much better fit than the normal distribution. A fact that becomes especially clear when one observes the differences in simulation outcomes for the normal based method and the Student-t method. Therefore, the normality assumption that researchers and practitioners often make in their simulation or valuation method can lead to erroneous conclusions.

The method that we proposed in this paper is an easy to implement alternative for using the normal distribution as the density function for innovations in Monte Carlo simulations. In this method, spikes are captured through the selection of distribution function. We claim this method to be a candidate to model the non-normal behaviour of electricity prices in addition to other models such as jump diffusion models or switching regimes models.

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