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FOUR ALTERNATIVE POLICIES TO RESTORE BALANCE OF PAYMENTS EQUILIBRIUM

BY J. TINBERGEN

Stimulated by Fleming's study on a related subject, the author compares four methods to eliminate balance of payments disequilibria between high-employment countries forming a closed group: (i) "discriminatory" import duties and subsidies, (ii) "discriminatory" duties only or the corresponding quantitative restrictions, (iii) nondiscriminatory duties, and (iv) devaluation or income adaptation. For each an "optimum version" is defined and chosen; they are compared as to (a) the loss in international trade and (b) the distribution of the "direct burden" between the countries (defined as the short-run loss in real expenditure). A number of rather specific simplifications are introduced, all tending to make the case as symmetric as possible with regard to countries and commodities. As a consequence of the high-employment hypothesis and of absence of production substitution, problems of optimum allocation of resources are ruled out; the approach is a short-run one. Only policies (i) and (iv) show no loss of trade, whereas the others do; but in the case of devaluation the "direct burden" is relatively heavier for the deficit countries than in the other three cases.

1. SETTING OF THE PROBLEM, ASSUMPTIONS MADE, AND PRACTICAL BACKGROUND

IN THIS paper I propose to compare—under assumptions to be specified and defended—four alternative policies to restore balance of payments equilibria. Let a group of m countries be given (forming together a closed group) with initial balance of payments deficits $\bar{D}^i$ ($i = 1, \cdots, m$) where

$$\sum_i \bar{D}^i = 0 \tag{1.1}$$

and hence some countries have negative $\bar{D}^i$, i.e., a surplus. As a rule it will be assumed that $\bar{D}^1 < \bar{D}^2 < \cdots < \bar{D}^m$. It will be assumed that in all four cases a policy of high employment will be pursued by all countries concerned, meaning that the total volume of production in each country is kept constant.

The policies to be considered are the following: I. *Discriminatory import duties and subsidies.* By this phrase I mean that each country imposes a tariff on its imports that is differentiated as to countries of origin ("discriminatory"). (When using the word discriminatory in the usual political sense we should not forget that it is doubtful whether the distinction between discrimination and nondiscrimination makes sense.) It is further assumed that the tariffs are so chosen as to maximise general welfare W of the group of countries (to be defined) under the side conditions that the balances of payments should be in equilibrium. As far as the optimum tariffs thus defined would turn out to be negative, it would mean that certain imports should be subsidized rather than taxed. This technique, which with other objectives, is actually applied in a number of countries, is therefore not excluded in this first policy considered, although the author very much doubts its practical possibility with the targets set here. It appears, however, that this case is an attractive theoretical case, useful for the understanding of the other, more practical policies.

II. *Discriminatory (positive) duties or import restrictions.* Here the same policy as under (I) is considered, but negative duties are excluded. Positive duties will restrict the volume of imports and the effect will, if the expenditure of the duties collected will satisfy a certain condition, be equivalent to a system of quotas that may be measured by the duties they replace.

III. *Nondiscriminatory import duties.* Here it is assumed that again only positive duties are imposed, but that no difference is made according to the origin of the imports. Still the objective is to eliminate the balance of payments deficits under the maximum level of *welfare.*

IV. *Devaluation or income adaptation.* This fourth policy consists of a general price decrease applied by each of the exporting countries to the extent made necessary by the targets indicated before.

The *assumptions* made are rather specific; therefore the paper only has a modest bearing. It is not intended as a basis for statistical testing but rather for teaching in that it tries—for reasons of simplification—to eliminate complications that, although important in practical problems, are not directly connected with the core of the questions at discussion. The main assumptions are: (1) a policy of high employment is pursued in all countries concerned, implying that production is not varied and that the problem of optimum allocation of resources is disregarded; (2) production does not use imported raw materials (or raw material suppliers are part of each country); from (1) and (2) it follows that there is no reason to expect changes in production cost and that the problems considered are mainly problems of distribution; (3) certain symmetry properties of the model are assumed that are not immediately connected with the problems at issue and that largely simplify the algebra; (4) the directive for finding, in each case of policy, the "best representative" of that case has been a welfare concept that is—as any such concept, subjective—and that in particular implies some degree of equality in satisfaction level between the countries studied.

These assumptions will be discussed in more detail at the places where they are introduced.

The *practical background* of the comparative study of these four policies may be seen in the desire to judge the methods that have, in the postwar years in particular, been used or advocated in order to equilibrate the balances of payments. The next objective is to derive devices for future use, in part for integration problems, although these problems are of a somewhat different structure. The immediate inducement to this study was the stimulating paper by M. Fleming.[1] Fleming follows a different method, in some aspects more general, in others less general; and the present paper may be considered in part as a comment on these latter aspects.

2. MODELS AND SYMBOLS USED

As the main instrument of our analysis a *macromodel* of a simple structure will be used in which it is assumed that each country *produces one good* which it supplies to all other countries and to its own home market. For country i the quantity supplied to country j will be indicated by x^{ij}, where the order of the indices indicates the direction of the movement of the commodities. All symbols (except those with a bar) indicate changes with respect to the initial situation; since a high-employment policy was assumed to be in operation before and after the adjustment, the sum total of all x^{ij}, representing production in country i, does not change:

$$\sum_{j=1}^{m} x^{ij} = 0. \tag{2.1}$$

In this sum, x^{ii} represents the quantity of i's produce retained for home consumption and investment; it equals

$$-\sum_{j \neq i} x^{ij},$$

where the inequality $j \neq i$ indicates that the summation should be extended over all values of j except i. Because of the high-employment policy exactly this quantity will be sold at the home market.

