A Simple Test for Causality in Volatility*

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Abstract

An early development in testing for causality (technically, Granger non-causality) in the conditional variance (or volatility) associated with financial returns, was the portmanteau statistic for non-causality in variance of Cheng and Ng (1996). A subsequent development was the Lagrange Multiplier (LM) test of non-causality in the conditional variance by Hafner and Herwartz (2006), who provided simulations results to show that their LM test was more powerful than the portmanteau statistic. While the LM test for causality proposed by Hafner and Herwartz (2006) is an interesting and useful development, it is nonetheless arbitrary. In particular, the specification on which the LM test is based does not rely on an underlying stochastic process, so that the alternative hypothesis is also arbitrary, which can affect the power of the test. The purpose of the paper is to derive a simple test for causality in volatility that provides regularity conditions arising from the underlying stochastic process, namely a random coefficient autoregressive process, and for which the (quasi-) maximum likelihood estimates have valid asymptotic properties. The simple test is intuitively appealing as it is based on an underlying stochastic process, is sympathetic to Granger's (1969, 1988) notion of time series predictability, is easy to implement, and has a regularity condition that is not available in the LM test.

Keywords: Random coefficient stochastic process, Simple test, Granger non-causality, Regularity conditions, Asymptotic properties, Conditional volatility.

JEL: C22, C32, C52, C58

1. Introduction

Although there have been many practical applications of testing causality (technically, Granger non-causality) of the conditional mean, especially in economics, there have been fewer applications of testing for causality in conditional higher moments, especially the variance or volatility associated with financial returns.

An early development was the portmanteau statistic of non-causality in variance of Cheng and Ng (1996). A subsequent development was the Lagrange Multiplier (LM) test of non-causality in the conditional variance (technically, in the conditional volatility) by Hafner and Herwartz (2006), who provided simulations results to show that their LM test was more powerful than the portmanteau statistic.

This result is not especially surprising as LM tests are typically more powerful than portmanteau tests, wherein the null hypothesis is well specified but the alternative is not so as to capture a wide range of departures from the null. On the other hand, the LM test is intended to have high power of a null hypothesis when the true value of the parameter is close to that given under the null.

While the LM test for causality proposed by Hafner and Herwartz (2006) is an interesting and useful development, it is nonetheless arbitrary. In particular, the specification on which the LM test is based does not rely on an underlying stochastic process, so that the alternative hypothesis is also arbitrary, which can affect the power of the test.

The purpose of the paper is to derive a simple test for causality in volatility that is sympathetic to Granger's (1969, 1988) notion of predictability using a VAR time series model, provides regularity conditions that arise from the underlying stochastic process, namely a random coefficient autoregressive process, and for which the (quasi-) maximum likelihood estimates have valid asymptotic properties.

The simple test is intuitively appealing as it is based on an underlying stochastic process, is sympathetic to Granger's notion of time series predictability, is easy to implement, and has a regularity condition that is not available in the LM test of Hafner and Herwartz (2006), which is based on an arbitrary specification.

The plan of the paper is as follows. Section 2 provides a simple test for causality in volatility, Section 3 compares the new test with the LM test of Hafner and Herwartz (2006), and Section 3 gives some concluding comments.

2. A Simple Test for Causality in Volatility

Consider the conditional mean of financial returns for commodity i, as follows:

$$y_{it} = E(y_{it}|I_{t-1}) + \varepsilon_{it} , \qquad i = 1, 2, \dots , m \tag{1}$$

where the returns, $y_{it} = \Delta log P_{it}$, represent the log-difference in financial commodity prices, P_t, I_{t-1} is the information set for all financial assets at time *t-1*, $E(y_{it}|I_{t-1})$ is the conditional expectation of returns, and ε_{it} is a conditionally heteroskedastic error term.

In order to derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks, ε_{it}, which may be written as a random coefficient autoregressive process, as follows:

$$\varepsilon_{it} = \phi_{it}\varepsilon_{it-1} + \phi_{jt}\varepsilon_{jt-1} + \eta_{it} , \quad i \neq j , \tag{2}$$

where

$\phi_{it} \sim iid(0, \alpha_i)$, $\alpha_i \geq 0$,
$\phi_{jt} \sim iid(0, \alpha_j)$, $\alpha_j \geq 0$,
$\eta_{it} \sim iid(0, \omega_i)$, $\omega_i \geq 0$,
$\eta_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ is the standardized residual,
h_{it} is the conditional volatility obtained by setting $\phi_{jt} = 0$ in (2), namely:

$$\varepsilon_{it} = \phi_{it}\varepsilon_{it-1} + \eta_{it} \tag{3}$$

which gives:

$$E(\varepsilon_{it}^2 | I_{t-1}) \equiv h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 . \tag{4}$$

The stochastic process given in equation (2) incorporates causality, so that the null hypothesis of non-causality holds when ϕ_{jt} = 0, which is equivalent to α_j = 0. The stochastic process can be extended to asymmetric conditional volatility models (see, for example, McAleer (2014)), and to give higher-order lags and a larger number of alternative commodities, namely up to *m*-1, but the symmetric bivariate process considered here is sufficient to focus the key ideas associated with causality.

