Options and higher-order risk premiums

by

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Options and higher-order risk premiums

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In this windy and rainy afternoon, just as many other days that I enjoyed in my warm office, I sat before the screen and tried to recall how I ended up here at this moment. Twenty years ago, I lived in Urumqi, the most remote city from any sea in the world. And this is why I became so obsessed with all sorts of fresh seafood in the open market in Rotterdam. Ten years ago, I was a bachelor student in public administration in Beijing. My passion for the ever-changing financial market and the beauty of mathematics leads me through all the difficulties that I encountered to change my major to finance, and later lead me to the Netherlands. Life is full of randomness and surprises, but I believe that one's true instinct for what she or he genuinely desires largely determines the long-run trend in life. As my PhD is about to finish soon, I would like to take this opportunity to thank people who helped me, supported me and encouraged me during the past years.

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1 Introduction

In 1900, the French mathematician Louise Bachelier (1870-1946) published his PhD thesis, titled "The Theory of Speculation". In Bachelier (1900), he constructed the first mathematical theory of continuous stochastic process (five years before Einstein), now called Brownian motion, and came up with an option pricing formula extremely close to the Black-Scholes formula. His work was largely forgotten for sixty years until Paul Samuelson and others realized that he had made a seminal contribution to the understanding of the dynamics of asset prices. In 1970s, Bachelier's insights on option pricing were refined and systemized by Black and Scholes (1973) and Merton (1973).

Modeling asset prices using Brownian motion has a long-lasting impact on the development of modern finance. As a powerful approximation of the asset price process most of the time, Brownian motion has been used in the modeling of the financial market, the practice of risk management, asset allocation and many other areas in finance. When the market goes to turmoil, however, the simple model may lead to underestimation of risk. Note that Brownian motion is a continuous process with independent and stationary increments that follow the normal distribution. It essentially captures the characteristics of the efficient market that the investors have incorporated all relevant information in prices and that the fluctuation of the prices is independent of its own history. Though Brownian motion has been widely used as a handy tool among academics and practitioners, much evidence has shown that the assumptions underlying the Brownian motion models may not be valid in the real world. There are at least four evidence not consistent with the assumptions of Brownian motion. First, financial time series usually exhibit skewed and fat-tailed distributions. Second, the dynamic of the financial time series is occasionally discontinuous, e.g. there are large movements in stock or index returns after the announcement of an unexpected event. Third, there exist many "anomalies" which cannot be easily explained by the efficient market hypothesis, for instance, the size effect, the value effect and the idiosyncratic volatility puzzle. Fourth, the volatility is time-varying rather than constant.

The chapters in this thesis relax the assumptions of Brownian motion in different ways and attempt to address the facts that are not consistent with the model of Brownian motion. In the second chapter coauthored with Chen Zhou, we consider a dynamic model that the stock returns have continuous (diffusion) and discontinuous (jump) components. Empirical literature[1] finds that idiosyncratic risks are priced in the expected returns of different assets. In the model of Chapter 2, we also add this feature and decompose the continuous and discontinuous components further into systematic and idiosyncratic counterparts. By considering a general pricing kernel with all underlying risk factors, we derive the expected return of individual stocks and decompose it into four risk premiums related to the four types of risks. Empirically, we estimate a model with dynamic volatility and dynamic jump intensities for 30 stocks and investigate the asset pricing consequences.

In the option market, professional traders often quote the options in terms of implied volatility, inverted from the option prices by the Black-Scholes formula. However, the implied volatility is calculated based on the assumption that the stock returns follow a log normal distribution. When the distribution exhibits heavy tail or skewness, which is usually the case in the financial

[1] For instance, Goyal and Santa-Clara (2003a), Ang et al. (2006a), Fu (2009), Huang et al. (2010) and Cao and Han (2013).

markets, it is not an accurate measure of volatility of the underlying risk neutral distribution. This is partly the reason why we observe a non-flat implied volatility surface for the index options across different maturities and strike prices. In the third chapter coauthored with Chen Zhou, we propose to use the maximum entropy method to estimate a single implied volatility measure from panels of option prices with different strike prices. The new nonparametric estimator is called the "entropy-based implied volatility". The method is free of distribution assumptions and can better reveal the information of the risk neutral distribution contained in the option prices . Numerical examples show that the maximum entropy method outperforms the Black-Scholes model and the model-free method in extracting the implied volatility. This phenomenon is more pronounced if the risk neutral distribution of the underlying asset deviates from the normal distribution, or if the number of available options is limited. In addition, the maximum entropy method allows for constructing confidence intervals around the implied volatility and extracting the implied skewness and kurtosis. We apply the maximum entropy method to the S&P500 index options for predicting future realized volatility. We empirically test the information content of the EBIV in predicting future monthly realized volatilities and index returns. Our empirical results point to the direction that the EBIV performs at least at a comparable level with the MFIV in different specifications. In many cases, its information content is of the highest among all available volatility measures, both in terms of in-sample fit and out-of-sample predictive power.

In the Black-Scholes model, the volatility parameter is constant and the European call and put options can be perfectly replicated by a portfolio of underlying asset and riskless bond. This implies that the return of a dynamically delta-hedged option portfolio should be equal to the risk free rate. However, Bakshi and Kapadia (2003) find that the dynamic delta-hedged strategy systematically underperforms zero. Hence, some additional risks are not hedged away after dynamic delta-hedging and the negative return may represent some sort of risk premium. Bakshi and Kapadia (2003) show that the negative delta-hedged option return is essentially an indicator of a negative variance risk premium. One economic interpretation is that volatility is unpredictable and positively related to the large negative movement of the market, so buyers of market volatility are willing to pay a premium for downside protection. If we consider the equity premium as the risk premium on the mean of the stock return, variance risk premium and jump risk premium are the higher order risk premium on the higher order moments of the stock return. The importance of understanding the variance risk premium lies not only in its practical usefulness, e.g. better prediction power for future return and volatility, but also in its role of further understanding the risk and return tradeoff as the central question in finance research.

The last chapter studies the determinants of the variance risk premium in the equity option market both theoretically and empirically. In this chapter, my coauthor Aurelio Vasquez and I use the return of a delta-hedged equity option portfolio as the proxy of the variance risk premium. We consider a stylized capital structure model with double-exponential jump diffusion process and derive the expected return of the delta-hedged option portfolio based on the model. The underlying asset process has to be modeled with a second random source, i.e. jumps follow a compound Poisson distribution, because otherwise the model cannot capture the fact that the delta-hedged equity option return is on average negative and represent the variance risk premium. There are two sources of the variance risk premium: stochastic volatility and unpredictable jumps. The jump model is chosen for simplicity and analytical reason. The model shows that the expected return of delta-hedged equity option portfolio is determined by two firm-level variables: financial leverage and asset volatility of the firm. Given other firm characteristics constant, the higher the leverage ratio, the higher the bankruptcy risk of the firm and the larger premium the option investors require to be compensated. The result also suggests that it is important to take into account the higher order polynomials of the determinants. Empirically, we find that these two structural variables can explain a large portion of the cross-sectional variation in the data and even subsume information in other determinants documented in the literature, such as idiosyncratic volatility and liquidity. The results from the double sorting portfolios are consistent with the theoretical implications. The empirical evidence also supports the nonlinear relation between the determinants and the delta-hedged equity option returns. These findings are robust across calls, puts and different moneyness levels.

To sum up, this thesis ventures beyond the traditional confines of Brownian motion and the assumption of log-normality in the finance literature, in both econometrics and economics ways. It not only involves the structural estimation of the parametric jump diffusion model (Chapter 2) and the non-parametric econometrics techniques (Chapter 3), but also provides the asset pricing implications on the higher order risk premiums (Chapter 2, 3 and 4) and the theoretical characterization of the variance risk premium on the equity options (Chapter 4). From these attempts, it is shown that venturing beyond normality helps us to further understand the patterns in both stock and option markets and the dynamics of the higher order risk premiums.

2 The decomposition of jump risks in individual stock returns

2.1 Introduction

Jumps in stock returns of individual firms are triggered by either systematic events or idiosyncratic shocks. During crisis events such as oil crisis in 1973, the black Monday in 1987, the dot-com crash in 2000 and the subprime crisis from 2007 to 2009, the financial market witnessed large jumps in most traded stocks. In addition, individual stocks may occasionally experience jumps due to firm specific events, such as earning surprises, merger and acquisition, etc. This chapter provides a new modeling framework for the individual stocks that allows for the estimation of time-varying systematic and idiosyncratic jump intensities and volatilities. From an asset pricing point of view, it is of both theoretical and practical importance to understand, how the systematic and idiosyncratic jump intensities can be estimated, and how they are related to the equity risk premium. This new model accommodates the joint dynamic structures of individual stock returns and the market returns, while allowing for jumps. Under such a framework, we estimate the dynamics of idiosyncratic and systematic jump intensities and volatilities for individual stocks and investigate the roles of different risks in the dynamics of equity premium.

We model the return innovation by a Generalized Autoregressive Conditional Heteroskedastic (GARCH)-jump mixture model in the spirit of Maheu et al. (2013). Maheu et al. (2013) only focuses on the market returns. We intend to investigate the dynamics of individual excess stock returns and allow the stock innovations to be affected by the market innovations. To be more specific, the market innovation has two components, which we call "market jump" and "market diffusion". The jump component follows a compound Poisson-normal distribution with autoregressive jump intensities. The diffusive component is governed by an asymmetric two-component GARCH process. The stock innovation has four components: "systematic jump" , "idiosyncratic jump", "systematic diffusion" and "idiosyncratic diffusion". The systematic jump in the stock innovation is triggered by the market jump with a certain probability. The systematic diffusion component loads on the market diffusion component governed by "beta", similar to that in the Capital Asset Pricing Model (CAPM). The idiosyncratic components are directed by similar dynamic structures as in the market components, but are independent from their systematic counterparts. To estimate the model, we provide a filter that can filter daily excess stock returns into large (jump) versus small (diffusion) components, as well as systematic and idiosyncratic counterparts in each of them.

In addition, our model allows for the decomposition of the dynamic equity premium by assuming a general pricing kernel with all underlying risk factors in the economy. The traditional CAPM suggests that the idiosyncratic risk is diversified away and not priced. However, empirical studies find that idiosyncratic risks not only matter in predicting the time series of stock market return[1], but it is also priced in the cross-section of stock returns[2]. The-

[1]For example, Goyal and Santa-Clara (2003b) and Guo and Savickas (2006).

[2]For example, Ang et al. (2006a), Fu (2009), Ang et al. (2009), Huang et al. (2010) and Stambaugh et al. (2015).

oretically, the pricing of idiosyncratic risks can be explained by the fact that investors in reality do not hold perfectly diversified portfolios. Levy (1978) and Merton (1987) show that under-diversified investors demand a return compensation for bearing idiosyncratic risk. In a asset pricing model based on prospect theory, where investors are loss averse over the fluctuations of their own stocks, Barberis and Huang (2001) also provide an explanation for the relation between expected returns and idiosyncratic risk. In this chapter, we include the idiosyncratic components of the stock innovations in the pricing kernel and test whether they are significantly priced in the dynamics of the equity premium. The specification of the pricing kernel is similar as that in Gourier (2016) and Bégin et al. (2016). The expected stock return can then be decomposed into four risk premiums: premiums on systematic and idiosyncratic diffusion risks and systematic and idiosyncratic jump risks, respectively.

We conduct a joint estimation strategy to identify different components in daily stocks returns from 15 stocks from 1963 to 2015. For each stock, we estimate the model for the market return and the stock return simultaneously. The main advantage is that the joint estimation helps us to identify the dynamics of the idiosyncratic components better than a two-stage estimation strategy, such as first estimating the model for market returns and then estimating for the stock returns conditional on the dynamics of market jump intensities and volatilities.

For our parameterization and sample, the idiosyncratic jump intensity and volatility account for a large amount of the total jump intensity and volatility for the stocks: idiosyncratic jump intensity contributes to 82.25% of the total jump intensity, and idiosyncratic variance contributes to 66.70% of the total variance on average. Over the time dimension, the contribution of idiosyncratic risks is declining, which implies that the firms are more and more affected by the systematic risks over the past 50 years. Further, all four types of risks are related to sizable premium in the expected return of individual stocks over time. The equity premium associated to idiosyncratic (jump) risks contribute to 57.18% (16.25%) of the total equity premium on average. Lastly, the cross-sectional difference in the expected stock returns in our sample can be explained by the difference in the model-implied systematic jump, idiosyncratic diffusive and idiosyncratic jump risk premiums.

The closest econometrics approach in the literature is Maheu et al. (2013). They estimate a GARCH-jump mixed model for the market returns with feature of time-varying equity premium and the role of jump in equity premium. We extent their framework to accommodate the estimation for stock returns, i.e. the need for estimating systematic and idiosyncratic counterparts in both jump and diffusive components. Our work is also related to Maheu and McCurdy (2004). They estimate the dynamics of jump and diffusive components in stock returns with constant equity premium. The difference between their model and ours is that we introduce the systematic and idiosyncratic counterparts in each components and consider their roles in the time-varying equity premium. On the technical side, we provide a procedure to filter out the four components in stock innovation. This is comparable to the one in Christoffersen et al. (2012), who estimated different specifications of dynamic jump model for the S&P 500 index.

This chapter complements the recent studies that intend to disentangle the four types of risks in equity premiums and in higher order risk premiums. Using stock return and option prices, Gourier (2016) and Bégin et al. (2016) conduct a joint estimation of both stock and option data to decompose the four risk premiums associated with systematic and idiosyncratic diffusive and jump risks. They both find that idiosyncratic risks contribute to more than 40% of the total equity premium on average after 1996 and that idiosyncratic risk mostly comes from the jump risk component. Using different data sets and methodology, we share some similar results. For instance, with only stock return data, we find that the idiosyncratic risks contribute to 57.18% of the total equity premium and idiosyncratic jump intensity plays a major role in the total jumps. To identify the dynamics of jump risks, we use 50 years of daily stock returns, which enables us to observe the evolution of the contribution of different risks over a long period. However, the option data are only available from 1996. In addition, to better capture the contribution of systematic risks in equity premium in a long time frame, we let the exposures of the stock to the market jump and diffusion risks to be time varying, related to the business cycle variable. In estimation methodology, we depart from the previous two papers in that we jointly estimate for the market return and the stock return to better identify the uncorrelated systematic and idiosyncratic components, while they first estimate the market parameters and then estimate the parameters for the individual stocks given the estimated market parameters. To the best of our knowledge, this is the first study to model dynamic jump and diffusion components, while considering the decomposition towards systematic and idiosyncratic risks based only on time series of stock returns.

The remainder of the chapter proceeds as follows. Section 2.2 presents our model setup and discusses the expected stock return under our model. Section 2.3 discusses the estimation methodology. Section 2.4 provides the data and the estimation results. Section 2.5 discusses the implications on asset pricing. Section 2.6 concludes. The technical derivations are postponed to the Appendix in Section 2.7.

2.2 Model

Our model builds on Maheu and McCurdy (2004) and Maheu et al. (2013). The former presents mixed GARCH-jump models for individual stocks, while the latter considers time-varying equity premium in the market returns. Differently, we aim at accommodating both systematic and idiosyncratic risks in individual stock returns. In section 2.2.1, we first present the model on market return process and the dynamics of volatility and jump intensity. Then we discuss the model on individual stock returns in section 2.2.2. Lastly, we specify a pricing kernel and derive the expression of expected returns on individual stocks in section 2.2.3.

2.2.1 Dynamics of market returns

We model the continuously compounded market return by the combination of a normally distributed diffusion component and a jump component. We first discuss the general structure of each component and then describe the time-varying dynamics in conditional variance and jump intensity.

Assume the following decomposition of the market return:

$$\text{Market: } R^m_{t+1} = log(\frac{S^m_{t+1}}{S^m_t}) = \alpha^m_{t+1} + y^m_{t+1} + z^m_{t+1}, \tag{2.1}$$

where S^m_{t+1} denotes the market price at the close of day $t+1$ and α^m_{t+1} is related to model-implied market equity premium expected for period $t+1$, given the information set Φ_t. We will derive the expression of α^m_{t+1} in Section 2.3. The log return is further driven by two stochastic processes: a jump component, y^m_{t+1} and a diffusive component, z^m_{t+1}. They are assumed to be independent conditional on the information available at time t. Due to the dynamic interaction between the two terms, these are not unconditionally independent.

The jump innovation is governed by a conditional Poisson jump-arrival process combined by a normal jump-size distribution. Define the discrete-valued number of jumps in the market return over the time period t to $t+1$ as N^m_{t+1}. The conditional distribution of N^m_{t+1} is a Poisson distribution with jump intensity $h_{y,t+1}$,

$$P(N^m_{t+1} = j|\Phi_t) = \frac{\exp(-h_{y,t+1})h^j_{t,t+1}}{j!}, \; j = 0,1,2,...$$

The conditional arrival rate of jumps, $h_{y,t+1}$ is the expected number of jumps for period $t+1$ given information at time t, that is,

$$h^m_{y,t+1} = E(N^m_{t+1}|\Phi_t).$$

As in Maheu and McCurdy (2004) and Maheu et al. (2013), we parameterize the dynamics of conditional jump intensity $h^m_{y,t+1}$ depending on past intensity and "news",

$$h^m_{y,t+1} = w^m_y + b^m_y h^m_{y,t} + a^m_y \zeta^m_t, \tag{2.2}$$

where w^m_y is the drift term, b^m_y measures persistence of jump intensity dynamic and a^m_y is the news-impact coefficient, associated with the jump innovation ζ^m_t, defined as follows. The jump intensity innovation ζ^m_t is the forecast update of the number of jumps N^m_t, when the information set at t becomes available:

$$\begin{aligned}\zeta^m_t &= E[N^m_t|\Phi_t] - E[N^m_t|\Phi_{t-1}] = E[N^m_t|\Phi_t] - h^m_{y,t} \\ &= \sum_{j=0}^{\infty} jP(N^m_t = j|\Phi_t) - h^m_{y,t}.\end{aligned} \tag{2.3}$$

The first part in Equation (2.3) gives the ex-post probability of j jumps at time t given information at t. It can be calculated as:

$$P(N^m_t = j|\Phi_t) = \frac{f(R^m_t|N^m_t = j, \Phi_{t-1})P(N^m_t = j|\Phi_{t-1})}{f(R_t|\Phi_{t-1})}, \tag{2.4}$$

where $f(.)$ refers to the conditional density of the market return. Note from this definition that the expectation of ζ^m_t conditional on information set Φ_{t-1} is zero. The jump intensity process is directed by the jump innovation rather than the squared-return innovations. This allows the the impact of time-varying jump intensity on expected variance dynamics to be different from that captured by the GARCH component of variance.

We assume that the jump size follows a normal distribution $N(\theta^m, (\delta^m)^2)$, where θ^m refers to the mean of jump size and $(\delta^m)^2$ is the variance, and the jumps occur independently. That is, the jump components in the return process are given by:

$$y^m_{t+1} = \sum_{j=1}^{N^m_{t+1}} x^j_{t+1},$$

where x^j_{t+1}, $j = 1, 2, \cdots$ are independently and identically distributed (i.i.d.) random variables drawn from $N(\theta^m, (\delta^m)^2)$. Therefore, the conditional mean and variance of the jump component y^m_{t+1} are $h^m_{y,t+1}\theta^m$ and $h^m_{y,t+1}((\theta^m)^2 + (\delta^m)^2)$, respectively. The conditional mean of market return is thus expressed as $E[R^m_{t+1}|\Phi_t] = \alpha^m_{t+1} + \theta^m hy_{y,t+1}$.

Further, the diffusion term z^m_{t+1} is assumed to follow a normal distribution $N(0, h^m_{z,t+1})$ with conditional variance $h^m_{z,t+1}$, i.e.

$$z^m_{t+1} = \sqrt{h^m_{z,t+1}}\epsilon^m_{t+1},\ \epsilon^m_{t+1} \sim \mathrm{N}(0,1),$$

where $h^m_{z,t+1}$ is governed by a two-component GARCH model with feedback from jumps. We adopt the specification in Maheu et al. (2013):

$$h^m_{z,t+1} = h^m_{z1,t+1} + h^m_{z2,t+1}, \tag{2.5}$$

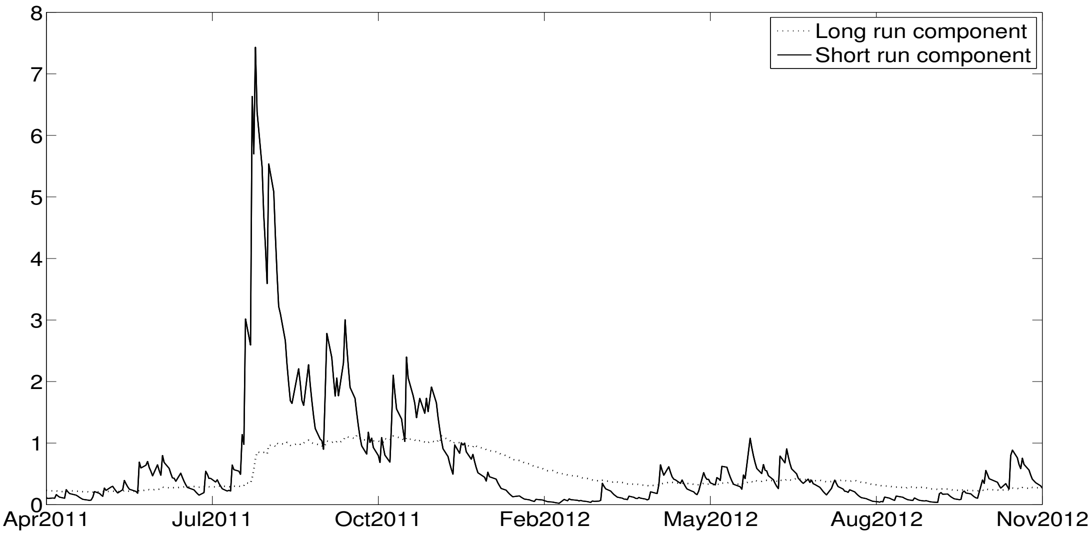
$$h^m_{z1,t+1} = w^m_z + b^m_{z1} h^m_{z1,t} + g^m_1(\Phi_t)(R^m_t - E[R^m_t|\Phi_{t-1}])^2 \tag{2.6}$$

$$h^m_{z2,t+1} = b^m_{z2} h^m_{z2,t} + g^m_2(\Phi_t)(R^m_t - E[R^m_t|\Phi_{t-1}])^2 \tag{2.7}$$

The long-run component is captured by $h_{z1,t+1}$ and the transitory moves are modeled by $h_{z2,t+1}$. The difference of the two models for

$$h_{z1,t+1}$$

Figure 2.1: Long run and short-run components of the diffusive variance (S&P 500)



Note: This figure shows the long-run and short-run components of the diffusive variance for market returns from 2011 to 2012.

and $h_{z2,t+1}$ is the parameter w_z^m, which capture the trend of the long-run component. Both Maheu et al. (2013) and our empirical analysis below confirm that it is essential to specify a two-component GARCH process to capture the diffusive volatility. It also helps to estimate jumps precisely. Otherwise, the noise and transitory part of the diffusive volatility could be potentially sorted as jumps[3].

The generalized news impact coefficient $g_i^m(\Phi_t)$ $(i = 1,2)$ for the ith GARCH

[3]We plot the long-run and short-run components from April 2011 to November 2012 in Figure 2.1, which shows that the long-run diffusive variance move more slowly than the short-run component.

component is given as,

$$g_i^m(\Phi_t) = \exp(\tau_{i1}^m + I_t^m(\tau_{i2}^m E[N_t^m|\Phi_t] + \tau_{i3}^m)),\ i = 1,2.$$

$$I_t^m = \begin{cases} 1 \text{ if } R_t^m - E[R_t^m|\Phi_{t-1}] < 0, \\ 0 \text{ otherwise.} \end{cases}$$

This model allows for asymmetric impact from good and bad news and feedback from jump innovations. The term $R_t^m - E[R_t^m|\Phi_{t-1}]$ is the total return innovation observable at time t. $E[N_t^m|\Phi_t]$ is the filtered number of jumps at time t given Φ_t.

Similar to Christoffersen et al. (2012), the filtered jump and diffusion terms, $\tilde{y}_t^m$ and $\tilde{z}_t^m$, can be obtained given parameters, which is illustrated in details in Appendix 2.7.4.

2.2.2 Dynamics of individual stocks returns

Next, we propose a model for the dynamics of individual stock returns, which involves the dependence between the return processes of individual stocks and the market. First, the individual stock return is modeled by a similar structure as that for the market return:

$$\text{Firm: } R_{t+1}^i = log(\frac{S_{t+1}^i}{S_t^i}) = \alpha_{t+1}^i + y_{t+1}^i + z_{t+1}^i, \tag{2.8}$$

For the jump component y_{t+1}^i, we assume that when there is a jump in the market, the probability that the market jump will trigger a jump in individual stock i is p^i. Further, there is an idiosyncratic jump process that is independent from the market jumps. In other words, conditional on having N_{t+1}^m jumps in the market, the number of individual jumps N_{t+1}^i equals to the sum of a binomial distributed random number $B^i(N_{t+1}^m, p^i)$ and an independently Poisson distributed random number N_{t+1}^ϵ with intensity $h_{y,t+1}^\epsilon$. Then the number of individual jumps N^i_{t+1} follows a Poisson distribution with intensity $h^i_{y,t+1}$ as:

$$h^i_{y,t+1} = p^i h^m_{y,t+1} + h^\epsilon_{y,t+1}. \tag{2.9}$$

The jump component in the individual return, y^i_{t+1}, is therefore given as,

$$y^i_{t+1} = \sum_{j=1}^{N^m_{t+1}} x^j_{i,t+1} 1_{\text{market jump j triggers a jump}} + \sum_{j=N^m_{t+1}+1}^{N^m_{t+1}+N^\epsilon_{t+1}} x^j_{i,t+1}$$

Here, the first part consists of the jumps triggered by the market jumps, while the second part consists of the idiosyncratic jumps that are independent from the market jumps. We assume that the jump size of individual stock $x^j_{i,t+1}$, $j = 0, 1, 2, \cdots$, are i.i.d. and follow a normal distribution $N(\theta^i, (\delta^i)^2)$. When a jump is triggered by a market jump, the size of the triggered jump and the corresponding market jump are assumed to be correlated with a correlation ϕ^i. The dynamics of $h^\epsilon_{y,t+1}$ follows a similar structure as in Equation (2.10):

$$h^\epsilon_{y,t+1} = w^i_y + b^i_y h^\epsilon_{y,t} + a^i_y \zeta^\epsilon_t. \tag{2.10}$$

in which the jump innovation term ζ^ϵ_t for period t is defined as

$$\zeta^\epsilon_t = E[N^\epsilon_t|\Phi_t] - E[N^\epsilon_t|\Phi_{t-1}] = E[N^\epsilon_t|\Phi_t] - h^\epsilon_{y,t}. \tag{2.11}$$

Notice that the ex-post expected number of idiosyncratic jumps is proportional to that of total jumps in the individual stock returns:

$$E[N^\epsilon_t|\Phi_t] = \frac{E[N^i_t|\Phi_t]\, h^\epsilon_{y,t}}{h^i_{y,t}}. \tag{2.12}$$

Finally, the ex-post expected number of total jumps in individual stock return $E[N^i_t|\Phi_t]$ can be calculated based on total jump intensity $h^i_{y,t}$ and conditional density of the stock return R^i_t:

$$E[N^i_t|\Phi_t] = \sum_{j=0}^{\infty} j \Pr(N^i_t = j|\Phi_t) = \sum_{j=0}^{\infty} j \frac{f(R^i_t|N^i_t = j, \Phi_{t-1}) \Pr(N^i_t = j|\Phi_{t-1})}{f(R^i_t|\Phi_{t-1})} \tag{2.13}$$

The jump innovation term ζ_t^ϵ is then determined by Equation (2.11) to (2.13).

Next, in the spirit of CAPM, we model the total diffusion component of the individual stock as the sum of systematic and idiosyncratic diffusion component:

$$z_{t+1}^i = \beta^i z_{t+1}^m + z_{t+1}^\epsilon, \tag{2.14}$$

where β^i is the factor loading of stock i on systematic diffusive risk and the idiosyncratic diffusive component z_{t+1}^ϵ follows a normal distribution $N(0, h_{z,t+1}^\epsilon)$. Further, z_{t+1}^m and z_{t+1}^ϵ are independent from each other.

The dynamic of idiosyncratic conditional variance has a parallel structure as that of the market. In addition, we assume that only the idiosyncratic innovation affects idiosyncratic conditional variance as follows:

$$h_{z,t+1}^\epsilon = h_{z1,t+1}^\epsilon + h_{z2,t+1}^\epsilon, \tag{2.15}$$

$$h_{z1,t+1}^\epsilon = w_z^i + b_{z1}^i h_{z1,t}^\epsilon + g_1(\Phi_t)(R_t^i - E[R_t^i|\Phi_{t-1}])^2 \tag{2.16}$$

$$h_{z2,t+1}^\epsilon = b_{z2}^i h_{z2,t}^\epsilon + g_2(\Phi_t)(R_t^i - E[R_t^i|\Phi_{t-1}])^2. \tag{2.17}$$

The generalized new impact coefficient $g_j(\Phi_t)$ $(j = 1,2)$ for the jth GARCH component allows for an asymmetric impact of good and bad idiosyncratic news:

$$g_j(\Phi_t) = \exp(\tau_{j1}^i + I_t(\tau_{j2}^i E[N_t^\epsilon|\Phi_t] + \tau_{j3}^i)),\ j = 1,2.$$

$$I_t = \begin{cases} 1 \text{ if } R_t^i - E[R_t^i|\Phi_t] < 0; \\ 0, \text{ otherwise.} \end{cases}$$

The dynamic of the idiosyncratic diffusive component is driven by the innovation of the stock return:

$$R_t^i - E[R_t^i|\Phi_t] = R_t^i - \alpha_t^i - \theta^i h_{y,t}^i.$$

The term $p^i\theta^i E[N_t^m|\Phi_t]$ represents the expected value of jumps triggered by the market, and is thus not regarded as idiosyncratic innovation. The $\beta^i \tilde{z}_t^m$ is the filtered systematic diffusive component in which $\tilde{z}_t^m$ can be obtained from the filtering procedure in Appendix 2.7.3. The $\alpha_t^i + \theta^i h_{y,t}^t$ is the conditional mean of the stock return.

The conditional variance of the diffusion component of the individual stock return can be expressed as,

$$h_{z,t+1}^i = (\beta^i)^2 h_{z,t+1}^m + h_{z,t+1}^\epsilon. \tag{2.18}$$

Under our model setup, the conditional variance of the individual stock return can be derived as:

$$Var(R_{t+1}^j|\Phi_t) = (\beta^i)^2 h_{z,t+1}^m + h_{z,t+1}^\epsilon + (p^i h_{y,t+1}^m + h_{y,t+1}^\epsilon)((\theta^i)^2 + (\delta^i)^2). \tag{2.19}$$

In this chapter, we only use the return data to estimate the jump and volatility dynamics of both the market and the stocks. Hence, long term time series from 1963 to 2015 that contain several extreme movements are used to identify the parameters that govern the dynamics of the infrequent jumps. When estimating from such a long time series, we allow the exposures of the individual stocks to the market risks, namely β^i and p^i, to vary with business condition in the spirit of conditional CAPM. Motivated by Avramov and Chordia (2006), we model the conditional β^i and p^i as:

$$\beta_t^i = \beta_1^i + \beta_2^i BC_t,$$
$$p_t^i = p_1^i + p_2^i BC_t,$$

Following Jagannathan and Wang (1996) and Avramov and Chordia (2006), we focus on the default spread as a proxy for the business condition variable BC_t, which has been shown to have the best predictive power for future business conditions in the literature. The default spread is defined as the yield differential between the Baa and Aaa corporate bonds.

2.2.3 Pricing Kernel and Expected Return

To facilitate our analysis of how various risk factors are priced, we introduce a parametric pricing kernel to price all four risk factors, including both volatility risk and the jump risk. With the assumed pricing kernel, we derive the expected returns of the individual stocks and the market. In the absence of arbitrage, a pricing kernel M_t is a positive stochastic process such that $M_t S_t$ is a martingale for any stock price process S_t. In a discrete-time setting, this condition is represented by the following identity:

$$E_t[\frac{M_{t+1}}{M_t}\frac{S_{t+1}}{S_t}] = 1.$$

Naik and Lee (1990) demonstrated that the market is incomplete when jumps are present in stock prices. Such market incompleteness implies the absence of a unique pricing kernel. To allow for the pricing of different risk factors, we adopt one candidate pricing kernel that prices the four sources of risks in our model on stock returns: systematic jump shock, systematic diffusion shock, idiosyncratic jump shock and idiosyncratic diffusion shock. We specify a standard log linear pricing kernel as:

$$\log(\frac{M_{t+1}}{M_t}) = -r - \mu_{t+1} - \Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{i=1}^{J} \Lambda^i z^{\epsilon}_{i,t+1} - \sum_{i=1}^{J} \Lambda^i y^{\epsilon}_{i,t+1}, \tag{2.20}$$

where r is risk-free rate, Λ^m, Λ^i, $i = 1, ..., J$ are related to the prices of different risk factors. Here, J denotes the total number of stocks in the economy. Recall that the terms z^m_{t+1} and y^m_{t+1} are market diffusive and jump components, and the terms $z^{\epsilon}_{i,t+1}$ and $y^{\epsilon}_{i,t+1}$ are idiosyncratic risk components in stock i, independent from the market risks. In such a pricing kernel, we implicitly assume that investors' portfolio are not well-diversified, and idiosyncratic diffusive and jump components are potentially priced. Hence, this general structure allows all possible risks in the market and individual stocks to be priced. In

Maheu et al. (2013), they specify a nonlinear pricing kernel to capture the risks due to dynamics of the higher order moments. More parameters would add the richness of the model, but also increase the difficulty of the estimation. To keep the simplicity of the model and to address the main research question in the paper, we use the linear model to specify the pricing kernel.

The coefficient μ_{t+1} is a normalizing constant to ensure that $E_t[\frac{M_{t+1}}{M_t}] = e^{-r}$. This implies that,

$$\mu_{t+1} = \log E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{i=1}^{J} \Lambda^i z^{\epsilon}_{t+1} - \sum_{i=1}^{J} \Lambda^i y^{\epsilon}_{t+1})].$$

We apply the pricing kernel in Equation (2.20) to price all stocks in the economy. In the absence of arbitrage, the following equations hold:

$$E_t[\frac{M_{t+1}}{M_t} e^{R^m_{t+1}}] = 1 \text{ and } E_t[\frac{M_{t+1}}{M_t} e^{R^i_{t+1}}] = 1, \tag{2.21}$$

for all i's. This leads to the following proposition on the expected returns of stocks. The proof is left to Appendix 2.7.5.

Proposition 2.1. *Under our model on the dynamics of the market and individual returns with the pricing kernel in Equation (2.20), the discrete-time conditional expected returns of the market and individual stock can be written as:*

$$\begin{aligned} E_t[\exp(R^m_{t+1})] &= \exp(r + \lambda_z h^m_{z,t+1} + \lambda_y h^m_{y,t+1}) \\ E_t[\exp(R^i_{t+1})] &= \exp(r + \beta^i \lambda_z h^m_{z,t+1} + p^i(e^a(1-e^b) + e^{\theta^i + \frac{1}{2}(\delta^i)^2} - 1) h^m_{y,t+1} + \\ &\quad \lambda_{zi} h^{\epsilon}_{z,t+1} + \lambda_{yi} h^{\epsilon}_{y,t+1}). \end{aligned}$$

where $a = -\Lambda^m \theta^m + \frac{1}{2}(\delta^m)^2 (\Lambda^m)^2$ *and* $b = \theta_i + \frac{1}{2}(\delta^i)^2 - \phi^i \delta^i \delta^m \Lambda^m$.
In addition, the parameters in the pricing kernel and in the dynamics of the market and individual stock returns should satisfy the following equations:

$$\lambda_z = \Lambda^m,\ \lambda_y = \xi^m(1) + \xi^m(-\Lambda^m) - \xi^m(1 - \Lambda^m),$$
$$\lambda_{zi} = \Lambda^i,\ \lambda_{yi} = \xi^i(1) + \xi^i(-\Lambda^i) - \xi^i(1 - \Lambda^i),$$

where $\xi^m(\psi) = exp(\theta^m \psi + \frac{(\delta^m)^2 \psi}{2}) - 1$ *and* $\xi^i(\phi) = exp(\theta^i \phi + \frac{(\delta^i)^2 \phi}{2}) - 1$.

From Proposition 2.1, we get the expected continuously compounded market return as:

$$\begin{aligned}\alpha_{t+1}^{m} &= r + (\Lambda^{m} - \frac{1}{2})h_{z,t+1}^{m} + (\xi^{m}(-\Lambda^{m}) - \xi^{m}(1-\Lambda^{m}))h_{y,t+1}^{m} \\ &= r + (\lambda_{z} - \frac{1}{2})h_{z,t+1}^{m} + (\lambda_{y} - \xi^{m}(1))h_{y,t+1}^{m}. \end{aligned} \tag{2.22}$$

For the individual stock i, the expected continuously compounded return is then:

$$\begin{aligned}\alpha_{t+1}^{i} = r + (\beta^{i}\Lambda^{m} - \frac{1}{2}(\beta^{i})^{2})h_{z,t+1}^{m} + p^{i}e^{a}(1-e^{b})h_{y,t+1}^{m} + \\ (\Lambda^{i} - \frac{1}{2})h_{z,t+1}^{\epsilon} + (\xi^{i}(-\Lambda^{i}) - \xi^{i}(1-\Lambda_{yi}))h_{y,t+1}^{\epsilon},\end{aligned}$$

Note that λ_z, λ_{zi}, λ_y and λ_{yi} are the market prices for loading on four types of risks, which are related to the parameters Λ^m and Λ^i in the pricing kernel. The expected stock return from our model can be interpreted in the following ways. First, with complete diversification and in the absence of jumps, only the first term in the expected return of individual stocks remains. The expected return derived under our model reduces to that from the CAPM.