The *demand* for x^{ij} will depend, we assume, on all prices prevailing in country j, and on income y^j:

$$x^{ij} = \xi^{ij} y^j - \epsilon^{ij} p^{ij} + \frac{\epsilon^{ij}}{m - 1} \sum_{h \neq i} p^{hj}. \tag{2.2}$$

Some characteristics of this assumption are: (a) it is a linear function of real income y^j and of prices; (b) it is such that a uniform price rise does not affect demand (negative and positive terms then cancel); and (c) it is as symmetric in the various prices as possible.

Without further assumptions as to $\bar{x}^{ij}$, ϵ^{ij} does not represent an elasticity.

[1] M. Fleming, "On Making the Best of Balance of Payments Restrictions on Imports," *Economic Journal*, Vol. 61, 1951, p. 48.

Since high-employment policy may, as a first approximation, also be translated by $y^j = 0$, the first term does not play any role in our problems. For the same reasons it is assumed that the only causes for changes in price will be found in the direct effects of import duties, subsidies, or devaluation (income adaptation). This hypothesis implies, however, that no raw materials are imported from the other countries; or at least that their prices are not affected by the policies considered. One might, in the latter case, think of raw materials being freely imported everywhere in the case of policies (I), (II), and (III), or of countries already integrated with their raw material suppliers.

The initial values of all prices are supposed to be one, which is a question of a choice of units. Further, certain *properties of symmetry*, not being of immediate significance for the problems but very much simplifying certain relations, will be assumed to exist. It will be assumed that the initial values of the trade between any two countries are almost equal and that the elasticities of demand are also almost equal making all $\epsilon^{ij} = \epsilon$. The first of these two assumptions implies that the countries are, from the economic point of view, of almost equal size. Any application to real problems would require a certain grouping of the countries into groups of almost equal size. Complete equality of trade is incompatible, however, with the existence of initial balance of payments deficits and our exact choice of initial values must be in accordance with those deficits. Such an exact choice will only be made for a special case to be discussed later.

For certain purposes, a *micromodel* will be used that is more general in that it assumes n goods to be bought by each country from every other country, but less general as to the form of the demand functions. It is assumed that the demand for any good in any country (other than its own goods) depends on the price only of the good considered.

Sometimes it seems worth while to make calculations for the case of *three countries only* if those for m countries turn out to be very complicated. There is some sense in using this case, the simplest where the exports of one country are not always equal to the imports of another, as in the two-country model. In particular the *nearly symmetry* hypotheses have been worked out in more detail for the case of three countries. Since $\bar{D}^1 < \bar{D}^2 < \bar{D}^3$ and $\sum \bar{D}^i = 0$, $\bar{D}^1$ will always be negative and $\bar{D}^3$ positive. Hence, the initial values of the x will have to satisfy the conditions:

$$\bar{x}^{12} + \bar{x}^{13} > \bar{x}^{21} + \bar{x}^{31} \tag{2.3}$$

$$\bar{x}^{12} + \bar{x}^{32} \simeq \bar{x}^{21} + \bar{x}^{23} \tag{2.4}$$

$$\bar{x}^{13} + \bar{x}^{23} > \bar{x}^{31} + \bar{x}^{32} \tag{2.5}$$

Since $\bar{x}^{13}$ appears twice on the *greater side*, $\bar{x}^{12}$ once on the *greater side* and once in the more *neutral* relation (2.4), $\bar{x}^{23}$ and $\bar{x}^{32}$ once on both sides, $\bar{x}^{21}$ and $\bar{x}^{31}$ only on the *smaller side* and $\bar{x}^{21}$ once also in the relation (2.4), it makes sense to assume that $\bar{x}^{13} > \bar{x}^{12} > \bar{x}^{23} = \bar{x}^{32} > \bar{x}^{21} > \bar{x}^{31}$, although this certainly is not necessary. In order to have a simple example with probable values it is therefore assumed that

$$\begin{aligned} &\bar{x}^{13} = 1 + 2\delta, \quad &&\bar{x}^{23} = 1, \quad &&\bar{x}^{21} = 1 - \delta, \\ &\bar{x}^{12} = 1 + \delta, \quad &&\bar{x}^{32} = 1, \quad &&\bar{x}^{31} = 1 - 2\delta; \end{aligned} \tag{2.6}$$

in which case we find:

$$\bar{D}^1 = -6\delta, \qquad \bar{D}^2 = 2\delta, \qquad \bar{D}^3 = 4\delta. \tag{2.7}$$

For a fair comparison between the four policies we will take, as a representative for each type of policy, an *optimum version* of each. This optimum version is the one where the instruments have such numerical values as to maximise *welfare*. To this end we want a *definition of welfare W*. It is only too well-known that such a definition must always contain an element of arbitrary choice, but it is equally obvious that practical policy cannot wait until this problem has been solved. The choice made here, following Fleming, is that total welfare W may be considered as the sum of welfare figures W^i for the individual countries and that $W^i = W^i(x^{1i}, x^{2i}, \cdots, x^{mi})$, whereas:[2]

$$\frac{\partial W^i}{\partial x^{ji}} = p^{ji}. \tag{2.8}$$

3. CRITERIA USED FOR COMPARISON

The comparison to be made between the effects of the policies defined above will be based on three criteria. The first is the *loss* $\bar{L}$ *of international trade* they cause, compared with the initial situation; i.e., the decrease in total turnover of international trade, valued at initial prices. This in fact seems to be a correct (inverse) measure of total satisfaction, given the fact that in all situations considered production remains the same and that hence a loss of international trade only means that certain

[2] Choosing the sum of the national welfare functions each obeying (2.8) as the general welfare function evidently would be hardly justifiable if the standard of life of the countries considered diverges considerably. As long as there is no correlation between this standard of life and the deficit D^i our choice may be seen as a further symmetry assumption only. That p^{ji} has been chosen as a measure for the social utility of good x^j to country i may be motivated by pointing out that the consumption of that good is restricted to the level where the marginal buyer pays that price for it.

goods are consumed elsewhere.[3] Of course it has to be admitted that the initial situation of free trade was a state of disequilibrium and that hence in the long run its prices will have to change. But it so happens that the effect of policy (IV), which maintains free trade, shows no loss of international trade and that therefore our criterion also measures the loss of international trade in comparison to that case, which is in long-term equilibrium.