The conditional volatility arising from equation (2) is given as:

$$E(\varepsilon_{it}^2 | I_{t-1}) \equiv h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \alpha_j \varepsilon_{jt-1}^2 . \tag{5}$$

Adding first-order lags of h_{it} and h_{jt} leads to a conditional specification that gives a simple test for causality in volatility that is sympathetic to Granger's (1969, 1988) notion of predictability, namely:

$$h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \alpha_j \varepsilon_{jt-1}^2 + \beta_i h_{it-1} + \beta_j h_{jt-1}, \tag{6}$$

in which $\alpha_i \geq 0$, $\alpha_j \geq 0$, $\beta_i \in (-1, 1)$, and $\beta_j \in (-1, 1)$. The model in equation (6) is a GARCH(1,1) model for commodity *i* with volatility spillovers from commodity *j*.

As the stochastic process follows a random coefficient autoregressive process, under normality (non-normality) of the random errors, the maximum likelihood estimators (quasi- maximum likelihood estimators) of the parameters will be consistent and asymptotically normal. For further details, see Ling and McAleer (2003) and McAleer et al. (2008), who provide general proofs of the asymptotic properties of multivariate conditional volatility models based on satisfying the regularity conditions in Jeantheau (1998) for consistency, and in Theorem 4.1.3 in Amemiya (1985) for asymptotic normality.

Therefore, a test for causality, or of Granger non-causality, is a test of the null hypothesis:

$$H_0: \alpha_j = \beta_j = 0 \,, \tag{7}$$

against the alternative hypothesis:

$$H_1: \alpha_j > 0, \; \beta_j = 0. \tag{8}$$

The test statistics follows an asymptotic $\chi^2(2)$ distribution under the null hypothesis. Note that the test is one-sided for α_j as it cannot be negative, though it can be conducted as a two-tailed test, as in Hafner and Herwartz (2006).

It is worth noting that the model of conditional volatility in equation (6) holds under both the null and the alternative hypotheses as it is a valid conditional volatility equation arising from the random coefficient autoregressive process in equation (2).

3. Comparison with the LM Test

Using the notation of this paper, the LM test of Hafner and Herwartz (2006) is based on the specification given as:

$$\varepsilon_{it} = \eta_{it} \sqrt{h_{it}} \sqrt{g_{jt}} \,, \tag{9}$$

where g_{jt} is effectively a GARCH(1,1) model for commodity j, namely:

$$g_{jt} = \omega_j + \alpha_j \varepsilon_{jt-1}^2 + \beta_j h_{jt-1} \,, \tag{10}$$

where ω_j is set arbitrarily to unity, and g_{jt} could be replaced by h_{jt} without loss of generality. The LM test is a test of the null hypothesis in equation (7), which is equivalent to $g_{jt} = 1$, against the alternative hypothesis:

$$H_1: \alpha_j \neq \beta_j \neq 0, \tag{11}$$

which is a two-sided test statistic, and is asymptotically distributed as $\chi^2(2)$ under the null hypothesis.

It is worth noting that, although the test of the null against the alternative based on equation (9) is statistically valid, it does not have a clear underlying stochastic process as it is a product of a definition of the standardized shocks of commodity i, η_{it}:

$$\varepsilon_{it} = \eta_{it} \sqrt{h_{it}} \,,$$

and, as stated above, the conditional volatility of commodity j, g_{jt}, which could be replaced by h_{jt} without loss of generality.

Moreover, the conditional expectation of ε_{it}^2, which is the conditional volatility of ε_{it} in equation (9), is given by:

$$h_{it} = h_{it}\, g_{jt},$$

which holds only under the null hypothesis in equation (7), in which $g_{jt} = 1$, whereas the specification underlying the simple test given in equation (6) holds under both the null and the alternative hypotheses.

4. Conclusion

An early development in testing for causality in conditional variance (technically the conditional volatility) associated with financial returns, was the portmanteau statistic for non-causality in variance of Cheng and Ng (1996). A subsequent development was the Lagrange Multiplier (LM) test of non-causality in the conditional variance by Hafner and Herwartz (2006), who provided simulations results to show that their LM test was more powerful than the portmanteau statistic.

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