Second, when the correlation between jump sizes in the market and individual stock returns ϕ^i is zero, the second term in the expected return of individual stocks is reduced to $p(e^a - 1)(1 - e^b)h_{y,t+1}^m$. Because the average jump size for the market return is generally estimated as negative in the literature with dynamic jump intensity,[4] the parameter a is therefore positive. The sign of the premium thus depends on the sign of the parameter b. As stated in Jiang and Oomen (2008), individual stock price jumps tend to be idiosyncratic and predominantly positive, presenting an interesting contrast to mostly negative jumps in market portfolios. Maheu and McCurdy (2004)

[4] Christoffersen et al. (2012) estimate the average jump size of SP 500 index as −0.174 from the DVSDJ model. In Maheu and McCurdy (2004), the jump size mean θ is significantly negative for the three indices (DJIA, Nasdaq 100 and TXX)

also find that the jump size mean is centered around zero for most of the individual firms. From the empirical evidence, we conjecture that θ^i is centered around zero or slightly higher than zero. Hence, b is positive, which implies that $(e^a - 1)(1 - e^b)$ is negative. Therefore, the expected return of the individual stock is decreasing with respect to the probability that market jump can trigger an individual jump p.

Third, when the correlation between jump sizes of market and individual stock returns ϕ^i is not zero, stocks whose jump sizes are more correlated with the market earn higher returns. Finally, idiosyncratic diffusive and jump risk premiums are included in the expected return because the pricing kernel allows all underlying risk factors n the economy. We will test their statistical and economic significance in our empirical study.

2.3 Estimation Methodology

In this section, we discuss our methodology to estimate the model described in Section 2.2. We apply a joint estimation strategy, i.e. to estimate the parameters for the market and the stock return dynamics together for each stock. An alternative method is to use a two-step estimation strategy: first estimate parameters in the market dynamics and then estimate the parameters in the individual stocks by substituting the estimated parameters and dynamics of $\hat{h}_y^m$ and $\hat{h}_z^m$ into the dynamics of h_y^i and h_z^i in the individual stock returns. There are two reasons why we prefer the joint estimation. First, in Equation (2.9), both $h_{y,t+1}^i$ and $h_{y,t+1}^\epsilon$ are latent processes. Therefore, it would be difficult to identify the parameters in the systematic and idiosyncratic component and make sure these are independent from each other in the second stage. Second, with the joint estimation methodology, the program can achieve a higher likelihood. For these two reasons, the joint estimation strategy is essential for identifying the parameters in the idiosyncratic and systematic components.

First, we provide the likelihood function for the market model. Given the parameters $\Theta^m = (\Lambda^m, w_z^m, b_{z1}^m, \alpha_{11}^m, \alpha_{12}^m, \alpha_{13}^m, b_{z2}^m, \alpha_{21}^m, \alpha_{22}^m, \alpha_{23}^m, w_y^m, b_y^m, a_y^m, \theta^m, \delta^m)$, we can get the time series of conditional market variance $h_{z,t+1}^m$ and market jump intensity $h_{y,t+1}^m$ by iterating from the starting time.

The likelihood function is given as follows. First, conditional on having $N_{t+1}^m = j$ jumps during time t to $t+1$, the jump component follows a normal distribution $N(j\theta^m, j(\delta^m)^2)$. The conditional density function for stock return can be written as:

$$f(R_{t+1}^m | N_{t+1}^m = j) = \frac{1}{\sqrt{2\pi(h_{z,t+1}^m + j(\delta^m)^2}} \exp \frac{(R_{t+1}^m - \alpha_{t+1}^m - j\theta^m)^2}{2(h_{z,t+1}^m + j(\delta^m)^2)}. \tag{2.23}$$

Second, since the number of jumps during time t and $t+1$ follow a Poisson distribution, we get that

$$\Pr(N_{t+1}^m = j) = \frac{(h_{y,t+1}^m)^j}{j!} \exp(-h_{y,t+1}^m). \tag{2.24}$$

Hence, the unconditional density of the return can be written as:

$$f(R_{t+1}^m) = \sum_{j=0}^{\infty} f(R_{t+1}^m | N_{t+1}^m = j) \Pr(N_{t+1}^m = j). \tag{2.25}$$

Then, the likelihood function can be constructed as:

$$L^m(R_{t+1}^m, \Theta^m) = \sum_{t=1}^{T} \log f(R_{t+1}^m), \tag{2.26}$$

where Θ^m includes 15 parameters for the market as stated above. We truncate the potential number of jumps in Equation (2.25) to a finite number. Maheu and McCurdy (2004) find that the conditional jump probability is zero for $N_{t+1} \geq 10$. The maximum number of jumps in a day is estimated as 5 in Christoffersen et al. (2012). Similarly, we also assume that the maximum number of jumps from t to $t+1$ is 5.

The likelihood function for the individual stock returns has the similar structure as that for the market returns. Substituting the subscript of Equation

(2.23) to Equation (2.26) from m to i, we have the likelihood function for the individual stocks:

$$L^i(R^i_{t+1},\Theta^i) = \sum_{t=1}^{T} \log f(R^i_{t+1}), \tag{2.27}$$

in which the parameter set Θ^i has 19 parameters: $\Theta^i = (\Lambda^i, w^i_z, b^i_{z1}, \alpha^i_{11}, \alpha^i_{12}, \alpha^i_{13}, b^i_{z2}, \alpha^i_{21}, \alpha^i_{22}, \alpha^i_{23}, w^i_y, b^i_y, a^i_y, \theta^i, \delta^i, \beta^i_1, \beta^i_1, p^i_1, p^i_2)$.

We estimate the 34 parameters by maximizing the sum of the likelihood function of the market returns and that of the stocks returns together for each stock. The joint likelihood function $L(R_{t+1},\Theta)$ is:

$$L(R_{t+1},\Theta) = L^m(R^m_{t+1},\Theta^m) + L^i(R^i_{t+1},\Theta^m,\Theta^i).$$

Based on the estimation result, the systematic jump and diffusion terms and the idiosyncratic jump and diffusion terms can be filtered out using the procedure in Appendix 2.7.4.

We are aware of the potential drawback that the estimation result for the market return may be different for each stock. As a justification of the methodology, in Section 2.4 we first estimate for the market alone by maximizing $L^m(R^m_{t+1},\Theta^m)$ and estimate jointly for the market and the stock by maximizing $L(R_{t+1},\Theta)$. We find that the estimated parameters of the market dynamic for all stocks are close to the estimated parameters from estimating the market return alone. In addition, we focus on the dynamics of the systematic and idiosyncratic components of the individual stocks in this chapter. To the best of our knowledge, joint estimation is the best solution to identify the parameters in each components of the stock return and to guarantee the correlation between idiosyncratic and systematic components as small as possible.

2.4 Estimation Results

This section first presents the data used in our empirical study and then the estimation results. Based on the results, we discuss the risk premiums related to the four potential risk factors.

2.4.1 Data

We use a dataset consisting of daily returns of the S&P500 index and 15 individual stock prices for the period Jan 3rd, 1963 to December 31st, 2015. The returns of the S&P500 index are regarded as the proxy of the market. These returns are adjusted for all applicable splits and dividend distributions and converted to continuously compounded daily returns. We have two criteria to select the stocks. First, the stocks are included in the S&P100 index by the end of 2015, which represent the largest and most established companies in the index. Second, the stocks are traded during the sample period from Jan 3rd, 1963 to December 31st, 2015. We select 15 stocks which satisfy the two criteria. The data are obtained from the Center for Research in Security Prices (CRSP) database.

Table 2.1 provides summary statistics for the daily continuously compounded returns of the 15 firms. The kurtosis ranges from 8.7 to 70.1, which shows strong evidence of non-normality in the stock returns. Such a non-normal feature calls for modeling jump risk. When plotting the time series of returns on S&P500 index in Figure 2.2, we observe evidence of discontinuous large changes reflecting jumps as well as the pattern of volatility clustering. These features call for modeling the dynamics in diffusive and jump risks. In order to get numerically stable estimates, we scale the daily return by 100, similar to Maheu et al. (2013). The details of the scaling procedure is presented in Appendix 2.7. The results in Table 2.1 to Table 2.6 are for the scaled

Table 2.1: Descriptive statistics for the daily stock return

	Mean	Std	Min	Max	Skewness	Kurtosis
ADM	0.029	2.048	-22.210	15.986	0.056	9.214
BAX	0.030	1.849	-30.504	16.465	-0.708	17.587
CL	0.031	1.621	-21.495	18.482	0.028	11.915
DD	0.013	1.600	-20.209	11.196	-0.151	9.119
DOW	0.022	1.832	-21.495	16.916	-0.222	11.403
EMR	0.027	1.588	-17.376	14.319	-0.144	10.092
GE	0.023	1.620	-19.251	17.984	-0.051	11.500
IBM	0.017	1.583	-26.119	12.351	-0.251	15.425
MMM	0.023	1.445	-30.114	10.899	-0.652	20.962
PEP	0.034	1.541	-15.448	14.951	0.021	8.919
PG	0.026	1.385	-37.687	20.029	-2.180	70.666
T	0.010	1.538	-23.920	20.837	-0.098	20.621
UTX	0.029	1.752	-33.213	12.789	-0.463	16.195
XOM	0.029	1.382	-26.726	16.455	-0.371	21.580
XRX	0.007	2.238	-29.801	32.963	-0.500	22.075
S&P500	0.009	1.017	-21.679	10.894	-0.9131	26.818

Table 2.1 presents the summary statistics for the daily returns of the 15 individual stocks and the S&P500 index. The dataset starts from January 3rd, 1963 and ends at December 31st, 2015. The daily returns are scaled by 100.

return.

2.4.2 Estimation Results for the market returns

Table 2.2 presents the estimated parameters for the returns on S&P500 index. In the column "Single estimation", we show the estimated parameters for estimating the market alone. First, we observe that the magnitude of jumps captured by the jump size estimates has an estimated mean -0.29 and variance 1.16. Both of the estimates are significant different from zero. In Figure 2.2(a), we observe that the negative jumps occur more frequently than the positive ones. This is in line with the positive market price of jump risk, since investors will be compensated for bearing potential negative price changes. Second, during the sample period, the jump arrival frequency matches the history of crisis. The pattern of $h^m_{y,t+1}$ in Figure 2.2(b) is consistent with the timing of the crises during the sample period. For example, the Asian crisis in 1997, the burst of internet bubble in 2000 and the recent subprime crisis in 2008. The unconditional expected value of the dynamic jump intensity $w_y^m/(1-b_y^m) = 0.15$ is comparable to the estimate in Maheu et al. (2013)[5]. The filtered jump component calculated using the estimated parameters and the procedure in Appendix 2.7.3 is presented in Figure 2.2(c).

Table 2.2: Parameter estimates for the market returns

	Single estimation		Joint estimation			
	parameters	t stats	mean	std	min	max
Λ^m	2.539	2.231	2.663	0.419	1.610	3.160
w_z^m	0.000	1.652	0.000	0.000	0.000	0.001
b_{z1}^m	0.976	61.341	0.976	0.003	0.971	0.978
α_{11}^m	-5.013	-23.387	-5.041	0.178	-5.268	-4.716
α_{12}^m	-1.611	-4.024	-1.354	0.350	-1.850	-0.780
α_{13}^m	0.954	2.963	0.905	0.415	0.059	1.378
b_{z2}^m	0.859	13.368	0.857	0.007	0.833	0.859
α_{21}^m	-14.008	-13.153	-13.854	0.233	-14.000	-13.041
α_{22}^m	-0.295	-3.442	-0.339	0.088	-0.479	-0.214
α_{23}^m	11.992	11.646	12.036	0.229	11.236	12.184
w_y^m	0.000	2.249	0.001	0.001	0.000	0.002
b_y^m	0.997	321.276	0.996	0.003	0.985	0.999
a_y^m	0.055	2.551	0.053	0.028	0.020	0.120
δ^m	-0.288	-3.421	-0.237	0.051	-0.302	-0.158
θ^m	1.156	11.420	0.983	0.123	0.805	1.164

Note: Table 2.2 shows the estimation results on the daily returns of S&P500 index, from January 1963 to December 2015. The column called "Single estimation" reports the results for estimating the market return alone with log likelihood 16052.803. The column called "Joint estimation" shows statistics of the 15 sets of estimated parameters for the market dynamics from joint estimation using both stock returns and market returns. In parentheses we report the t statistics. Note that we report estimation results with the return data multiplied by 100.

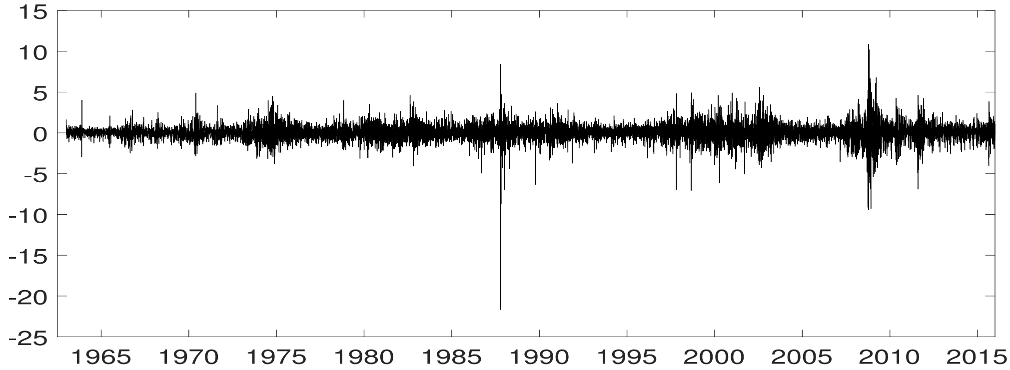
From the estimation on the market return, we observe positive price on systematic risk, with the estimated market price of systemic risk Λ^m 2.54 statistically larger than zero. Figure 2.3 illustrates the time varying equity premium for the market returns. The average equity premium is 0.05, indicated by the dotted line.

In the columns called "Joint estimation" in Table 2.2, we report the estimated market parameters from the joint estimation. Since we jointly estimate

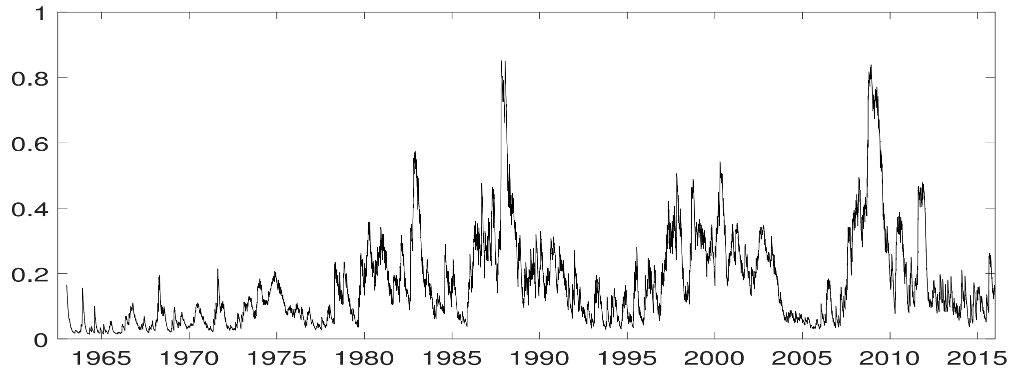
[5]The unconditional jump intensity can be estimated differently in different models due to specification of variance dynamics and intensity dynamics. In the DVSDJ model in Christoffersen et al. (2012), the estimate is around 0.025 for S&P500 and in Maheu and McCurdy (2004), it is 0.135 for DJIA.

Figure 2.2: Conditional jump intensity and filtered jump component (S&P500)

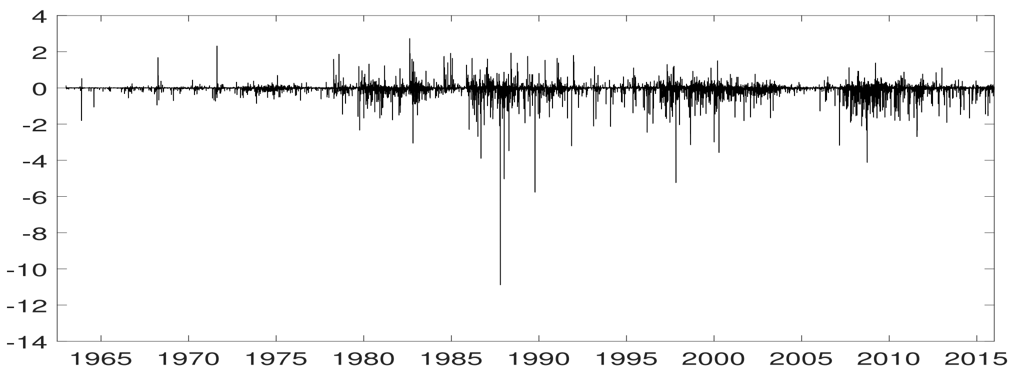
(a) Daily return



(b) Conditional jump intensity

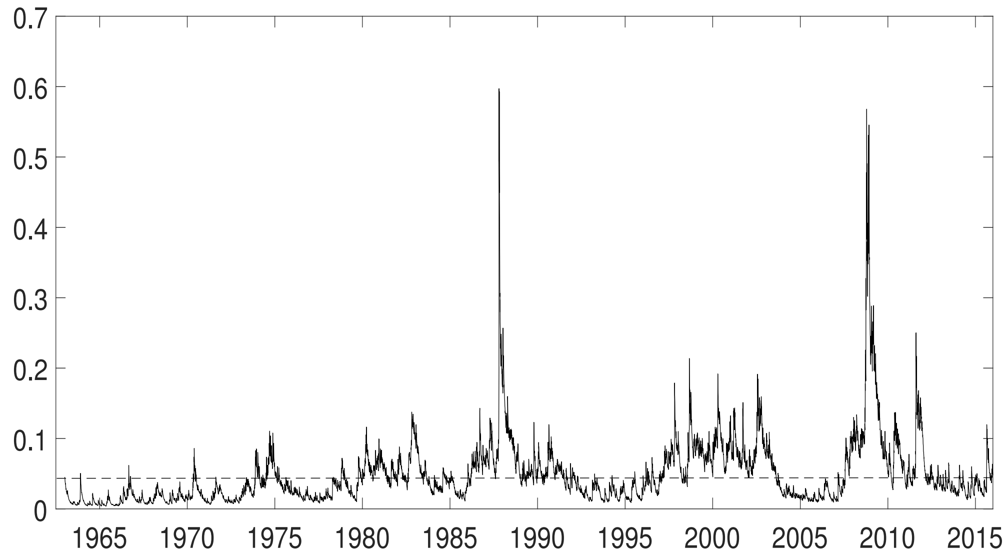


(c) Jump Component



Note: Figure (a) plots the daily return of the S&P 500 index from Jan 1963 to December 2015. By estimating the model in Section 2.2.1, we filter out the conditional jump intensity $h^m_{y,t+1}$ and plot in Figure (b). The filtered jump component is presented in Figure (c). The returns are scaled by 100.

Figure 2.3: Conditional equity premium (S&P 500)



Note: This figure shows the estimated time series of the daily conditional equity premium from 1962 to 2015. The dotted line represents the level of the unconditional mean of the equity premium.

for the stock and market returns, the estimation results for the market daily returns are not exactly the same for each stock. From the table, it shows that the results from joint estimation are close to that from the single estimation. The means of all parameters estimated from maximizing the joint likelihood are within the 95% confidence interval of the estimates from maximizing the market returns alone. The evidence suggest that the joint estimation methodology provides solid estimates for the market parameters. We ex-ante choose the joint estimation because of the advantages discussed ealier in this section, but from the comparison we conclude that the joint estimation is not such a necessity for this chapter.

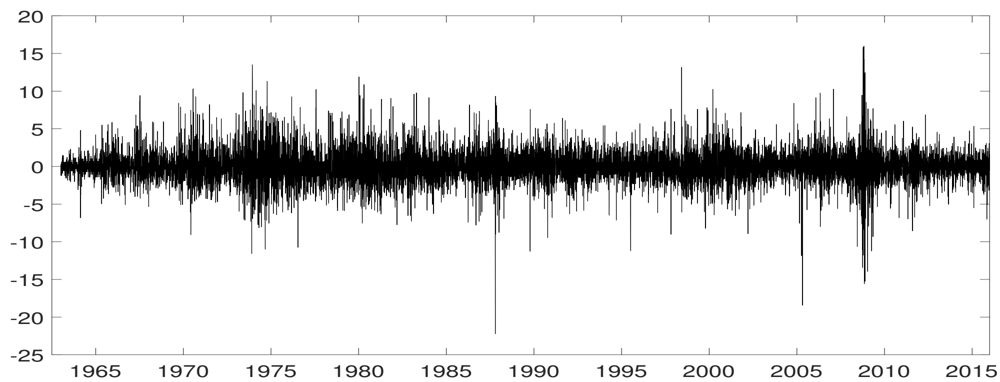
2.4.3 Estimation Results for the individual returns

The model for individual stock returns is estimated for the 15 stocks in our sample. We first show several general features of the model in Figure 2.4 to Figure 2.6 by taking the stock ADM (Archer Daniels Midland Company) as an example. We present the original times series plot for the daily returns of ADM in Figure 2.4(a). The decomposition of the total conditional jump intensity is shown in Figure 2.4(b) and (c). The systematic part contributes to 13.14% of the total conditional jump intensity on average. Hence, it is important to model idiosyncratic jump risk and study whether it is priced. When comparing diffusive and jump risks, we decompose the total conditional variance given in Equation 2.19 into the conditional variances of the diffusive and jump components. The decomposition of the total conditional variance is shown in Figure 2.5(b) and (c), respectively. On average, the contribution of conditional variance of the jump component to the total conditional variance is 45.66%.

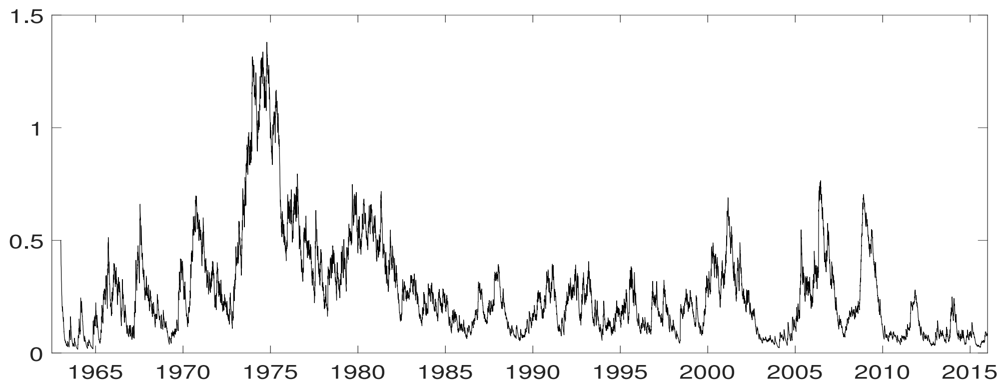
Using the filtering procedure in Appendix 2.7.4 and the estimated parameters, we filter out the systematic and idiosyncratic jump and diffusion components for ADM and show the plots in Figure 2.6. The solid lines are jump components and the dotted lines are diffusion components. The figures show that the model for individual stocks in Section2.2.2 can capture large negative and positive outliers which are important for modeling the heavy-tailed distribution of the stock returns. In addition, it shows that the model succeeds to differentiate the idiosyncratic and systematic components. For instance, the correlation between systematic and idiosyncratic diffusive components is 0.17 and the correlation between systematic and idiosyncratic jump components is 0.33. The patterns in these figures agree with the findings in Ornthanalai (2014) and Li et al. (2008) that the daily return data favor small-sized jumps that occur frequently over the current practice that typically model jumps as

Figure 2.4: Decomposition of conditional jump intensity (ADM)

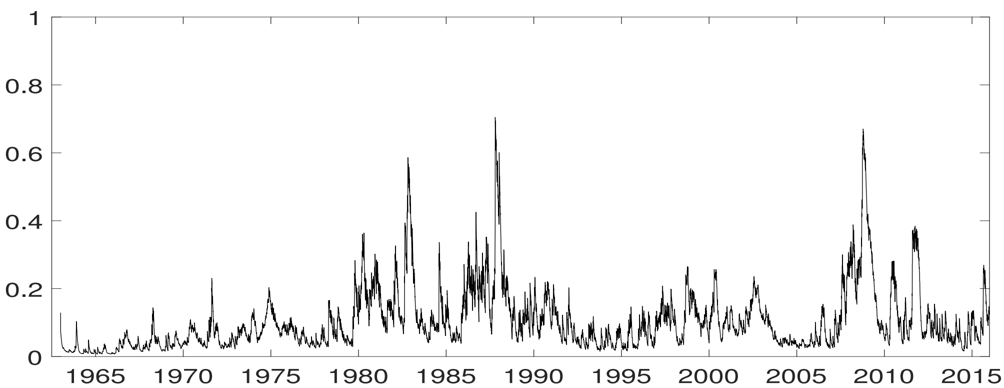
(a) Daily return



(b) Conditional idiosyncratic jump intensity



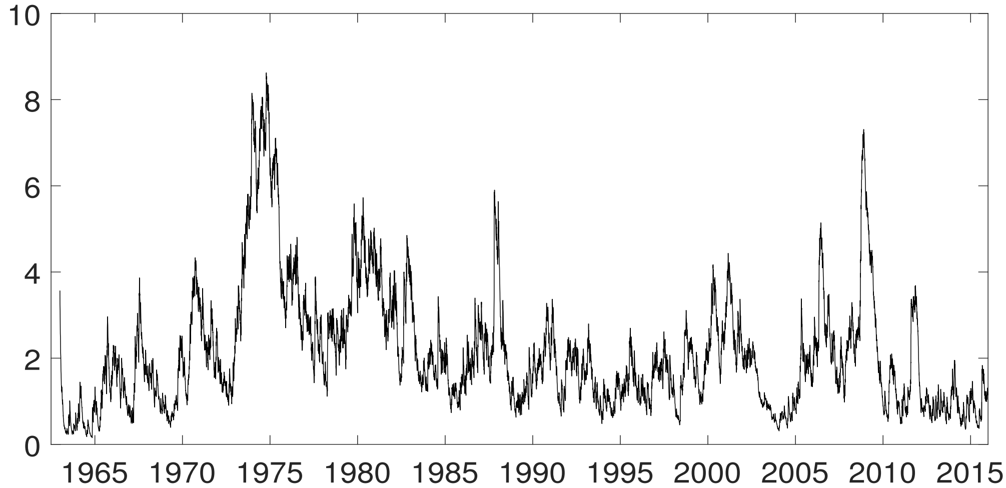
(c) Conditional systematic jump intensity



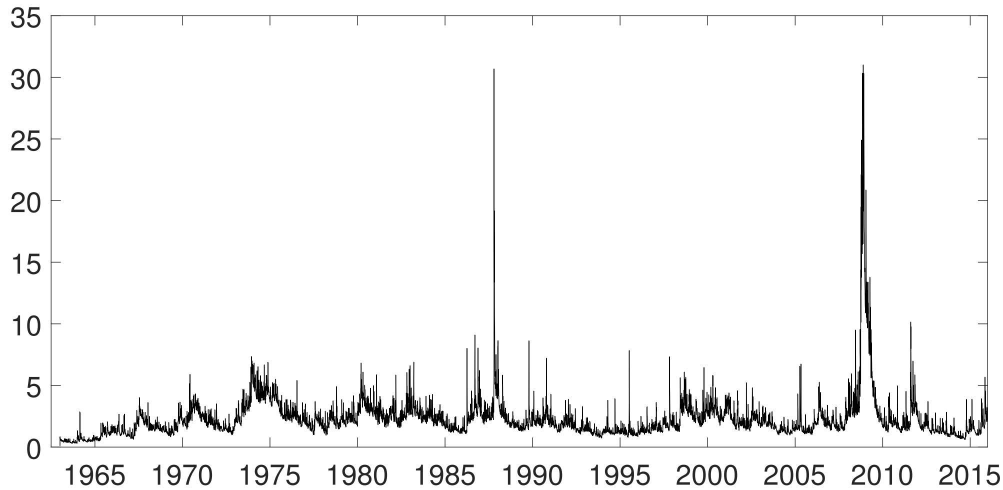
Note: Figure (a) plots the daily return of ABT from Jan 1963 to December 2015. Estimating the joint model for ADM in Section 2.2.2, we filter out the conditional jump intensity $h^i_{y,t+1}$ and decompose it into two parts: idiosyncratic jump intensity $h^{\epsilon}_{y,t+1}$ and systematic jump intensity $\beta^i_t h^m_{y,t+1}$. They are plotted in Figure (b) and (c).

Figure 2.5: Decomposition of conditional variance (ADM)

(a) Jump part of the conditional variance



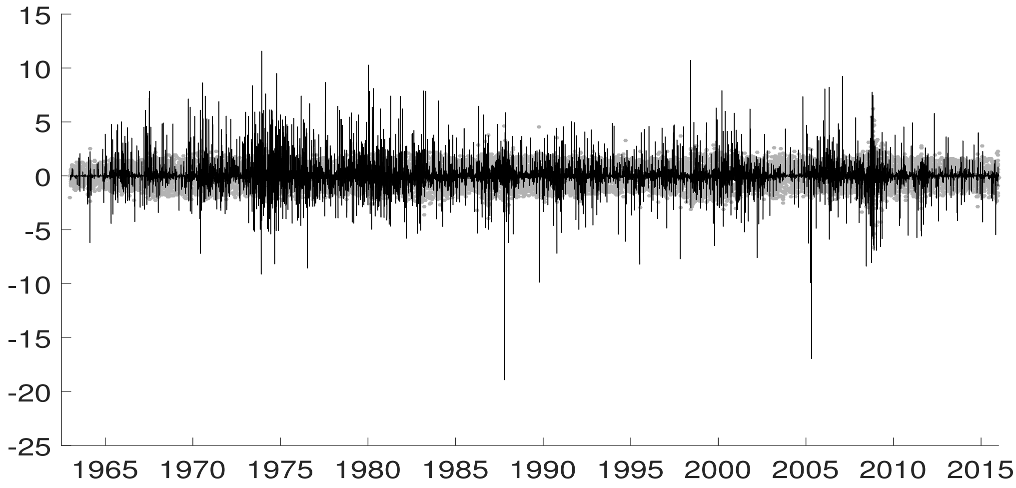
(b) Diffusive part of the conditional variance



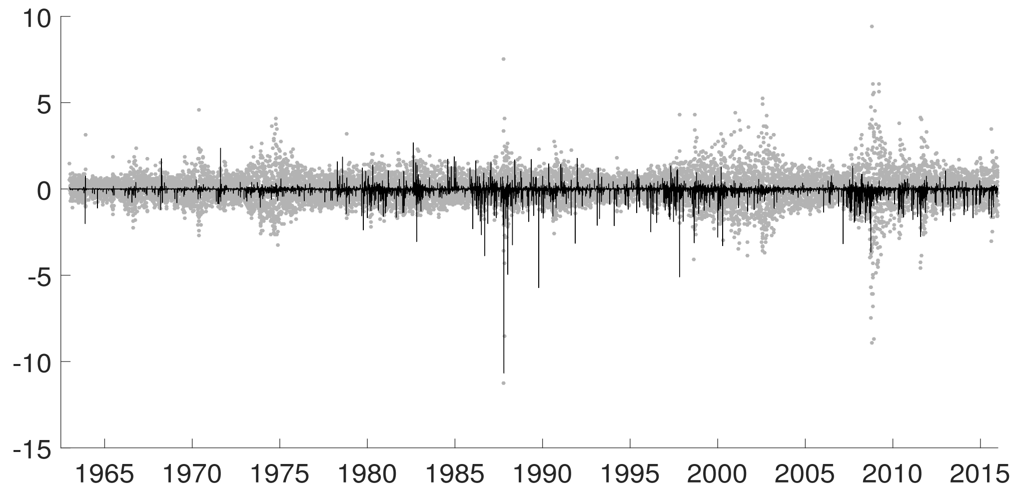
Note: We decompose the total conditional variance of ADM given in Equation 2.19 into the diffusive and jump variance components. The diffusive and jump components of the total variance are given in Figure (a) and (b).

Figure 2.6: Filtered diffusion and jump components (ADM)

(a) Filtered idiosyncratic jump and diffusion components for ADM



(b) Filtered systematic jump and diffusion components for ADM



Note: This figure shows the filtered jump and difussion components of ADM stock returns using the procedure in Appendix 2.7.4. Figure (a) shows filtered idiosyncratic jump (z_t^e) and difussion components (y_t^e) and Figure (b) shows systematic jump and diffusion components. The solid lines plots the jump components and the the gray markers represent the diffusion components.

large and rare event.

Next, we discuss the estimates of the model parameters. Since we present the summary statistics for the market parameters in Table 2.2, we only present the parameters for the idiosyncratic components and the exposure parameters to the market risks in Table 2.3 and Table 2.4 for each stock. From the cross-sectional comparison, we obtain the following stylized facts. First, there is evidence on the persistence of conditional jump intensity for all firms. The persistent parameters, b_y^i, for the jump intensity are significantly positive from 0.874 to 0.999. [6] Second, the importance of revision to the conditional idiosyncratic jump intensity is similar as that to the conditional systematic jump intensity. The parameter a_y^i, which captures the the effect of the most recent intensity residual (the change in the conditional forecast of number of jumps due to last day's information) ranges from 0.036 to 0.362. The estimated a_y^m for the market is 0.055.

Third, the negative idiosyncratic return innovation has significantly positive effect on the long-run and short-run conditional diffusive variance: α_{13}^i and α_{23}^i are both positive for 14 out of 15 stocks. The filtered number of jumps $E[N_t|\Phi_t]$, on the other hands, has negative effect on the long-run conditional diffusive variance for all stocks, and positive effect on the short-run conditional diffusive variance for 11 out of 15 stocks. This means that the jump innovations increase the conditional diffusive variance in the short run and decrease that in the long run.

Fourth, the jump characteristics of the individual stocks are different from that of market jumps. The estimated jump size mean θ^i are negative for 2 out of 15 stocks, and positive for the other firms. This result is consistent with

[6] The estimated persistent parameters are very close to 1, but they are all significantly different from 1 at the 5% level. The unit-root test rejects the null hypothesis of non-stationary jump intensity series. From the figure that shows the time series of the jump intensity, we observe that a shock does not have a persistent effect on future jump intensities.

Jiang and Yao (2013), who find that individual stock price jumps tend to be idiosyncratic and predominantly positive, presenting an interesting contrast to mostly negative jumps in market portfolios. The result also supports Duffee (1995)'s conjecture that there is a negatively skewed market factor and an idiosyncratic firm factor, which is positively skewed. Further, the volatility of the idiosyncratic jump size δ^i for individual firms are all higher than that for the market. We present estimates of θ^i and δ^i for each stock in Table 2.3 and Table 2.4

The dependence structure between individual stocks and the market return is captured by the loadings on the two types of systematic risks, β_t^i and p_t^i. The summary statistics of the two variables are shown for each individual stocks in Table 2.5. From Table 2.3 and Table 2.4, we find that there are 13 out of 15 stocks with β_t^i positively related to the default spread (β_2^i is positive) and 4 out of 15 stocks with p_t^i positively related to the default spread. In general, for most stocks the time-varying exposure to the market risk is countercyclical: stock returns of those firms react more to the market returns during economy downturns than during upturns. Two exceptions in our sample are Exxon Mobil (XOM) and Colgate-Palmolive Company (CL), whose exposure to the market risk is negatively related to the default spread ($\beta_2^i < 0$ and $p_2^i < 0$). These two stocks are generally considered as defensive stocks. Exxon Mobil, a large oil producer with light leverage is expected to react less to the market movement during a recessionary period. In addition, Colgate-Palmolive is less sensitive to the market movement during bad times than during good times, because it is a geographically diversified nondurable consumer brand,

Finally, Figure 2.8 present the time-varying contribution of idiosyncratic jump intensity to the total jump intensity (Figure 2.8(a)) and the time-varying contribution of the idiosyncratic volatility to the total volatility (Figure 2.8(b)) for the 15 stocks on average. Idiosyncratic jump intensity (variance) contribution to 82.25% (66.70%) of the total jump intensity (variance). The declining patterns in the two figures show that the systematic jump intensity and volatility play increasingly important roles during the past 50 years.

2.5 Pricing jump risk in the expected stock returns

In this section, we analyze how the four types of risks are priced in the cross-sectional of the expected stock returns. First, we calculate the four premia on the four types of risks and investigate their contribution to the total expected stock return during the sample period. Second, we sort the stocks according to the four risk premia and construct portfolios representing stocks with low to high premia. By comparing portfolio performance over the entire sample period, we check whether the four types of risks are priced in our sample of expected stock returns.

Note that in order to calculate the expected stock return in Proposition 2.1, we need to first calculate the parameters in the pricing kernel, Λ^m and Λ^i. From Proposition 1, we get that:

$$\lambda_z = \Lambda^m, \ \lambda_y = \xi^m(1) + \xi^m(-\Lambda^m) - \xi^m(1 - \Lambda^m),$$

Hence, λ_y and λ_z are calculated based on the estimated Λ^m from the joint model. Similarly, we can calculate the idiosyncratic risk parameters λ_{yi} and λ_{zi} for individual stocks using estimated Λ^i.