The second criterion to be used is the *distribution of the burden between the countries*. By this concept we mean nothing but the set of figures indicative of total consumption of (and investment in) goods in each country, disregarding the fact of changing foreign assets. The concept may be said to represent the *direct burden* on each country—i.e., the decrease in the volume of goods at their disposal during a time unit—and only to be a correct criterion for short-run views.

The third criterion we shall use is the *stability condition* that distinguishes devaluation from the other policies. For each policy, a stable new equilibrium will only be attained if the elasticities of demand fulfill certain requirements. Policies for which this condition is more stringent have less chance of being applicable, which is in itself a disadvantage. This may also be formulated by saying that such policies tend to make heavier disturbances necessary in the internal economy of the country considered.

4. DISCRIMINATORY IMPORT DUTIES AND SUBSIDIES

As announced already in Section 2, we will present our main argument with the help of our *macromodel*.

Let a positive or negative duty be levied of t^{ij} in country j on products from country i, bringing prices to the level $1 + t^{ij}$. The demand for product i in country j will then become:

$$x^{ij} = -\epsilon t^{ij} + \frac{\epsilon}{m-1} \sum_{h \neq i} t^{hj}, \tag{4.1}$$

$$x^{ii} = -\sum_{k \neq i} x^{ik}. \tag{4.2}$$

Assuming that these duties and subsidies be levied in such a way as to maximize *welfare* with the side conditions that equilibrium in the

[3] It might be asked why *welfare* was not taken as the criterion. This was in fact tried; it turned out to be a very complicated job to calculate the welfare change and as a first approximation it was equal in all four cases. "Loss in international trade" in a sense appears to be a more sensitive measure. In its turn it is, however, less appropriate as a basis for the "optimum version" of each policy, since it is a linear function of the t's.

balances of payments be restored, we may calculate their level with the help of the well-known Lagrange multipliers λ from the equations:

$$\frac{\partial W}{\partial t^{ij}} + \sum_k \lambda_k \frac{\partial D^k}{\partial t^{ij}} = 0 \qquad (i, j = 1, \cdots, m), \tag{4.3}$$

$$D^i = -\bar{D}^i \qquad (i = 1, \cdots, m). \tag{4.4}$$

Now we assumed (Section 2) that $W = \sum W^i$ $(i = 1, \cdots, m)$, where W^i only depends on x^{hi} $(h = 1, \cdots, m)$, i.e., the x consumed in country i. Further, t^{ij} directly influences only the demand for all imports into j, x^{hj} $(h = 1, \cdots, m$ but $h \neq j)$ and, via x^{hj}, through (4.2), x^{hh}, except x^{jj} which only depends on the import duties in the other countries. Hence

$$\frac{\partial W}{\partial t^{ij}} = \sum_{h \neq j} \frac{\partial W^j}{\partial x^{hj}} \frac{\partial x^{hj}}{\partial t^{ij}} + \sum_{h \neq j} \frac{\partial W^h}{\partial x^{hh}} \frac{\partial x^{hh}}{\partial t^{ij}}. \tag{4.5}$$

Here

$$\frac{\partial W^j}{\partial x^{hj}} = p^{hj} = 1 + t^{hj} \quad \frac{\partial W^h}{\partial x^{hh}} = 1, \tag{4.6}$$

and

$$\frac{\partial x^{hj}}{\partial t^{ij}} = \begin{cases} \dfrac{\epsilon}{m-1} & \text{for } h \neq i \quad (4.7) \\ -\epsilon & \text{for } h = i \quad (4.8) \end{cases}$$

$$\frac{\partial x^{hh}}{\partial t^{ij}} = \frac{-\epsilon}{m-1} \quad \text{for } h \neq i, j \tag{4.9}$$

$$\frac{\partial x^{ii}}{\partial t^{ij}} = \epsilon \qquad \frac{\partial x^{jj}}{\partial t^{ij}} = 0. \tag{4.10}$$

Therefore

$$\frac{\partial W}{\partial t^{ij}} = \sum_{h \neq i,j} (1 + t^{hj}) \frac{\epsilon}{m-1} - (1 + t^{ij})\epsilon - \sum_{h \neq i,j} \frac{\epsilon}{m-1} + \epsilon$$

or

$$\frac{\partial W}{\partial t^{ij}} = -t^{ij}\epsilon + \sum_{h \neq i,j} t^{hj} \frac{\epsilon}{m-1}. \tag{4.11}$$

The balance-of-payments deficit

$$D^k = \sum_{h \neq k} x^{hk} - \sum_{h \neq k} x^{kh}. \tag{4.12}$$

The import duty t^{ij} only influences the demand for x^{hj} $(h \neq j)$; hence we have:

$$\frac{\partial D^k}{\partial t^{ij}} = -\frac{\partial x^{kj}}{\partial t^{ij}} \quad \text{if} \quad k \neq j, \tag{4.13}$$

where this equals

$$\frac{\epsilon}{m-1} \quad \text{if} \quad k \neq i, \quad \text{and} \quad \epsilon \quad \text{if} \quad k = i, \tag{4.14}$$

and

$$\frac{\partial D^j}{\partial t^{ij}} = \sum_{h \neq j} \frac{\partial x^{hj}}{\partial t^{ij}} = -\epsilon + \frac{m-2}{m-1}\epsilon = -\frac{\epsilon}{m-1}. \tag{4.15}$$