2.5.1 Decomposing the expected stock return

To understand the economic significance of the four risk premiums related to systematic and idiosyncratic jump risks and systematic and idiosyncratic jump risks, we decompose the model-implied expected return into four corresponding risk premiums and evaluate their contributions. We aggregate idiosyncratic diffusive and jump premiums into the idiosyncratic risk premium

Table 2.3: Estimated parameters for the stock returns

	ADM	BAX	CL	DD	DOW	EMR	GE	IBM
Λ_z^i	2.421	2.717	2.457	0.274	1.672	4.347	1.376	1.003
	(1.876)	(3.142)	(2.113)	(0.184)	(1.499)	(2.315)	(0.855)	(0.734)
w_z^i	0.000	0.000	0.003	0.041	0.001	0.002	0.000	0.005
	(4.403)	(0.321)	(1.669)	(1.598)	(0.615)	(2.13)	(0.077)	(2.906)
b_{z1}^i	0.985	0.972	0.967	0.783	0.977	0.971	0.945	0.966
	(4.426)	(831.788)	(175.454)	(5.67)	(454.626)	(171.757)	(95.781)	(242.418)
α_{11}^i	-4.829	-4.418	-4.797	-2.938	-5.085	-9.501	-4.296	-4.430
	(-13.5)	(-37.569)	(-18.955)	(-14.09)	(-13.435)	(-0.965)	(-11.252)	(-18.567)
α_{12}^i	-4.661	-3.417	-0.610	-0.727	-1.111	-0.786	-0.604	-2.515
	(-2.583)	(-4.858)	(-2.42)	(-5.753)	(-2.199)	(-4.209)	(-2.686)	(-2.976)
α_{13}^i	0.882	1.315	0.825	0.637	1.462	5.998	1.319	0.853
	(2.738)	(5.518)	(1.772)	(2.201)	(2.497)	(0.63)	(2.68)	(3.318)
b_{z2}^i	0.688	0.641	0.306	0.051	0.324	0.000	0.303	0.795
	(0)	(7.745)	(3.463)	(0.057)	(2.273)	(0.007)	(1.74)	(14.067)
α_{21}^i	-14.147	-14.294	-2.715	-15.862	-15.314	-15.135	-14.674	-14.311
	(-0.168)	(-1.351)	(-13.556)	(-0.056)	(-0.185)	(-0.101)	(-0.245)	(-1.057)
α_{22}^i	-0.618	-0.009	0.339	0.450	1.494	1.618	0.504	-0.414
	(-0.03)	(-0.256)	(1.457)	(0.161)	(3.074)	(3.108)	(0.902)	(-2.071)
α_{23}^i	11.817	11.648	0.202	10.135	10.683	10.862	11.324	11.678
	(0.01)	(1.134)	(0.716)	(0.036)	(0.135)	(0.073)	(0.199)	(0.903)
w_y^i	0.001	0.002	0.000	0.001	0.005	0.035	0.001	0.000
	(2.118)	(8.497)	(1.006)	(0.156)	(2.491)	(1.933)	(2.088)	(3.133)
b_y^i	0.999	0.989	0.998	0.998	0.971	0.875	0.998	0.996
	(690.339)	(686.008)	(113.016)	(94.485)	(94.282)	(15.86)	(290.105)	(115.194)
a_y^i	0.078	0.066	0.041	0.061	0.208	0.373	0.044	0.025
	(5.82)	(5.51)	(3.574)	(0.356)	(3.793)	(4.215)	(7.469)	(62.857)
θ_i	0.255	0.173	0.187	0.193	0.064	0.134	0.112	0.157
	(2.878)	(1.08)	(4.25)	(1.545)	(1.389)	(2.97)	(1.804)	(1.357)
δ^i	2.368	2.519	1.781	1.472	2.153	1.352	1.316	2.973
	(21.332)	(25.105)	(25.21)	(3.349)	(13.702)	(14.796)	(20.889)	(12.633)
β_1^i	-1.908	-0.472	-0.094	-0.220	-0.149	0.015	-0.153	-0.742
	(0.717)	(-3.536)	(-2.804)	(-0.461)	(-2.784)	(0.841)	(0.089)	(-4.423)
β_2^i	0.521	-0.187	-0.381	0.129	-0.042	-0.095	-0.008	0.071
	(1.212)	(1.739)	(-0.741)	(1.054)	(4.257)	(0.175)	(0.748)	(3.305)
p_1^i	-1.722	0.010	437.937	-2.717	-4.634	8.907	-2.700	1.715
	(-0.102)	(1.081)	(4.128)	(-0.077)	(-0.937)	(1.473)	(-0.19)	(1.223)
p_2^i	2.472	-2.669	-651.153	2.143	2.699	-7.118	-2.956	-5.128
	(-0.051)	(-1.676)	(-4.211)	(0.102)	(0.863)	(-1.472)	(-0.285)	(-2.273)
lgl	42745.685	41547.098	39435.002	39338.968	40701.499	39324.554	39071.487	38957.459

Note: Table 2.3 shows the estimation results on the daily returns of the 8 out of 15 stocks, from January 1963 to December 2015. T statistics are shown in the parentheses. lgl is the maxmized log likelihood for each stock.

Table 2.4: Estimated parameters for the stock returns (continued)

	MMM	PEP	PG	T	UTX	XOM	XRX
Λ_z^i	1.259	3.583	2.258	0.272	2.148	2.879	0.992
	(0.802)	(2.293)	(2.076)	(0.306)	(1.718)	(1.562)	(1.335)
w_z^i	0.000	0.049	0.006	0.022	0.004	0.085	0.013
	(0.595)	(2.55)	(2.431)	(3.513)	(1.123)	(3.658)	(3.039)
b_{z1}^i	0.989	0.759	0.932	0.798	0.953	0.726	0.957
	(716.75)	(12.914)	(114.195)	(23.665)	(94.559)	(13.187)	(174.391)
α_{11}^i	-5.073	-3.010	-3.925	-2.569	-4.230	-2.918	-4.020
	(-45.912)	(-10.199)	(-19.826)	(-16.366)	(-14.625)	(-14.261)	(-20.192)
α_{12}^i	-18.320	-0.243	-1.053	-0.133	-1.423	-0.098	-1.136
	(-2.281)	(-1.646)	(-4.783)	(-0.833)	(-2.584)	(-1.109)	(-3.516)
α_{13}^i	0.838	0.718	1.366	-0.045	0.935	0.434	0.683
	(2.67)	(2.603)	(6.233)	(-1.795)	(2.62)	(0.629)	(2.634)
b_{z2}^i	0.000	0.000	0.168	0.000	0.398	0.108	0.276
	(0)	(-0.373)	(1.158)	(0.001)	(3.223)	(0.609)	(2.416)
α_{21}^i	-14.636	-16.188	-14.480	-14.942	-14.559	-16.544	-14.277
	(-0.173)	(-0.091)	(-0.265)	(-0.198)	(-0.281)	(-0.145)	(-74.179)
α_{22}^i	0.560	1.613	0.436	0.850	0.604	1.908	-0.716
	(1.025)	(1.674)	(2.281)	(7.364)	(2.152)	(1.675)	(-2.157)
α_{23}^i	11.350	9.806	11.454	11.056	11.437	9.455	11.708
	(0.135)	(0.056)	(0.212)	(0.151)	(0.222)	(0.083)	(63.075)
w_y^i	0.003	0.000	0.000	0.000	0.000	0.002	0.000
	(2.572)	(0.722)	(1.916)	(0.631)	(1.926)	(1.424)	(16.509)
b_y^i	0.980	1.000	0.998	0.999	0.999	0.993	0.997
	(267.539)	(448.247)	(159.528)	(236.416)	(100.171)	(282.223)	(196.02)
a_y^i	0.356	0.066	0.037	0.049	0.036	0.128	0.019
	(3.914)	(6.855)	(9.241)	(9.965)	(4.665)	(2.733)	(6.958)
θ_i	0.017	0.084	0.107	0.180	0.176	-0.028	-0.213
	(0.867)	(2.559)	(1.993)	(4.948)	(2.805)	(-0.816)	(-0.976)
δ^i	1.785	1.378	1.638	1.457	1.918	1.239	4.191
	(18.417)	(32.815)	(21.345)	(29.234)	(16.81)	(13.729)	(27.807)
β_1^i	-0.347	-0.401	-0.824	-0.672	-0.093	-0.230	-0.063
	(-3.382)	(-4.239)	(-5.952)	(-7.034)	(-0.191)	(-1.802)	(-0.054)
β_2^i	-0.030	-0.100	-0.054	0.133	-0.042	-0.018	-0.123
	(0.704)	(0.252)	(2.648)	(2.982)	(0.845)	(-0.425)	(0.927)
p_1^i	0.682	-3.119	33.411	0.317	-0.320	-0.506	-0.313
	(0.914)	(-1.971)	(0.676)	(0.122)	(-0.196)	(-0.264)	(-1.71)
p_2^i	-0.400	2.216	-45.767	-1.776	-0.132	-0.181	-1.174
	(-0.849)	(1.804)	(-0.718)	(-0.606)	(-0.099)	(-0.162)	(-1.318)
lgl	38251.068	38828.660	36857.448	37229.895	40849.123	37444.384	42899.222

Note: Table 2.4 shows the estimation results on the daily returns of the 8 out of 15 stocks, from January 1963 to December 2015. T statistics are shown in the parentheses. lgl is the maxmized log likelihood for each stock.

Table 2.5: Summary statistics of the time varying β_t^i and p_t^i

	β_t^i mean	std	min	max	p_t^i mean	std	min	max
ADM	0.262	0.080	0.174	0.903	0.652	0.172	0.278	0.999
BAX	0.516	0.041	0.326	0.589	0.088	0.063	0.000	0.306
CL	0.623	0.095	0.242	0.809	0.171	0.374	0.000	1.000
DD	0.918	0.057	0.835	1.257	0.380	0.190	0.114	0.991
DOW	0.825	0.015	0.745	0.850	0.178	0.188	0.022	0.991
EMR	0.921	0.038	0.730	0.985	0.729	0.332	0.000	0.999
GE	0.852	0.003	0.836	0.857	0.005	0.005	0.000	0.026
IBM	0.513	0.017	0.487	0.610	0.082	0.100	0.000	0.531
MMM	0.686	0.009	0.638	0.700	0.567	0.045	0.330	0.636
PEP	0.605	0.026	0.473	0.649	0.318	0.191	0.081	0.990
PG	0.415	0.010	0.363	0.431	0.248	0.397	0.000	1.000
T	0.587	0.037	0.532	0.810	0.205	0.094	0.003	0.442
UTX	0.873	0.016	0.788	0.900	0.388	0.014	0.315	0.411
XOM	0.780	0.006	0.746	0.790	0.334	0.018	0.243	0.363
XRX	0.828	0.044	0.612	0.903	0.234	0.020	0.142	0.342

Note: Table 2.5 shows summary statistics of the time varying β_t^i and p_t^i for each stock. Note that we report estimation results with the return data multiplied by 100.

for each stock. The average contribution of the idiosyncratic risk premium across the 15 stocks is presented in Figure 2.7(a). We find that on average the idiosyncratic risk premium contributes to 57.18% of the model-implied expected return. This confirms that idiosyncratic risks are economically important pricing factors in the expected stock return over time.

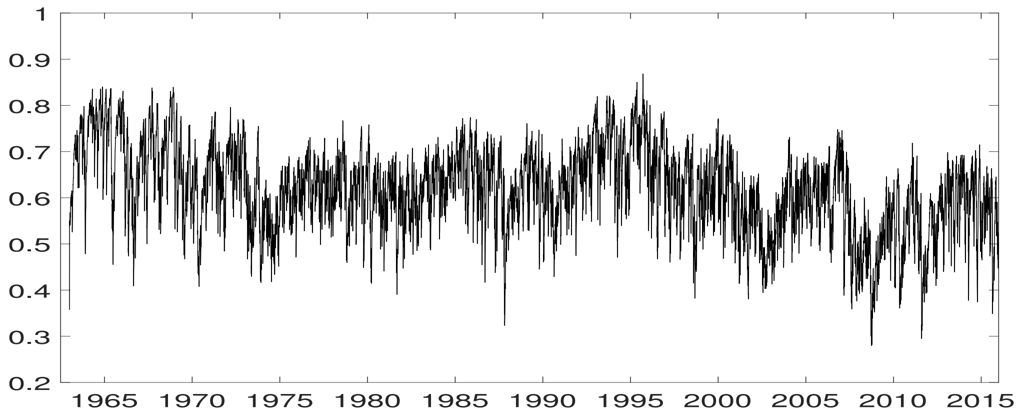
Further, the idiosyncratic risk premiums decrease dramatically during crises, i.e. in the Asian crisis in 1997 and in the recent subprime crisis in 2008. This can be explained by the fact that during the crisis, systematic events, such as the bankruptcy of Lehman Brothers, drive the stock prices more than idiosyncratic events, such as earning surprises. Therefore systematic risk premium accounts more during the crises.

We also aggregate the systematic and idiosyncratic jump risk premiums into a jump premium. We show the average contribution of the jump risk premium across the 15 stocks to the total expected return in Figure 2.7(b). The jump risk premium accounts for almost 16% of the model-implied expected returns. In addition, it remains around a stable level over our sample period.

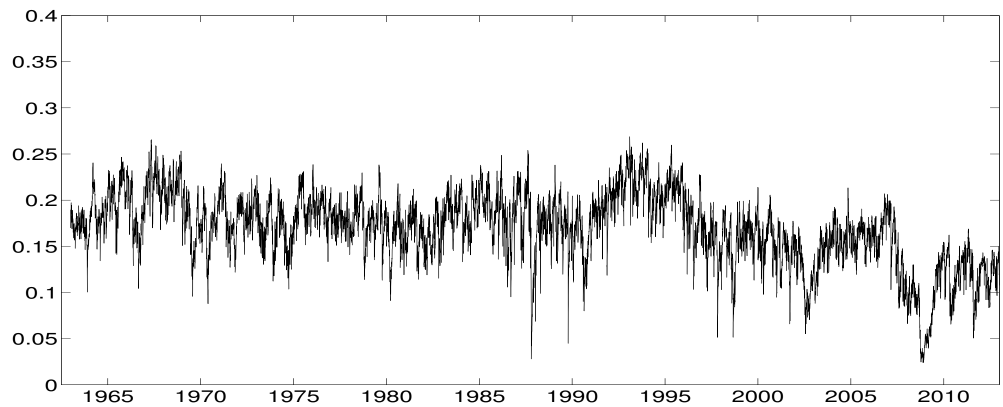
This shows that prices of jump risks are of great economic importance in studying the expected stock return.

Figure 2.7: Decomposition of risk premium over time

(a) Idiosyncratic risk contribution



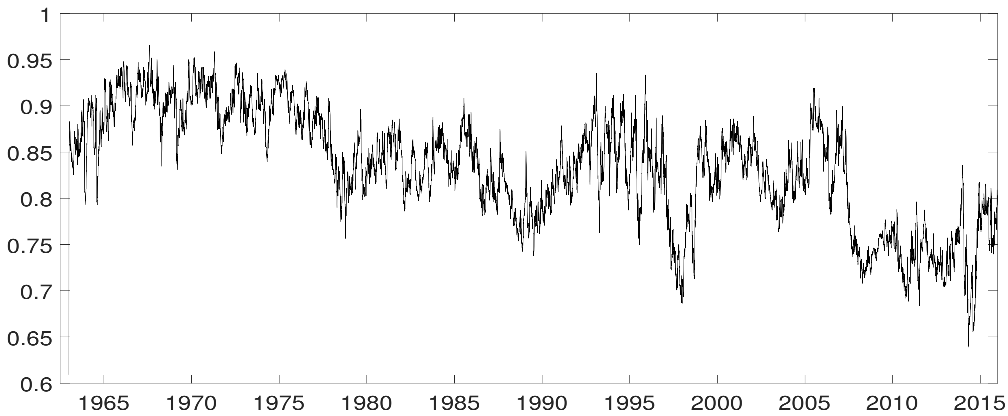
(b) Jump risk contribution



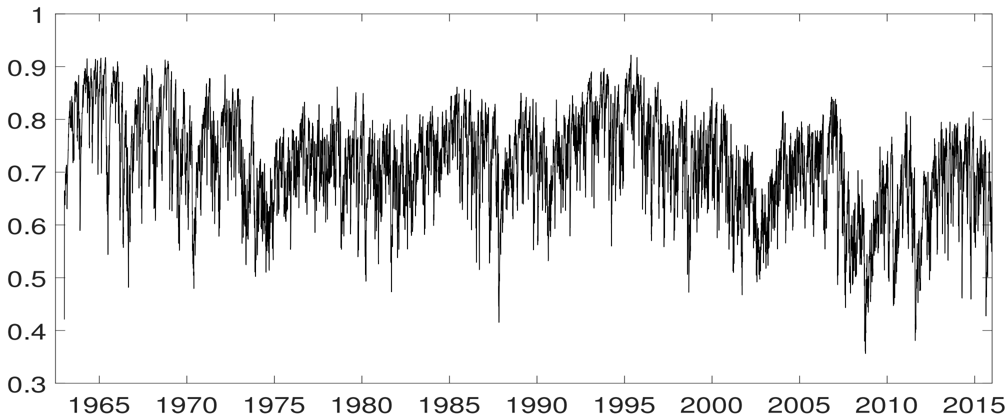
Note: We first calculate the four risk premia for each stock and take the cross-sectional average on each day. We aggregate the idiosyncratic diffusive and jump risk premia and show their contribution in the total expected return in Figure (a). We aggregate the systematic and idiosyncratic jump risk premia and show their contribution in the expected stock return in Figure (b).

Figure 2.8: Decomposition of jump intensity and volatility over time

(a) Idiosyncratic jump intensity contribution



(b) diosyncratic volatility contribution



Note: Figure (a) shows the average contribution of idiosyncratic jump intensity in the total jump intensity (hy_t^i/hy_t) for all stocks. Figure (b) shows the average contribution of idiosyncratic diffusive volatility in the total diffusive volatility (hy_t^i/hy_t) for all stocks.

2.5.2 Portfolio performance

If the risk premiums that we recover represent the reward for bearing risk, stocks with a higher risk premium should have higher expected returns than their peers. To check whether this is the case, we conduct a portfolio performance analysis, by sorting based on the estimated risk premiums. Denote the systematic diffusive risk, systematic jump risk, idiosyncratic diffusive risk, and idiosyncratic jump risk as SD, SJ, ID and IJ. When the information on the conditional premiums is available at t, we use them to sort the stock return one day ahead at $t+1$. In this sense, the sorting is a "pseudo out-of-sample" approach. It is not completely out-of-sample because we use all information over the whole sample to estimate the model. The stocks are sorted into five portfolios with 3 stocks in each of them, according to the risk premium on each type of risks. In each portfolio, we assign equal weights to the stocks and calculate the portfolio return over our sample period.

Table 2.6: Portfolio performance

	Total Premium	SD	SJ	ID	IJ
1	0.033	0.058	0.039	0.042	0.043
2	0.038	0.045	0.047	0.027	0.053
3	0.061	0.082	0.069	0.077	0.068
4	0.051	0.055	0.044	0.042	0.061
5	0.141	0.081	0.087	0.137	0.096
5-1	0.108	0.023	0.049	0.095	0.053
t stat	3.462	0.791	1.750	3.119	1.926
p value	0.000	0.215	0.040	0.001	0.027

Note: We denote the systematic diffusive risk, systematic jump risk, idiosyncratic diffusive risk, and idiosyncratic jump risk as SD, SJ, ID and IJ. The stocks are sorted in ascending order into five portfolios with 3 in each of them, according to the risk premia on each type of risks. We also sort the stocks according to the total risk premia. In each portfolio, we assign equal weights to the stocks and calculate the portfolio return over the sample period. The annualized portfolio returns for the constructed portfolios are presented in the first five rows in Table 2.6. The last two rows report t-statistics and corresponding p-values when testing the null hypothesis that the difference between the return of the fifth portfolio and the first portfolio is equal to zero. The t-statistics are calculated based on the Newey-West standard error.

As shown in Proposition 2.1, the sorting based on the SD and the SJ premiums will be identical to the sorting based on the level of β and $p(1-e^b)$, respectively. Thus, if the market exposures are assumed to be constant, then the sorting based on the two risk premiums remain constant throughout the sample period. We relax this restriction by making the exposure parameters vary with the default spread as a proxy of the business condition. Hence, the sorting for market risk premium is time varying. Sorting based on the

ID and IJ premiums are also time-varying because the conditional idiosyncratic volatility, $h^{\epsilon}_{z,t}$, and conditional idiosyncratic jump intensity, $h^{\epsilon}_{y,t}$, vary over time. The annualized portfolio returns for the constructed portfolios are calculated in the first five rows in Table 2.6. The sixth row reports the difference between the average returns of the fifth and the first portfolios. The last two rows report t-statistics and corresponding p-values when testing the null hypothesis that the difference is equal to zeros. The t-statistics are calculated based on the Newey-West standard error.

From Table 2.6, we find that when we sort the total conditional equity premium of the stocks, the future return increases from 1.7% from the first quintile to 12.2% in the fifth quintile. This supports the claim that the setup of model that it is able to capture the variation of the equity premium both over time and across different stocks. When we sort the stocks based on the ID and IJ premiums, the average returns are increasing from the low to high premium portfolios. The difference between the average returns of the fifth and the first portfolios are 8.6% and 5.7%, respectively, and both of these are statistically significant at the 5% confidence level. Hence, the idiosyncratic diffusive and jump risks are both priced in the cross-section of expected stock returns in our sample. However, when the stocks are sorted based on the SD, the average returns of the three portfolio return are not lining up in a particular order. Further, the difference between the average returns between the average returns of the third and the first portfolio is negative and not significantly different from zero. As a robustness check, we sort the stocks according to beta obtained from OLS regression, the results are similar as sorting on SD. This suggests that the result is not due to systemic estimation error in SD. When we sort the stocks on SJ, the systematic jump risk premium, however, the difference between the fifth and first portfolio return is statistically significant at 5%.

This observation that systematic diffusive risk is not priced in our sample contradicts the classic CAPM. Due to the complexity of the model in this chapter, we only estimate the model for 15 stocks over a 50 years time period. The use of small sample of stocks in this chapter may limit the generality of these results. Alternatively, it is possible that the result is in line with the low-volatility anomaly, which has been found in the United States over an 85-year period and in global markets for at least the past 20 years. The low-volatility anomaly says that portfolios of low-volatility and low-beta stocks have produced higher risk-adjusted returns than portfolios with high-volatility stocks in most markets studied, for instance, by Blitz and Van Vliet (2007), Blitz et al. (2013), Baker et al. (2011), and Frazzini and Pedersen (2014). Our results shed light on the low volatility anomaly by showing that the anomaly derives from the systematic diffusive risk rather than the systematic jump risk.

To summarize, when we decompose the model-implied expected return to the four risk premiums, we find that both the systematic and idiosyncratic risk premiums are of economic significance over time. When sorting the stocks based on the four risk premiums, we find that systematic jump risk, idiosyncratic diffusive risk and idiosyncratic jump risk are priced in the cross-section of the expected stock return.

2.6 Conclusion

In this chapter, we propose a novel econometric framework for modeling the jump risk in individual stock returns. It distinguishes not only jump and diffusion components, but also systematic and idiosyncratic components. All four types of risks along with their associated risk premiums are time-varying. The model also allows for time-varying loadings on systematic diffusive and jump risks. Consequently, we decompose the stock return and study different sources of risk, especially jump risks. The study addresses two questions: (1) How to estimate the four sources of time varying risks in a jump diffusion model for individual stock returns? (2) How different types of risks are priced in equity premium over time and in the cross-section?

We estimate the model with dynamic conditional variance and dynamic jump intensity on 15 stocks returns. We find that (1) the model is able to identify different types of risks only using daily stock returns; (2) Idiosyncratic jump intensity and idiosyncratic diffusive variance account a large amount in the total jump intensity and diffusive variance, i.e. on average 82.25% and 66.7% respectively. The contribution of systematic risks increases over the past 50 years. For the pricing of risks, we find that systematic jump, idiosyncratic diffusive and idiosyncratic jump risk are significantly priced in the cross-section of expected stock returns in our sample.

2.7 Appendix

2.7.1 Scaling Returns

Empirically, we find that we need to scale the daily return R^m and R^i to get numerically stable estimates. We only present the scaling procedure for the market returns. Individual stock returns can be scaled in a similar pattern. Suppress the time index for convenience of notation, and recall from section 2 that

$$
\begin{aligned}
R^m &= \alpha^m + y^m + z^m, \\
\alpha^m &= r + (\lambda_y^m - \xi^m(1))h_y^m + (\lambda_z - 0.5)h_z^m,
\end{aligned}
$$

where $\xi^m(\phi) = exp(\theta^m\phi + \frac{(\delta^m)^2\phi}{2})$, and y^m follows a compound Poisson distribution with parameters (h_y^m, θ^m and δ^m). Scaling R^m by 100 and denoting the scaled return by R_{100}^m:

$$
R_{100}^m = \alpha_{100}^m + y_{100}^m + z_{100}^m,
$$

in which parameters with subscript 100 are for the scaled returns R^m_{100}:

$$\alpha^m_{100} = 100\alpha^m = 100(r + (\lambda^m_y - \xi^m(1))h^m_y + (\lambda_z - 0.5)h^m_z),$$
$$y^m_{100} = 100y^m \text{ and } z^m_{100} = 100z^m.$$

One can verify that $z^m_{100} \sim N(0, h^m_{100,z})$, where $h^m_{100,z} = 100^2 h^m_z$, and y^m_{100} follows a compound Poisson distribution with parameters $(h^m_y, \theta^m_{100}, \delta^m_{100})$, where $\theta^m_{100} = 100\theta_m$ and $\delta^m_{100} = 100\delta^m$. The original return is thus written as:

$$R^m = \frac{\alpha^m_{100}}{100} + \frac{y^m_{100}}{100} + \frac{z^m_{100}}{100}.$$

Similarly, the log linear pricing kernel is:

$$log(\frac{M_{t+1}}{M_t}) = -\frac{r_{100}}{100} - \frac{\mu_{100}}{100} - \Lambda_z \frac{z^m_{100}}{100} - \Lambda_y \frac{y^m_{100}}{100} - \sum_{i=1}^{J} \Lambda_{zi} \frac{z^\epsilon_{100,i}}{100} - \sum_{i=1}^{J} \Lambda_{yi} \frac{y^\epsilon_{100,i}}{100}.$$

In the absence of arbitrage, we have the following equality:

$$E_t[\frac{M_{t+1}}{M_t} e^{R^m}] = 1.$$

Henceforth, the expression for equity premium in terms of scaled terms is:

$$\alpha^m_{100} = r_{100} + (\Lambda_z - 0.5)\frac{h^m_{100,z}}{100} + 100(\xi^m_{100}(1 - \Lambda_y) - \xi_{100}(-\Lambda_y))h^m_y,$$

where $\xi^m_{100}(\phi) = exp(\frac{\theta^m_{100}}{100}\phi + \frac{(\delta^m_{100})^2\phi}{20000}) - 1$.

2.7.2 Derivation for conditional $\beta^i_{t|t-1}$

We assume in the chapter that the factor loading for market diffusion risk β^i and the probability p that the market jump triggers a jump in the individual stock return are constant during the estimation period. However, the general exposure of individual stocks to the market risk given information set at time $t-1$, defined as

$$\beta^i_{t|t-1} = \frac{cov(R^m_t, R^i_t|\Phi_{t-1})}{var(R^i_t|\Phi_{t-1})},$$

changes over time. We derive the part $cov(R_t^m, R_t^i|\Phi_{t-1})$ in this section since $var(R_t^i|\Phi_{t-1})$ is given in formula (2.19).

Th covariance between market return and individual stock return can be written as two parts:

$$\begin{aligned} cov(R_t^m, R_t^i|\Phi_{t-1}) &= cov(\alpha_t^m + y_t^m + z_t^m, \alpha_t^m + y_t^m + z_t^m|\Phi_{t-1}) \\ &= cov(y_t^m, y_t^i|\Phi_{t-1})) + cov(z_t^m, z_t^i|\Phi_{t-1})). \end{aligned} \tag{2.28}$$

From definition we know that $z_t^i = \beta^i z_t^m + z_t^\epsilon$, hence the second part in formula (2.28) is equal to $\beta h_{z,t}^m$. If we assume that the correlation between jump size of the individual stock and the market is zero, the covariance between the two jump terms can be derived as:

$$\begin{aligned} cov(y_t^m, y_t^i|\Phi_{t-1}) &= cov(\sum_{j=1}^{N_t^m} x_{m,t}^j, \sum_{j=1}^{N_t^m} x_{i,t}^j 1_m + \sum_{j=N_t^m+1}^{N_t^m+N_t^\epsilon} x_{i,t}^j|\Phi_{t-1}) \\ &= E(\sum_{j=1}^{N_t^m} x_{m,t}^j \sum_{j=1}^{N_t^m} x_{i,t}^j 1_m) - E(\sum_{j=1}^{N_t^m} x_{m,t}^j)E(\sum_{j=1}^{N_t^m} x_{i,t}^j 1_m) \\ &= p^i\theta^m\theta^i(h_{y,t}^m - (h_{y,t}^m)^2). \end{aligned}$$

Henceforth, the conditional $\beta_{t|t-1}^i$ can be expressed as:

$$\beta_{t|t-1}^i = \frac{\beta^i h_{z,t}^m + p^i\theta^m\theta^i(h_{y,t}^m - (h_{y,t}^m)^2)}{(\beta^i)^2 h_{z,t+1}^m + h_{z,t+1}^\epsilon + (p^i h_{y,t+1}^m + h_{y,t+1}^\epsilon)((\theta^i)^2 + (\delta^i)^2)}.$$

2.7.3 Filter jump and diffusion terms from the market return

Given the 15 parameters for the market return: $\Theta^m = (\Lambda^m, w_z^m, b_{z1}^m, \alpha_{11}^m, \alpha_{12}^m, \alpha_{13}^m, b_{z2}^m,$ $\alpha_{21}^m, \alpha_{22}^m, \alpha_{23}^m, w_y^m, b_y^m, a_y^m, \theta^m, \delta^m)$, we get the time series of conditional variance $h_{y,t}^m$ and conditional jump intensity $h_{z,t}^m$[7]. Next, we discuss how to filter out the

[7] We set starting value of the jump intensity $h_{y,1}^m$ to the unconditional value as $E[h_{y,t}^m] = \frac{w_y}{1-b_y}$ and the starting value of the conditional variance as $var(data) - h_{y,1}^m((\theta^m)^2 + (\delta^m)^2)$, where $var(data)$ is the variance of the market return.

normal component of the return z_t. The filtration of z_t^m involves solving the expectation $\hat{z}_t^m = E[z_t^m|\Phi_t]$. Note that if market return and number of jump are known at time t, we can express z_t^m as:

$$z_t^m(R_t^m, N_t^m = j) = \sqrt{\frac{h_{z,t}^m}{h_{z,t}^m + j(\delta^m)^2}}(R_t^m - \alpha_t^m - j\theta^m).$$

The expectation $E[z_t^m|\Phi_t]$ can then be solved using the following summation:

$$\hat{z}_t^m = E[z_t^m|\Phi_t] = \sum_{j=0}^{\infty} z_t^m(R_t^m, N_t^m = j)\Pr(z_t^m, N_t^m = j),$$

where $\Pr(z_t^m, N_t^m)$ is the joint probability of z_t^m and $n_t^m = j$ given that R_t^m is known. Using Bayes' rule, we can write the filtering density $\Pr(z_t^m, N_t^m)$ as:

$$\Pr(z_t^m, N_t^m) = \Pr(z_t^m|R_t^m, N_t^m = j)\Pr(N_t^m = j). \tag{2.29}$$

The second term on the right-hand of Equation (2.29) is given by Equation (2.4), and the first term is the probability of of z_t^m given that R_t^m and $N_t^m = j$ are known. Hence, we can write the expected ex post normal component of the return as

$$\begin{aligned}\hat{z}_t^m &= \sum_{j=0}^{\infty} z_t^m(R_t^m, N_t^m = j)\Pr(z_t^m|R_t^m, N_t^m = j)\Pr(N_t^m = j) \\ &= \sum_{j=0}^{\infty} \frac{h_{z,t}^m}{h_{z,t}^m + j(\delta^m)^2}(R_t^m - \alpha_t^m - j\theta^m)\Pr(N_t^m = j).\end{aligned}$$

Once $\hat{z}_t^m$ is known, we can directly infer the filtered jump term $\hat{y}_t^m$ by noting that $\hat{y}_t^m = R_t^m - \alpha_t^m - \hat{z}_t^m$. The time series of filtered $\hat{z}_t^m$ and $\hat{N}_t^m$ from estimated parameters are used in the procedure of maximizing likelihood function for individual stocks.

2.7.4 Filter jump and diffusion terms from individual stock returns

Given the parameters for the market return and stock return, Θ^m and Θ^i, the time series of $h_{y,t}^i$, $h_{z,t}^i$, $h_{y,t}^\epsilon$, and $h_{z,t}^\epsilon$ can be computed according to the dynamics in Equations (2.9), (2.18), (2.10) and (2.15). In this section, we discuss how to filter out the unobserved diffusion components and jump components for individual stocks.

Similar as filtering procedure for the market, if the number of jumps in the individual stock return $n_t^i = j$ and stock return R_t^i are known at time t, we can express z_t^i as:

$$z_t^i(R_t^i, n_t^i = j) = \sqrt{\frac{h_{z,t}^i}{h_{z,t}^i + j(\delta^i)^2}}\,(R_t - \alpha_t^i - j\theta^i),$$

where the expression for α_t^i is given in Appendix 2.7.5. Since $z_t^i(R_t^i, n_t^i = j)$ depends on the discrete number of jumps $n_t^i = j$, the expectation $E_t[z_t^i]$ can be solved by summing up all possible number of jumps:

$$\hat{z}_t^i = \sum_{j=0}^{\infty} \Pr(n_t^i = j | R_t^i)\; z_t^i(R_t^i, n_t^i = j).$$

Once $\hat{z}_t^i$ is known, we can infer $\hat{y}_t^i$ from the relation that $\hat{y}_t^i = R_t^i - \mu_t^i - \hat{z}_t^i$, given the information at time t. After we obtain $\hat{y}_t^i$ and $\hat{z}_t^i$, the filtered idiosyncratic jump component $\hat{y}_t^\epsilon$ and $\hat{z}_t^\epsilon$ can be calculated by,

$$\tilde{z}_t^\epsilon = \tilde{z}_t^i - \beta \tilde{z}_t^m,\;\; \tilde{y}_t^\epsilon = \tilde{y}_t^i - p\tilde{h}_{y,t}^m \theta^i.$$

2.7.5 Expected Return for the market and individual stocks

In this section, we provide the proof for Proposition 2.1. In the absence of arbitrage, the martingale condition in Equation (2.21) should be satisfied for the market index and individual stock return. First, substituting the pricing kernel in Equation (2.20) and market dynamic in Equation (2.1) into the Equation $E_t[\frac{M_{t+1}}{M_t} e^{R_{t+1}^m}] = 1$, we have:

$$\frac{E_t[\exp(-r - \Lambda^m z_{t+1}^m - \Lambda^m y_{t+1}^m - \sum_{j=1}^J \Lambda^j z_{j,t+1}^\epsilon - \sum_{j=1}^J \Lambda^j y_{j,t+1}^\epsilon + \alpha_m + z_{t+1}^m + y_{t+1}^m)]}{E_t[\exp(-\Lambda^m z_{t+1}^m - \Lambda^m y_{t+1}^m - \sum_{j=1}^J \Lambda^j z_{t+1}^j - \sum_{j=1}^J \Lambda^j y_{t+1}^j)]} = 1.$$

Since y^m_{t+1} and z^m_{t+1} are independent, $E_t[exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1})]$ can be calculated as:

$$E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1})] = exp(\frac{1}{2}(\Lambda^m)^2 h^m_{z,t+1} + h^m_{y,t+1}(\exp(-\Lambda^m\theta^m + \frac{1}{2}(\Lambda^m)^2(\delta^m)^2) - 1)),$$

where we use the moment generating function of normal distribution and compound Poisson distribution. Since there is no correlation between z^m_{t+1}, y^m_{t+1}, $z^\epsilon_{j,t+1}$ and $y^\epsilon_{j,t+1}$, we can get the expression for α_m from Equation (2.22):

$$\alpha_m = r + (\Lambda^m - \frac{1}{2})h^m_{z,t+1} + (\xi(-\Lambda^m) - \xi(1-\Lambda^m)h^m_{y,t+1},$$

where $\xi(\phi) = \exp(\theta\phi + \frac{\delta^2\phi}{2}) - 1$.

Next, if we substitute the pricing kernel in Equation (2.20) and the dynamics of individual stock return in (2.8) into the equation $E_t[\frac{M_{t+1}}{M_t}e^{R^i_{t+1}}] = 1$, we have:

$$\frac{E_t[\exp(-r - \Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^\epsilon_{j,t+1} - \sum_{j=1}^J \Lambda^j y^\epsilon_{j,t+1} + \alpha_i + z^i_{t+1} + y^i_{t+1})]}{E_t[\exp(-\Lambda^m z^m_{t+1} - \Lambda^m y^m_{t+1} - \sum_{j=1}^J \Lambda^j z^j_{t+1} - \sum_{j=1}^J \Lambda^j y^j_{t+1})]} = 1. \tag{2.30}$$

The nominator can be written as:

$$e^{-r+\alpha_i} E_t[\exp((-\Lambda^m + \beta)z^m_{t+1})] E_t[\exp(-\Lambda^m y^m_{t+1} + y^i_{t+1}(ph^m_{y,t+1}))] \times E_t[-\sum_{j\neq i}^J \Lambda^j z^\epsilon_{j,t+1} - \sum_{j\neq i}^J \Lambda^j y^\epsilon_{i,t+1} - (\Lambda^i - 1)z^\epsilon_{i,t+1}) - (\Lambda^i - 1)y^\epsilon_{i,t+1})].$$

What we focus on is the second part: $E_t[\exp(-\Lambda^m y^m_{t+1} + y^i_{t+1}(ph^m_{y,t+1}))]$. The jump components for the individual and the market are $y^m_{t+1} = \sum_{j=0}^{N^m_{t+1}} x^m_j$ and $y^i_{t+1} = \sum_{j=0}^{N^i_{t+1}} x^i_j$ respectively. For each market jump, the probability that it triggers a jump in individual stock return is p. Conditionally on N^m_{t+1} jumps in the market, we assume there are T jumps in the individual stocks which are triggered by market and N_ϵ idiosyncratic jumps independent with the market.