Equations (4.3) therefore become:

$$-\epsilon t^{ij} + \frac{\epsilon}{m-1} \sum_{h \neq i,j} t^{hj} + \epsilon \lambda_i - \frac{\epsilon \lambda_j}{m-1} - \frac{\epsilon}{m-1} \sum_{k \neq i,j} \lambda_k = 0,$$

which may be written

$$(t^{ij} - \lambda_i + \lambda_j) - \frac{1}{m-1} \sum_{h \neq i,j} (t^{hj} - \lambda_h + \lambda_j) = 0 \tag{4.16}$$

$$(i, j = 1, \cdots, m; i \neq j)$$

These equations, $m^2 - m$ in number, each of them containing only the t^{hj} with the same j, i.e. for one country, may be divided into m groups, each for one j. Each group then contains $m - 1$ independent equations for the $m - 1$ unknowns, and their solution can easily be seen to be:

$$t^{ij} = \lambda_i - \lambda_j. \tag{4.17}$$

The values of λ_i have to be found from equations (4.4) which may be given the form

$$D^i = \sum_{j \neq i} x^{ji} - \sum_{j \neq i} x^{ij} = \sum_{j \neq i} \left\{ -\epsilon t^{ji} + \frac{\epsilon}{m-1} \sum_{h \neq j} t^{hi} \right\} - \sum_{j \neq i} \left\{ -\epsilon t^{ij} + \frac{\epsilon}{m-1} \sum_{h \neq i} t^{hj} \right\} = -\bar{D}^i. \tag{4.18}$$

Using only complete sums and deducting the appropriate items where incomplete sums are required, we get

$$-\sum_j t^{ji} + \frac{m-1}{m-1} \sum_h t^{hi} - \frac{1}{m-1} \sum_j t^{ji} + \sum_j t^{ij} - \frac{1}{m-1} \left(\sum_j \sum_h t^{hj} - \sum_h t^{hi} - \sum_j t^{ij} \right) = -\frac{\bar{D}^i}{\epsilon} \tag{4.19}$$

or

$$\frac{m}{m-1}\sum_j t^{ij} - \frac{1}{m-1}\sum_h \sum_j t^{hj} = -\frac{\bar{D}^i}{\epsilon}. \tag{4.20}$$

We could, if we wished, replace the t's by the expressions (4.17); we would then have m equations in the λ's, which add up to the identity $0 = 0$, meaning that one λ may be chosen arbitrarily. In fact, only the differences interest us. For certain purposes, however, it is even better to adhere to equations (4.20) in the t's.

As we have set out in Section 3, we are particularly interested in the *loss L in international trade* caused by this policy and in the changes in consumption of the individual countries.

L may be composed by adding the losses L^i for each country i in imports, where

$$L^i = -\sum_{j \neq i} x^{ji}$$

which, according to (4.19), after division by ϵ equals

$$\sum_j t^{ji} - \sum_h t^{hi} + \frac{1}{m-1}\sum_j t^{ji}.$$

Hence

$$\frac{L}{\epsilon} = +\frac{1}{m-1}\sum_i \sum_j t^{ji} = +\frac{1}{m-1}\sum_i \sum_j (\lambda_j - \lambda_i) = 0. \tag{4.21}$$

There appears to be *no loss in international trade.*

The change C^i in *consumption* calculated according to our device (Section 3) equals

$$C_i = \sum x^{ji} = \sum_{j \neq i} x^{ji} + x^{ii} = \sum_{j \neq i} x^{ji} - \sum_{j \neq i} x^{ij} = D_i = -\bar{D}_i,$$

i.e., *the change in consumption is equal to the initial balance-of-payments surplus,* a statement which is valid for all cases where import and export prices are not changed.

After having presented our argument with the help of the macromodel, we may add a few considerations based on the *micromodel.* As the reader will remember, the assumption was made for this model that more than one product is supplied from each country j to any other country i, these products being numbered with a lower index k but that the demand for good k only depends on its own price. Since again incomes are supposed to be constant we now have

$$x_k^{ji} = -\epsilon_k^{ji} t_k^{ji} \quad (i, j = 1, \cdots, m; k = 1, \cdots, n). \tag{4.22}$$

Welfare W^i now depends on mn quantities x_k^{ji} and $\partial W^i / \partial x_k^{ji} = 1 + t_k^{ji}$

whereas $\partial x_k^{ji}/\partial t_k^{ji} = -\epsilon_k^{ji}$. Hence

$$\frac{\partial W^i}{\partial t_k^{ji}} = -(1 + t_k^{ji})\,\epsilon_k^{ji}. \tag{4.23}$$

Apart from exerting a direct influence only on x_k^{ji}, t_k^{ji} indirectly influences x_k^{jj} because $x_k^{jj} = -\sum_{i \neq j} x_k^{ji}$, by virtue of which we have

$$\frac{\partial W^j}{\partial t_k^{ji}} = \epsilon_k^{ji}. \tag{4.24}$$

It follows that

$$\frac{\partial W}{\partial t_k^{ji}} = \frac{\partial W^i}{\partial t_k^{ji}} + \frac{\partial W^j}{\partial t_k^{ji}} = -t_k^{ji}\epsilon_k^{ji}. \tag{4.25}$$