Conditional on the information of N_m, T and N_ϵ, y^i_{t+1} and y^m_{t+1} follow normal distributions: $N((T+N_\epsilon)\theta_i, (T+N_\epsilon)\delta_i^2)$ and $N(N_m\theta_m, N_m\delta_m^2)$, respectively. Assume that the correlation between x_i which are triggered by the market and x_m is ϕ, and then the conditional covariance between y^i_{t+1} and y^m_{t+1} is $T\phi\delta_i\delta_m$. Using the moment generating function for binomial distribution, the conditional expectation can be written as:

$$\begin{aligned} &E_t[\exp(-\Lambda^m y^m_{t+1} + y^i_{t+1}(ph^m_{y,t+1}))|N_m, T] \\ =&\exp((-\Lambda^m\theta_m + \frac{1}{2}\delta_m^2(\Lambda^m)^2)N_m + (\theta_i + \frac{1}{2}\delta_i^2 - \phi\delta_i\delta_m\Lambda^m)T). \end{aligned}$$

While T and N_m is still correlated, we use the fact that T follows binomial distribution $B(p, N_m)$ conditional on N_m. Let $a = -\Lambda^m\theta_m + \frac{1}{2}\delta_m^2(\Lambda^m)^2$ and $b = \theta_i + \frac{1}{2}\delta_i^2 - \phi\delta_i\delta_m\Lambda^m$, and use the law of iterated expectation, the unconditional expectation of $\exp(aN_m + bT)$ can be expressed as:

$$\begin{aligned} E_t[E_t[\exp(aN_m + bT)|N_m]] &= E_t[\exp(aN_m)E_t[\exp(bT)|N_m]] \\ &= E_t[\exp(aN_m + \log(1-p+pe^b)N_m)] \\ &= \exp(h^y_m((1-p+pe^b)e^a - 1). \end{aligned}$$

Substituting everything back to the Equation (2.30) and taking log of the two sides, we have

$$\begin{aligned} \alpha_i = r &+ (\beta\Lambda^m - \frac{1}{2}\beta^2)h^m_{z,t+1} + pe^a(1-e^b)h^m_{y,t+1} + (\Lambda^i - \frac{1}{2})h^\epsilon_{z,t+1} \\ &+ (\xi(-\Lambda^i) - \xi(1-\Lambda^i))h^\epsilon_{y,t+1}. \end{aligned}$$

If we let $\lambda_z = \Lambda^m$, $\lambda_{zi} = \Lambda_{zi}$ and $\lambda_{yi} = \xi(1) + \xi(-\Lambda_{yi}) - \xi(1-\Lambda_{yi})$, the expression for the discrete-time equity premium can be expressed as:

$$\begin{aligned} E_t[\exp(R^i_{t+1})] = \exp(r &+ \beta\lambda_z h^m_{z,t+1} + p(e^a(1-e^b) + e^{\theta_i + \frac{1}{2}\delta_i^2} - 1)h^m_{y,t+1} \\ &+ \lambda_{zi}h^\epsilon_{z,t+1} + \lambda_{yi}h^\epsilon_{y,t+1}). \end{aligned} \tag{2.31}$$

We can see that the expected stock return is increasing in β and ϕ because $\lambda_z > 0$ and $\lambda_y > 0$ and it is increasing in p when $e^a(1-e^b) + e^{\theta_i + \frac{1}{2}\delta_i^2} - 1 > 0$.

The price of market diffusive risk λ_z and market jump risk λ_y can be obtained from estimating the model for the market index and the price of equity-specific diffusive risk can be estimated from the model for each stock. The form of expected stock return in Equation (2.31) is comparable with the continuous time expression in Yan (2011).

3 | The entropy-based implied volatility and its information content

3.1 Introduction

In financial markets, investors use options to hedge their positions against unfavorable future movements of asset prices. Consequently, option prices reflect investors' perceptions on the likelihood of having such movements. Risk measures implied by option prices can therefore be informational superior to their historical counterparts in forecasting the risk of the underlying asset. With a large literature emphasizing on the information content of option implied risk measures, little efforts have been devoted to examine whether these implied measures actually capture the characteristics of the risk neutral distribution. The situation is even more in doubt when the method of estimating option implied risk measures is based on certain parametric assumptions without empirical validation. An example reflecting this critique is the working horse methodology in practice, the Black-Scholes (BS) formula. Estimating the implied volatility by the BS formula (Black and Scholes (1973)) based on options with different strike prices results in the well-known volatility smile or smirk. This is against the uniquely defined volatility in the underlying Gaussian model. Furthermore, Neumann and Skiadopoulos (2013) show that the implied skewness calculated from S&P500 index options is consistently negative and the implied kurtosis is always higher than three during the period from 1996 to 2010. All empirical evidence points to the fact that the risk neutral distribution observed in the financial market is inconsistent with the Gaussian assumption in the BS formula. Therefore, the Black-Scholes implied volatility (BSIV) may not capture the volatility of the risk neutral distribution accurately. This critique may apply to any parametric method for estimating the implied volatility.

In this chapter, we investigate a non-parametric method, the maximum entropy (ME) method, for estimating the option implied risk measures. The estimated implied volatility using the ME method is called the entropy-based implied volatility (EBIV). We show at least four advantages of the ME method. First, the ME method does not rely on parametric models while allowing the data to determine the shape of the risk neutral distribution. Second, different from the model-free method in Britten-Jones and Neuberger (2000) and Bakshi et al. (2003), the ME method does not require a large number of options with strike prices covering a wide range. Even with limited number of options, this method can produce more accurate estimates than the BSIV and the model-free implied volatility (MFIV). Third, the ME method allows for calculating implied skewness and implied kurtosis. Last but not least, the ME method allows for constructing confidence intervals around the implied volatility by utilizing a nonparametric analog of likelihood ratio statistics proposed by Kitamura and Stutzer (1997).

Using non-parametric methods to extract risk measures of the risk neutral distribution has been studied extensively in the literature, in particular the so-called model-free method. This stream of literature started from the pioneer work of Britten-Jones and Neuberger (2000) and Bakshi et al. (2003), with following-up works in Dennis and Mayhew (2002), Jiang and Tian (2005), Bali and Murray (2013), Neumann and Skiadopoulos (2013), and DeMiguel et al. (2014). They show that the expected variance under the risk neutral measure can be approximated by a linear combination of European call and put option prices with strikes spanning the full range of possible values for the underlying asset at maturity. Consequently, this method makes the implied volatility tradable on the market. Jiang and Tian (2005) show that the truncation error and the discretionary error of the model-free method under the stochastic volatility and random jump (SVJ) model are admissible under certain parameter specifications. However, these errors tend to be larger when the underlying distribution is more negatively skewed, when the available number of options is limited and when the market is more volatile.

This chapter also contributes to the literature on testing the information content of implied risk measures. Several studies find that the implied volatility is superior to the historical volatility of the underlying asset in predicting future realized volatility; see Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Christensen and Prabhala, Fleming (1998), Blair et al. (2001) and Busch et al. (2011). In addition, DeMiguel et al. (2014) show that using the implied risk measures can improve the selection of mean-variance portfolios which leads to a better out-of-sample performance. We test the information content of the EBIV, and compare it with that of the BSIV and MFIV.

The ME method for extracting option implied risk measures is closely related to the principle of maximum entropy proposed in Buchen and Kelly (1996). Buchen and Kelly (1996) find that given simulated option prices at different strikes, estimating the risk neutral distribution by maximizing the entropy can accurately fit the true risk neutral density. In this chapter, we apply this method to obtain the estimated risk neutral distribution first, and then calculate characteristics of estimated distribution, such as the EBIV, implied skewness and kurtosis. Different from Buchen and Kelly (1996), we focus on the implied risk measures rather than the full risk neutral distribution. The empirical goal of this study is to compare the estimation error and information content of the EBIV to the other alternatives such as the BSIV and MFIV. Lastly, we provide a novel methodological contribution for constructing confidence intervals around the EBIV based on Kitamura and Stutzer (1997). This study is also related to Stutzer (1996) in which options are priced using the maximum entropy method. By contrast, we conduct the reverse procedure to extract information of the risk neutral distribution from option prices.

This paper has three main contributions. First, the proposed estimators of the implied volatility, skewness and kurtosis are more accurate than their counterparts using the BS formula and the model-free method. In particular, when the risk neutral distribution exhibits heavy-tailedness and negative skewness, the EBIV is more accurate than the BSIV and MFIV. If the number of available options is reduced or the true volatility increases, the estimation error of MFIV becomes more salient while the EBIV remain robust. Second, to the best of our knowledge, this paper is the first to construct confidence intervals around implied volatility using the ME method. The coverage ratios of the constructed confidence intervals are found to be close to the confidence levels. Third, using the prices of S&P500 index options, we provide both in-sample and out-of-sample evidence that the EBIV performs better than the BSIV and MFIV in forecasting future realized volatility. In particular, the superior performance of the EBIV is more pronounced in high volatility regimes. In addition, the variance risk premium calculated from the EBIV performs comparably or better than that based on BSIV or MFIV in forecasting future stock returns.

The remainder of the chapter proceeds as follows. Section 3.2 discusses the estimation of the option implied risk measures using the ME method. Section 3.3 compares the accuracy of different implied risk measures and shows the coverage ratio of the confidence intervals around the EBIV. The information content of different implied volatilities are compared in Section 3.4. Section 3.5 concludes.

3.2 The entropy-based implied volatility

We first introduce the ME method for extracting the risk neutral distribution from option prices. The implied volatility is consequently calculated from the extracted risk neutral distribution. An important feature of this method is that it allows for constructing the confidence interval around the implied volatility as explained in Section 3.2.2

3.2.1 The maximum entropy method

The ME method is a non-parametric method for estimating the risk neutral distribution with the following intuition. The absence of arbitrage guarantees the existence of a risk neutral probability measure under which the price of any security equals to the expectation of its discounted payoffs. By considering existing options prices as constraints for the underlying risk neutral distribution, one may search for the distribution that maximizes the entropy while obeying all constraints. The optimal distribution is then regarded as the estimated risk neutral distribution. In the reminder of the chapter, all probability measures refer to the risk neutral probability measure.

Let X_t represent the gross return of a stock at the expiry time t in the future. Denote S_0 as the current price of the stock. At time 0, the value of a call option with strike price K equals to the expectation of its discounted payoff at time t as follows:

$$C = \mathbb{E}[\max(S_0 X_t - K, 0)]/r_t, \tag{3.1}$$

where r_t is the gross risk free rate from time 0 to t. In a discrete-state setting, we assume that there are n possible states of X_t, denoted as $X_{t1}, \cdots, X_{tn}$, with probabilities $q_1, \cdots, q_n$ respectively. In addition, we require $q_i > 0$ and

$\sum_{i=1}^{n} q_i = 1$. The pricing equation (3.1) can be rewritten as:

$$C = \sum_{i=1}^{n} q_i(\max(S_0 X_{ti} - K, 0))/r_t.$$

A similar pricing equation can be correspondingly established for put options.

The number of possible states is usually much larger than the number of available options. Consequently, the pricing equations on available options are not sufficient to uniquely determine the underlying risk neutral distribution. Buchen and Kelly (1996) show that if the pricing equations are regarded as constraints on the risk neutral distribution, by maximizing the entropy, defined as

$$\ell_{ET} = -\sum_{i=1}^{n} q_i log(q_i),$$

a unique optimal distribution can be obtained. Since the entropy measures the amount of missing information, the optimal distribution is the least prejudiced distribution compatible with the given constraints. For statistical inference, there is no reason to prefer any other distribution, if the only available information is the pricing equations (Buchen and Kelly (1996)).

More specifically, suppose there are k_1 call options with strike price $K_c(j)$ and option price $C(j)$, $j = 1, \cdots, k_1$. In addition, there are k_2 put options with strike price $K_p(j)$ and option price $P(j)$, $j = 1, \cdots, k_2$. Then the constraints based on the call and put options are:

$$C(j) = \sum_{i=1}^{n} q_i(\max(S_0 X_{ti} - K_c(j), 0))/r_t,\ j = 1, ..., k_1 \tag{3.2}$$

$$P(j) = \sum_{i=1}^{n} q_i(\max(K_p(j) - S_0 X_{ti}, 0))/r_t,\ j = 1, ..., k_2 \tag{3.3}$$

$$S_0 = \sum_{i=1}^{n} q_i S_0 X_{ti}/r_t \tag{3.4}$$

$$\sum_{i=1}^{n} q_i = 1,\ q_i > 0,$$

To present the constraints in a concise manner, we express the $k_1 + k_2 + 1$ constraints in equations (3.2), (3.3) and (3.4) as:

$$\sum_{i=1}^{n} q_i g_j(X_{ti}) = 0, j = 1, ..., k, \tag{3.5}$$

where $k = k_1 + k_2 + 1$. The constrained optimization problem is to maximize the entropy ℓ_{ET} with the $k+1$ constrains. If we include both at-the-money call and put options as constraints, the put call parity implies the constraint in Equation (3.2). In that case, the constraint (3.2) should be removed.

The Lagrange function associated with the constrained optimization problem is:

$$\mathcal{L} = \sum_{i=1}^{n} q_i log(q_i) + \gamma(\sum_{i=1}^{n} q_i - 1) + \lambda'(\sum_{i=1}^{n} q_i g(X_{ti})),$$

where $\gamma \in \mathbb{R}$ and $\lambda \in \mathbb{R}^m$ are the Lagrange multipliers, $g(X_{ti}) = (g_1(X_{ti}), ..., g_k(X_{ti}))^T$. The first order conditions for $\mathcal{L}$ are solved by:

$$\hat{q}_i = \frac{\exp(\hat{\lambda}' g(X_{ti}))}{\sum_{i=1}^{n} \exp(\hat{\lambda}' g(X_{ti}))}, i = 1, ..., n, \tag{3.6}$$

$$(\hat{\lambda}_1, ..., \hat{\lambda}_k) = \arg\min \sum_{i=1}^{n} \exp(\lambda' g(X_{ti})), \tag{3.7}$$

where λ_j is the Lagrange multiplier of the jth constraint in equation (3.5). Notice that the estimated $\hat{q}_i$ is presented as a function of the Lagrange multipliers which are uniquely solved from minimizing a strictly convex function.

After estimating the risk neutral probabilities associated to the predetermined states, the EBIV is calculated as:

$$EBIV = V^Q = \sqrt{\sum_{i=1}^{n} \hat{q}_i (log(X_{ti}) - \mu^Q)^2}, \mu^Q = \sum_{i=1}^{n} \hat{q}_i log(X_{ti}).$$

We choose to calculate the EBIV of the continuously compounded returns rather than the discrete return in order to compare it later with the BSIV, because the BSIV is also based on the continuously compounded return.

In the literature, the entropy is also named as the Kullback-Leibler divergence measure, which is a member of the Cressie-Read divergence family. In fact, taking any member in the Cressie-Read divergence family as the objective function results in a non-parametric method for estimating a probability distribution under given constraints. A notable example of such a method is the so-called empirical likelihood (EL) method, in which the log likelihood is considered as the objective function. However, there are at least two reasons why the ME method is preferred over the EL method. Empirically, the ME method provides a robust performance with respect to the variation in the possible states. Regardless whether we simulate states from a certain distribution, or enforce a series of equally distanced values as states, the estimated risk neutral distribution remains robust as long as the chosen states cover the range of the strike prices. On the contrary, the result following the EL method may change substantially once varying the choice of the states[1]. Theoretically, Theorem 1 in Schennach (2007) shows that the EL method suffers from a dramatic degradation of its asymptotic properties under even the slightest amount of misspecification.

3.2.2 Constructing the confidence interval of the EBIV

An important feature of the ME method is that it facilitates the construction of a confidence interval around the EBIV. The procedure of constructing the confidence interval follows an intuition similar to hypothesis testing. Roughly speaking, by considering the null hypothesis that the implied volatility equals to a certain value around the point estimate, one may perform a likelihood ratio with confidence level α. Such a hypothesis would be rejected for values that are far off the point estimate. Conversely, values that are not rejected will form the confidence interval at the confidence level $1-\alpha$. A rigorous

[1] Simulation results on the comparison of the two methods are upon request.

description of this idea is given as follows.

First, given the level of the mean μ_Q, consider a hypothesis testing problem as $H_0: V^Q = V_0^Q$, where V_0^Q is a given level of volatility to be tested. Kitamura and Stutzer (1997) proposed the following likelihood ratio testing statistics:

$$LR_T = 2n[\log M(V_0^Q) - \log M(\hat{V}^Q)].$$

The two terms $M(V_0^Q)$ and $M(\hat{V}^Q)$ are defined as follows. The term $M(\hat{V}^Q) = \frac{1}{n}\sum_{i=1}^n \exp(\hat{\lambda}' g(X_{ti}))$ is the minimized value of the function (3.7) under the k constraints in (3.5). The term $M(V_0^Q) = \frac{1}{n}\sum_{i=1}^n \exp(\tilde{\lambda}' g(X_{ti}) + \tilde{\lambda}_{k+1} g_{k+1}(X_{ti}))$ is the minimized value of a different optimization problem

$$(\tilde{\lambda}_1, ..., \tilde{\lambda}_k, \tilde{\lambda}_{k+1}) = \arg\min \sum_{i=1}^n \exp(\lambda' g(X_i) + \lambda_{k+1} g_{k+1}(X_i)),$$

with the initial k constraints in (3.5) and an additional $(k+1)$-th constraint

$$\sum_{i=1}^n q_i g_{k+1}(X_{ti}) = 0,$$

where $g_{k+1}(X_{ti}) = (X_{ti} - \mu_Q)^2 - (V_0^Q)^2$.

It is shown that under the null hypothesis, $LR_T \xrightarrow{d} \chi_1^2$ as $n \to \infty$. Consequently, one may vary the value of V_0^Q around the estimated implied volatility $\hat{V}^Q$ and search for the values for which the null is not rejected under a given confidence level α. Since LR_T is increasing for $V_0^Q > \hat{V}^Q$ and decreasing for $V_0^Q < \hat{V}^Q$, there must exist two values V_L^Q and V_H^Q such that H_0 is rejected for $V_0^Q < V_L^Q$ and for $V_0^Q > V_H^Q$ while H_0 is not rejected for $V_0^Q \in [V_L^Q, V_H^Q]$. Then the interval $[V_L^Q, V_H^Q]$ is regarded as the confidence interval of V^Q with the given confidence level α. Obviously, we have that $\hat{V}^Q \in [V_L^Q, V_H^Q]$.

In this procedure, we assume that the mean of the continuous compounded return μ^Q is fixed when varying the constraint based on V_0^Q. Such an assumption is partially supported by the fact that the mean of the discrete return is fixed provided that both at-the-money call and put option prices are available. According to the put-call parity, the mean of the discrete stock return is derived as:

$$\sum_{i=1}^{n} q_i X_{Ti} = \frac{(C_{atm} - P_{atm})r_t + 1}{S_0},$$

where C_{atm} and P_{atm} are at-the-money call and put option prices. Approximately, we regard the mean of the continuously compounded return as fixed at $\sum_{i=1}^{n} \hat{q}_i log(X_{Ti})$ when we vary the value of V_0^Q. In the simulation, we do observe that the means of the discrete returns and the continuously compounded returns are close with the difference at a negligible magnitude.

3.3 The performance of the EBIV: a numerical study

In this section, we compare the performance of the ME method, the model-free method, and the Black-Scholes (BS) model for backing out implied volatility from option prices. In Section 3.3.1, we layout the technical details on how we use the BS model and the model-free method. The data generating process used for the simulation study is given in Section 3.3.2. Finally, in Section 3.3.3, we demonstrate that the ME method is more accurate than the other two methods when there are less number of options available and when the underlying distribution is heavy-tailed with non-zero skewness.

3.3.1 Three methods for backing out implied volatility

The most conventional method for backing out implied volatility from option prices is to use the BS model. We first calculate the implied volatilities from all available option prices, and then take the average as the estimate of the implied volatility, denoted as the BSIV.

When the underlying return distribution deviates from log-normal, taking the average of the BS implied volatilities may not be an efficient way to aggregate information across different strike prices. Britten-Jones and Neuberger (2000) and Bakshi et al. (2003) propose the MFIV which is independent of the pricing models. It is derived entirely from no-arbitrage conditions and can be considered as a linear combination of European call and put option prices with strikes spanning the full range of possible values for the underlying asset at maturity. The MFIV is defined as follows:

$$MFIV = e^{rT}V - \mu^2,$$
$$V = \int_{S_0}^{\infty} \frac{2(1-\ln[\frac{K}{S_0}])}{K^2} C(K,T)dK + \int_0^{S_0} \frac{2(1+\ln[\frac{S_0}{K}])}{K^2} P(K,T)dK,$$

where $C(K,T)$ ($P(K,T)$) is the call (put) option price with strike price K and maturity T and μ is the mean of the risk neutral return, which can also be replicated by an option portfolio. The details for calculating μ are given in Appendix 3.6.1. V is defined as $V = E^Q[e^{-rT}R_T^2]$. In a discrete setting, the term V can be approximated as:

$$V \approx \sum_{i=1}^{m} \frac{2(1-\ln[\frac{K_i}{S_0}])}{K_i^2} C(K_i,T)(K_i - K_{i+1}) + \sum_{j=m+1}^{n} \frac{2(1+\ln[\frac{S_0}{K_j}])}{K_j^2} P(K_j,T)(K_j - K_{j+1}),$$

where $K_1 > K_2 > \cdots > K_m > S = K_{m+1} > K_{m+2} > \cdots > K_n > K_{n+1} = 0$ are the strike prices of the available options.

Practically, since the number of available options is limited, we apply a curve-fitting method to interpolate and extrapolate the prices of the unavailable options as follows. Available option prices are first mapped to implied volatilities using the BS model. For unavailable options with strike prices within the available range, following Bates (1991) and Jiang and Tian (2005), we use cubic spines to interpolate their implied volatilities. For unavailable options with strike prices beyond the available range, we use the end-point implied volatility as their implied volatilities. Then we use the BS model to transform the obtained implied volatilities for unavailable options back to option prices. Eventually we have the option prices with moneyness ranging from 0.35 to 1.65 with an interval 0.002. All the prices of these options are used for calculating the MFIV.

Note that there are different definitions of the model-free implied volatility. The VIX index, disseminated by the by the Chicago Board of Options Exchange (CBOE), is constructed in accordance with Britten-Jones and Neuberger (2000). The VIX is defined in the following way:

$$VIX^2 = E^Q[\int_0^T (\frac{dS_t}{S_t})^2 dt]$$

The definition is not completely consistent with the definition of the EBIV. Hence, in the numerical analysis, we compare the performance of EBIV with the MFIV, which definition is consistent with the definition of EBIV.

3.3.2 The underlying risk neutral distributions in the numerical study

We consider four data generating processes to generate the underlying continuously compound returns. We start by assuming that stock price follows a geometric Brownian motion under the risk neutral measure:

$$dS_t = rS_t dt + \sigma S_t dw_t,$$

where S_t is the stock price at time t, r is the risk-free rate, σ is the constant instantaneous volatility of the process and dw_t is the increment in a standard Wiener process. Throughout the section, we employ an annual risk-free rate r at 5%, an annual volatility σ at 20% (or 40%) and the initial stock price S_0 at 100. Under this model, the risk neutral distribution of the continuously compounded T-year returns $\ln(R_T)$ is normally distributed:

$$\ln(R_T) \sim N((r - \frac{1}{2}\sigma^2)T, \sigma^2 T). \tag{3.8}$$

The mean of the risk neutral distribution, $(r - \frac{1}{2}\sigma^2)T$, ensures that the expectation of R_T is e^{rT} under the risk neutral measure. Note that the BS model and the model-free method are derived based on this assumption. These two methods should provide accurate estimates for the implied volatility if 3.8 reflects reality.

Next, we consider distributions deviating from the normal distribution, in particular, the Student-t and the skewed Student-t distributions. More specifically, the continuously compound return is given as

$$\ln(R_T) \sim (r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\epsilon. \tag{3.9}$$

For the random term ϵ, we first employ the standardized Student-t distribution with 5 degree of freedom, and then the standardized skewed Student-t distribution ($skewt(\eta,\lambda)$) proposed in Hansen (1994). The skewed Student-t distribution has mean 0, variance 1, a degree of freedom η and skewness parameter λ. We use two sets of parameters: $\eta = 5$, $\lambda = -0.3$ and $\eta = 5$, $\lambda = -0.7$. The latter is more negatively skewed than the former.

Notice that although the mean return is comparable with that in (3.8), the pricing equation, $ER_T = e^{rT}$, does not hold if ϵ follows the non-normal distributions, though it remains approximately true.

3.3.3 Results

Based on the risk neutral distributions specified in Section 3.3.2, we calculate the call and put option prices using numerical integration for several moneyness with one month to expiration. Results for other expiration horizons are provided in the robustness check in Section 3.3.4. The call and put option prices with strike price K and maturity T are calculated by numerical integration:

$$C(K,T) = \int_{K/S_0}^{\infty} (S_0 R_T - K) f(R_T) dR_T / r_T \tag{3.10}$$

$$P(K,T) = \int_{0}^{K/S_0} (K - S_0 R_T) f(R_T) dR_T / r_T \tag{3.11}$$

where $f(R_T)$ is the density function of R_T, r_T is the risk-free rate that is used to discount payoff of the options.

Following Bakshi et al. (2003), we only consider out-of-the-money (OTM) options and at-the-money (ATM) options because in-the-money options are less traded in the option market, while their prices can be derived from the put-call parity under the no-arbitrage condition. Consequently, they do not provide additional information for extracting the implied volatility.

We consider different ranges of strike prices which result in different estimation accuracy. In the first case, we specify the moneyness (K/S_0) of call options from 1 to 1.15 with equal interval 0.025, and the moneyness of put options from 0.85 to 1 with the same interval. There are 14 options in total. In the second case, we reduce the number of available options and only consider six options: call options with moneyness 1, 1.05, 1.1 and put options with moneyness 0.95, 0.975, 1. By comparing the two cases, we evaluate the performance of the three methods with different number of available options. The calculated option prices with the chosen moneynesses under different distributions are reported in Table 3.1.

Table 3.2 reports the estimated implied volatilities under different distributions using the three methods. The first row shows the true volatility of the underlying distribution and the second row shows the number of options. Under the normal distribution, all three methods provide very accurate estimates, while for other risk neutral distributions, each method has some estimation errors. We calculate the relative improvements of the MFIV and EBIV compared to the BSIV in the parenthesis, which is defined as $\frac{|XXIV-TrueVolatility|}{|BSIV-TrueVolatility|}$, where XXIV is either MFIV or EBIV.

From Table 3.2, we observe that under the normal distribution, all three methods provide accurate estimates. However, when the underlying distribution is heavy-tailed or negatively skewed, the EBIV estimates are closer to the true value than both the BSIV and MFIV. The improvement is substantial.

Although the MFIV performs better than the BSIV under heavy-tailed or skewed distributions, the estimation error increases when the underlying dis-

Table 3.1: Option prices under different risk neutral distributions

(a) Panel A: $\sigma = 0.2$

	Moneyness(K/S_0)	lognormal	Student t	Skewt1	Skewt2
Call	1.15	0.020	0.078	0.020	0.000
	1.125	0.057	0.125	0.038	0.001
	1.1	0.148	0.210	0.080	0.003
	1.075	0.349	0.373	0.188	0.022
	1.05	0.744	0.691	0.474	0.237
	1.025	1.435	1.292	1.146	1.002
	1	2.512	2.333	2.336	2.310
Put	0.85	0.003	0.029	0.062	0.093
	0.875	0.015	0.054	0.105	0.149
	0.9	0.061	0.107	0.184	0.242
	0.925	0.193	0.222	0.329	0.402
	0.95	0.504	0.469	0.598	0.675
	0.975	1.106	0.979	1.086	1.137
	1	2.096	1.917	1.922	1.899

(b) Panel B: $\sigma = 0.4$

	Moneyness(K/S_0)	lognormal	Student t	Skewt1	Skewt2
Call	1.15	0.730	0.811	0.394	0.041
	1.125	1.049	1.063	0.598	0.131
	1.1	1.479	1.410	0.921	0.400
	1.075	2.046	1.882	1.417	0.947
	1.05	2.774	2.521	2.140	1.779
	1.025	3.688	3.368	3.123	2.886
	1	4.805	4.456	4.367	4.247
Put	0.85	0.353	0.396	0.580	0.702
	0.875	0.613	0.602	0.814	0.944
	0.9	1.003	0.913	1.138	1.266
	0.925	1.552	1.369	1.582	1.691
	0.95	2.289	2.017	2.180	2.248
	0.975	3.231	2.897	2.967	2.966
	1	4.390	4.036	3.976	3.879

Note: This table reports call and put option prices with different moneynesses under different risk neutral distributions. Panel A reports results for $\sigma = 0.2$ and Panel B reports for $\sigma = 0.4$. Risk neutral distributions of the continuously compounded stock returns follow the normal, the Student-t or two skewed Student-t distributions as specified in (3.8) and (3.9). The degree of freedom of the Student-t and the two skewed Student-t distributions is 5. For the two skewed Student-t distributions, the skewness parameters are -0.3 and -0.7. The risk-free rate is 5%, K is the strike price, S_0 is the initial stock price 100, and the standard deviation of the underlying risk neutral distributions σ is 0.2.

Table 3.2: Comparison across the three methods, one-month maturity

	Volatility σ	0.2	0.2	0.4	0.4
	Option No.	14	6	14	6
Normal	BSIV	0.200	0.200	0.400	0.400
	MFIV	0.200	0.200	0.400	0.400
	EBIV	0.200	0.202	0.402	0.413
Student-t	BSIV	0.211	0.192	0.385	0.373
	MFIV	0.198	0.195	0.387	0.374
		(0.209)	(0.628)	(0.896)	(0.979)
	EBIV	0.199	0.196	0.393	0.393
		(0.139)	(0.466)	(0.444)	(0.264)
Skewt1	BSIV	0.206	0.192	0.374	0.368
	MFIV	0.197	0.197	0.383	0.366
		(0.433)	(0.342)	(0.657)	(1.082)
	EBIV	0.198	0.196	0.391	0.391
		(0.322)	(0.558)	(0.361)	(0.269)
Skewt2	BSIV	0.195	0.187	0.350	0.359
	MFIV	0.196	0.196	0.375	0.355
		(0.818)	(0.334)	(0.501)	(1.106)
	EBIV	0.197	0.193	0.384	0.387
		(0.668)	(0.541)	(0.308)	(0.310)

Note: This table reports the estimated implied volatility calculated from 14 or 6 options with one-month maturity by the Black-Scholes formula (BSIV), the model-free method (MFIV) and the maximum entropy method (EBIV) under different risk neutral distributions. The first row provides the true volatilities of the underlying risk neutral distribution. The second row presents number of options used in calculating implied volatilities. The moneynesses of the 14 options range from 0.85 to 1.15 and the moneynesses of the 6 options range from 0.95 to 1.05, both with equal interval 0.025. The degree of freedom of the Student-t and the two skewed Student-t distributions is 5. For the two skewed Student-t distributions, the skewness parameters are -0.3 and -0.7, for "Skew1" and "Skew2" respectively. The estimation improvements of the MFIV and the EBIV compared to the BSIV are presented in parenthesis, which is defined as $\frac{|XXIV-TrueVolatility|}{|BSIV-TrueVolatility|}$, where XXIV is either EBIV or MFIV.

tribution is more negatively skewed. However, the estimation improvement of the EBIV remains robust across different specifications. When we decrease the available number of options from 14 to 6 or increase the true volatility from 0.2 to 0.4, the better performance of EBIV compared to MFIV becomes more evident. For example, an estimation error of the MFIV under the $skewt(5, -0.7)$ distribution with 6 options and true volatility 0.4 is even larger than that of the BSIV, while the EBIV has an estimation error as low as 31% of that of the BSIV. The non-robust performance of the MFIV may be attributed to the increase of the truncation error and extrapolation error, whereas the ME method does not suffer from such a problem.

Table 3.3: Coverage rates of the confidence intervals around the EBIV

volatility		Normal	Student t	Skewt1	Skewt2
0.2	95%	91.21%	91.40%	92.30%	93.10%
	90%	85.00%	85.50%	86.60%	92.30%
0.4	95%	92.39%	91.60%	93.40%	92.50%
	90%	88.78%	86.70%	87.60%	84.50%

Note: This table reports the coverage rates of confidence intervals under different risk neutral distributions. The upper panel is for $\sigma = 0.2$ under 95% and 90% confidence levels and the lower panel is for $\sigma = 0.4$. The degree of freedom of the Student-t and the two skewed Student-t distributions is 5. For the two skewed Student-t distributions, the skewness parameters are -0.3 and -0.7, for "Skew1" and "Skew2" respectively. The details of constructing the confidence interval of the EBIV is in Section 3.2.2.

An additional advantage of the ME method is that one may construct the confidence interval around the estimate of the implied volatility. Table 3.3 shows the coverage rate of the EBIV confidence interval under the four distributions with six option prices. This analysis is conducted as follows. We simulate 10000 states from the true distribution for 100 times[2]. For each set of simulated states, we first calculate option prices, and then construct the

[2]When simulating the states, kurtosis of the simulated sample might differ from the true value in many simulations. The reason is that sample kurtosis is very sensitive to extreme

confidence interval for the implied volatility. Next, we examine the confidence interval covers the true volatility, i.e. whether the true volatility falls into the constructed interval. Across the 100 simulations, we count the number of "coverage" and divide that number by 100. From Table 3.3, we find that the coverage rates of the confidence interval are close to the confidence levels under all four distributions.

3.3.4 Robustness and discussion

Results from Section 3.3.3 show that the EBIV aggregates information in the option prices more efficiently than the MFIV, when the underlying distribution exhibits heavy tails and when the number of options is limited. We now conduct robustness check to ensure the generality of our findings.

In Table 3.4 and Table 3.5, we provide the estimated implied volatilities based on option prices with three-month and one-year maturity, respectively. In addition to the 14 or 6 options with symmetric moneyness, we consider asymmetric moneyness with 3 call options (moneyness ranging from 1 to 1.05) and 7 put options (moneyness ranging from 0.85 to 1), with equal interval 0.025. This is to reflect that there are more available put options than call options traded in the market. Column 5 an 8 in Table 3.4 and Table 3.5 report the results when having asymmetric moneyness. Results in these two tables show that the EBIV still has a better estimation accuracy than the MFIV and BSIV for longer maturity options.

The second generalization is to consider more complex data generating process for the risk neutral distribution, for example, the SVJ model. Further, with such data generating process, the risk neutral distribution possesses non-

observations. If there are no extreme observations in the simulated sample, sample kurtosis is downward biased. To alleviate this bias, we only consider the simulated sample if the sample kurtosis is higher than 80% of the true kurtosis.

Table 3.4: Comparison across the three methods: 3-month maturity

	Volatility	0.2	0.2	0.2	0.4	0.4	0.4
	Option No.	14	6	3+7	14	6	3+7
Normal	BSIV	0.200	0.200	0.200	0.400	0.400	0.400
	MFIV	0.200	0.200	0.200	0.400	0.400	0.400
	EBIV	0.201	0.202	0.200	0.404	0.417	0.405
student t	BSIV	0.195	0.192	0.200	0.384	0.376	0.379
	MFIV	0.194	0.193	0.204	0.386	0.376	0.382
		(1.086)	(0.908)	(8.784)	(0.862)	(0.995)	(0.854)
	EBIV	0.197	0.196	0.197	0.395	0.397	0.393
		(0.515)	(0.453)	(6.329)	(0.328)	(0.131)	(0.343)
skewt1	BSIV	0.192	0.188	0.199	0.368	0.361	0.370
	MFIV	0.194	0.191	0.208	0.379	0.363	0.378
		(0.746)	(0.762)	(10.454)	(0.664)	(0.941)	(0.714)
	EBIV	0.197	0.196	0.197	0.391	0.393	0.391
		(0.343)	(0.359)	(3.486)	(0.274)	(0.185)	(0.302)
skewt2	BSIV	0.184	0.177	0.190	0.341	0.341	0.354
	MFIV	0.191	0.186	0.208	0.367	0.346	0.369
		(0.534)	(0.581)	(0.763)	(0.563)	(0.902)	(0.659)
	EBIV	0.195	0.193	0.195	0.384	0.384	0.385
		(0.290)	(0.310)	(0.471)	(0.281)	(0.262)	(0.329)

Note: This table reports the estimated implied volatility calculated from 14 or 6 options with three-month maturity by the Black-Scholes formula (BSIV), the model-free method (MFIV) and the maximum entropy method (EBIV) under different risk neutral distributions. The first row provides the true volatilities of the underlying risk neutral distribution. The second row presents number of options used in calculating implied volatilities. The moneynesses of the 14 options range from 0.85 to 1.15 and the moneynesses of the 6 options range from 0.95 to 1.05, both with equal interval 0.025. The degree of freedom of the Student-t and the two skewed Student-t distributions is 5. For the two skewed Student-t distributions, the skewness parameters are -0.3 and -0.7, for "Skew1" and "Skew2" respectively. The estimation improvements of the MFIV and the EBIV compared to the BSIV are presented in parenthesis, which is defined as $\frac{|XXIV - TrueVolatility|}{|BSIV - TrueVolatility|}$, where XXIV is either EBIV or MFIV.