For D^i we have

$$\begin{aligned} D^i &= \sum_{j\neq i}\sum_k x_k^{ji} - \sum_{j\neq i}\sum_k x_k^{ij} \\ &= -\sum_{j\neq i}\sum_k \epsilon_k^{ji} t_k^{ji} + \sum_{j\neq i}\sum_k \epsilon_k^{ij} t_k^{ij} = -\bar{D}^i, \end{aligned} \tag{4.26}$$

from which we deduce $\partial D^j/\partial t_k^{ji} = -\epsilon_{ks}^{ji}$, $\partial D^j/\partial t_k^{ji} = \epsilon_k^{ji}$, and $\partial D^h/\partial t_k^{ji} = 0$, $(h \neq i, j)$. The optimum values of the t_k^{ji} are therefore given by the equations

$$\frac{\partial W}{\partial t_k^{ji}} + \sum_h \lambda_h \frac{\partial D^h}{\partial t_k^{ji}} = -\epsilon_k^{ji} t_k^{ji} - \epsilon_k^{ji}(\lambda_i - \lambda_j) = 0 \tag{4.27}$$

or

$$t_k^{ji} = \lambda_j - \lambda_i\,. \tag{4.28}$$

We therefore find, in this case, essentially the same result as in (4.17), and in addition that t_k^{ji} is independent of k. The import duties must be equal for all goods supplied by country j to country i. This result is at variance with Fleming's device that the duties should be proportional to what he calls the responsiveness $R = \partial(px)/\partial x$. Fleming's result is however somewhat more general; ours is only valid with perfect competition. The main conclusion that the duties have to depend on the *order of strength* of the countries is common to all approaches, however.

5. DISCRIMINATORY (POSITIVE) DUTIES OR IMPORT RESTRICTIONS

We now assume that boundary conditions be imposed to the effect that *only positive values of t^{ij} are accepted*, which may either be interpreted literally as positive duties (excluding subsidies) or as representing import restrictions the extent of which is measured by the duties that would be necessary in order to attain the same effect.[4] This comes

[4] This involves certain hypotheses as to the way in which the proceeds of these duties are spent, which we need not specify here.

to (i) leaving out of our system of maximum conditions (4.16) a certain number of these equations, say, those for $t^{i'j'}$, where the values of i' and j' are as yet unknown and (ii) putting equal to zero these same $t^{i'j'}$. From the remaining equations the other t^{ij} can be determined; they have to fulfill the condition that they are all positive. It appears that this condition can be satisfied in different ways, depending on the data of the problem. As a *first case*, we shall treat *Fleming's solution*.

In this case[5] we are able to place the countries in an order of decreasing λ's: $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \cdots \geqslant \lambda_m$ and then put equal to zero all t's with $j' < i'$, i.e., with *falling* indices. There remain the t's with *rising* indices. The symmetry of the equations is somewhat disturbed and the new system may be described as follows.

No change occurs in the group of equations relating to the import duties of the last country m, since these duties all have rising indices. Hence these t^{im} remain equal to $\lambda^i - \lambda^m$, which are always positive. For any country $n < m$, the t^{in} with $i \geqslant n$ vanish and the group of equations is

$$(t^{1n} - \lambda_1 + \lambda_n) - \frac{1}{m-1}(t^{2n} - \lambda_2 + \lambda_n) \cdots - \frac{1}{m-1}(t^{n-1,n} - \lambda_{n-1} + \lambda_n) - A = 0$$

$$-\frac{1}{m-1}(t^{1n} - \lambda_1 + \lambda_n) + (t^{2n} - \lambda_2 + \lambda_n) \cdots - \frac{1}{m-1}(t^{n-1,n} - \lambda_{n-1} + \lambda_n) - A = 0$$

$$-\frac{1}{m-1}(t^{1n} - \lambda_1 + \lambda_n) - \frac{1}{m-1}(t^{2n} - \lambda_2 + \lambda_n) \cdots + (t^{n-1,n} - \lambda_{n-1} + \lambda_n) - A = 0,$$

where

$$A = +\frac{1}{m-1}(-\lambda_{n+1} + \lambda_n) + \frac{1}{m-1}(-\lambda_{n+2} + \lambda_n) \cdots + \frac{1}{m-1}(-\lambda_m + \lambda_n);$$

that is, to say, the terms in which the t's are equal to zero. The solution

[5] I am indebted to Messrs. J. Hartog and L. M. Koyck for a stimulating discussion out of which the following results emerged. They are not, however, responsible for any mistakes that still might be involved.

is easily found; writing s^{in} for $t^{in} - \lambda_i + \lambda_n$, a for $1/(m - 1)$, we have

$$\begin{aligned} s^{1n} - as^{2n} - as^{3n} \cdots - as^{n-1,n} &= A \\ - as^{1n} + s^{2n} - as^{3n} \cdots - as^{n-1,n} &= A \\ - as^{1n} - as^{2n} - as^{3n} \cdots + s^{n-1,n} &= A \end{aligned} \tag{5.1}$$

from which it is clear that $s^{1n} = s^{2n} = \cdots s^{n-1,n} = A/[1 - a(n - 1)]$. Elimination of the auxiliary symbols yields:

$$t^{in} = \lambda_i - \lambda_n + \frac{(m - n)\lambda_n - \sum_{n+1}^{m} \lambda_h}{m - n + 1}, \qquad (i < n). \tag{5.2}$$

These expressions are always positive since $\lambda_i > \lambda_n$ and $\lambda_n > \lambda_h$,

$(h = n + 1, \cdots, m)$.

There remains the problem to find the λ's; these are functions of the given initial deficits $\bar{D}^i$ which we have to determine by substituting equations (5.2) into the demand equations (4.1) and the latter into (4.12). We shall only illustrate the procedure for the case of *three countries.*

For $m = 3$ we find that now of the six import duties t^{ij} only t^{12}, t^{13}, and t^{23} remain different from zero. The duties in country 3 remain as before:

$$t^{13} = \lambda_1 - \lambda_3, \qquad t^{23} = \lambda_2 - \lambda_3, \tag{5.3}$$

whereas the one for country 2 becomes

$$t^{12} = \lambda_1 - \lambda_2 + \tfrac{1}{2}(\lambda_2 - \lambda_3). \tag{5.4}$$

The balance-of-payments deficits are, in the case where $\epsilon = 2$:

$$\begin{aligned} D^1 &= 2t^{12} + 2t^{13} - t^{23} = -\bar{D}^1 \\ D^2 &= -t^{12} - t^{13} + 2t^{23} = -\bar{D}^2 \\ D^3 &= -t^{12} - t^{13} - t^{23} = -\bar{D}^3. \end{aligned} \tag{5.5}$$

Combining (5.3), (5.4), and (5.5) we find

$$\begin{aligned} 4\lambda_1 - 2\lambda_2 - 2\lambda_3 &= -\bar{D}^1 \\ -2\lambda_1 + 2\tfrac{1}{2}\lambda_2 - \tfrac{1}{2}\lambda_3 &= -\bar{D}^2 \\ -2\lambda_1 - \tfrac{1}{2}\lambda_2 + 2\tfrac{1}{2}\lambda_3 &= -\bar{D}^3, \end{aligned} \tag{5.6}$$

and

$$\lambda_2 - \lambda_3 = \frac{-\bar{D}^2 + \bar{D}^3}{3}, \tag{5.7}$$

$$\lambda_1 - \lambda_2 = \frac{5}{12}\bar{D}^2 + \frac{1}{12}\bar{D}^3. \tag{5.8}$$

We cannot find the absolute values of the λ's, since the equations (5.5) and (5.6) add up to an identity $0 = 0$ and are therefore dependent; but we only need the differences between the λ's. If we like, we may transform (5.7) and (5.8) into expressions with $\bar{D}^1$ and without one of the other $\bar{D}$'s, since $\bar{D}^1 + \bar{D}^2 + \bar{D}^3 = 0$. From (5.7) it is clear that the order of countries 2 and 3 is unambiguous; always $\lambda_2 > \lambda_3$, since $\bar{D}^3 > \bar{D}^2$. The order of 1 and 2, however, need not coincide with the order of their $\bar{D}$'s; it may be that $\lambda_1 < \lambda_2$, since the sign of $\bar{D}^2$ may be negative.

Here we are confronted with the fact that Fleming's solution is not always possible. If

$$5\,\bar{D}^2 + \bar{D}^3 < 0$$

we are not able to put the countries in the order assumed; neither are we able to do so by interchanging countries 1 and 2.

This brings us back to the general problem where the number of countries need not be three. It would be somewhat lengthy to give a complete discussion. The following remarks may, however, be an indication of the situation. Dependent on the data it will be possible either to have only a smaller number of positive t's than in the Fleming solution or, sometimes, to have a larger number. There are, however, certain limits. On the one hand, it is always possible to have $m - 1$ positive t's, one of the possibilities being the one of the nondiscriminatory duties (cf. Section 6). On the other hand, it follows from the previous case (cf. Section 4) that not all the t's can be positive. More specifically, since the equations for the t's can be grouped according to the tariff-imposing countries, and the group of t's relating to one imposing country depends on the λ's in a way independent from the way in which the t's in any other imposing countries do, it follows that at least one of the t's in countries other than the weakest has to be taken equal to 0. Otherwise, the t's of such a country would be determined by the equations (4.17) and a number of them (where $i > j$) would be negative. There may be situations, it seems, where one t taken equal to zero would suffice. It may be easily shown that if for each country i only the duty with respect to the weakest country, viz., t^{mi} be taken equal to zero, the other duties become:

$$t^{ji} = \lambda_j - \lambda_m\,.$$

This is always positive again if we succeed in finding a set of λ_h's falling in value while h increases. The conditions to be fulfilled by the $\bar{D}^h$ in order that this applies will presumably be stricter than in Fleming's case. For the three-country model it can easily be shown that they sometimes can be met.

If, for countries i ($i \neq m, m - 1$), both t^{im} and $t^{i,m-1}$ are assumed to be zero, one finds, for the other t's

$$t^{ji} = \lambda_j - \frac{\lambda_{m-1}}{2} - \frac{\lambda_m}{2},$$

which will always be possibly positive, provided that the λ_h's meet the condition of falling with rising h's. It seems probable that this condition may be shown to be less stringent than in the previous case, etc. Thus depending on the set of $\bar{D}^h$, there may be found a different number of t's that can be chosen positive, with a minimum of $m - 1$ and a maximum of $m^2 - 2m + 1$.

Having discussed the problem of the optimum values of (positive) import duties we have now to enquire what the consequences are for the *loss L in international trade* and the changes in *welfare*. From (4.21) we see that, in our first case

$$\frac{L}{\epsilon} = + \frac{1}{m-1} \sum_i \sum_j t^{ji}, \qquad (j < i). \tag{5.9}$$

Since all t's are now positive, there is, with this type of policy, always a loss in international trade.

As an illustration, we calculate the loss for the three-country case. From (5.3), (5.4), (5.7), and (5.8) we find

$$\begin{aligned} L = t^{12} + t^{13} + t^{23} &= \lambda_1 - \lambda_2 + \tfrac{1}{2}(\lambda_2 - \lambda_3) \\ &+ \lambda_1 - \lambda_3 + \lambda_2 - \lambda_3 = \bar{D}^3 = -\bar{D}^1 - \bar{D}^2. \end{aligned} \tag{5.10}$$

For the special case this amounts to

$$L = 4\delta. \tag{5.11}$$

Also in the cases where Fleming's solution does not apply, formula (4.21) shows that there will always be a loss in international trade.

The *distribution of the burden* between the various countries, represented by the values of C^i, the change in *consumption*, is the same here as in the previous case for the reasons given in Section 4.

Finally, a few remarks may again be added on the solution of our present problem for the *micromodel*. Because of the simplifying hypothesis made for this model, that demand only depends on the price of the article considered, the solution is simpler here than in the macromodel. This will at once be clear from equations (4.27), where each t_k^{ji} is determined independently from all the others. In our present problem this means, that although a number of the t's are to be zero, this does not influence the equations for the other t's. Equation (4.28) therefore remains valid for all positive t's and it says that if we are again able to put the countries in the order of rising λ's all t's with *rising* indices will be admissible. Of course, the numerical values of the λ's are now different from those in the case of positive and negative t's. They have to be found from the side conditions which it is not difficult to express as functions of λ^i. This may be left to the reader.