Table 3.5: Implied volatilities estimated using the three methods: 1-year maturity

	Volatility	0.2	0.2	0.2	0.4	0.4	0.4
	Option No.	14	6	3+7	14	6	3+7
Normal	BSIV	0.200	0.200	0.200	0.400	0.400	0.400
	MFIV	0.200	0.200	0.200	0.400	0.400	0.400
	ETIV	0.204	0.202	0.200	0.407	0.423	0.407
student t	BSIV	0.188	0.192	0.198	0.392	0.386	0.388
	MFIV	0.190	0.192	0.199	0.390	0.381	0.387
		(0.850)	(0.933)	(0.598)	(1.363)	(1.328)	(1.037)
	EBIV	0.197	0.197	0.197	0.401	0.406	0.399
		(0.256)	(0.402)	(1.181)	(0.101)	(0.452)	(0.072)
skewt1	BSIV	0.191	0.193	0.201	0.366	0.363	0.368
	MFIV	0.193	0.193	0.203	0.382	0.368	0.382
		(0.844)	(0.977)	(3.253)	(0.536)	(0.867)	(0.572)
	EBIV	0.200	0.199	0.201	0.399	0.404	0.398
		(0.040)	(0.086)	(0.651)	(0.036)	(0.112)	(0.049)
skewt2	BSIV	0.188	0.183	0.193	0.331	0.334	0.342
	MFIV	0.190	0.187	0.200	0.366	0.346	0.369
		(0.840)	(0.810)	(0.054)	(0.486)	(0.812)	(0.546)
	EBIV	0.193	0.196	0.198	0.389	0.393	0.390
		(0.542)	(0.253)	(0.316)	(0.163)	(0.102)	(0.175)

Note: This table reports the estimated implied volatility calculated from 14 or 6 options with one-year maturity by the Black-Scholes formula (BSIV), the model-free method (MFIV) and the maximum entropy method (EBIV) under different risk neutral distributions. The first row provides the true volatilities of the underlying risk neutral distribution. The second row presents number of options used in calculating implied volatilities. The moneynesses of the 14 options range from 0.85 to 1.15 and the moneynesses of the 6 options range from 0.95 to 1.05, both with equal interval 0.025. The degree of freedom of the Student-t and the two skewed Student-t distributions is 5. For the two skewed Student-t distributions, the skewness parameters are -0.3 and -0.7, for "Skew1" and "Skew2" respectively. The estimation improvements of the MFIV and the EBIV compared to the BSIV are presented in parenthesis, which is defined as $\frac{|XXIV-TrueVolatility|}{|BSIV-TrueVolatility|}$, where XXIV is either EBIV or MFIV.

trivial skewness and kurtosis. We can thus examine the performance of the ME method for estimating implied skewness and kurtosis. The SVJ model has been applied for pricing options in Bakshi et al. (1997) and for illustrating truncation errors of model-free implied volatility in Jiang and Tian (2005). The model is specified as:

$$\begin{aligned}\frac{dS_t}{S_t} &= \sqrt{V_t}dW_t + J_t dN_t - \mu_J \lambda dt,\\ dV_t &= (\theta_v - \kappa_v V_t)dt + \sigma_v \sqrt{V_t} dW_t^v,\\ dW_t dW_t^v &= \rho dt,\end{aligned}$$

where N_t follows a homogeneous Poisson process with jump intensity λ and $\ln(1+J_t)$ follows a normal distribution $N(\ln(1+\mu_J) - \frac{1}{2}\sigma_J^2, \sigma_J^2)$. If $\lambda = 0$, the model reduces to the Heston (1993) model. We choose the parameters as $\kappa_v = 1$, $\sigma_v = 0.25$, $\rho = 0$, $\lambda = 0.5$, $\theta_v = V_0 \kappa_v$, $V_0 = 0.1854^2$. In addition, to evaluate the impact of the jump process, we choose two sets of parameters for the jumps: (1) $\mu_J = -1.75$, $\sigma_J = 0.5$, (2) $\mu_J = -0.075$, $\sigma_J = 2.5$, correspondingly. The volatility is 0.203 for the first set of parameters and 0.453 for the second set of parameters.

Since the unconditional return distribution is unknown in this case, it is necessary to conduct pre-simulations to obtain option prices and the true skewness and kurtosis. First, we simulate 21 daily returns to get one monthly return, and repeat this 100,000 times. Second, option prices, the true volatility, skewness and kurtosis are calculated based on these simulated monthly return.

With the obtained option prices, we estimate the implied volatility by the BS model, the model-free method and the ME method, and compare the estimates with the true volatility. In addition, we calculate the implied skewness and kurtosis using the model-free method and the ME method. The results for options with one-month maturity are reported in Table 3.6: the second column for each parameter set reports the simulated moments and the column 3-5 reports the implied moments estimated from different methods. The results show that the ME method gives the most accurate estimation in all cases, and is especially robust when the unconditional distribution has higher volatility, more negative skewness and higher kurtosis.

Table 3.6: Implied moments for the stochastic volatility and jump (SVJ) model)

Parameter set I				Parameter set II			
	volatility	skewness	kurtosis		volatility	skewness	kurtosis
true value	0.203	-0.401	3.179	true moments	0.453	-0.554	3.53
BS	0.2	-	-	BS	0.445	-	-
MF	0.205	-0.274	2.924	MF	0.427	-0.059	1.76
ET	0.203	-0.404	3.207	ET	0.454	-0.543	3.435

Note: This table provides a comparison between the three methods, the Black-Scholes formula (BS), the model-free (MF) method and the entropy-based (EB) method, when the option prices are simulated from the stochastic volatility and jump (SVJ) model (details in Section 3.3.4). For the two sets of parameters given in Section 3.3.4, we present the true moments of the underlying distribution in the first row and the implied moments calculated from different methods in the second to fourth rows. To determine the true moments of the risk neutral distributions with the two parameter sets, 100,000 monthly returns are simulated by aggregating each 21 daily returns simulated from the SVJ model. Option prices and the true moments are calculated based on the simulated monthly return. There are 14 options in total, with one-month maturity. The moneynesses of the options range from 0.85 to 1.15 with equal interval 0.025.

Lastly, we check the robustness of the ME method by focusing on its fundamental step: the estimated risk neutral distribution. For the original four data generate processes, we compare the sample of the simulated distribution (blue bars) and the estimated density produced by the ME method (red lines) in Figure 1. More specifically, the estimated risk neutral densities in these figures are estimated from 14 options with one year maturity. The figures show that the risk neutral density estimated by the ME method matches the true density in all four cases. Option prices with different moneynesses essentially provide information on different parts of the distribution.

To conclude, the ME method provides more accurate estimates of option implied volatility than the BS model and the model-free method. Our main findings are robust to the choice of different number of options, maturities and data generating process. In addition, the ME method can also be applied for other higher moments of the risk neutral distribution, because it provides an accurate estimation for the risk neutral distribution.

3.4 The information content of EBIV

In this section, we conduct an empirical analysis to explore the information content of the EBIV using the S&P500 index option traded in the Chicago Board Options Exchange (CBOE). We first review the basic statistical properties of the various volatility measures and then investigate their relative performance as predictors for the subsequent realized volatilities of the underlying S&P500 index. Further, we analyze the forecasting power of the variance risk premia derived from different implied volatilities on the subsequent returns of the S&P500 index.

The volatility measures we include in the empirical analysis are: lagged realized volatility (RV), Black-Scholes implied volatility (BSIV), VIX provided by CBOE, model-free implied volatility (MFIV) and entropy-based implied volatility (EBIV). We include both the VIX and the MFIV because the construction methodologies of these two measures are different as illustrated in Section 3.3. In addition, the VIX measure is not truly model free because the derivation of this measure involves assumptions of the underlying process. While Jiang and Tian (2005) and Carr and Wu (2009a)) argue that jumps are unlikely to create sizable biases in VIX, this view has been revised in Carr et al. (2012), Andersen et al. (2015) and Martin (Forthcoming). The problem is that price jumps induce a discrepancy between the fair value of future cumulative squared returns and VIX^2, even in the continuous sampling limit. The MFIV is more consistent to the model-free manner because the method does not depend on distribution assumption on the underlying process.

3.4.1 Data

Our sample period covers from January 1996 to August 2014. We get the S&P500 index price data from The Center for Research in Security Prices database. We obtain the S&P500 index options data from the Ivy DB database of OptionMetrics. Continuously-compounded zero-coupon interest rates are also obtained from OptionMetrics as a proxy for the risk-free rate. From the CBOE, we get daily levels of the newly calculated VIX index[3] and match them with the trading days on which options with one month expiration are traded.

Our analysis is conducted based on call and put options quoted on the S&P500 index with 30 days expiration. We choose one-month maturity because the options with one month to expire are more actively traded than with other maturities. From January 1996 to February 2007, in each month, there is only one day on which options with 30 days expiration are traded. From March 2007, there are several such days in each month. To avoid the overlapping problem described in Christensen and Prabhala, Christensen et al. (2001) and Jiang and Tian (2005), we select one date in each month from 2006 to 2014, such that the time intervals between any two adjacent dates are the closest to 30 days. With this procedure, there are 222 selected dates in total. Midpoints of the bid-ask spread are used as the option prices instead of the actual trade prices. This follows Jackwerth (2000) who demonstrates that measurement of risk neutral distribution is not sensitive to the existence of spreads.

Table 3.7 presents the descriptive statistics of the out-of-the-money call and

[3] Although the CBOE changed the methodology for calculating the VIX in September 2003, these have backdated the new index using the historical option prices.

Table 3.7: Descriptive statistics of the S&P500 index options with 1 month expiration

(a) Panel A: Call options

K_c/S	1	1.025	1.05	1.075	1.1	1.125	1.15
Mean	24.62	11.38	4.78	2.23	1.28	0.93	0.77
Variance	80.02	65.66	33.11	15.39	7.36	4.35	2.28
Skewness	1.11	1.66	3.15	5.26	7.17	8.66	9.73
Maximum	7.69	1.28	0.40	0.40	0.40	0.40	0.40
Minimum	64.85	53.25	45.20	36.80	29.30	24.50	19.30
obs.	222	222	222	196	123	57	33

(b) Panel B: Put options

K_p/S	0.85	0.875	0.9	0.925	0.95	0.975	1
Mean	1.96	2.69	3.91	5.78	8.92	14.38	24.10
Variance	9.06	13.37	20.66	30.97	47.59	66.61	80.86
Skewness	4.57	4.12	3.41	2.83	2.25	1.66	0.99
Maximum	0.40	0.40	0.40	0.53	1.33	3.65	8.25
Minimum	25.20	30.05	34.45	40.35	47.35	55.25	62.65
obs.	222	190	216	220	221	221	222

Note: This table reports descriptive statistics of the S&P500 call and put options with 1 month expiration from January 1996 to August 2014. The options are selected by the procedure illustrated in Section 3.4.1. The first row in Panel 3.7a (Panel 3.7b) shows moneynesses of the out-of-the-money call (put) options, where K_c (K_p) are exercise prices of the call (put) options, S is the current price of the S&P500 index. The last row labelled "obs" shows the number of observations in each moneyness category.

put options. We apply several filters to select the options. First, option quotes less than 3/8 are excluded from the sample. Such low prices may not reflect the true option value due to proximity to tick size. Second, options with zero open interest are excluded from the sample. Third, following Aït-Sahalia and Lo (1998) and Bakshi et al. (2003), we exclude in-the-money options, because they have less liquidity than out-of-the-money options.

For BSIV, we calculate the mean of the Black-Scholes implied volatility using all available option prices after the filtering procedure.

We use all available option prices with moneyness between 0.85 to 1.15 to calculate the MFIV. We interpolate and extrapolate the prices of the unavailable options using the method discussed in Section 3.3.1. Eventually we have option prices with moneyness ranging from 0.35 to 1.65. All the prices of these options are used for calculating the MFIV.

By contrast, we do not use all option prices in the ME method. To calculate the EBIV, we select options with moneyness that are closest to the moneyness ranging from 0.85 to 1.15 with equal interval 0.025. The reason is that with more option prices as constraints, the ME method may run into numerical difficulties as follows. If the covariance matrix of the constraints in equation (3.5), $cov(g_i(X_t), g_j(X_t))$, is close to singular, then the numerical solution for the Lagrange multipliers becomes unstable (Buchen and Kelly (1996)).

On each selected trading day, we also calculate the historical realized volatility (RV) in the previous month. Following Christensen and Prabhala, we adopt the realized volatility over the 30 calendar days proceeding the current observation dates as the lagged realized volatility RV_t. It is computed as the sample standard deviation of the daily index returns:

$$RV_t = \sqrt{\frac{1}{30}\sum_{i=1}^{30}(r_{t,i} - \bar{r}_t)^2},$$

where $\bar{r}_t = \frac{1}{30}\sum_{k=1}^{30} r_{t,k}$, and $r_{t,i}$, $i = 1, ..., 30$, are the log index returns on the

30 days preceding to the selected trading day t. All of the volatility measures are expressed in annual terms to facilitate interpretation.

3.4.2 Descriptive statistics of different volatility measures

Table 3.8 reports descriptive statistics of the five measures of volatility: RV, VIX, BSIV, MFIV and EBIV. Table 3.9 shows the correlation matrix of these measures. We first observe that the mean of the four implied volatility measures, VIX, BSIV, MFIV and EBIV, are comparable. All of them exceed the mean of the realized volatility measure RV by about 24%, which is in line with the positive volatility risk premium. Second, the four implied volatility measures are highly correlated, with all correlation coefficients above 0.99. VIX is more correlated with MFIV and EBIV than with BSIV. This may be a consequence of the fact that the three share the same nonparametric feature by construction.

Table 3.8: Descriptive statistics of different measures of volatility

	Mean	Median	Std. Dev.	Skewness	Kurtosis	Maximum	Minimum
RV	0.172	0.145	0.101	2.739	14.158	0.784	0.055
VIX	0.217	0.200	0.093	2.460	13.122	0.809	0.102
BSIV	0.219	0.204	0.078	2.994	17.341	0.765	0.130
MFIV	0.209	0.195	0.085	2.460	13.349	0.758	0.103
EBIV	0.210	0.194	0.089	2.257	11.343	0.745	0.097

Note: This table reports the descriptive statistics for volatility measures RV, VIX, BSIV, MFIV and EBIV. RV is the realized volatility of the preceding 30 days defined in 3.12. VIX is the volatility index provided by CBOE. BSIV is the average Black-Scholes implied volatility of all available option prices after filter procedure. MFIV is calculated based on Appendix 3.6.1. The details of calculating EBIV is in 3.2.1 and 3.4.1. Statistics are reported for the full sample from January 1996 to August 2014. In all tables and figures, the volatility measures are annualized and given in decimal form.

In Figure 3.1, we plot the estimated MFIV and EBIV from 1996 to 2014. Figure 3.1a shows their strong comovement during the period. From Figure 3.1b, we observe that in most of the time, the differences between the EBIV and the MFIV are negative and small. Occasionally, the differences can be positive and large. This is consistent with the results in Table 3.4 and 3.5:

Table 3.9: Correlation matrix of different measures of volatilities

	RV	VIX	BSIV	MFIV	EBIV
RV	1.000	0.734	0.735	0.735	0.735
VIX	0.734	1.000	0.994	0.998	0.998
BSIV	0.735	0.994	1.000	0.996	0.993
MFIV	0.735	0.998	0.996	1.000	0.998
EBIV	0.735	0.998	0.993	0.998	1.000

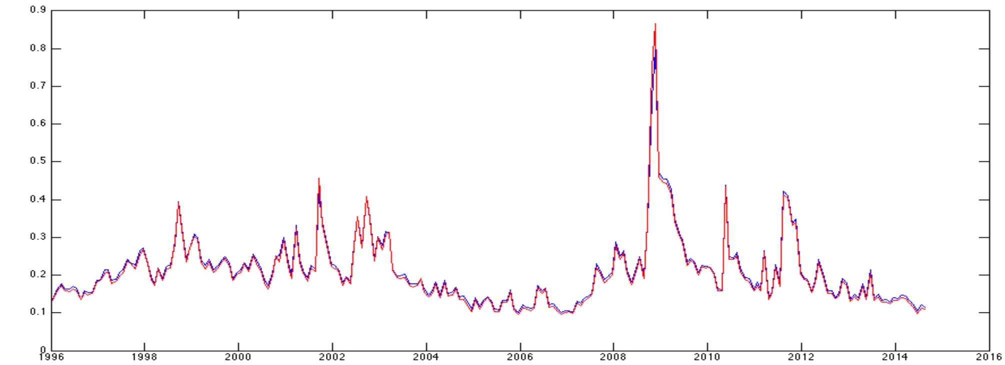
Note: This table reports the correlation coefficients across volatilities measures: RV, VIX, BSIV, MFIV and EBIV. RV is the realized volatility of the preceding 30 days defined in 3.12. VIX is the volatility index provided by CBOE. BSIV is the average Black-Scholes implied volatility of all available option prices after filter procedure. MFIV is calculated based on Appendix 3.6.1. The details of calculating EBIV is in 3.2.1 and 3.4.1. The sample period is from January 1996 to August 2014.

when the number of the put options is more than the number of call options, the MFIV may overestimate the true volatility under the low volatility regime ($\sigma = 20\%$). Conversely, the underestimation occurs under the higher volatility regime ($\sigma = 40\%$). To further illustrate this observation, Figure 3.1c plots the spread between the EBIV and the MFIV against the BSIV. By regarding the BSIV as an indication of high and low volatility regime, this scatter plot further demonstrates that in a higher volatility regime, the spread is also higher. We attribute this phenomenon to the fact that the model-free method produces less accurate estimates of implied volatility when the market condition becomes more volatile.

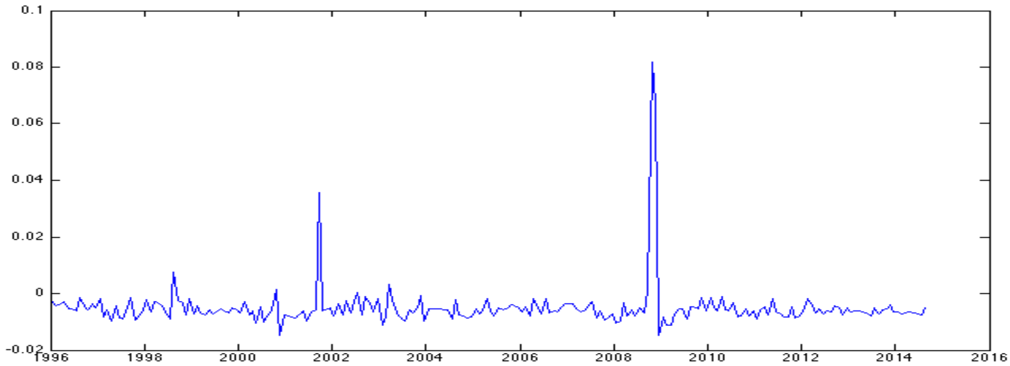
Figure 3.2a presents the confidence interval for the estimated EBIV and Figure 3.2b shows the length of the confidence interval over time. Lastly, in Figure 3.3, we provide the estimated risk neutral distributions on four example dates using the ME method. From the figures, we observe that the estimated risk neutral distributions may differ across different market environments.

Figure 3.1: Comparison between the MFIV and the EBIV

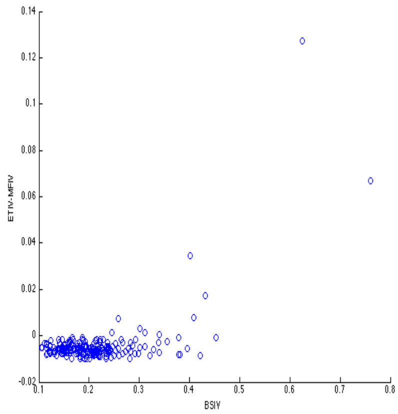
(a) EBIV and MFIV



(b) Difference between EBIV and MFIV



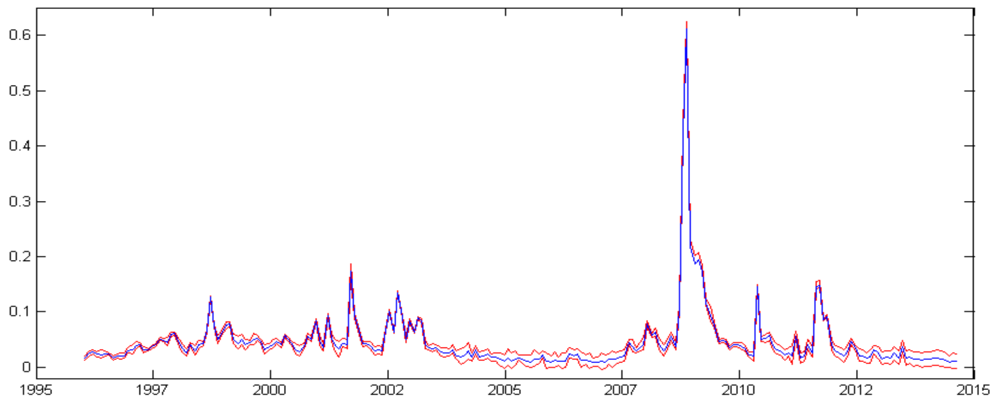
(c) BSIV and MFIV-EBIV



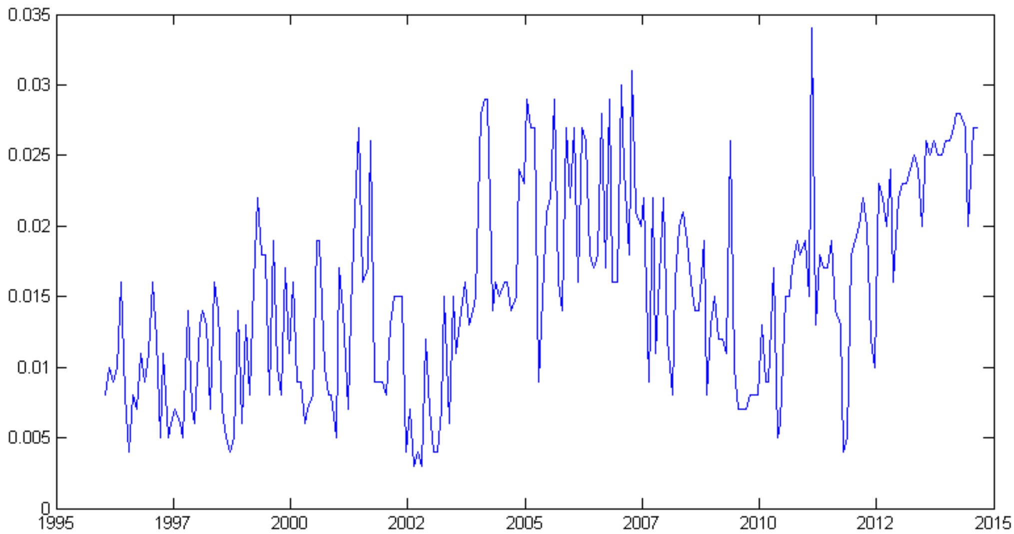
Note: This figure shows the comparison between the MFIV and the EBIV from January 1996 to August 2014. The blue line in 3.0a represents the MFIV and the red line represents the EBIV. Figure 3.0b shows the time series of the spread between EBIV and MFIV. Figure 3.0c shows the scatter plot of the BSIV (horizontal axis) against the spread between MFIV and EBIV (vertical axis).

Figure 3.2: Confidence interval using the maximum entropy method

(a) Confidence interval for EBIV



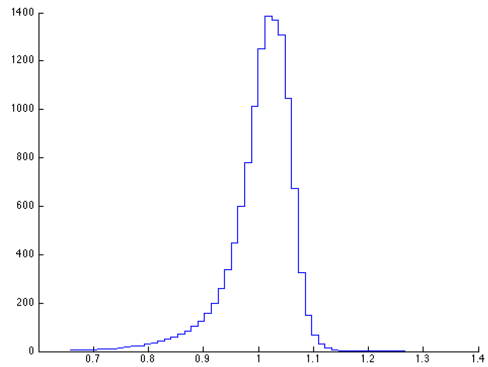
(b) Length of the confidence interval



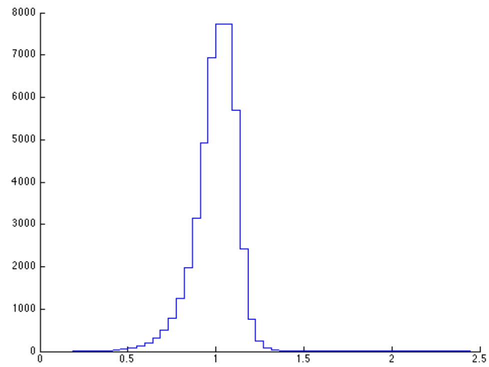
Note: The red lines in Figure 3.1a are the upper and lower bounds of the confidence interval around the estimated EBIV with a confidence level 95%. The blue line in Figure 3.1a represents the point estimate of the EBIV. Figure 3.1b shows the time series of the lengths of the confidence intervals.

Figure 3.3: Estimated risk neutral distributions on four selected dates

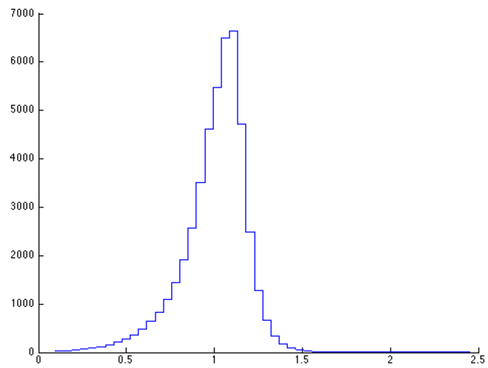
(a) June 18th, 1998



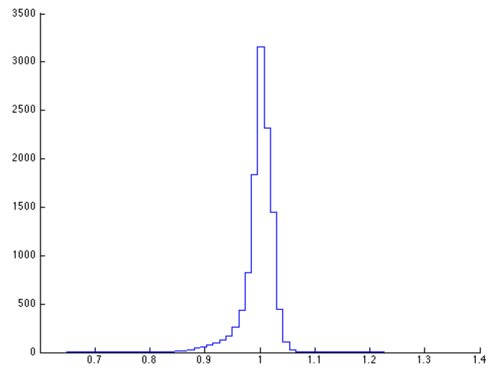
(b) September 20th, 2001



(c) October, 23rd, 2008



(d) June 19th, 2014



Note: This figure shows the estimated risk neutral distributions using the maximum entropy method on four selected dates.

3.4.3 Forecasting the stock market volatility

Prior research has extensively analysed the information content of the BSIV on predicting the future realized volatility. In particular, recent studies seem to agree on the informational superiority of the BSIV compared to historical volatility. In this chapter, we assess the predictive power of the EBIV, and compare it to the other implied volatility measures.

To nest previous research within our framework, we use five competing volatility measures: RV_t, $BSIV_t$, $MFIV_t$, VIX_t and $EBIV_t$ to forecast the realized volatility in the next period RV_{t+1}. To explore the predictive ability of the implied volatility measures, we first include each of them within an in-sample regression separately. We run the following regression

$$RV_{t+1} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+1},$$

with different predictors $x_{i,t} \in I = \{RV_t, BSIV_t, MFIV_t, EBIV_t, VIX_t\}$. The R^2 of these regressions captures the proportion of total variation in the ex-post realized volatility explained by the predictors.

We also employ encompassing regressions to investigate whether EBIV has additional predictive information compared with other volatility measures. To mitigate the multicollinearity problem, we first regress $EBIV_t$ on the other volatility measures and get the error term. Then, we run the bivariate regression of the volatility measures and the error term from the first step. The regressions are specified as follows:

$$\text{First Step: } EBIV_t = \alpha_{EBIV,i} + \beta_{EBIV,i} X_{i,t} + \epsilon_{EBIV,X_i,t}$$
$$\text{Second Step: } RV_{t+1} = \alpha_i + \beta_{1,i} X_{i,t} + \beta_{2,i} \epsilon_{EBIV,X_i,t} + \epsilon_t,\ i \neq 5,$$

where $X_{i,t}$ an element from I, which is different from $EBIV_t$.

Furthermore, we investigate whether other volatility measures have additional information content in predicting the future realized volatility, compared with $EBIV_t$. Similarly, we estimate the following two-step regressions:

$$\text{First Step: } X_{i,t} = \alpha_{i,EBIV} + \beta_{i,EBIV} EBIV_{i,t} + \epsilon_{X_i,EBIV,t}$$

$$\text{Second Step: } RV_{t+1} = \alpha_i + \beta_{1,i} EBIV_{i,t} + \beta_{2,i} \epsilon_{X_i,EBIV,t} + \epsilon_t,\ i \neq 5,$$

Table 3.10 summarizes the results of both univariate and the second-step encompassing regressions for the realized volatility in the next month. First, from the estimation results of univariate regressions in Panel A Table 3.10, we observe that the adjusted R^2 is the highest when using the EBIV compared to other implied volatility measures. Second, in the bivariate regressions with volatility measures and the uncorrelated EBIV residuals, the coefficients of the EBIV residual term are all statistically significant at 10% in Panel B. This indicates that the EBIV can explain some variations in the future realized volatility that other volatility measures cannot explain. Third, when we include EBIV in the predictive regression, none of the additional information in RV, BSIV, VIX and MFIV is statistically significant at 10%. In Panel C, the coefficients of EBIV are all significant at 1%, while the error terms of the other volatility measures regressing on EBIV are not significant. The results indicate that the EBIV plays a dominant role in explaining the variations of the future realized volatility. In summary, the evidence suggests that, among all the implied volatility measures, the EBIV explains the most variation in the next month realized volatility with the highest in-sample fit. It is also notable that even if the MFIV uses more options as inputs, its information content does not outweigh that of EBIV.

We then turn to the out-of-sample evidence reported Table 3.11. We use moving window of 100 observations preceding to the period to be forecasted as the estimation window in the regression. Consequently, the remaining 122 months are the forecasting period. We use the mean squared forecasting error (MSFE) as the overall measure of forecasting accuracy. We choose MSFE for two reasons. First, it is the most widely used loss function in the volatility forecasting literature. Second, Patton (2011) shows that it is one of the loss function that yields inference that is invariant to the choice of units of measurement. If we denote $\widehat{RV}_t$ as a forecast for RV_t, the MFSE is formally defined as,

$$MFSE = \frac{\sum_{t=101}^{222}(\widehat{RV}_t - RV_t)^2}{122}$$

To compare the out-of-sample performance of the competing implied volatility measures, we compute the Diebold and Mariano (1995) and West (1996) (DMW) statistic for testing the null of equal predictive ability) ($MFSE_j = MFSE_i$) against the alternative that the competing measure has a lower MSFE than the baseline measure ($MFSE_j > MFSE_i$). In Panel A Table 3.11, we report the full-sample MFSE ratio and DMW statistics in the parenthesis, The MFSE ratio is defined as follows:

$$MFSE(X_i, X_j) = \frac{MFSE(X_j)}{MFSE(X_i)}$$

where X_i represent the implied volatility measures on the first column and X_j represent those on the first row. Panel A shows that in the full sample, the rank of out-of-sample MFSE is RV, BSIV, VIX, MFIV and EBIV from the largest to the smallest. In terms of the significance of DMW statistics, we can see that the out-of-sample performance of EBIV is significantly better than VIX and MFIV at 5%. For other pairs of competing implied volatility measures, we do not observe any statistical significance.

Since the EBIV is particularly accurate in backing out the implied volatility during high volatility periods, we divide the monthly forecast of future volatility into three subsamples by sorting the BSIV preceding to the forecasted month in ascending order. Panel B, C and D in Table 3.11 report the results in these three subperiods, indicated by the “Low volatility period”, “Medium volatility period” and “high volatility period” columns. First, in the low volatility period, all implied volatility measures significantly outperform the lagged realized volatility measure (RV) at 5%. All the model free implied volatility measures outperform the BSIV at 10%. MFIV has the best performance among all competing measures. Second, in the medium volatility period, VIX has the best out-of-sample predictive power. Third, in the high volatility regime, the forecasting error of EBIV is smaller than the other four implied volatility measures, but EBIV only significantly outperforms VIX and MFIV.

We also examine whether the out-of-sample performance has significant improvement when we include additional variables in the predictive regression. We calculate two MSFE ratios: $MFSE(\{X\},\{X,\epsilon_{EBIV,X}\})$ and $MFSE(\{EBIV\},\{EBIV,\epsilon_{X,EBIV}\})$. The first one is used to investigate the out-of-sample performance after we add the uncorrelated error term of EBIV in the univariate predictive regression of the other implied volatility measures. The second one aims at analyzing whether there is incremental out-of-sample performance when we add the other implied volatility measures to the univariate regression of the uncorrelated EBIV error term. To assess the relative predictive power of two nested models, we use the MSFE-adjusted statistic suggested in Clark and West (2007). We summarize the MSFE ratio and the DMW statistics of the encompassing models in Table 3.12. The table shows that the out-of-sample performance of the univariate model for RV, BSIV and VIX improved significantly after we add the uncorrelated EBIV term in the model. However, adding the uncorrelated EBIV term decreases the out-of-sample performance of MFIV. The table also shows that the out-of-sample prediction performance of EBIV cannot be improved by including information in other implied volatility measures.

As a further robustness check, we provide in-sample and out-of-sample results based on the log-level of the realized volatilities in Table 3.13 and Table 3.14. In this log volatility specification, we find that the EBIV and MFIV have the highest in-sample adjusted R^2 among all the implied volatility measures. The additional information in EBIV has significant predictive power that RV

Table 3.10: Predict the realized volatility: volatility regressions

Panel A: Univariate regressions of the 5 volatility measures				
	α	β_1	β_2	adj.R^2
RV	0.050***	0.680***	-	0.486
	(3.681)	(7.544)	-	
BSIV	-0.024	0.917***	-	0.554
	-(1.428)	(9.982)	-	
VIX	-0.003	0.805***	-	0.554
	-(0.221)	(9.929)	-	
MFIV	-0.006	0.823***	-	0.559
	-(0.419)	(9.879)	-	
EBIV	0.005	0.787***	-	0.564
	(0.397)	(10.401)	-	

Panel B: Bivariate regressions (volatility measures and uncorrelated EBIV residuals)				
	α	β_1	β_2	adj.R^2
RV + $\epsilon_{EBIV,RV}$	0.050***	0.679***	0.655***	0.571
	(6.285)	(11.873)	(5.944)	
BSIV + $\epsilon_{EBIV,BSIV}$	-0.029***	0.926***	1.675*	0.570
	-(2.631)	(15.179)	(1.570)	
VIX + $\epsilon_{EBIV,VIX}$	-0.003	0.803***	1.424**	0.570
	-(0.292)	(16.099)	(2.332)	
MFIV + $\epsilon_{EBIV,MFIV}$	-0.006	0.822***	1.082*	0.567
	-(0.596)	(15.026)	(1.722)	

Panel C: Bivariate regressions (EBIV and uncorrelated residuals of other measures)				
	α	β_1	β_2	adj.R^2
EBIV + $\epsilon_{RV,EBIV}$	0.006	0.785***	0.136	0.571
	(0.641)	(14.952)	(1.124)	
EBIV + $\epsilon_{BSIV,EBIV}$	0.006	0.785***	-0.941	0.570
	(0.637)	(14.760)	-(0.804)	
EBIV + $\epsilon_{VIX,EBIV}$	0.006	0.785***	-0.662	0.570
	(0.643)	(15.850)	-(1.074)	
EBIV + $\epsilon_{MFIV,EBIV}$	0.006	0.785***	-0.313	0.567
	(0.629)	(14.791)	-(0.486)	

Note: This table reports the results for predicting future realized volatility using different measures of volatility. All regressions are based on monthly non-overlapping observations. The dependent variable is the realized volatility in the next month defined in equation (3.12). Robust t-statistics are reported in parentheses taking into account the heteroscedastic and autocorrelated error structure.

Table 3.11: Out-of-sample Diebold-Mariano-West test

Panel A: Full sample

	BSIV	VIX	MFIV	EBIV
RV	0.928	0.922	0.915	0.890
	(0.754)	(0.858)	(0.963)	(1.229)
BSIV		0.993	0.986	0.959
		(0.263)	(0.485)	(1.154)
VIX			0.993	0.966*
			(0.924)	(1.678)
MFIV				0.973*
				(1.779)

Panel B: Low volatility period

	BSIV	VIX	MFIV	EBIV
RV	0.342***	0.317***	0.316***	0.317***
	(5.284)	(5.589)	(5.676)	(5.845)
BSIV		0.926***	0.925**	0.927
		(2.996)	(2.187)	(1.573)
VIX			0.999	1.001
			(0.032)	(-0.040)
MFIV				1.002
				(-0.051)

Panel C: Medium volatility period

	BSIV	VIX	MFIV	EBIV
RV	0.986	0.956	0.961	0.959
	(0.163)	(0.539)	(0.505)	(0.503)
BSIV		0.970***	0.975	0.973**
		(2.269)	(1.486)	(1.919)
VIX			1.005	1.003
			(-0.456)	(-0.364)
MFIV				0.998
				(0.219)

Panel D: High volatility period

	BSIV	VIX	MFIV	EBIV
RV	0.975	0.986	0.971	0.928
	(0.165)	(0.093)	(0.196)	(0.474)
BSIV		1.011	0.996	0.952
		(-0.292)	(0.082)	(0.899)
VIX			0.985	0.942**
			(1.170)	(2.101)
MFIV				0.956**
				(2.006)

Note: This table reports the MFSE ratio ($\frac{MFSE_j}{MFSE_i}$) and DMW statistics in the parenthesis, where i represent the implied volatility measures on the first column and j represent those on the first row. Diebold and Mariano (1995) and West (1996) (DMW) statistics are computed to test the null of equal predictive ability) ($MFSE_j = MFSE_i$) against the alternative that the competing model has a lower MSFE than the baseline model ($MFSE_j > MFSE_i$).

Table 3.12: Diebold-Mariano-West test: nested models

	RV	BSIV	VIX	MFIV
$MFSE(\{X\},\{X,\epsilon_{EBIV,X}\})$	0.894***	0.983***	0.986***	1.052
	(3.956)	(2.231)	(2.061)	(-0.383)
$MFSE(\{EBIV\},\{EBIV,\epsilon_{X,EBIV}\})$	1.005	1.024	1.021	1.082
	(0.506)	(1.196)	(-0.382)	(-0.335)

Note: This table reports the MFSE ratio and the DMW statistics for nested models. The MFSE ratio is defined as $MFSE(X_i, X_j) = \frac{MFSE(X_j)}{MFSE(X_i)}$ and the DMW statistics is calculated after adjusting for nested models suggested in Clark and West (2007).

and BSIV do not have, but it is not significant in the regression of VIX and MFIV. When we regress the EBIV and the uncorrelated other implied volatility measures, none of these measures are significant. In addition, in Table 3.15, it is robust that the smallest out-of-sample MFSE is still obtained by using the EBIV in the full sample. However, it is not significantly better than MFIV in the full sample and in the high volatility period.

Finally, we conduct a robustness check by using a broader choice of strike prices, i.e. moneyness ranging from 0.5 to 1.5. The quantitative results remain valid[4]. An additional observation is that the adjusted R^2 when using the BSIV as a sole regressor becomes smaller when we incorporate more options. It shows that using the BS formula is not efficient for integrating information in a large number of option prices. By contrast, the adjusted R^2 of using the EBIV as the sole regressor increases in this case. Therefore, the ME method can better integrate the information contained in multiple option prices.