6. NONDISCRIMINATORY IMPORT DUTIES

By nondiscriminatory import duties we shall understand duties r^i which are the same for imports of different provenance into the same country. We add the condition that at least one country shall impose no duties at all. This means that only $m - 1$ unknowns r^i $(i = 2, \cdots, m)$ are now introduced and that prices $p^{ji} = r^i$ for any j. Since the number of side conditions is also equal to $m - 1$, these conditions will determine the r's without allowing any room for making W a maximum. They will run as follows:

$$\begin{aligned} D^i = \sum_{j \neq i} x^{ji} - \sum_{j \neq i} x^{ij} &= \sum_{j \neq i} \left(-\epsilon r^i + \sum \frac{\epsilon}{m-1} r^i \right) \\ &\quad + \sum_{j \neq i} \left(\epsilon r^j - \sum \frac{\epsilon}{m-1} r^j \right) = -\bar{D}^i, \end{aligned} \tag{6.1}$$

where the sums inside the brackets are to be extended over $m - 2$ competing import goods and hence are equal to $(m - 2)\,\epsilon r^i/(m - 1)$ and $(m - 2)\epsilon r^j/(m - 1)$ respectively. Hence

$$-\epsilon r^i + \sum_{j \neq i} \epsilon \frac{r^j}{m-1} = -\bar{D}^i$$

or

$$-\frac{m}{m-1} r^i + \frac{1}{m-1} \sum_{j=1}^{m} r^j = -\frac{\bar{D}^i}{\epsilon}. \tag{6.2}$$

As a cross check we find that the sum total of these m equations yields the identity $0 = 0$. From $r^1 = 0$ it follows that

$$\frac{1}{m-1} \sum r^j = -\frac{\bar{D}^1}{\epsilon}$$

and therefore

$$\frac{m}{m-1} r^i = -\frac{\bar{D}^1}{\epsilon} + \frac{\bar{D}^i}{\epsilon}. \tag{6.3}$$

The duties appear to be proportional to the difference of the deficit of the country concerned with the deficit of country 1, the strongest country.

The *loss L of international trade* now amounts to

$$L = -\sum_i \sum_{j \neq i} x^{ji} = +\sum_i \epsilon r^i = -(m-1)\bar{D}^1. \tag{6.4}$$

For our special three-country model this amounts to $+12\delta$, which is considerably more than with discriminatory import duties, where $L = 4\delta$. Generally speaking the loss amounts, in the three-country case, to $-2\,\bar{D}^1$ as against $-\bar{D}^1 - \bar{D}^2$ with discriminatory duties. Since $\bar{D}^2 > \bar{D}^1$, it follows that

$$L^{discr} < L^{nondiscr}, \tag{6.5}$$

the loss is less if discrimination is applied.

The *distribution of the burden* between the countries is the same as in Section 4.

7. DEVALUATION

In this case, no import duties are levied, but export prices are decreased by t^i by country i. The important difference between this and the previous cases is that the prices in the balances of payments are now no longer equal to one, but to $1 - t^i$.

Also in the case of devaluation there are only $m - 1$ unknowns, viz., the price decreases t^i effectuated by country i, where t^1 will be taken equal to zero beforehand. Hence here again only the side conditions determine the solution. They are

$$D^i = \sum_{j \neq i} x^{ji} - \sum_{j \neq i} x^{ij} - \sum_{j \neq i} t^j + (m - 1)t^i = -\bar{D}^i, \tag{7.1}$$

where

$$x^{ji} = \epsilon t^j - \frac{\epsilon}{m - 1} \sum_{h \neq j} t^h. \tag{7.2}$$

Elaboration of these formulae yields

$$\sum_{j \neq i} \left(\epsilon t^j - \frac{\epsilon}{m - 1} \sum_{h \neq j} t^h \right) - \sum_{j \neq i} \left(\epsilon t^i - \frac{\epsilon}{m - 1} \sum_{h \neq i} t^h \right)$$

$$- \sum_{j \neq i} t^j + (m - 1)t^i = \epsilon \sum t - \epsilon t^i - \frac{\epsilon}{m - 1} \sum_{j \neq i} (\sum t - t^j)$$

$$- (m - 1)\, \epsilon t^i + \frac{\epsilon}{m - 1} \sum_{j \neq i} (\sum t - t^i) - \sum t + t^i + (m - 1)\, t^i$$

$$= \epsilon \sum t - \epsilon t^i - \epsilon \sum t + \frac{\epsilon}{m - 1} (\sum t - t^i) - (m - 1)\, \epsilon t^i + \epsilon \sum t$$

$$- \epsilon t^i - \sum t + t^i + (m - 1)t^i = -\bar{D}^i.$$

This may be written

$$(\epsilon m - m + 1)\, (\sum t - mt^i) = -\, (m - 1)\, \bar{D}^i. \tag{7.3}$$

Taking $i = 1$, we can express $\sum t$ as a function of $\bar{D}^1$ and substitute this in all the other equations, which gives us

$$t^i = \frac{m-1}{\epsilon m - m + 1} \frac{\bar{D}^i - \bar{D}^1}{m}. \tag{7.4}$$

As in (6.3) we here find that the degree of devaluation required for country i is proportional to the difference in deficit between country i and country 1. In order, however, that a stable solution be obtained, i.e., a positive value for t^i, the elasticity has to obey the condition

$$\epsilon m - m + 1 > 0 \quad \text{or} \quad \epsilon > 1 - \frac{1}{m}. \tag{7.5}$$

This condition corresponds to the well-known statement that the elasticity has to surpass some critical value, that depends on the details of the model. It represents a drawback for devaluation as an instrument of restoring equilibrium.