To sum up, EBIV has the smallest out-of-sample forecasting error among all the competing implied volatility measures. During the low, medium and high volatility period, the differences of the forecasting errors between EBIV and other implied volatility measures are statistically significant in terms of DMW statistics in some specifications. The extra information in EBIV significantly improves the out-of-sample performance of RV, BSIV and VIX, while the extra information in other implied volatility measures does not significantly improve the out-of-sample performance of EBIV.

3.4.4 Forecasting stock market returns

The theoretical model in Bollerslev et al. (2009a) suggest that variance risk premium (VRP) may serve as a predictor for future returns. The variance risk premium is defined by the difference between the ex ante risk-neutral

[4] Regression results and out-of-sample analysis are available upon request.

Table 3.13: Predict the realized volatility: log volatility regressions

Panel A: Univariate regressions of the 5 volatility measures				
	α	β_1	β_2	adj.R^2
RV	-0.617***	0.690***	-	0.461
	(-5.237)	(11.817)	-	
BSIV	-0.038	1.169***	-	0.587
	(-0.379)	(19.924)	-	
VIX	-0.293***	0.996***	-	0.589
	(-2.849)	(17.583)	-	
MFIV	-0.271***	1.008***	-	0.593
	(-2.652)	(17.860)	-	
EBIV	-0.305***	0.969***	-	0.593
	(-3.146)	(18.550)	-	

Panel B: Bivariate regressions (volatility measures and uncorrelated EBIV residuals)				
	α	β_1	β_2	adj.R^2
RV + $\epsilon_{EBIV,RV}$	-0.617***	0.690***	0.924***	0.596
	(-6.286)	(14.800)	(9.510)	
BSIV + $\epsilon_{EBIV,BSIV}$	-0.038	1.169***	1.131*	0.596
	(-0.344)	(18.404)	(1.744)	
VIX + $\epsilon_{EBIV,VIX}$	-0.293	0.996***	1.366	0.596
	(-3.106)	(19.116)	(1.544)	
MFIV + $\epsilon_{EBIV,MFIV}$	-0.271	1.008***	0.461	0.596
	(-2.708)	(18.262)	(0.345)	

Panel C: Bivariate regressions (EBIV and uncorrelated residuals of other measures)				
	α	β_1	β_2	adj.R^2
EBIV + $\epsilon_{RV,EBIV}$	-0.305***	0.969***	0.042	0.596
	(-3.209)	(18.874)	(0.471)	
EBIV + $\epsilon_{BSIV,EBIV}$	-0.305***	0.969***	-0.197	0.596
	(-3.085)	(18.212)	(-0.255)	
EBIV + $\epsilon_{VIX,EBIV}$	-0.305***	0.969***	-0.410	0.596
	(-3.220)	(18.975)	(-0.454)	
EBIV + $\epsilon_{MFIV,EBIV}$	-0.305***	0.969***	0.529	0.596
	(-3.078)	(18.133)	(0.382)	

Note: This table reports the results for predicting future log realized volatility using different measures of log volatility. All regressions are based on monthly non-overlapping observations. The dependent variable is the realized volatility in the next month defined in equation (3.12). Robust t-statistics are reported in parentheses taking into account the heteroscedastic and autocorrelated error structure.

Table 3.14: Out-of-sample Diebold-Mariano-West test: log volatility regressions

Panel A: Full sample	BSIV	VIX	MFIV	EBIV	Panel B: Low volatility period	BSIV	VIX	MFIV	EBIV
RV	0.772***	0.762***	0.760***	0.751***	RV	0.513***	0.491***	0.493***	0.485***
	(3.330)	(3.387)	(3.468)	(3.601)		(4.435)	(4.592)	(4.559)	(4.727)
BSIV		0.986	0.984	0.973	BSIV	0.000	0.957	0.962	0.945
		(0.639)	(0.657)	(1.060)			(1.605)	(1.005)	(1.401)
VIX			0.998	0.986	VIX			1.005	0.987
			(0.261)	(1.242)				-(0.216)	(0.528)
MFIV				0.988	MFIV				0.982
				(1.394)					(1.039)
Panel C: Medium volatility period	**BSIV**	**VIX**	**MFIV**	**EBIV**	**Panel D: High volatility period**	**BSIV**	**VIX**	**MFIV**	**EBIV**
RV	0.863	0.830*	0.833*	0.830*	RV	0.867	0.920	0.905	0.881
	(1.343)	(1.695)	(1.779)	(1.745)		(0.924)	(0.456)	(0.549)	(0.669)
BSIV		0.961*	0.964	0.962	BSIV		1.061	1.043	1.016
		(1.693)	(1.303)	(1.450)			(-1.058)	(-0.700)	(-0.224)
VIX			1.003	1.001	VIX			0.983	0.957*
			(-0.227)	(-0.056)				(1.237)	(1.771)
MFIV				0.997	MFIV				0.974
				(0.236)					(1.611)

Note: This table reports the MFSE ratio ($\frac{MFSE_j}{MFSE_i}$) and DMW statistics in the parenthesis, where i represent the implied volatility measures on the first column and j represent those on the first row. Diebold and Mariano (1995) and West (1996) (DMW) statistics are computed to test the null of equal predictive ability) ($MFSE_j = MFSE_i$) against the alternative that the competing model has a lower MSFE than the baseline model ($MFSE_j > MFSE_i$).

Table 3.15: Diebold-Mariano-West test: nested models

	RV	BSIV	VIX	MFIV
$MFSE(\{X\},\{X,\epsilon_{EBIV,X}\})$	0.894***	0.983**	0.986**	1.052
	(3.956)	(2.231)	(2.061)	(-0.383)
$MFSE(\{EBIV\},\{EBIV,\epsilon_{X,EBIV}\})$	1.005	1.024	1.021	1.082
	(0.506)	(1.196)	(-0.382)	(-0.335)

Note: This table reports the MFSE ratio and the DMW statistics for nested models. The MFSE ratio is defined as $MFSE(X_i, X_j) = \frac{MFSE(X_j)}{MFSE(X_i)}$ and the DMW statistics is calculated after adjusting for nested models suggested in Clark and West (2007).

expectation of the future return variation over the $[t, t+1]$ time interval and the ex post realized return variation over the $[t-1, t]$ time interval: $VRP_t = IV_t - RV_t$. Note that we implicitly assume that RV_t is a martingale process and RV_t measures the expection of realized variance $E_t[RV_{t+1}]$. The advantage is that this VRP measure does not depend on the specification of the forecast model for the future variance. Instead, it is completely model-free. In this chapter, we intend to compare the performance of VRP using different methods for backing out the implied variance. We use univariate regressions to examine the in-sample fit and out-of-sample forecasting performance.

Denote the ex-post return for month $t+1$ as R_{t+1}, the regressions take the form:

$$R_{t+1} = \alpha_i + \beta_i x_{i,t} + \epsilon_{i,t+1}, \tag{3.12}$$

where $x_{i,t}$ is one of the item in $I = \{VRP_{BS,t}, VRP_{MF,t}, VRP_{EB,t}\}$, $VRP_{BS,t} = BSIV_t^2 - RV_t^2$ and $VRP_{MF,t}$ and $VRP_{EB,t}$ are calculated based on $MFIV_t$ and $EBIV_t$ in a similar way.

Table 3.16 reports the results of predicting future monthly returns. In all regressions, the estimated slope coefficients associated with the VRP measures are significant at 5% confidence level. In addition, VRP_{EB}explains more variations in future monthly returns than VRP_{BS} and VRP_{MF} with the highest R^2 at 7.1%. The out-of-sample setup is similar to that in Section 3.4.3. The only difference is that the variable to be forecasted here is the monthly stock return R_t instead of the realized volatility RV_t. In the out-of-sample results, VRP_{EB} performs as good as VRP_{MF}. The VRP_{EB} and the VRP_{MF} both perform better than VRP_{BS} and they forecast more accurately in the Medium and High volatility regimes.

All our empirical results point to the direction that the EBIV performs at least comparable with the MFIV in different specifications. In many cases, its information content is of the highest among all available volatility measures,

Table 3.16: Predict the monthly returns using variance risk premium

	In-Sample Estimation			Out-of-sample MFSE			
	α	β_1	$adj.R^2$	All days	Low	Medium	High
VRP_{BS}	0.002	0.353***	0.051	2.421E-03	6.325E-04	2.236E-03	4.409E-03
	(0.427)	(3.916)					
VRP_{MF}	0.000	0.411***	0.065	2.385E-03	6.620E-04	2.243E-03	4.242E-03
	-(0.040)	(4.430)					
VRP_{EB}	0.000	0.425***	0.071	2.389E-03	6.589E-04	2.242E-03	4.259E-03
	(0.046)	(4.273)					

Note: This table reports the results for predicting future monthly return using different variance risk premia. VRP_{BS} is the variance risk premium calculated by the difference between $BSIV^2$ and realized variance in the last month RV^2, as defined in equation (3.12). VRP_{MF} and VRP_{EB} are variance risk premium calculated based on MFIV and EBIV in a similar way. The sample period extends from January 1996 to August 2014. In the panel In-sample Estimation, all regressions are based on monthly non-overlapping observations. The dependent variable is S&P500 index return in the next month. Robust t-statistics are reported in parentheses taking into account the heteroscedastic and autocorrelated error structure. In the panel Out-of-sample MFSE, the forecasting are conducted based on a moving window of 100 observations preceding to the period to be forecasted. Besides the results for "All Days" in the forecasting period, the whole sample is further split into three sub-samples by sorting the BSIV in the month preceding to the forecasting period in ascending order, which results in three regimes: "Low", "Medium" and "High". The RMSEs are then calculated within each sub-sample.

both in terms of in-sample fit and out-of-sample predictive power.

3.5 Conclusion

This chapter provides the first comprehensive investigation on the option implied volatility estimated by the ME method. The ME method extracts the risk neutral distribution of an asset, given a set of option prices at different strikes. The EBIV is then calculated based on the estimated risk neutral distribution. Compared to parametric methods such as the BS model, the ME method does not depend on any parametric assumption. Compared to the MFIV, proposed by Bakshi et al. (2003), the ME method does not require many options with strike prices spanning the full range of possible values for the underlying asset at expiry. Therefore, the ME method combines the advantages in the model-free and the parametric methods: on the one hand, it aggregates information in multiple options with different strikes efficiently; on the other hand, it produces accurate estimates even if the number of options is limited. Lastly, it allows for constructing confidence interval around the estimated implied volatility thanks to a nonparametric analog of the likelihood ratio test.

With numerical examples, we show that the EBIV has a lower estimation error than the BSIV and the MFIV, particularly when the underlying distribution exhibits heavy tail and non-zero skewness. With limited number of available options or under high volatility level, the accuracy of the EBIV remains robust while the estimation error of the MFIV gets higher. The confidence interval around the EBIV has a coverage ratio that is close to the correct confidence level across various numerical examples. These findings are robust to the choice of different number of options, maturities and data generating process.

We remark that the ME method also yields estimators for other higher moments of the risk neutral distribution, such as skewness and kurtosis. The estimators perform better than their counterparts when using the model-free method. A potential explanation is that the ME method provides an accurate estimation for the risk neutral distribution.

Using the S&P500 index options, we empirically test the information content of the EBIV in predicting future monthly realized volatilities and index returns. Our empirical results point to the direction that the EBIV performs at least at a comparable level with the MFIV in different specifications. In many cases, its information content is of the highest among all available volatility measures, both in terms of in-sample fit and out-of-sample predictive power.

A potential drawback of the ME method is that the tail region of the estimated risk neutral distribution largely depends on the options with the highest and lowest strike prices. Given limited number of available options, the estimated density can be less accurate for that part. This may be the reason why estimating implied skewness and kurtosis is less accurate than implied volatility. Improving the estimation of the tail region of the risk neutral density and the option implied skewness and kurtosis is left for future research. The main purpose of this chapter in my thesis is to model the systematic and idiosyncratic components of jumps in the individual stock return.

3.6 Appendix

3.6.1 Calculation of the Model-free implied moments

The calculation of the model-free option implied moments follows from Bakshi et al. (2003). Let the t-period continuous compounded return be given by: $R_t = \ln[S_t] - \ln[S_0]$. The fair values of the mean, volatility, cubic and quartic contract at time 0 are defined as:

$$M(0,t) = E[e^{-rt}R_t],\ V(0,t) = E[e^{-rt}R_t^2],\ W(0,t) = E[e^{-rt}R_t^3], \text{ and } X(0,t) = E[e^{-rt}R_t^4].$$

To simplify the notations, we ignore the time period information in the parenthesis in the following equations, for instance $V = V(0,t)$. Further, under the risk neutral measure, the values M, V, W and X can be replicated by the option prices as,

$$M = 1 - e^{-rt} - \frac{1}{2}V - \frac{1}{6}W - \frac{1}{24}X,$$
$$V = \int_S^\infty \frac{2(1-\ln[\frac{K}{S_0}])}{K^2}C(K,t)dK + \int_0^S \frac{2(1+\ln[\frac{S_0}{K}])}{K^2}P(K,t)dK,$$
$$W = \int_S^\infty \frac{6\ln[\frac{K}{S}] - 3(\ln[\frac{K}{S_0}])^2}{K^2}C(K,t)dK - \int_0^S \frac{6\ln[\frac{K}{S}] + 3(\ln[\frac{S_0}{K}])^2}{K^2}P(K,t)dK,$$
$$X = \int_S^\infty \frac{12(\ln[\frac{K}{S}])^2 - 4(\ln[\frac{K}{S_0}])^3}{K^2}C(K,t)dK - \int_0^S \frac{12(\ln[\frac{K}{S})^2] + 4(\ln[\frac{S_0}{K}])^3}{K^2}P(K,t)dK.$$

The t-period risk neutral return mean μ, volatility $MFIV$, skewness $MFIS$, and kurtosis $MFIK$ are given as

$$\mu = e^{rt}M,$$
$$MFIV = \sqrt{e^{rt}V - \mu^2}$$
$$MFIS = \frac{e^{rt}W - 3\mu e^{rt}V + 2\mu^3}{(e^{rt}V - \mu^2)^{3/2}},$$
$$MFIK = \frac{e^{rt}X - 4\mu e^{rt}W + 6e^{rt}\mu^2 V - 3\mu^4}{(e^{rt}V - \mu^2)^2}.$$

4 | Firm leverage and equity option returns

4.1 Introduction

The notion that options are not redundant assets has been widely accepted in financial economics (e.g. Buraschi and Jackwerth (2001) and Jones (2006)). In the past two decades, the equity option market in the United States has experienced exponential growth. The average daily trading volume (open interest) of equity options has increased from 0.79 (18.23) million in 1996 to 14.81 (263.57) million in 2015. In light of the tremendous growth in this market, understanding the determinants of the equity option returns becomes increasingly relevant.

Recent studies find that several factors are related to the equity option returns, e.g., the difference between historical realized volatility and at-the-money implied volatility (Goyal and Saretto (2009)), idiosyncratic volatility of the underlying stock (Cao and Han (2013)), option illiquidity (Christoffersen et al. (2014)) and volatility term structure (Vasquez (Forthcoming)). In addition to these market-based factors, how does the firm's structural variables suggested in the Merton-type capital structure model affect the equity option returns? Do they play an additional, or even a more fundamental role in explaining the cross-sectional variation of the equity option return? To answer these questions, this chapter aims to examine the relation between firm's structural characteristics and the expected equity option returns from the viewpoint of a capital structure model. Subsequently, we investigate the explanatory power of these characteristics using cross-sectional equity option data in the US market.

In this chapter, we consider the delta-hedged equity option portfolio, consisting of a long option position, dynamically delta-hedged by a short position in the stock, such that the portfolio is therefore not sensitive to the small movements in the underlying stock. The portfolio is not exposed to risks except for variance risk and jump risk. Bakshi and Kapadia (2003) show that the sign and magnitude of this portfolio return are closely related to the variance risk premium. While much of the existing knowledge about the variance risk premium is based on the index options, e.g. Bakshi and Kapadia (2003), Todorov (2010) and Bollerslev et al. (2009b), the variance risk premium of the individual stocks is less well understood. A natural question is, which firm characteristics are related to the variance risk premium of the individual stocks? Structural models following Merton (1974) imply that all contingent claims written on a single firm's asset or cash flow should be priced according to the same source of risk factor. Hence, the theoretical determinants that affect equity risk or credit risk of the firm, such as financial leverage and asset volatility, may also affect higher order risk premium of the stock, i.e. the variance risk premium.

Consider two firms with the same asset processes, but different leverage ratios. They are both exposed to the market volatility risk and/or market jump risk. The firm with higher leverage ratio is more exposed to the market volatility and market jump risk, and has a higher default probability than the firm with lower leverage ratio. If the price of volatility risk and jump risk is negative, as suggested Bakshi and Kapadia (2003) and Carr and Wu (2009b), the delta-hedged option returns should on average be negative and more negative for the firm with higher leverage ratio.

To formalize the idea, we derive the expected return of a delta-hedged option portfolio based on the capital structure model developed by Chen and Kou (2009). In this model, the dynamic of the asset value of a firm follows a double-exponential jump diffusion process. The firm's capital structure consists of equity and perpetual debt with constant coupon payments. When the asset value hits a certain threshold, the firm declares bankruptcy. Based on this framework, we find that the expected return of delta-hedged equity option portfolio relates to several firm-level structural variables: the variance of the jump component in the asset process, the leverage ratio of the firm and the level of bankruptcy trigger. The result implies, after dynamically hedging out the option exposure to the underlying stocks, the portfolio return is still driven by the determinants of the stock returns. The reason is that, due to the exposure to variance risk or jump risk, the effect of the firms' characteristics on the stock returns is inherited to the variance risk premium of the firm. Furthermore, simulations of the model show that the relation between the determinants and the portfolio returns is nonlinear. This implies that it is important to take into account the higher order polynomials of the determinants when we analyze the relation between the theoretical determinants and the expected return of the delta-hedged option portfolios.

There are two common sources of variance risk: the presence of stochastic volatility and the occurrence of unanticipated jumps. We use the jump diffusion-type of model instead of the stochastic volatility model for several reasons. First, using capital structure model with jump diffusion process has advantage in pricing contigent claims on the firm, because when a large drop of the firm value is possible, bankruptcy can happen by surprise and short-term credit spread can be generated at a more reasonable level than the traditional diffusion models. For this reason, Cremers et al. (2008), Collin-Dufresne et al. (2012) and Huang and Huang (2012) adopt the double-exponential jump diffusion process in a capital structure model to explain the pricing of term structure of credit spreads and CDO tranches of corporate debt. Second, a closed form for the equity value of the firm is available under the jump diffusion model when the jump size follows an exponential distribution. The explicit form of joint distribution of default time and default trigger is not possible under the stochastic volatility model, i.e. in Barsotti (2011) and McQuade (2013). We believe that it is important to have the analytic form of the equity value to further analyze the relation between leverage and return of options on equity. Third, the two types of risks explain the expected delta-hedged return with similar implications. In both models, there is one extra stochastic component that cannot be completely hedged away by delta-hedging. In this context we focus on modeling the jump risk to analyze the relation between firm's structural characteristics and the delta-hedged option returns.

We use the delta-hedged option returns as a proxy of the variance risk premium of the individual stocks, instead of a more direct measure: the difference between the realized variance and the implied variance for two reasons. First, due to liquidity reason, the available number of equity options for a firm is usually less than six in one day. Xiao and Zhou (2015) shows that the approximation error of the risk neutral variance can be huge when the number options is limited. The delta-hedged option portfolio requires only one option for each stock on the same day. Second, it is easier to implement trading strategies using delta-hedged equity option portfolios, rather than trading directly on the variance risk premium of the individual stocks.

To test the implications of the model, we examine the cross-section of equity option returns in the US market. We pick one call (or put) option on each optionable stock that has a maturity about one month and evaluate the return of the portfolio that buys one call (or put) and daily delta-hedges with the underlying stock. The empirical results are supportive of the model implications. First, the delta-neutral strategy that buys equity options and hedges with the underlying stock significantly underperforms zero. On average, the strategy loses about 1.97% of the starting value of the portfolio in one month. Second, after controlling for other firm characteristics such as firm size, the delta-hedged option return is decreasing with leverage ratio and asset volatility. The result of double sorting portfolios based on asset volatility and leverage ratio shows that the returns from quintile 1 to quintile 5 along both sorting criteria exhibit a monotonic trend, which is consistent with the theory. Third, using the short-term debt ratio as a proxy for the level of the bankruptcy trigger, we find that the delta-hedged option portfolios in firms with a higher short-term debt ratio exhibit significantly more negative returns than those of firms with mainly long-term debt financing. Fourth, we find evidence of the nonlinear relationship between the two determinants and the delta-hedged option returns. The coefficients of the higher order polynomials of the determinants are statistically significant in explaining the cross-sectional variation of delta-hedged option returns.

This chapter contributes to several strands of the literature. First, it adds to a growing literature that studies the cross section of delta-hedged equity option returns. Previous papers have identified several market-based factors that affect the delta-hedged return in the cross section of equity options[1]. Most results from this literature are motivated by volatility-related option mispricing and option liquidity. However, how the firm's structural characteristics affect the delta-hedged option return have not attracted sufficient attention. This chapter departs from these papers along several dimensions. First, our findings augment the literature by showing that the financing decision of the firm plays a sizable role in generating cross-section variations in delta-hedged equity option returns. To the best of our knowledge, this paper is the first one to identify theoretical determinants of delta-hedged equity

[1] See Goyal and Saretto (2009), Cao and Han (2013), Vasquez (Forthcoming), Christoffersen et al. (2014) and Cao et al. (2016).

option returns or the equity variance risk premium.

Second, compared with the empirical research in this field, this paper provides a framework to explain the proposed relation, such that the interaction of the different structural parameters and the delta-hedged gain can be understood in a structural model. Third, the results of the existing research generally cannot be explained by usual risk factor models, whereas the theoretical model and the empirical results in this paper are in general consistent within a risk-based framework. Our paper is also related to González-Urteaga and Rubio (Forthcoming), who find that the market volatility risk premium and the default premium are key determinants risk factors in the cross-sectional variation of average volatility risk premium. However, they consider a representative set of portfolios rather than the equity options on individual stocks. The focus of our paper is different from theirs.

Second, this paper contributes to the literature on the impact of leverage on the prices or returns of different assets. The notion that equity is a call option on the firm's asset goes back to Merton (1974). Following this philosophy, Geske (1979) models equity options as compound options on firm's asset, but the firm is not allowed to declare bankruptcy before the debt matures. Toft and Prucyk (1997) propose an equity option pricing model that allows for taxes and bankruptcy and show that firm's leverage and debt covenants affect option values and implied volatility skew. Ericsson et al. (2009) show empirical evidence that leverage and volatility are important determinants of credit default swap premia. More recently, Geske et al. (2016) study the impact of leverage on the pricing of equity options. However, the models in these papers only assume one dimension of randomness in the undelying asset. Hence, they cannot explain the negative delta-hedged option returns. The model in this chapter differs from this stream of literature in that there are two independent sources of randomness in the underlying asset process, such that a more realistic level of delta-hedged gain of the equity option portfolio can be generated.

The remainder of the chapter is organized as follows. Section 4.2 presents the capital structure model. It develops and interprets the pricing formulas for options on levered equity. Section 4.3 describes the data and the summary statistics. Section 4.4 presents empirical results of double sorting portfolios and cross-sectional multivariate regressions that control for various firm-specific variables including size, idiosyncratic volatility and liquidity. It also investigates time series properties of delta-hedged option returns. Section 4.6 concludes.

4.2 Pricing Options on leveraged equity with endogenous default and jump risk

As illustrated in Todorov (2010), there are two sources of market variance risk: the presence of stochastic volatility and the occurrence of unanticipated jumps. Hence, to generate a non-zero variance risk premium for the individual stocks in an equilibrium capital structural model, we need to introduce both type of risk in the pricing kernel and in the asset returns. In the literature, Merton (1974) provides the first structural model of the capital structure of the firm. The key assumptions in this chapter are that the asset of the firm follows a diffusion process and the firm can only default when the debt matures. If we make the same assumption of default, but allow the firm's asset to follows stochastic volatility or jump-diffusion process, then the equity of the firm can be considered as a European call option and the explicit form of equity is possible under both process.

By allowing the creditors to take over the firm when its value falls below a certain threshold, Black and Cox (1976) relax the restrictive assumption by Merton (1974) on default. They make the default time of the firm uncertain and equity becomes a down-and-out call option on the firm's asset. This more flexible and realistic setting has been adopted in many different applications. The downside is that when the asset process becomes more complex, the equity value of the firm is not always available in explicit form. For example, McQuade (2013) and Barsotti (2011) only provide a semi-analytic solution of the equity value when assets dynamic is described by stochastic volatility. When the asset process is characterized by exponential jumps, however, Zhou (2001), Hilberink and Rogers (2002), and Chen and Kou (2009) show that it is possible to obtain an explicit form of equity value with endogenous default. The advantage of the capital structure model using jump diffusion process is that when a large drop of the firm value is possible, bankruptcy can happen by surprise and short-term credit spread can be generated at a more reasonable level than the traditional diffusion models. For this reason, Cremers et al. (2008), Collin-Dufresne et al. (2012) and Huang and Huang (2012) adopt the double-exponential jump diffusion process in a capital structure model to explain the pricing of term structure of credit spreads, CDO tranches of corporate debt and long-dated S&P 500 index options.

Based on the evidence provided in the literature, we believe that the capital structure model with the double exponential jump-diffusion process not only provides us a framework to analyze the connection between firm's capital structure and non-zero delta-hedged equity option return, but it is also able to capiture the realistic pricing of different contigent claims of the firm.

This section discusses the details of the capital structure model with double-exponential jumps in the spirit of Chen and Kou (2009), Collin-Dufresne et al. (2012) and Huang and Huang (2012). In Section 4.2.1 and Section 4.2.2, we present the asset model, change of measure and the valuation of equity and debt of the firm. Then, the expected return of the delta-hedged equity option portfolio is derived in Section 4.2.3. The intuition of the propositions and numerical examples are provided in Section 4.2.4 and Section 4.2.5. The implications of the model are then used to motivate the empirical analysis.

4.2.1 Asset model

To derive the expected gain of the delta-hedged equity option portfolio, we need to use both the physical distribution and the risk neutral distribution of the asset value. In this chapter, we specify a general pricing kernel with jump diffusion process and idiosyncratic risks. The specification is in line with an incomplete market in which idiosyncratic risks cannot be completely hedged. The consumption c_t follows a jump-diffusion process:

$$\frac{dc_t}{c_t} = \mu^m dt + \sigma^m dW_t^m + d(\sum_{i=1}^{N_t^m}(J_i^m - 1)), \tag{4.1}$$

where $\{N_t^m, t \geq 0\}$ is a Poisson process with jump intensity λ^m and and $\{J_i^m\}$ is a sequence of independent identically distributed nonnegative random variables such that $Y = \ln(J_i^m)$ has a double-exponential density,

$$f_Y(y) = p_u^m \eta_u^m e^{-\eta_u^m y} \mathbb{1}_{y \geq 0} + p_d^m \eta_d^m e^{\eta_d^m y} \mathbb{1}_{y<0},\ \eta_u^m > 1, \eta_d^m > 1, p_u^m + p_d^m = 1. \tag{4.2}$$

To be more specific, Y has a mixed distribution:

$$Y = \begin{cases} x^+ & \text{with probability } p_u^m \\ -x^- & \text{with probability } p_d^m. \end{cases}$$

where x^+ and x^- are exponential random variables with means $\frac{1}{\eta_u^m}$ and $\frac{1}{\eta_d^m}$. The subscription m indicates the parameters that drive the aggregate consumption, which is considered as a proxy of the market factor.

Consider the utility function of the special form $U(c_t) = \frac{c_t^\alpha}{\alpha}$ if $0 < \alpha < 1$. A representative investor maximizes a utility function of the consumption process c_t. It has been shown in Kou (2002) that, when the consumption process follows the jump diffusion process in Equation (4.1), the equilibrium price of a derivative on this asset is given by the discounted expectation of the payoff under the risk neutral measure Q.

The Radon-Nikodym derivative for change-of-measure, $dQ/dP = Z_t/Z_0$, is a martingale under P,

$$Z_t = e^{rt} c_t^{\alpha-1} = exp(-\lambda^m \zeta^{(\alpha-1)} - \frac{1}{2}(\sigma^m)^2(\alpha-1)^2 + \sigma^m(\alpha-1)W_t^m)\prod_{i=1}^{N_t^m} J_i^m \quad (4.3)$$

where $\zeta^{(\alpha)}$ is given by

$$\zeta^{(\alpha)} = E[(J_i^m)^\alpha - 1] = E[e^{\alpha Y} - 1] = \frac{p_u^m \eta_u^m}{\eta_u^m - \alpha} + \frac{p_d^m \eta_d^m}{\eta_d^m + \alpha} - 1. \quad (4.4)$$

After specifying the pricing kernel, we consider a firm whose asset value V_t follows a double exponential jump-diffusion process under the physical measure,

$$\frac{dV_t}{V_t^-} = \mu dt + \sigma dW_t + d(\sum_{i=1}^{N_t}(J_i - 1)), \quad (4.5)$$

where $dW_t = \rho dW_t^m + \sqrt{1-\rho^2} dW_t^\epsilon$, $\rho \in [0,1)$, W_t^m and W_t^ϵ are independent standard Brownian processes. The number of jumps in the firm's asset value is equal to the number of systematic jumps: $N_t = N_t^m$, with jump intensities $\lambda = \lambda^m$. The jump size in the firm's asset process is driven by the systematic jumps: $J_i = J_{mi}^\beta$, where β is the sensitivity of jumps in the firm's asset process to the systematic jumps. It follows a double exponential Poisson distribution with probability $p_u = p_u^m$ to jump up and probability $p_d = p_d^m$ to jump down. The means of the positive and negative jump sizes are $\frac{1}{\eta_u} = \frac{\beta}{\eta_u^m}$ and $\frac{1}{\eta_d} = \frac{\beta}{\eta_d^m}$ respectively. Note that in this model, the idiosyncratic jump and diffusion risks are not priced.

Using the Radon-Nikodym derivative in Equation (4.3) and the Girsanov theorem with jump diffusion process, we can transform the asset process from the physical measure P in Equation (4.5) to the risk neutral measure Q:

$$\frac{dV_t}{V_t^-} = (r - \lambda^Q(E^Q(J_i - 1)))dt + \sigma dW^Q + d(\sum_{i=1}^{N_t^Q}((J_i^Q) - 1)), \quad (4.6)$$

where W_t^Q is a new Brownian motion under Q, $W_t^Q = W_t - \rho\sigma_m(\alpha-1)t$. N_{mt}^Q is a new Poisson process with jump intensity $\lambda^Q = \lambda + \lambda_m \xi^{(\alpha-1)}$. The jump size J_i^Q is specified as $J_i^Q = (J_{mi}^Q)^\beta$, where J_{mi}^Q are independent identically distributed random variables with a new density under Q:

$$f_{J_{mi}}^Q(x) = \frac{1}{1+\xi^{(\alpha-1)}} x^{\alpha-1} f_{J_{mi}}(x). \tag{4.7}$$

Under the risk neutral measure, the J_i^Q follows a new double exponential Poisson process with parameters p_u^Q, p_d^Q, η_u^Q and η_d^Q:

$$\eta_u^Q = \eta_u - \alpha + 1, \ \eta_d^Q = \eta_d + \alpha - 1,$$
$$p_u^Q = \frac{p_u\eta_u}{(\xi^{(\alpha-1)}+1)(\eta_u-\alpha+1)}, \ p_d^Q = \frac{p_d\eta_d}{(\xi^{(\alpha-1)}+1)(\eta_d+\alpha-1)}.$$

The firm's asset return process in Equation (4.5) and(4.6) are in fashion of CAPM model. The diffusive component of the asset return is correlated with the market's diffusive component by ρ. Different from the standard continous version of the CAPM model, we allow the market to exhibit jumps and the individual firm asset to be triggered by this systematic jump. The loading of the firm's jump size on the systematic jump size is denoted by β. Note that the systematic jump is the key feature that this model can generate sizable delta-hedged equity option returns. Except for allowing systematic jumps, Collin-Dufresne et al. (2012) also allow idiosyncratic jumps in the asset returns to match the short-term crefit spread. However, in the standard equilibrium models, the idiosyncratic risk can be completely diversified away. Hence, not priced in the risk premiums. For that reason, in the context of explaining the delta-hedged option return (variance risk premium) of the individual firms, we do not consider idiosyncratic jump risks in the asset returns.

4.2.2 Debt, equity and market value of the firm

The firm pays a nonnegative coupon, c, per instant of time when the firm is solvent. Let V_B denote the level of asset value at which bankruptcy is declared.

The bankruptcy occurs at time $\tau = inf\{t \geq 0 : V_t \leq V_B\}$. Upon default, the firm loses $1 - \alpha_d$ of V_τ, leaving debt holders with value $\alpha_d V_\tau$ and stockholders with nothing. Note that V_τ may not be equal to V_B due to jumps. The debt value, equity value and the level of bankruptcy trigger are given in Appendix 4.7.1.

4.2.3 Delta-hedged returns of options on the levered equity

In this subsection, we turn to the valuation of options written on the levered equity and the derivation of delta-hedged option returns. The value of an European option wittern on equity $S(V;V_B)$ at time 0 maturing at t, with strike price K can be expressed as:

$$O(0,t;K) = e^{-rt} E^Q[\text{Payoff} \times 1_{\tau \geq t}],$$

where τ is the stopping time when asset value of the firm hits the bankruptcy trigger the first time and Q represents the risk neutral measure. The payoff for the call options at maturity is $\max(S_t(V_t;V_B) - K, 0)$ and $\max(K - S_t(V_t;V_B), 0)$ for the put options.

To remove the impact of the underlying stock movement on the option returns, we consider the return on a portfolio of a long position in an option, hedged by a short position in the underlying stock, such that the portfolio is not sensitive to the movement of the underlying stock prices. The delta-hedged gain, $\Pi_{0,t}$, is defined as the gain or loss on a delta-hedged option position, subtract the risk free rate earned by the portfolio:

$$\Pi_{0,t} = O_t - O_0 - \int_0^t \Delta_u dS_u - \int_0^t r(O_u - \Delta_u S_u) du, \tag{4.8}$$

where $\Delta_t = \frac{\partial O_t}{\partial S_t}$, r is the constant risk free rate. By Ito's lemma, under the physical distribution, the option price can be written as,

$$O_t = O_0 + \int_0^t \frac{\partial O}{\partial u} du + \int_0^t \frac{\partial O_u}{\partial S_u} dS_u^c + \frac{1}{2} \int_0^t \frac{\partial^2 O_u}{\partial S_u^2} dS_u^c dS_u^c + \sum_{0<u<t} (O(S_u) - O(S_{u-})). \tag{4.9}$$

where dS^c_u is the continuous part of dS_u. The last part in (4.9) sums up the movement of the option price due to the discontinuous jumps from time 0 to t. Therefore,$O(S_u)$ is the option price evaluated at S_u which is the stock price immediately after a jump and $O(S_{u-})$ is the option price evaluated just before the jump.

The equity value is a function of the asset value given by (4.22) in the Appendix. The stochastic process of the equity value can also be obtained by Ito's lemma:

$$dS = \frac{\partial S}{\partial V} dV^c + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}\sigma^2 V^2 dt + d\sum_{i=1}^{N_t}(S(V) - S(V_-)), \tag{4.10}$$

where $S(V) - S(V_-) = (VJ_i + a(VJ_i)^{-\gamma_1} + b(VJ_i)^{-\gamma_2}) - (V + aV^{-\gamma_1} + bV^{-\gamma_2})$. The subscripts of S and V are ignored for simplicity.

Under the risk neutral measure Q, the process of the equity value can be rewritten as,

$$dS^Q = \mu_S^Q dt + \sigma_S^Q dW_t^Q + d\sum_{i=1}^{N_t^Q}(S(V) - S(V_-)). \tag{4.11}$$

On the one hand, we can obtain μ_S^Q and σ_S^Q by substituting the risk neutral process of V_t.[2] On the other hand, since equity value is a convex function of the asset value, the discounted equity price process should be a martingale under the risk neutral measure. Hence, $\mu_S^Q = rS - \lambda^Q E^Q[S(V) - S(V-)]$. The discounted option price process $e^{-rt}O_t$ is also a martingale under Q, the integro-partial differential equation of the option price O_t is:

$$rO_t = \frac{\partial O}{\partial t} + \frac{\partial O}{\partial S}\mu_S^Q + \frac{1}{2}\frac{\partial^2 O}{\partial S^2}(\sigma_S^Q)^2 + \lambda^Q E^Q[O(S) - O(S_-)]. \tag{4.12}$$

Combining Equations (4.9) and (4.12), the option price can be rewritten as,

$$O_t = O_0 + \int_0^t \frac{\partial O}{\partial S} dS^c + \int_0^t (rO - \frac{\partial O}{\partial S}\mu_S^Q - \lambda^Q E^Q[O(S) - O(S_-)])dt$$
$$+ \sum_{0<u<t}(O(S_u) - O(S_{u-})). \tag{4.13}$$

[2] $\mu_S^Q = (r - \lambda^Q(E^Q(J_i - 1)))\frac{\partial S}{\partial V}V_t + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}\sigma^2 V_t^2$, $\sigma_S^Q = \sigma V_t \frac{\partial S}{\partial V}$.

Therefore, the expected delta-hedged gain is equal to:

$$
\begin{aligned}
E(\Pi_t) &= E(O_t - O_0 - \int_0^t \frac{\partial O}{\partial S} dS_u - \int_0^t r(O - \frac{\partial O}{\partial S} S du)) \\
&= \int_0^t \{-\lambda^Q E^Q[O(S) - O(S_-)] + \lambda^Q E^Q[(S(V) - S(V_-))\frac{\partial O}{\partial S}] \\
&- \lambda E[(S(V) - S(V_-))\frac{\partial O}{\partial S}] + \lambda E[O(S) - O(S_-)]\} dt.
\end{aligned} \tag{4.14}
$$

The following proposition shows the relation between delta-hedged gains, the jump risk premium, and the option gamma. The proof of Proposition 1 is provided in Appendix 4.7.2 and Appendix 4.7.3.

Proposition 4.1. *Let the firm's asset price process under the physical and risk neutral measures follows the dynamics given in Equations (4.5) and (4.6), and the equity value of the firm is given by Equation (4.22) in the Appendix. Then, the expected delta-hedged gain can be expressed as,*

$$
E(\Pi_t) \approx \int_0^t \frac{1}{2} \frac{\partial^2 O_u}{\partial S_u^2} (\frac{\partial S_u}{\partial V_u} \frac{V_u}{S_u})^2 (\lambda E[J-1]^2 - \lambda^Q E^Q[J-1]^2) S_{u-}^2 du. \tag{4.15}
$$

where $\frac{\partial^2 O}{\partial S^2}$ *represents the gamma of the option,* $E[.]$ *is the expectation operator under the physical measure, and* $E^Q[.]$ *is the expectation operator under the risk neutral measure.*

The expected delta-hedged gain $E(\Pi_t)$ *is negative, if and only if* $\frac{p_u}{\eta_u^3} - \frac{p_d}{\eta_d^3} < 0$. *Furthermore, if* $E(\Pi_t)$ *is negative,*

(1) it is decreasing with the firm's leverage ratio $\frac{c}{rV}$ *and the variance of the jump component in the firm's asset process:* $\lambda E[J^2]$.

(2) it is more negative for firms with exogenous (higher) bankruptcy trigger than those with endogenous (lower) bankruptcy trigger.

One sufficient but not necessary condition for negative expected delta-hedged gain is that the absolute value of the negative jump size is larger than the positive jump size on average, $1/\eta_d > 1/\eta_u$, and the expected jump size is less than zero: $E[Y] = \frac{p_u}{\eta_u} - \frac{p_d}{\eta_d} < 0$. It means that, with continuous trading, the expected delta-hedged gain is negative provided that the jump size is on average negative, and there are occasionally price discontinuities ($\lambda > 0$). The results hold for both call options and put options. The intuition is explained in more detail in Section 4.2.4. The expected gain is positively correlated with option gamma, the second order derivarive of the option price over the underlying stock price. For the options with the same underlying asset, this suggests that the expected delta-hedged gain is the most negative for at-the-money options.

4.2.4 Implications of the Propositions

Note that the call option price is a strictly convex function of the underlying stock price. Consider an at-the-money call option at t_0, the option price is C_0. The underlying stock price and the strike price are both 100. If the stock price suffers a negative jump at t_1 from 100 to 92, then the option price drops from C_0 to C_1. However, the positive gain of the delta-hedge position $-\frac{\partial C_0}{\partial S_0}(S_1 - S_0)$ exceeds the loss in the option value, because option price is a convex function of the underlying price. Similarly, after positive jumps, the gain of the delta-hedged call option is positive. If the stock price change is small enough, and the stock return can be approximated as a diffusion process, then the gain of delta-hedged option position should be zero on average. However, if we assume that discontinuous jumps occur sometimes in the stock return, then the movement of stock return cannot be hedged out completely. The second order derivative of the option price over the underlying stock price leaves us with the gamma risk.

The reasoning can also be applied to the put options. Therefore, the delta-hedged gains for call and put option are both positive after unexpected jumps. If the negative jumps in the stock prices are more frequent than the positive ones, and if the average absolute size of negative jumps is larger than the positive ones, the gain of delta-hedged option position is then negatively related to the underlying stock return. Usually, the stock returns comove with the market return in the same direction, so the expected gain of the delta-hedged option position is negative, as the investors pay a premium to hedge against the undesired jump risk.

Note that in the equilibrium, only the systematic risks are priced in the expected return. Hence, based on our setting for jumps in asset process of individual firms in (4.5), the necessary condition for negative delta-hedged portfolio gain in our model is that the systematic jump is on average negative, there are more negative systematic jumps and the firm's exposure to the systematic jumps are positive. These conditions are in general consistent with findings in Todorov (2010) and Bollerslev and Todorov (2011).

Furthermore, the delta-hedged option gain is related to the variance risk premium (VRP, defined as the difference between variance of the stock return under the physical measure and that under the risk neutral measure) (Proof in Appendix 4.7.2):

$$E(\Pi_t) \approx \frac{1}{2}\frac{\partial^2 O}{\partial S^2} \times VRP_{0,t} \times S_0^2. \tag{4.16}$$

There are two implications that follow from (4.16). First, as the option gamma is positive, a negative (positive) variance risk premium implies that $E(\Pi_t)$ will be negative (positive). A negative variance risk premium is consistent with the notion that volatility rises with the negative jumps. Bollerslev and Todorov (2011) show that realized variance is priced due to its correlation with large negative jumps. The recent evidence in the variance swap market documented by Dew-Becker et al. (2015) further confirms the finding. The model framework in this chapter is consistent with the recent findings. Second, as the option gamma is the largest for at-the-money options, the absolute value of $E(\Pi_t)$ is also the largest for the same underlying stock. However, after scaling the value of $E(\Pi_t)$ by the initial investment $O - \frac{\partial O}{\partial S} S_0$, the absolute return may not be the highest for at-the-money options. This may help to explain the empirical results in Section 4.4.

Proposition 4.1 states that even after controlling the impact of the underlying stock movement by dynamic delta-hedging, the expected scaled option return is still related to the structural characteristics of the underlying firm. From the derivation in Appendix 4.7.3, we know that the scaled return is linked to the firm's characteristics through the variance risk premium. For firms with the same asset processes, the variance risk premium is determined by the sensitivity of the equity value with respect to the asset value, $\beta = \frac{\partial S}{\partial V} \frac{V}{S}$. In the model, the equity beta β is expressed as:

$$\beta = \frac{\partial S}{\partial V} \frac{V}{S} = 1 + \frac{(1-\kappa)c}{rS} - \frac{(\gamma_1 + 1)S^{D_1}}{S} - \frac{(\gamma_2 + 1)S^{D_2}}{S}, \tag{4.17}$$

Here, following Gomes and Schmid (2010), we use $S^{D_1} = aV^{-\gamma_1}$ and $S^{D_2} = bV^{-\gamma_2}$ to denote the value of the default option. The second term comes from the discontinuous jump term of the asset process. These two terms capture the effect of leverage on returns. If the bankruptcy trigger is exogenously determined by the strict-worth covenant, a is less than zero and decreases in the coupon value c. In this case, the equity risk is apparently increasing in the leverage ratio. Furthermore, the default option increases the equity risk. If the bankruptcy risk is endogenously determined by the equity holders, it is possible that a is larger than zero and increases in the coupon value. The default option lowers down the equity risk because the trigger is given by maximizing the equity value. However, the effect is not large enough to compensate the effect on risk of levering up equity cash flow. Overall, the risk of equity is increasing in firm's leverage ratio.

4.2.5 Simulations and implications of the model

In this subsection, we provide the numerical results of this model and examine the relation between the structural characteristic of the firm and the delta-hedged option returns. The details of the simulation procedure are illustrated in Appendix 4.7.4. Table 4.1 presents the parameter sets used in the simulations.

Table 4.1: Parameter sets used in the simulations

Panel A: Common Parameters

Parameters	σ	κ	r	α	V_0	ρ	a	σ_1
Value	0.25	0.35	0.04	0.9	100	0.5	0.2	0.2

Panel B: Parameters in the jump component

Parameters	p_u	p_d	η_u	η_d	λ
Value	0.4	0.6	8	4	0/ 0.5/ 1

Note: This table presents parameters sets for simulating the delta-hedged option returns. Panel A shows value of the common parameters. σ is the asset volatility of the firm, κ is the tax rate, r is the risk free rate, α is the percentage of the asset value that the debt holders can get upon bankruptcy, V_0 is the initial asset value of the firm, ρ is the correlation between diffusion terms in the asset process and in the consumption process, a is the risk aversion coefficient in the representative investor's utility function (power function). σ_1 is the volatility of the consumption process. Panel B shows value of parameters in the jump component of the firm's asset process. p_u is the probability that the asset return has a positive jump, p_d is the probability that the asset return has a negative jump, $1/\eta_u$ is the absolute mean of the upward jump size, and $1/\eta_d$ is the absolute mean of the downward jump size.

Panel 4.1a shows the value of the common parameters. We use asset volatility $\sigma = 0.25$, the median asset volatility of the US firms reported in Choi and Richardson (2015) and Correia et al. (2014). The a is the risk aversion coefficient in the representative investor's utility function (power function). The value of a is obtained from Bliss and Panigirtzoglou (2004), who estimate the risk aversion coefficient of the power function from S&P 500 index options. The risk-free rate is assumed to be 4%, and the initial asset value is assumed to be 100. Panel B shows the value of parameters in the jump component of the firm's asset process. p_u is the probability that the asset return has a positive jump, p_d is the probability that the asset return has a negative jump, $1/\eta_u$ is the absolute mean of the upward jump size, and $1/\eta_d$ is the absolute mean of the downward jump size.

In Figure 4.1a, we assume that the stocks have negative jumps on average ($p_u = 0.4, \eta_u = 8, \eta_d = 4$). The horizontal axis gives the level of the book leverage ratio c/rV. we compute the delta-hedged gains after one month scaled by the square of the initial stock price, for different book leverage ratio and for different jump intensities ($\lambda = 0$, 0.5, and 1). The vertical axis shows the delta-hedged gains scaled by the square of the initial stock price. In Figure 4.1b, we show the delta-hedged gains scaled by the initial investment. As these two figures show similar patterns, we scaled the delta-hedged gains by the initial investment to understand better about the portfolio return.

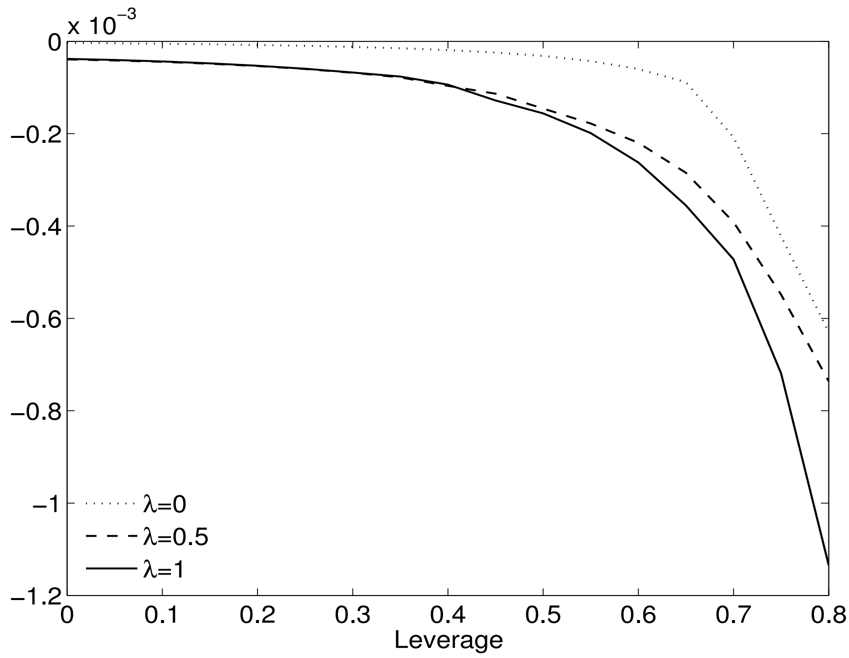
Figures 4.1a and 4.1b show the nonlinear relationship between leverage and scaled delta-hedged gain. The nonlinear relation is similar as the relation between leverage and stock return in Doshi et al. (2016). In addition, the relationship between these two variables becomes highly convex at high levels of leverage. Hence, it is important to take this non-linearity into account when examining the effect of leverage on scaled delta-hedged option gain. In the empirical part of this chapter, we will consider these patterns of non-linearity and show that they play crucial roles when we examine the determinants of delta-hedged option return.

Based on the propositions and simulations, we form three hypotheses below and test these in Section 4.4.

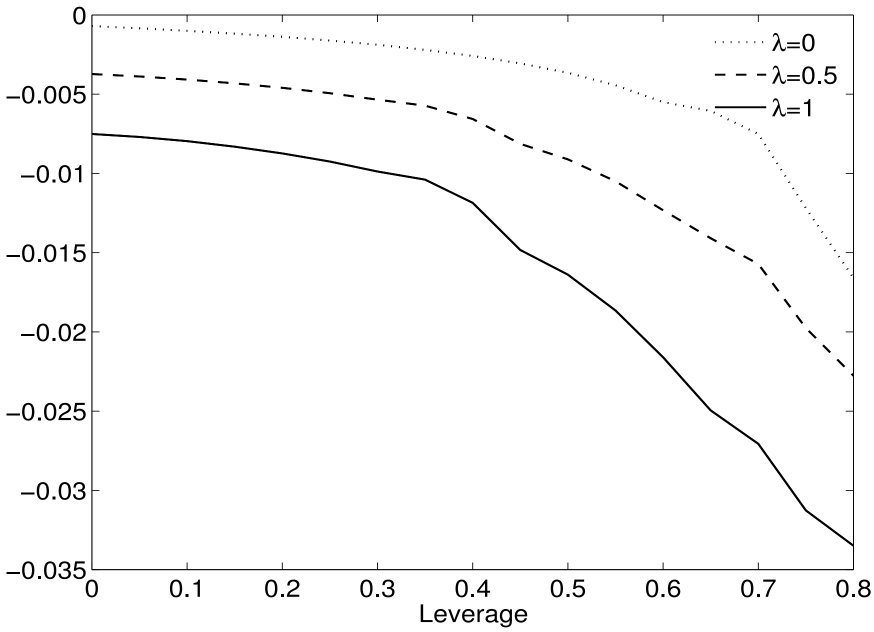
Hypothesis 1: The delta-hedged equity option return is negatively related with firm's leverage ratio and asset volatility, ceteris paribus.

Figure 4.1: Leverage and scaled delta-hedged option gain

(a) Scaled by square of stock price



(b) Scaled by the initial investment



Note: This figure shows the relation between leverage and scaled delta-hedged gain generated by the model in Section 4.2.3, when the jump size is negative on average. The top panel presents scaled delta-hedged gain generated from the model for different leverage and jump intensity

Hypothesis 2: The delta-hedged equity option return is more negative for options on equity in a firm where debt is protected by net-worth covenants than for options on equity in a firm with debt withour these covenants, ceteris paribus.

Hypothesis 3: Higher order polynomials of the leverage and asset volatility significantly affect the delta-hedged option returns.

4.3 Data

4.3.1 Option data and delta-hedged option return

The data on equity options are from the OptionMetrics Ivy DB database. The dataset contains information on the entire U.S. equity option market except for the financial firms, from January 1996 to August 2014. The data fields include daily closing bid and ask quotes, trading volume, open interest, implied volatility and the option's greeks computed by OptionMetrics based on standard market conventions. The IVs and greeks are calculated using a binomial tree model using Cox et al. (1979). Continuously-compounded zero-coupon interest rates are also obtained from OptionMetrics as a proxy for the risk-free rate.

Several filters are applied to select the options. First, to mitigate the problem of early exercise feature of American options, we select short-maturity options with expiration from 25 days to 35 days. Only at-the-money and out-of-the-money options are included. In addition, the options are included only if the underlying stock does not pay dividends during the remaining life of the option. Second, prices that violate arbitrage bounds are eliminated. Third, all observations are eliminated if the following conditions apply: (i) the ask is lower than or equal to the bid, (ii) the bid is equal to zero, (iii) the spread is lower than the minimum tick size (equal to 0.05 for option trading below 3 and 0.10 in any other cases), (iv) there is no volume or open interest.

After the filtering procedure, we select options under four categories: at-the-money (ATM) call, out-of-the-money (OTM) call, ATM put and OTM put. For each month, we select one option for each firm under each category, with moneyness closest to a specified value and maturity closest to 30 days. The specified value is 1 for ATM call and put, 0.95 for OTM call, and 1.05 for OTM put.

The delta-hedged option portfolio is constructed by holding a long position in an option, hedged by a short position in the underlying stock, such that the exposure of the option to the movement of the underlying stock is removed as much as possible. The definition of delta-hedged option gain follows Bakshi and Kapadia (2003). Let $C_{t,t+\tau}$ represents the price of an European call option at time t maturing at $t+\tau$ with strike price K. Denote the corresponding option delta by $\Delta_{t,t+\tau}$, and $\Delta_{t,t+\tau} = \frac{\partial C_{t,t+\tau}}{\partial S_t}$. The delta-hedged gain $\Pi_{t,t+\tau}$ is the gain or loss on a delta-hedged option position, deducting the risk-free rate earned by the net investment. In continuous time, delta-hedged call option gain is,

$$\Pi_{t,t+\tau} = C_{t+\tau} - C_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r_u (C_u - \Delta_u S_u) du.$$

where r_u is the annualized risk-free rate at time u. Consider a portfolio of a call option that is hedged discretely N times over the period $[t, t+\tau]$, where the hedged is rebalanced at each dates t_n, $n = 0, 1, ...N-1$. The discrete delta-hedged call option gain up to the maturity date $t+\tau$:

$$\begin{aligned} \Pi_{t,t+\tau} = {} & \max(S_{t+\tau} - K, 0) - C_t \\ & - \sum_{n=0}^{N-1} \Delta_{t_n} [S_{t_{n+1}} - S_{t_n}] - \sum_{n=0}^{N-1} r_n (C_t - \Delta_{t_n} S_{t_n}) \frac{\tau}{N}. \end{aligned} \quad (4.18)$$

The definition for delta-hedged put option gains is similar as in Equation (4.18), except that the option price and delta are for the put options and the payoff of the put options is $\max(K - S_{t+\tau}, 0)$. To make the delta-hedged gains comparable across stocks, we scale the delta-hedged call option gain $\Pi_{t,t+\tau}$ by $\Delta_t S_t - C_t$ and by $P_t - \Delta_t S_t$ for the put options. In section 4.4, we refer to the scaled delta-hedged option gain $\Pi_{t,t+\tau}/(\Delta_t S_t - C_t)$ as the delta-hedged call option return.

From Proposition 4.1, we know that one determinant of the delta-hedged option return is the volatility of the jump component in the asset process. However, it is difficult to disentangle the volatility of jump component and diffusion component of the asset process using empirical data. Hence, we use asset volatility as a proxy. Following Correia et al. (2014), we define the asset volatility as implied volatility of the equity option $\times$ (1-leverage ratio).

Christoffersen et al. (2014) document the illiquidity premia in the equity option market. To control the effect of liquidity, we define the option illiquidity measure as the relative bid-ask spread:

$$IL^o = \frac{2(O_{bid} - O_{ask})}{O_{bid} + O_{ask}}$$

where O_{bid} is the highest closing bid price and O_{ask} is the lowest closing ask price.

4.3.2 Stock and balance sheet data

Stock prices and the realized volatility are retrieved from the OptionMetrics database. The realized volatility is calculated over the past 30 calendar days, using a simple standard deviation calculation on the logarithm of the close-to-close daily total return. The idiosyncratic volatility is defined as the standard deviation of the error term of the Fama-French three-factor model estimated using the daily stock returns over the previous month. The definition follows Ang et al. (2006b) and Cao and Han (2013).

The balance sheet data are obtained from Compustat database. Fama and French (1992) suggests that size is a potential risk factor, and it is reasonable to control size in the cross section of option returns. Firm size is defined as the natural logarithm of the firm's asset value on the balance sheet. The book leverage ratio is calculated as the sum of total debt (data item: LTQ) and the par value of the preferred stock (data item: PSTKQ), minus deferred taxes and investment tax credit (data item: TXDITCQ), divided by total asset (data item: ATQ). The financial firms are excluded for that their financing decisions cannot be explained by the conventional capital structure models.

Following Toft and Prucyk (1997), we use the maturity structure of the firm's debt as a proxy for the existence of net-worth hurdles, more specifically, the ratio of long-term debt due in one year plus notes payable to total debt. Leland (1994) argues that short-term debt can be associated with an exogenous bankruptcy trigger that equals the market value of debt. Long-term debt results in an endogenous bankruptcy point which is below its exogenous counterpart. Intuitively, this indicates that firms with a large proportion of debt due in the immediate future must pass a net-worth hurdle. Otherwise, they are unable to roll over their debt. Firms primarily financed by long-term debt need not overcome such a strict net-worth hurdle.

4.3.3 Summary statistics

After calculating the delta-hedged option returns, we merge the equity options data with their underlying stock information and the balance sheet data. The final data sample have 221,743 observations for ATM calls, 201,474 for OTM calls, 183,893 for ATM puts and 170,716 for OTM puts. Table 4.2 shows that the means of the delta-hedged options for call and put options are both -1.97% with a standard deviation 0.09. The average moneyness of the chosen options is 0.98 with a standard deviation of 0.03. The days to maturity ranges from 26 to 33 days across different months, with an average of 31 days. The detailed information for the delta-hedged option returns under the four categories are presented in Table 4.3.

4.4 Cross sectional analysis of delta-hedged option return

This section presents results of Fama-French regression results, tests several potential explanation of the results and reports some robustness checks.

4.4.1 Average delta-hedged option return

Table 4.3 presents time series average of delta-hedged option returns for individual stocks. It shows that the average delta-hedged return for ATM (OTM) call option is -1.72% (-2.25%) and -1.76% (-2.20%) for ATM (OTM) put options. Table 4.3 also reports the results of t-test for the time series mean of firms' delta-hedged option returns. There are 5809 firms in the ATM call option category. About 92% of them have negative average delta-hedged returns and 13% of them have significantly negative delta-hedged returns. In contrast, only 5 out of the 5809 firms have significantly positive delta-hedged returns. Results for the other three categories shows similar patterns.

4.4.2 Delta-hedged option returns, size and leverage

We study the relation between book leverage and delta-hedged option returns using monthly Fama-Macbeth regressions. For Table 4.4 to 4.6, the dependent variable for month t is the scaled return of delta-hedged ATM call option held until maturity, where the maturity of the options is about one month. All independent variables in the regression are predetermined at time t. The key variable of interest is the book leverage of the underlying firm. Table 6 provides robustness checks and results for the put options.

Table 4.2: Summary statistics of option data

Variable	Mean	Std. Dev.	10th Pctl	25th Pctl	Median	75th Pctl	90th Pctl
Panel A: Call options							
Delta-hedged return until maturity(%)	-1.97	9.28	-9.31	-5.18	-2.03	0.52	4.40
Moneyness=S/K	0.98	0.03	0.94	0.96	0.98	1.00	1.01
Days to maturity	30.96	2.47	26	30	32	33	33
Relative bid-ask spread	0.21	0.20	0.05	0.09	0.15	0.26	0.43
Implied volatility (IV)	0.47	0.24	0.23	0.30	0.42	0.58	0.79
Delta	0.46	0.11	0.30	0.38	0.47	0.54	0.59
Panel B: Put options							
Delta-hedged return until maturity(%)	-1.97	7.91	-8.28	-4.70	-2.05	0.04	3.29
Moneyness=S/K	1.02	0.03	0.99	1.00	1.02	1.05	1.06
Days to maturity	30.89	2.49	26	30	32	33	33
Relative bid-ask spread	0.19	0.18	0.05	0.08	0.14	0.24	0.40
Implied volatility (IV)	0.49	0.25	0.24	0.31	0.43	0.60	0.81
Delta	-0.40	0.10	-0.53	-0.47	-0.40	-0.32	-0.26
Panel C: Other variables							
Book leverage (BL)	0.48	0.24	0.16	0.29	0.48	0.64	0.83
Size=log(asset value)	7.64	2.02	5.14	6.18	7.52	8.96	10.33
Long term debt due in one year	0.03	0.09	0.00	0.00	0.00	0.03	0.08
Long term debt due in five years	0.17	0.26	0.00	0.00	0.10	0.26	0.47
Realized volatility (RVol)	0.46	0.32	0.19	0.26	0.38	0.57	0.83
Idiosyncratic volatility (IVol)	0.38	0.28	0.14	0.20	0.31	0.48	0.71
VRP	-0.03	2.79	-0.20	-0.04	0.02	0.07	0.17

Note: This table reports the descriptive statistics of delta-hedged option returns for the pooled data. The data sample period is from January 1996 to August 2014. For call options, delta-hedged return until maturity is calculated as delta-hedged gain scaled by $(\Delta S - C)$, where Δ is the Black-Scholes option delta, S is the underlying stock price and C is the price of call option. For put options, it is scaled by $(P - \Delta S)$. Delta-hedged gain is the change in the value of a portfolio consisting of one long option position, daily hedged by the underlying stock, so that the portfolio is not sensitive to the stock price movement. Moneyness is the ratio of stock price over option strike price. Days to maturity is the calenda days until option expiration. Relative bid-ask spread is the difference between bid and ask option price divided by the avereage of bid and ask price. Implied volatility (IV), delta and vega are provided by OptionMetrics based on Black-Scholes model. Realized volatility is the standard deviation of the daily stock return during the past 30 days. Idiosyncratic volatility (IVol) is the standard deviation of the residuals of Fama-French three factor model estimated using the daily return over the previous month. Size is the logrithm of the firm's asset. Book leverage is the sum of total debt and the par value of the preferred stock, minus deferred taxes and investment tax credit, divided by total asset.

Table 4.3: Summary statistics of delta-hedged option returns

		Obs.	Mean	Std.Dev.	No. of firms	mean < 0	$t < -2$	mean > 0	$t > 2$
Call	ATM	221,743	-1.72	8.60	5809	5342	762	467	5
	OTM	201,474	-2.25	9.96	5793	5255	726	538	8
Put	ATM	183,893	-1.76	6.82	5807	5150	691	657	7
	OTM	170,716	-2.20	8.93	5676	4991	728	685	6

Note: This table reports summary statistics of delta-hedged returns for call and put options under the at-the-money (ATM) and out-of-the-money (OTM) categories. The third to sixth columns represent number of observations, mean, standard deviation, and number of firms. The column mean $< 0(> 0)$ reports the number of firms with mean of the delta-hedged returns less (more) than zero. The column $t < -2(> 2)$ reports the number of firms with t statistics of delta-hedged returns less (more) than two.

Table 4.4: Delta-hedged option returns and book leverage

	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5	MODEL6
Intercept	-0.020	-0.037	-0.032	0.008	0.022	0.011
	(-9.77)	(-13.34)	(-12.86)	(4.43)	(9.17)	(2.94)
BL	0.005	-0.008	-0.007	-0.005	-0.041	-0.042
	(2.41)	(-3.64)	(-2.87)	(-2.41)	(-12.00)	(-12.49)
SIZE_A		0.003				0.001
		(13.03)				(5.33)
SIZE_S			0.003			
			(14.54)			
IV				-0.049		
				(-13.82)		
IV_a					-0.078	-0.069
					(-14.05)	(-11.50)
Average adj. R^2	0.004	0.013	0.015	0.038	0.032	0.035

Note: This table reports the average coefficients (t statistics) from monthly cross-sectional Fama-MacBeth regressions of at-the-money delta-hedged call option returns. The sample period covers data from January 1996 through August 2014. BL (Book leverage) is the sum of total debt and the par value of the preferred stock, minus deferred taxes and investment tax credit, divided by total asset. Size_a is the logrithm of the firm's asset. Size_s is the logrithm of the firm's market capitalization. IV is the Black-Scholes option implied volatility. IV_a is the firm's asset volatility, which is calculated as IV*(1-BL). Reported are coefficients and Dama-MacBeth t-statistics with Newy-West correction for serial correlation.

The univariate regression of delta-hedged option return on book leverage in Model 1 of Table 4.4 shows that the relation between the two is positive. However, when the firm size measured by asset value (Model 2) or by capitalization (Model 3) or implied volatility of the underlying stock (Model 4) are controlled in the regression, the relation becomes significantly negative. It confirms the theoretical finding that, the negative relation exists only in similar firms in all respects except that their book leverages are different. For firms with similar sizes, firms with higher leverage have lower delta-hedged returns. Compared to large firms, smaller firms usually have lower leverage ratio and higher asset volatility, which may lead to lower delta-hedged returns. This is one possible explanation why univariate regression shows positive relation between the leverage ratio and the delta-hedged option returns.

The significant negative relation between delta-hedged option returns and the leverage ratio is robust to different control variables. Note that when controlling for the asset volatility instead of implied volatility, the average estimated coefficient of book leverage (-0.041 in Model 5 and -0.042 in Model 6) and the corresponding t statistics are larger than that in other regressions. Following Correia et al. (2014), the asset volatility is calculated as $IV \times (1 - BL)$. The result that controlling for asset volatility is the most efficient to establish the relation between the delta-hedged option return and book leverage also supports the theoretical model.

4.4.3 Controlling for volatility misestimation, idiosyncratic volatility and option illiquidity

In the recent literature, several variables have been found to be important determinants of delta-hedged option returns. Goyal and Saretto (2009) link the delta-hedged option returns to the difference between historical realized volatility and at-the-money implied volatility. They are motivated by the volatility misestimation and option mispricing. Cao and Han (2013) find negative relation between the delta-hedged option returns to idiosyncratic volatility, consistent with market imperfections and constrained financial intermediaries. In a recent paper, (Christoffersen et al., 2014) report that an increase in option illiquidity decreases the current option price and predicts higher expected delta-hedged option returns. In Table 4.5, we control for the idiosyncratic volatility, option illiquidity and volatility deviation to examine whether they can explain the negative relation between delta-hedged option returns and book leverage. We find that the relation is robust after including these other control variables.

In Table 4.5, idiosyncratic volatility (IVol) is calculated as the standard deviation of the residuals of Fama-French three factor model estimated using the daily return over the previous month. Systematic volatility (SysVol) is the square root of (Vol^2-$IVol^2$), where Vol is the standard deviation of the stock return in the past month. Consistent with Cao and Han (2013), we find a negative relation between delta-hedged option returns and idiosyncratic volatility in Model 1 of Table 4.5. When both idiosyncratic volatility and systematic volatility are included in the regression in Model 1, the idiosyncratic volatility plays a significant role, with estimated coefficient -0.025 and t statistics -10.34. In Model 2 of Table 4.5, after controlling for the idiosyncratic volatility and systematic volatility, the estimated coefficient of book leverage is negative and significant. It shows that the negative relation between book leverage and delta-hedged option return cannot be explained by the limits to arbitrage or market imperfections.

In Model 3 of Table 4.5, following Goyal and Saretto (2009), we measure the volatility deviation as the log difference of historical volatility (Vol) and implied volatility (IV). This variable has a significantly positive coefficient, which is consistent with Goyal and Saretto (2009). More importantly, after controlling for this proxy of volatility-related option mispricing, the coefficient for book leverage remains statistically significant. Thus, volatility-related mispricing does not explain the result either.

Model 4 of Table 4.5 further controls for option illiquidity, measured as the difference between bid and ask option price divided by the average of bid and ask price. The result shows that on average, the coefficient of option illiquidity is significant and positive, consistent with the illiquidity premia in the equity option market by Christoffersen et al. (2014). In the presence of option illiquidity, the coefficient of book leverage is still negative and significant in Model 4. Moreover, including the asset volatility (IV_a) in Model 5 makes the magnitude of the estimated coefficient and t statistics of book leverage larger, and that of idiosyncratic volatility smaller. In addition, we control for the stock return during the life of the options in Model 4 and Model 5. The coefficients of the stock return are not significant in both regressions, suggesting that the implemented delta-hedging strategy is efficient and makes the portfolio not sensitive to the underlying stock price movement.

4.4.4 Delta-hedged return and the covenant effect

The model of this chapter predicts that the delta-hedged option return of a firm with protected debt is more negative than that of a firm with unprotected debt. As suggested by (Toft and Prucyk, 1997), the maturity structure of the firm's debt can be used as a proxy for the existence of net-worth hurdles. Leland (1994) argues that short-term debt can be associated with an exogenous bankruptcy trigger V_b that equals the market value of debt on the issue date. Long-term debt, on the other hand, results in an endogenous bankruptcy point which is significantly below this value. Intuitively, this indicates that firms with a large proportion of debt due in the immediate future must pass a net-worth hurdle. Otherwise they are unable to roll over their debt. We,

Table 4.5: Controling for idiosyncratic volatility and option illiquidity

	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	-0.009	-0.023	-0.008	-0.048	0.003
	(-7.62)	(-9.47)	(-2.33)	(-4.56)	(0.3)
IVol	-0.025	-0.018	-0.033		-0.001
	(-10.34)	(-7.65)	(-10.13)		(-0.46)
SysVol	0.003	0.002	-0.008		
	(1.03)	(0.73)	(-1.76)		
Size_A		0.002	0.001	0.003	0.001
		(7.55)	(3.29)	(13.26)	(5.31)
BL		-0.006	-0.006	-0.009	-0.045
		(-3.26)	(-3.03)	(-4.87)	(-12.91)
Vol deviation			0.022		
			(7.09)		
Option Illiquidity				0.016	0.008
				(3.01)	(1.68)
Stock return				-0.005	-0.006
				(-0.53)	(-0.64)
IV_a					-0.073
					(-12.26)
Average adj. R^2	0.021	0.027	0.045	0.067	0.091

Note: This table reports the average coefficients (t statistics) from monthly cross-sectional Fama-MacBeth regressions of at-the-money delta-hedged call option returns. The sample period covers data from January 1996 through August 2014. Idiosyncratic volatility (IVol) is the standard deviation of the residuals of Fama-French three factor model estimated using the daily return over the previous month. Systematic volatility (SysVol) is the square root of (Vol^2-$IVol^2$), where Vol is the standard deviation of the stock return in the past month. Size_a is the logrithm of the firm's asset. Book leverage (BL) is the sum of total debt and the par value of the preferred stock, minus deferred taxes and investment tax credit, divided by total asset. Vol deviation is calculated as the log difference between VoL and IV. OptionIll is the difference between bid and ask option price divided by the avereage of bid and ask price. Stock return is the stock return of the underlying firm until maturity. IV_a is the firm's asset volatility, which is calculated as IV*(1-BL). Reported are coefficients and Dama-MacBeth t-statistics with Newy-West correction for serial correlation.

therefore, use the ratio of long-term debt due in one year to total debt as the first covenant proxy, CVNT1. The ratio of long-term debt due within five years to total debt is the second covenant proxy, CVNT5.

Table 4.6: Delta-hedged return and the covenant effect

	Call options				Put options			
	ATM		OTM		ATM		OTM	
Intercept	0.003	0.004	-0.001	0.000	0.008	0.008	0.011	0.012
	(0.84)	(0.95)	(-0.22)	(0.09)	(3.17)	(3.26)	(3.05)	(3.17)
BL	-0.007	-0.006	-0.010	-0.009	-0.004	-0.004	-0.006	-0.005
	(-3.29)	(-2.99)	(-4.06)	(-3.79)	(-2.60)	(-2.32)	(-2.19)	(-2.01)
Size_A	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000
	(1.72)	(1.58)	(1.75)	(1.64)	(0.23)	(0.15)	(0.52)	(0.64)
IV	-0.046	-0.046	-0.049	-0.050	-0.052	-0.052	-0.059	-0.059
	(-10.29)	(-10.40)	(-10.39)	(-10.49)	(-17.67)	(-17.94)	(-14.77)	(-15.35)
Option Illiquidity	0.002	0.002	0.005	0.006	0.002	0.003	0.002	0.002
	(-0.43)	(-0.48)	(-1.65)	(-1.81)	(-0.68)	(-0.82)	(-0.53)	(-0.64)
cvnt1	-0.010		-0.013		-0.007		-0.011	
	(-2.73)		(-2.74)		(-1.61)		(-1.68)	
cvnt5		-0.005		-0.006		-0.004		-0.005
.		(-4.26)		(-4.11)		(-3.30)		(-3.13)
Average adj. R^2	0.043	0.043	0.036	0.036	0.056	0.056	0.044	0.044

Note: This table reports the average coefficients (t statistics) from monthly cross-sectional Fama-MacBeth regressions of at-the-money delta-hedged call option returns. The sample period covers data from January 1996 through August 2014. Book leverage (BL) is the sum of total debt and the par value of the preferred stock, minus deferred taxes and investment tax credit, divided by total asset.Size_A is the logrithm of the firm's asset. Implied volatility (IV) is provided by OptionMetrics based on Black-Scholes model. Relative bid-ask spread is the difference between bid and ask option price divided by the avereage of bid and ask price. Stock return is the stock return of the underlying firm until maturity. CVNT1 is the ratio of long term debt due in one year, divided by total long term debt. CVNT5 is the ratio of long term debt due within five years, divided by total long term debt. Reported are coefficients and Dama-MacBeth t-statistics with Newy-West correction for serial correlation.

Table 4.6 reports the results of regressing the delta-hedged option return on the book leverage (BL) and the covenant proxies (CVNT1 and CVNT5). The regressions are estimated for four samples of delta-hedged option returns: at-the-money (ATM) call, out-of-the-money (OTM) call, at-the-money

(ATM) put and out-of-the-money put. First, we find that, in the four samples, the estimated coefficient of the covenant proxies are significantly less than zero after controlling for book leverage (BL), the firm size, implied volatility and option illiquidity. The estimated coefficients range from -0.004 to -0.013 and the t statistics range from -1.61 to -4.26.

Second, short-term covenant proxy (CVNT1) has a larger effect on the delta-hedged option returns than the longer-term covenant proxy (CVNT5). This pattern shows in all four samples, for instance, the estimated coefficient of cvnt1 (-0.010) is twice as large as that of cvnt5 (-0.005) in the ATM call options category. This can be explained that long term debt due in the nearer future places a stricter net-worth covenant than the one that is due in the further future. Thus, for firms with a similar leverage ratio and other characteristics, the effect of cvnt1 on delta-hedged option returns is larger than that of cvnt5. In addition, the magnitude of covenant proxy effect is larger for out-of-the-money options than at-the-money options. This is true for both call and put options. Overall, the results presented in Table 4.6 support the hypothesis predicted in the theoretical model.