The *loss L in international trade* may be calculated from the first sum in (7.1) which has to be summed for all countries. According to the elaboration this equals the expression

$$-\epsilon t^i + \frac{\epsilon}{m-1}\left(\sum t - t^i\right) = -\epsilon \frac{m}{m-1} t^i + \frac{\epsilon}{m-1} \sum t \tag{7.6}$$

for country i and hence, in total amounts to

$$-\epsilon \frac{m}{m-1} \sum t - \frac{m\epsilon}{m-1} \sum t = 0.$$

There is *no loss of international trade.*

The *distribution of the burden* according to our previously given definition may be calculated in the following way:

$$D^i = C^i - \sum_{j \neq i} t^j + (m-1)t^i = -\bar{D}^i$$

or

$$C^i = -\bar{D}^i + \sum t - mt^i, \tag{7.7}$$

which, with (7.3), becomes

$$C^i = -\frac{\epsilon m}{\epsilon m - m + 1} \bar{D}^i. \tag{7.8}$$

Since the value of the denominator is, for *acceptable* ϵ's, smaller than that of the numerator, the distribution of the burden is more uneven than in the other cases. The strongest country wins more and the weakest loses more.

8. SUMMARY AND CONCLUSIONS

We may summarise our findings in the following table:

EFFECT OF FOUR ALTERNATIVE METHODS TO RESTORE BALANCE OF PAYMENTS EQUILIBRIUM

Policy	Discriminatory Import duties (or restrictions)		Nondiscriminatory Import duties (or restrictions)	Devaluation or Income Adaptation
	and subsidies	only		
Effects	I	II	III	IV
Loss of international trade......	0	$\frac{\epsilon}{m-1} \Sigma \Sigma t^{ji} (> 0)$	$-(m-1)\overline{D}^1 (> 0)$	0
(three countries)......	0	$-\overline{D}^1 - \overline{D}^2$ [b]	$-2\overline{D}^1$	0
(three countries, special case)...	0	4δ [b]	12δ	0
Burden on country i..	$\overline{D}^i$	$\overline{D}^i$	$\overline{D}^i$	$\frac{\epsilon m}{\epsilon m - m + 1} \overline{D}^i$
Extent of measures..	[a]	[a]	$r^i = \frac{m-1}{m} \frac{\overline{D}^i - \overline{D}^1}{\epsilon}$	$t^i = \frac{m-1}{m} \frac{\overline{D}^i - \overline{D}^1}{\epsilon m - m + 1}$
Stability condition..	$\epsilon > 0$	$\epsilon > 0$	$\epsilon > 0$	$\epsilon > 1 - \frac{1}{m}$

[a] This has not been calculated explicitly.
[b] This is true in Fleming's case.

From this table it appears that devaluation has, in comparison to the two practically possible systems of restriction II and III, the important advantage of a zero *loss in international trade,* meaning that the optimum use of resources will be better approached than with import duties or restrictions. It has the two disadvantages of *putting a heavier burden* on the *consumption* (cf. Section 3) of *the weaker countries* and of being bound to the condition that elasticities of demand should be large enough.

Of the two systems of restriction, it is clear that *discriminatory restriction reduces the loss of international trade,* as has been pointed out in particular by Frisch.

Method I, where also subsidies are applied and the *symmetry* is therefore maintained, is perhaps only of a theoretical interest, since, as has been said already, the application of subsidies by the strong countries in order to help the weak ones would not seem probable. It is theoretically interesting, however, since it represents a sort of a bridge between the other policies. It has in common with devaluation that there is no loss of international trade, and it has even the advantages over devaluation that the burden is distributed more evenly and that there is no difficulty with low elasticities.

These conclusions may be placed against a somewhat more general background which, however, necessarily must also be more speculative. A first remark may be made referring to the choice between devaluation, which implies the maintenance or even introduction of free trade, and the policies of restriction. The advantage of free trade (and hence devaluation or income adaptation in order to restore balance of payments) is evident, particularly so for long-term policies, since it implies the optimum use of resources. The disadvantage of being bound to higher elasticities also counts less in the long run since long-term elasticities are, generally speaking, higher than short-term. Restrictive policies on the other hand may be considered to be appropriate for the cure of temporary and large disequilibria as caused by catastrophes.

A second remark may be added referring to the choice between detailed and general policies. Cases I and II once more illustrate that detailed policies (i.e. policies with a large number of instruments) sometimes are superfluous. In fact, the result that $t^{ij} = \lambda^i - \lambda^j$ is, in various cases, considered the optimum solution, after all means that we do not use the many degrees of freedom we have at our disposal. We reduce the $m^2 - m$ values of the t's to only $m - 1$ values of the differences between m λ's. This will always happen (and is the basis of all defences of free-pricing systems) if the number of our political instruments exceeds the number of our targets. Detailed policies, in a sense are only necessary if there are a large number of targets to be attained that cannot be given a general covering formula. In practical matters this will often be so again for short-run policies, when all sorts of boundary conditions will have to be taken into account as a consequence of rigidities. As an example the maintenance of high employment may be cited. In the long run this is a single target since in the long run workers are mobile from one industry to another. In the short run it is a multiple target since there is lack of mobility. Only then, and hence temporarily, are detailed policies necessary.

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[Footnotes]

[1] **On Making the Best of Balance of Payments Restrictions on Imports**
M. Fleming
The Economic Journal, Vol. 61, No. 241. (Mar., 1951), pp. 48-71.
Stable URL:
http://links.jstor.org/sici?sici=0013-0133%28195103%2961%3A241%3C48%3AOMTBOB%3E2.0.CO%3B2-K