4.4.5 The nonlinear effect of book leverage and asset volatility

Consider the mechanics of the model captured by Figure 4.1, which raises two important issues. First, the relation between the determinants (leverage and asset volatility) and returns is likely to be highly nonlinear. Any return regression that includes leverage as a regressor will therefore need to specify higher-order polynomials of leverage. A second problem is that the role of leverage differs on whether the delta-hedged option return is positive or negative. This is evident from Figure 4.1. If the firm's delta-hedged option return (variance risk premium) is positive, leverage will increase the return, and the

Table 4.7: The nonlinear effect of book leverage and asset volatility

	MODEL1	MODEL2	MODEL3	MODEL4
IV_a	-0.077		-0.07	
	(-13.85)		(-12.43)	
$BL \times 1_{ret>0}$	0.067	0.017	0.238	-0.034
	(19.21)	(6.57)	(20.96)	(-5.60)
$BL \times 1_{ret<0}$	-0.084	-0.058	-0.167	-0.044
	(-19.78)	(-18.06)	(-15.66)	(-7.15)
$IV\_a \times 1_{ret>0}$		0.098		0.128
		(12.79)		(14.76)
$IV\_a \times 1_{ret<0}$		-0.14		-0.136
		(-35.91)		(-35.30)
$BL^2 \times 1_{ret>0}$			-0.261	0.067
			(-18.42)	(8.8)
$BL^2 \times 1_{ret<0}$			0.117	-0.018
			(11.98)	(-3.67)
SIZE_a			0.001	0.001
			(6.03)	(5.63)
Option Illiquidity			-0.003	-0.001
			(-0.76)	(-0.19)
IVol				-0.006
				(-4.44)
Average adj. R^2	0.256	0.402	0.315	0.409

Note: This table reports the average coefficients (t statistics) from monthly cross-sectional Fama-MacBeth regressions of at-the-money delta-hedged call option returns. The sample period covers data from January 1996 to August 2014. IV_a is the firm's asset volatility, which is calculated as IV*(1-BL). BL is book leverage. $1_{ret<0}$ ($1_{ret>0}$) is a dummy variable which is equal to one when the stock return is positive (negative). Idiosyncratic volatility (IVol) is the standard deviation of the residuals of Fama-French three factor model estimated using the daily return over the previous month. Size_a is the logrithm of the firm's asset. Option Illiquidity is the difference between bid and ask option price divided by the avereage of bid and ask price. Reported are coefficients and Dama-MacBeth t-statistics with Newy-West correction for serial correlation.

first-order leverage term will be estimated with a positive coefficient, but if the delta-hedged option return is negative, higher leverage will show up with a negative coefficient. If we ignore this and regress the resulting sample of negative and positive levered returns on leverage, the resulting estimates may not be informative regarding the role of leverage.

We explore these issues in Table 4.7, in which we regress the delta-hedged returns on leverage, asset volatility, interaction terms and higher order terms of these two determinants. The two determinants are interacted with a dummy variable $1_{ret>0}$ ($1_{ret<0}$), which is equal to one when the equity return is positive (negative). In Model 1, the coefficient of the interaction term $BL \times 1_{ret>0}$ ($BL \times 1_{ret<0}$) is significantly positive (negative). After including the interaction term of asset volatility with dummies in Model 2, the coefficients of all interaction terms are consistent with the theory, as expected. In Model 3 and 4, we include higher order interaction terms, which are all significant. The adjusted R^2 also increases drastically from about 10% in Table 5 to more than 40% in Model 4 Table 7. Interestingly, the size effect remains statistically significant. The idiosyncratic volatility effect remains, but the magnitude decreases. The effect of option illiquidity does not exist after including the structural variables, leverage and asset volatility.

4.5 Leverage-based trading strategy

We now investigate the cross-sectional relation between delta-hedged option returns using portfolio sorting approach. This section confirms the Fama-Macbeth regression results in the previous section, propose a leverage-based trading strategy and examine the impact of trading cost on the profitability of the trading strategy.

As in Section 4.4, for each optionable stock, we choose an option with a time-to-maturity closest to 30 days for each of the four option categories:

ATM call, OTM call, ATM put and OTM put. At the end of each month, we first sort stocks with traded options into five quintiles based on their sizes, (or asset volatility) and then, within each size quintile, we further sort the stocks by their book leverage ratio into five quintiles. In each size quintile, the trading strategy buys the delta-hedged options on stocks ranked in the bottom leverage quintile and sells the delta-hedged options on stocks ranked in the top leverage quintile. The delta-hedged options are rebalanced every day based on their delta and held until maturity. The delta-hedged option returns are calculated in the same way as in Section 4.4.

4.5.1 Double sorts on size and leverage

Table 4.8 reports the equal-weighted average return of 25 portfolios for delta-hedge call and put options. Each portfolio consists of selling delta-hedged options on stocks located in a given quintile sorted by size and leverage. Different from the summary statistics in Table 4.2 and Table 4.3, the returns are positive on average in Table 4.8, because the short positions of the delta-hedged options are considered in the trading strategy. Table 4.8 also reports in the "5-1" column the difference in the average return of the top and bottom book leverage quintile in each size quintile. The t-statistics for the time series of "5-1" portfolios are computed using a Newey-West correction for serial correlation using 2 lags for monthly returns.

Panel A of Table 4.8 reports the results for monthly delta-hedged returns on ATM call options. Panel A shows that the 5-1 portfolios which sell the delta-hedged calls with the highest leverage ratio and buy the ones with the lowest leverage ratio earn a significant positive return from size quintile 1 to size quintile 4. From Panel B to Panel D, all the 5-1 portfolios earn positive returns on average, with most of them statistically significant. In general, the effect of book leverage on the delta-hedged option return is decreasing with

Table 4.8: Returns of selling delta-hedged options: Double sorting on size and leverage

	1-BL	2	3	4	5-BL	5-1	t-stat
Panel A: ATM Call							
1-size	2.28	2.44	2.32	2.41	2.72	0.44***	2.78
2	1.68	1.61	1.72	2.05	2.1	0.42***	4.19
3	1.64	1.53	1.38	1.69	2.03	0.39***	4.16
4	1.45	1.38	1.32	1.64	1.66	0.22**	2.36
5-size	1.18	1.09	1.09	1.24	1.07	-0.1	0.8
Panel B: OTM Call							
1-size	2.7	2.8	2.81	2.98	3.57	0.88***	4.81
2	2.17	2.11	2.44	2.57	2.53	0.36**	2.56
3	2.08	1.92	2.01	2.28	2.57	0.49***	4.01
4	1.93	1.68	1.96	2.27	2.38	0.45***	3.87
5-size	1.66	1.64	1.64	1.65	1.78	0.12	0.89
Panel C: ATM Put							
1-size	2.5	2.51	2.31	2.38	2.99	0.49***	4.79
2	1.88	1.82	1.73	1.97	2.01	0.13	1.46
3	1.73	1.32	1.48	1.67	2.15	0.42***	5.4
4	1.55	1.36	1.38	1.44	1.67	0.12	1.21
5-size	1.12	1.07	1.1	1.19	1.21	0.09	1
Panel D: OTM Put							
1-size	2.85	2.8	2.79	2.86	3.61	0.76***	5.25
2	2.01	2.02	1.99	2.34	2.36	0.35***	2.83
3	2.13	1.75	1.81	2.09	2.61	0.48***	3.83
4	1.94	1.55	1.71	1.8	2.28	0.34***	3.35
5-size	1.51	1.55	1.54	1.69	1.76	0.24**	2.04

Note: This table reports the average returns pf delta-hedged options on stocks of different size and leverage level. At the end of each month, the optionable stocks are first sorted into five quintiles based on their asset size, and then within each size quintile, they are further sorted into five quintiles by leverage ratio. The results for ATM call, OTM call, ATM put and OTM put are presented in Panel A to Panel D. The t statistics are corrected for seriabl correlation (Newey-West correction with 2 lags for monthly return). The sample dates from January 1996 to August 2014.

the firms' asset sizes. As firms grow larger, they have better opportunities to issue more debt. In that case, leverage ratio becomes a less important indicator for bankruptcy. If the bankruptcy risk premium is considered as a dominant component in the delta-hedged option return, the effect of book leverage is smaller in larger firms. Moreover, the effect of book leverage on OTM delta-hedged options is stronger than that on ATM delta-hedged options. For example, in the first size quintile, the average 5-1 portfolio return of delta-hedged ATM call options is 0.44 with t statistics 2.78, while for OTM call options, the return is 0.88 with t statistics 4.81.

Interestingly, Vedolin (2012) find relatively weak evidence for the relation between financial leverage and variance risk premium. One explanation is that the firm characteristics are not controlled in the analysis. The implication of the theoretical model in this chapter is that for two otherwise same firms, higher leverage contributes to lower delta-hedged option returns. Hence, controlling for the firm characteristics is essential for disentangling the relation between leverage and delta-hedged option return.

4.5.2 Double sorts on asset volatility and leverage

In the previous section, we use Fama-Macbeth regressions to show that the negative relation between delta-hedged option return and leverage ratio is more evident after controlling for firms' asset volatility. Table 4.9 uses the conditional double sort to confirm the finding. At the end of each month, we first sort the stocks into five quintiles by their asset volatility, calculated as $IV \times (1 - BL)$, where IV is the implied volatility and BL is the book leverage. Within each asset volatility quintile, the stocks are further sorted into five quintiles by their book leverage. The equal weighted average returns of selling the delta-hedged options on the stocks in each quintile are reported in Table 4.9.

Table 4.9: Returns of selling delta-hedged options: Double sorting on asset volatility and leverage

	1-BL	2	3	4	5-BL	5 _1	t-stat
Panel A: ATM Call							
1-Asset Vol	0.75	1.04	1.41	1.41	1.63	0.88***	7.92
2	0.89	1.03	1.11	1.61	2.29	1.40***	14.19
3	1.13	1.21	1.45	1.67	2.62	1.48***	14.63
4	1.22	1.53	1.58	1.8	2.7	1.48***	12.48
5-Asset Vol	2.26	2.44	2.22	2.48	3.23	0.98***	6.69
Panel B: OTM Call							
1-Asset Vol	1.25	1.45	2.07	2.15	2.28	1.03***	8.55
2	1.17	1.5	1.73	2.22	2.83	1.66***	10.07
3	1.62	1.85	2.05	2.4	2.99	1.36***	10.17
4	1.74	1.98	2.1	2.31	3.24	1.50***	10.6
5-Asset Vol	2.64	2.81	2.72	2.97	4.03	1.39***	8.41
Panel C: ATM Put							
1-Asset Vol	0.66	0.95	1.36	1.39	1.84	1.18***	14.61
2	0.85	0.9	1.17	1.48	2.3	1.45***	17.87
3	1.12	1.18	1.47	1.71	2.5	1.37***	14
4	1.34	1.52	1.64	1.94	2.83	1.49***	14.21
5-Asset Vol	2.48	2.57	2.4	2.59	3.33	0.85***	5.96
Panel D: OTM Put							
1-Asset Vol	1	1.3	1.85	2	2.37	1.36***	12.9
2	1.35	1.35	1.48	1.97	2.87	1.53***	14.91
3	1.54	1.52	1.87	2	2.82	1.29***	9.52
4	1.52	1.66	1.99	2.08	3.48	1.96***	14.66
5-Asset Vol	2.87	2.99	2.54	3.12	3.77	0.90***	5.32

Note: This table reports the average returns pf delta-hedged options on stocks of different asset volatility and leverage level. At the end of each month, the optionable stocks are first sorted into five quintiles based on their asset volatility, and then for each size quintile, they are further sorted into five quintiles by leverage ratio. The results for ATM call, OTM call, ATM put and OTM put are presented in Panel A to Panel D. The t statistics are corrected for seriabl correlation (Newey-West correction with 2 lags for monthly return). The sample dates from January 1996 to August 2014.

Table 4.9 shows that, in all 4 Panels of different option categories and all asset volatility quintiles, selling delta-hedged options on high leverage stocks significantly outperforms selling delta-hedged options on low leverage stocks. The average outperformance ranges from 0.85% to 1.96%. Consistent with the theory, investors selling delta-hedged options with higher asset volatility get higher returns than that with lower asset volatility. For instance, the delta-hedged returns in the fifth asset volatility quintile are always larger than that in the first asset volatility quintile in Panel A to Panel D. In addition, the effect of book leverage on delta-hedged options is larger for OTM options than ATM options and larger for put options than for call options. We also construct value-weighted portfolios as robustness check in Table 4.10. It shows relatively smaller magnitude than in Table 4.9, but the differences between portfolio 5 and 1 are still strongly significant. The reason is that small firms receive smaller weights than the large firms and the relation between leverage and portfolio returns is the strongest in the smallest firms.

4.6 Conclusion

How does the Merton-type structural model explain the cross-sectional variation of equity option return? This chapter argues in a jump-diffusion capital structure model that, firm's leverage ratio and asset volatility are two determinants of the expected return of delta-hedged equity options. We first derive the expected return of the delta-hedged equity option based on a capital structure model, in which the asset value of a firm is driven by a double exponential jump-diffusion process. In the model, the expected return of the delta-hedged equity option is closely linked to option gamma and the variance risk premium of the underlying firm, which is related to firm's financial characteristics. Furthermore, the theory suggests that the relation between the determinants and the delta-hedged equity option returns is nonlinear. Empir-

Table 4.10: Robustness Check: Double sorting on asset volatility and leverage (value weighted portfolios)

	1-BL	2	3	4	5-BL	5 _1	t-stat
Panel A: ATM Call							
1-Asset Vol	0.75	1.01	1.36	1.32	1.38	0.63***	5.23
2	0.91	1.01	1.08	1.59	2.14	1.23***	12.35
3	1.14	1.21	1.44	1.63	2.53	1.39***	14.05
4	1.19	1.52	1.56	1.73	2.63	1.45***	11.59
5-Asset Vol	1.96	2.21	2.01	2.32	3.07	1.11***	7.6
Panel B: OTM Call							
1-Asset Vol	1.24	1.42	2.01	2.01	2.01	0.77***	6.24
2	1.2	1.49	1.68	2.19	2.7	1.51***	9.12
3	1.64	1.85	2.04	2.38	2.9	1.26***	9.23
4	1.7	1.95	2.07	2.23	3.12	1.42***	9.1
5-Asset Vol	2.34	2.62	2.44	2.79	3.81	1.48***	8.21
Panel C: ATM Put							
1-Asset Vol	0.66	0.91	1.28	1.28	1.62	0.96***	11.29
2	0.87	0.89	1.18	1.47	2.22	1.35***	16.38
3	1.14	1.19	1.45	1.69	2.43	1.29***	13.44
4	1.3	1.48	1.61	1.92	2.74	1.44***	12.3
5-Asset Vol	2.26	2.4	2.27	2.43	3.11	0.85***	5.19
Panel D: OTM Put							
1-Asset Vol	1.01	1.25	1.75	1.92	2.15	1.14***	10.16
2	1.37	1.31	1.51	1.97	2.76	1.39***	13.38
3	1.58	1.51	1.86	1.99	2.74	1.16***	8.72
4	1.44	1.6	1.92	2.01	3.38	1.94***	12.39
5-Asset Vol	2.68	2.84	2.4	2.93	3.46	0.77***	3.79

Note: This table reports the average returns pf delta-hedged options on stocks of different asset volatility and leverage level. At the end of each month, the optionable stocks are first sorted into five quintiles based on their asset volatility, and then within each size quintile, they are further sorted into five quintiles by leverage ratio. The results for ATM call, OTM call, ATM put and OTM put are presented in Panel A to Panel D. The t statistics are corrected for seriabl correlation (Newey-West correction with 2 lags for monthly return). The sample dates from January 1996 to August 2014.

ically we find that these two structural variables can explain a large portion of the cross-sectional variation in the data and even subsume information in other determinants documented in the literature, such as idiosyncratic volatility and liquidity. The results of double sorting exercise are consistent with the theory. There is also evidence of the nonlinear relation between the determinants and the delta-hedged equity option returns: the determinants affect positive and negative returns differently. These findings are robust across calls, puts, and different moneyness levels.

Overall, this chapter explores one channel, i.e. financial decision of the firm, that differentiates the pricing of variance risk premium of individual stocks. The model indicates that the first-order equity risk can transfer directly to higher-order risks such as the variance risk and jump risk. There are at least two dimensions of research that can be explored in the future. The first dimension is to consider the investment channel and the leverage channel simultaneously. The interaction of the two channels is able to explain more empirical patterns in the equity option market. The second dimension for further research is to extend the model and accommodate more complex capital structures, e.g. security provisions and conversion rights. The extended model can examine how the heterogeneity of firm's debt structure affects firms' default incentives and the expected return of delta-hedged equity options. These questions are left for future research.

4.7 Appendix

4.7.1 Derivation of debt and equity value of the firm

In this subsection, to simplify the notation, we drop the superscript Q in the parameters, i.e. using p_u, p_d, η_u and η_d instead of p_u^Q, p_d^Q, η_u^Q and η_d^Q. To compute the total debt and equity values, one needs to compute the distribution of the default time τ and the joint distribution of V_τ and τ. Kou and Wang (2003) show that the analytical solutions for these distributions depend on the roots of the following equation:

$$r = -(r - \frac{1}{2}\sigma^2 - \lambda\xi)x + \frac{1}{2}\sigma^2 x^2 + \lambda(\frac{p_u\eta_u}{\eta_u - x} + \frac{p_d\eta_d}{\eta_d + x} - 1), \tag{4.19}$$

which has exactly four roots γ_1, γ_2, $-\gamma_3$ and $-\gamma_4$, with

$$0 < \gamma_1 < \eta_d < \gamma_2,\ 0 < \gamma_3 < \eta_u < \gamma_4.$$

Based on the distribution of default time and the joint distribution of default threshold and default time, the value of total asset, debt and equity value of the firm can then be obtained. The total market value of the firm is the firm asset value plus the tax benefit and minus the bankruptcy cost, which depend on the asset value of the firm V and the bankruptcy threshold V_B:

$$\begin{aligned} v(V, V_B) &= V + E[\int_0^\tau \kappa\rho P e^{-rt} dt] - (1-\alpha_d)E[V_\tau e^{-r\tau}] \\ &= V + \frac{\kappa c}{r}(1 - d_1(\frac{V_B}{V})^{\gamma_1} - d_2(\frac{V_B}{V})^{\gamma_2}) - (1-\alpha_d)V_B(c_1(\frac{V_B}{V})^{\gamma_1} + c_2(\frac{V_B}{V})^{\gamma_2}), \end{aligned} \tag{4.20}$$

where $c_1 = \frac{\eta_d - \gamma_1}{\gamma_2 - \gamma_1}\frac{\gamma_2 + 1}{\eta_d + 1}$, $c_2 = \frac{\gamma_2 - \eta_d}{\gamma_2 - \gamma_1}\frac{\gamma_1 + 1}{\eta_d + 1}$, $d_1 = \frac{\eta_d - \gamma_1}{\gamma_2 - \gamma_1}\frac{\gamma_2}{\eta_d}$, and $d_2 = \frac{\gamma_2 - \eta_d}{\gamma_2 - \gamma_1}\frac{\gamma_1}{\eta_d}$. The value of total debt at time 0 is the sum of the expected coupon payment before bankruptcy and the expected payoff upon bankruptcy:

$$\begin{aligned} D(V; V_B) &= E[\int_0^\tau e^{-rt} c dt + \alpha_d e^{-r\tau} V_\tau] \\ &= \frac{c}{r}(1 - d_1(\frac{V_B}{V})^{\gamma_1} - d_2(\frac{V_B}{V})^{\gamma_2}) + \alpha_d V_B(c_1(\frac{V_B}{V})^{\gamma_1} + c_2(\frac{V_B}{V})^{\gamma_2}), \end{aligned} \tag{4.21}$$

The total equity value is the difference between the total asset value and the total debt value,

$$S(V;V_B) = v(V;V_B) - D(V;V_B) \quad (4.22)$$
$$= V + aV^{-\gamma_1} + bV^{-\gamma_2} - \frac{(1-\kappa)c}{r},$$

where $a = \frac{(1-\kappa)cd_1}{r}V_B^{\gamma_1} - c_1 V_B^{\gamma_1+1}$ and $b = \frac{(1-\kappa)cd_2}{r}V_B^{\gamma_2} - c_2 V_B^{\gamma_2+1}$.
The bankruptcy trigger V_B is either exogenously given by a net worth covenant, i.e. a covenant triggers bankruptcy when the asset value hits the threshold $V_B = \frac{c}{r\alpha_d}$, where α_d is the portion of asset value the debt holders can get upon default. The bankruptcy trigger can also be determined endogenously if it is within the equity holder's discretion to declare banktuptcy. For a fixed coupon level c, Chen and Kou (2009) derive the optimal choice of V_B^* by maximizing the total equity values:

$$V_B^* = \frac{\epsilon c}{r}, \text{ where } \epsilon = \frac{(1-\kappa)(d_1\gamma_1 + d_2\gamma_2)}{c_1\gamma_1 + c_2\gamma_2 + 1}. \quad (4.23)$$

Whether the default trigger is determined exogenously (protected debt) or endogenously (unprotected debt) has impact on the pricing of the firm's equity value, and furthermore on the pricing of options on the firm's levered equity. Empirically, it is possible to use balance sheet data to approximate the protective net-worth covenant. For instance, the term structure of the firm's debt can be used as a proxy for the existence of net worth hurdle. Leland (1994) shows that the short term debt can be associated with an exogenous bankruptcy triger that equals the market value of debt on the issue date. Long term debt results in an endogenous trigger which is significantly below the previous one. This implies that a firm with a large portion of long-term debt due in the immediate future faces a net-worth hurdle. Otherwise they are not able to renew the credit line.

4.7.2 Proof of Proposition 1

The first part in Equation (4.14) can be expanded using the Taylor expansion:

$$E^Q[O(S) - O(S_-)] \approx E^Q[\frac{\partial O}{\partial S}(S - S_-) + \frac{1}{2}\frac{\partial^2 O}{\partial S^2}(S - S_-)^2]. \tag{4.24}$$

Similarly, under the physical measure, we approximate the expected change of the option price until the second order:

$$E[O(S) - O(S_-)] \approx E[\frac{\partial O}{\partial S}(S - S_-) + \frac{1}{2}\frac{\partial^2 O}{\partial S^2}(S - S_-)^2]. \tag{4.25}$$

Substituting Equation (4.24) and (4.25) into Equation (4.14), we get Equation (4.15) in Proposition 4.1. Using Taylor expansion, the change of stock price in jump times can be further expanded. The quadratic term in Equation (4.1) is approximately equal to,

$$(S(V) - S(V_-))^2 \approx (\frac{\partial S}{\partial V}(V - V_-) + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}(V - V_-)^2)^2 \tag{4.26}$$

There is quadratic, cubic and quatic terms in the above formula. Since higher order terms play a less important role, we only consider the first order term such that Equation (4.15) is simplified as,

$$E(\Pi_t) \approx \int_0^t \frac{1}{2}\frac{\partial^2 O_u}{\partial S_u^2}(\frac{\partial S_u}{\partial V_u})^2(\lambda E[V_u - V_{u-}]^2 - \lambda^Q E^Q[V_u - V_{u-}]^2)du.$$

Note that the option price is a strictly convex function of the underlying asset price and option gamma $\frac{\partial^2 O}{\partial S^2}$ is positive for both call and put options. $\frac{\partial S}{\partial V}$ is also positive because stock price S is a call option on the firm's asset V. Recall the expressions of the jump intensity of the asset value under physical and risk neutral measure λ and λ^Q, and density of the jump size under the physical and risk neutral measure in Section 4.2.1 and substitute them in Equation (4.15),

$$\lambda E[V - V_-]^2 - \lambda^Q E^Q[V - V_-]^2 \tag{4.27}$$

$$= \lambda(\xi^{(2)} + \xi^{(0)} - 2\xi^{(1)} - (\xi^{(\alpha+1)}) + \xi^{(\alpha-1)} - 2\xi^{(\alpha)}))V_-^2. \tag{4.28}$$

where $\xi^{(x)}$ is a function of x given in Equation (4.4) and α ($0 < \alpha < 1$) is the risk aversion coefficient in the utility function. Let $f(x) = \xi^{(x)}$. To show that (4.27) is less than zero, we have to prove $f'(\alpha+1) + f'(\alpha-1) - 2f'(\alpha) < 0$. In other words, $f'(x)$ is a concave function for $0 < x < 1$. To prove this, we calculate the third derivative of $f(x) = \xi^{(x)}$:

$$\frac{\partial^3 f(x)}{\partial x^3} = \frac{6\eta_u^4\eta_d^4\,(p_u/\eta_u^3 - p_d/\eta_d^3)}{(\eta_u - \alpha)^4(\eta_d + \alpha)^4}$$

If the parameters in the above equation satisfies the following two conditions, then $f'(x)$ is a concave function of x. The first condition is that the absolute value of the negative jump size is larger than the positive jump size on average, that is, $1/\eta_d > 1/\eta_u$. The second condition is that the expected jump size is less than zero: $E[y] = \frac{p_u}{\eta_u} - \frac{p_d}{\eta_d} < 0$. When the parameters of the underlying asset process follows the above two conditions, the expected delta-hedged option return is negative.

Next, we derive the relation between $E(\Pi_t)$ and the variance risk premium over the time period 0 to t. The variance of $\log(S_t)$ is measured by its quadratic variation (QV). For a period from time 0 to t, it is given by,

$$[\log(S),\log(S)]_{(0,t]} = \int_0^t \left(\frac{\partial S_s}{\partial V_s}\frac{V_s}{S_s}\sigma\right)^2 ds + \sum_{0<s\leq t}\left(\frac{S_s - S_{s-}}{S_s}\right)^2.$$

The randomness in QV generates variance risk. As the randomness in this model comes from the jumps in the stock price, only the jump part contributes to the variance risk premium. The variance risk premium (VRP) of the stock is defined as the wedge between the expected quadratic variation under the physical measure and the risk neutral measure. Thus, the VRP over the time period $(0,t]$ is,

$$\begin{aligned} VRP_{0,t} &= E^P[[\log(S),\log(S)]_{(0,t]}] - E^Q[[\log(S),\log(S)]_{(0,t]}] \\ &\approx \int_0^t \left(\frac{1}{S_u}\right)^2\left(\frac{\partial S_u}{\partial V_u}\right)^2 \left(\lambda E[V_u - V_{u-}]^2 - \lambda^Q E^Q[V_u - V_{u-}]^2\right) du. \end{aligned}$$

The second step uses the Taylor expansion in Equation (4.26). If the time interval is small enough, we loosely have the following relation between VRP and expected delta-hedged gain:

$$E(\Pi_t) \approx \frac{1}{2}\frac{\partial^2 O}{\partial S^2} \times VRP_{0,t} \times S_0^2.$$

4.7.3 Proof of Proposition 2

If we assume that the maturity of the option is short enough, then the relation between VRP and the expected delta-hedged gain can be rewritten as,

$$E(\Pi_t)/S_0^2 \approx \frac{1}{2}\frac{\partial^2 O}{\partial S^2} \times VRP.$$

The scaled delta-hedged gain $E(\Pi_t)/S_0^2$ is related to the capital structure of the firm through the variance risk premium, especially from the term: $(\frac{1}{S}\frac{\partial S}{\partial V})^2$. We will prove that this term is increasing in the coupon value (c) of the firm. If the book leverage ratio of the firm is approximated as $\frac{c}{rV}$, then the absolute value of the scaled $E(\Pi_t)$ is increasing in the book leverage, for the same level of asset value.

The partial derivative of the equity value S with respect to the asset value V is:

$$\frac{\partial S}{\partial V}(V; V_B) = 1 - a\gamma_1 V^{-\gamma_1 - 1} - b\gamma_2 V^{-\gamma_2 - 1},\ \gamma_1 > 0,\ \gamma_2 > 0,$$

in which $a = \frac{(1-\kappa)cd_1}{r}V_B^{\gamma_1} - c_1 V_B^{\gamma_1+1}$ and $b = \frac{(1-\kappa)cd_2}{r}V_B^{\gamma_2} - c_2 V_B^{\gamma_2+1}$. The parameters c_1, d_1, c_2, d_2, γ_1 and γ_2 given in Section 4.2.2 are only related to the parameters in the asset process, not to the capital structure of the firm. As $\frac{1}{S}$ is increasing in c, $\frac{1}{S}\frac{\partial S}{\partial V}$ will be definitely increasing in c, if $\frac{\partial S}{\partial V}$ is increasing in c,

$$\frac{\partial S/\partial V}{\partial c} = -\gamma_1 V^{-\gamma_1 - 1}\frac{\partial a}{\partial c} - \gamma_2 V^{-\gamma_2 - 1}\frac{\partial b}{\partial c}, \tag{4.29}$$

The sign of the above expression depends on several factors. Two situations, whether the firm faces an exogenous or endogenous trigger, are considered. In the first case, firm's debt is protected by a strict net-worth covenant. This covenant triggers bankruptcy when the asset value V hits the threshold $V_B = \frac{c}{r\alpha}$. In the second case, the bankruptcy trigger is determined endogenously by the debt holders. As showed in Equation (4.23),

$$V_B^* = \frac{\epsilon c}{r}, \text{ where } \epsilon = \frac{(1-\kappa)(d_1\gamma_1 + d_2\gamma_2)}{c_1\gamma_1 + c_2\gamma_2 + 1}. \tag{4.30}$$

In both situations, $V_B = xc$ where x is a constant. Substituting into the expression of a, we have,

$$\frac{\partial a}{\partial c} = (\frac{(1-\kappa)d_1}{r} - c_1 x)x^{\gamma_1}c^{\gamma_1+1}.$$

Note that Equation (4.29) mainly depends on the sign of the first term, because the second term plays a less important role here ($0 < \gamma_1 < \eta_d < \gamma_2$). As $V_B > V_B^*$ and $x > x^*$, it follows that,

$$\frac{\partial S}{\partial V}(V;V_B) > \frac{\partial S}{\partial V}(V;V_B^*),$$
$$\frac{\partial S/\partial V}{\partial c}(V;V_B) > \frac{\partial S/\partial V}{\partial c}(V;V_B^*).$$

Hence, the absolute scaled delta-hedged gain is higher for the firms with a strict net-worth covenant than for those without it; after increasing the leverage ratio, the change in the absolute scaled delta-hedged gain is also higher for the firms with strict net-worth covenant.

From the above derivation, we know that the term $\frac{1}{S}\frac{\partial S}{\partial V}$ for the firms with strict net-worth covenant is more likely to increase with the coupon value c than those without. It can be shown that even for firms with endogenous bankruptcy trigger, with reasonable parameter assumptions, the term $\frac{1}{S}\frac{\partial S}{\partial V}$ is increasing in c. The proof is available upon request.

4.7.4 Details of the simulation procedure

Based on the parameters in Table 1, we first simulate the diffusion and the jump component of the firm's asset process under the physical measure and the risk neutral measure for 10,000 times. Note that the volatility of the diffusion term is constant, and volatility risk is not priced in this model. Hence, the diffusion terms are the same under the physical and the risk neutral measure.

Second, starting from the initial asset value $V_0 = 100$, we simulate 10,000 paths of daily returns. In each path, there are 21 daily returns, consisting of the daily returns in one month.

Third, for different level of leverage ratio, the equity value of the firm is then calculated based on Equation 4.22. The daily value are available both under the physical and under the risk neutral measure.

Fourth, the equity option values is the discounted average of the payoff of the option at the end of the month under the risk neutral measure. In this numerical example, we only consider at-the-money call option, i.e. the strike price of the option is equal to the initial stock price.

Finally, we construct a portfolio consisting of buying a call equity option and daily delta-hedging the underlying stock. The share of the stock is approximated as the delta, the first order derivative of the option price with respect to the stock price under the Black-Scholes model.

Summary

Modeling asset prices using Brownian motion has a long-lasting impact on the development of modern finance. However, it has been shown that much empirical evidence in the real-world financial markets are not consistent with the assumptions underlying the Brownian motion models. The chapters in this dissertation relax the assumptions of Brownian motion in different ways and study the implications of those deviations.

In the second chapter coauthored with Chen Zhou, we consider a dynamic model that the stock returns have continuous (diffusion) and discontinuous (jump) components. We further decompose these two components into systematic and idiosyncratic counterparts. By considering a general pricing kernel with all underlying risk factors, we derive the expected return of individual stocks and decompose it into four risk premiums related to the four types of risks. Empirically, we estimate the model jointly for daily stock returns and market returns and investigate the asset pricing consequences. We find that the idiosyncratic jump intensity contributes about 82.25% of the total jump intensity on average, and idiosyncratic variance contributes about 66.7% of the total diffusive variance, while the contribution of idiosyncratic risks decreases over the past 50 years. By sorting the stocks into five quintiles by estimated risk premiums, we find that the systemic jump risks, idiosyncratic diffusive risks and idiosyncratic jump risks are significantly priced in the cross section of our sample.

When the underlying distribution exhibits heavy tail or skewness, the Black-Scholes implied volatility is not an accurate estimate of the underlying risk neutral volatility. In the third chapter coauthored with Chen Zhou, we propose to use the maximum entropy method to estimate a single implied volatility measure from panels of option prices with different strike prices, called the "entropy-based implied volatility". The method is free of distribution assumptions and can better reveal the information of the risk neutral distribution contained in the option prices. Numerical examples show that the maximum entropy method outperforms the Black-Scholes model and the model-free method in extracting the implied volatility. Empirical evidence shows that the entropy-based implied volatility contains more information content in forecasting the future realized volatility than other volatility measures. If the volatility of the underlying process is stochastic or if there are discontinuous movements in the underlying process, the investors would pay insurance to hedge against the unfavorable volatility risk. In the last chapter with Aurelio Vasquez, we use the return of a delta-hedged equity option portfolio as the proxy of the variance risk premium and study the determinants of the variance risk premium in the equity option market both theoretically and empirically. In this chapter, we consider a stylized capital structure model with double-exponential jump diffusion process and derive the expected return of the delta-hedged option portfolio based on the model. Under realistic assumptions, the model shows that the expected return of delta-hedged equity option portfolio is negatively related to firm leverage, asset volatility, and debt covenant. Empirically, we find that a high leverage portfolio of delta-hedged calls under-performs a low leverage portfolio by a statistically significant 0.63% to 1.45% per month after controlling for asset volatility. The results are robust to other determinants of options returns such as idiosyncratic volatility, historical minus implied volatility, and option liquidity.

Nederlandse samenvatting (Summary in Dutch)

Het modeleren van prijzen van financiële instrumenten, zoals aandelen en obligaties, met een Brownse beweging heeft een langblijvend effect gehad op de ontwikkeling van moderne financiële theorië. Er is echter veel empirisch bewijs uit de financiële markten dat niet consistent is met de aannames van de Brownse beweging. In de hoofdstukken van deze dissertatie worden de aannames van de Brownse beweging op verschillende manieren afgezwakt en worden de implicaties van deze afwijkingen bestudeerd.

In het tweede hoofdstuk, geschreven met Chen Zhou, gebruiken we een dynamisch model waarin aandelenrendementen diffuse en discontinue sprong componenten bevatten. We ontbinden deze twee componenten in systematische en idiosyncratische tegenhangers. Door een algemene prijzingskernel te gebruiken met alle onderliggende risicofactoren, kunnen we de verwachte rendementen van individuele aandelen afleiden en deze ontbinden in vier risicopremies gerelateerd aan de vier risicotypes. In een empirische toepassing schatten we een gezamenlijk model voor dagelijkse aandelenrendementen en marktrendementen en bestuderen we de gevolgen voor financiële prijsstrategieën. We constateren dat de idiosyncratische sprongintensiteit gemiddeld gezien goed is voor circa 82.25% van de totale sprongintensiteit. De idiosyncratische variantie goed is voor circa 66.70% van de totale diffusie variantie, terwijl de bijdrage van het idiosyncratische risico is afgenomen in de laatste 50 jaar. Door het rangschikken van de aandelen in vijf kwintielen op basis van geschatte risicopremies, constateren we dat systeemsprongrisico's (systemic jump risks), idiosyncratische diffuse risico's, en idiosyncratische sprongrisico's significant worden geprijsd in de cross-sectionele steekproef.

Wanneer de onderliggende kansverdeling dikke staarten heeft of scheef is, is de Black-Scholes geïmpliceerde volatiliteit geen nauwkeurige schatter voor de onderliggende risico-neutrale volatiliteit. In het derde hoofdstuk, geschreven met Chen Zhou, stellen we een maximum entropy methode voor om een enkel geïmpliceerde volatiliteitsmaatstaf te schatten van panels van optieprijzen met verschillen uitoefenprijzen, genaamd de 'entropy-based implied volatility'. Deze methode maakt geen aannames over kansverdelingen en is beter in het onthullen van de risico-neutrale kansverdeling van optieprijzen. Numerieke voorbeelden laten zien dat de maximum entropy methode beter presteert dan het Black-Scholes model en de modelvrije methode in het afleiden van de geïmpliceerde volatiliteit. Empirisch bewijs laat zien dat de entropy-gebaseerde geïmpliceerde volatiliteit meer informatie bevat voor het voorspellen van toekomstige gerealiseerde volatiliteit dan andere volatiliteitsmaatstaven.

Wanneer de volatiliteit van het onderliggende proces stochastisch is of wanneer het onderliggende proces discontinue bewegingen bevat, zouden de investeerders verzekering willen betalen om te hedgen tegen ongewenste volatiliteitsrisico. In het laatste hoofdstuk, geschreven met Aurelio Vasquez, gebruiken we het rendement van een delta-hedged optieportefeuille als proxy voor de risicopremie voor variantie en bestuderen we de determinanten van deze risicopremie in de optiemarkt zowel theoretisch als empirisch. In dit hoofdstuk gebruiken we een gestileerd kapitaalstructuur model met een dubbel-exponentieel sprongdiffusie proces en leiden we het verwachte rendement af van het delta-hedged optieportefeuille gebaseerd op het model. Onder realistische aannames laat het model zien dat het verwachte rendement op het delta-hedged optieportefeuille negatief gecorreleerd is met de financieringshefboom van bedrijven, de volatiliteit van prijzen van financiële instrumenten, en leningsovereenkomsten. In een empirische toepassing constateren we dat een portefeuille van delta-hedged calls met een hoge hefboom slechter presteert dan een portefeuille met een lage hefboom met een statistisch significant 0.63% tot 1.45% per maand na het controleren voor volatiliteit in de prijzen van de financiële instrumenten. De bevindingen zijn robuust tegen het controleren voor andere determinanten van optierendementen zoals idiosyncratische volatiliteit, historische minus geïmpliceerde volatiliteit, en de liquiditeit van opties.

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