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SOME MEASUREMENTS OF ELASTICITIES OF SUBSTITUTION[1]

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INTRODUCTION

So far, when measuring elasticities of demand, most econometricians have concentrated upon the plain elasticity of total demand for a given commodity. For many important problems we should, in addition, like to know something of "partial elasticities," as I might provisionally call them. I am thinking of the elasticity of the demand for imported units only of a given commodity; or of the elasticity of the demand, for a given good, as far as supplied by one specific country. In both cases the demand relates to part of a market only; in the first instance total sales relate to all units of the given good supplied by imports as well as by home production; in the second instance total sales consist of the sales by the country considered as well as by all competing countries.

Of course it is a question of words, as Triffin has rightly pointed out,[2] whether we shall speak of a part of a market or a separate market, in any given case. The border line between different commodities, and hence between given markets, cannot be drawn unambiguously. Instead of saying that we consider two or more parts of one market, we may say that we are considering two or more immediately competing markets. Nevertheless, our statement does have a real meaning, which is that we cannot neglect, in a number of practical cases, the influence of competing prices. We have to know the dependence of demand on more than one price.

Knowledge of these "partial elasticities" is important for a number of problems of international trade: (1) what is the influence of a given change in exchange rates on the volume of employment and on the balance of payments or (ii) what decline in internal prices is necessary in order to effectuate a given payment in foreign currency? The transfer problem especially has fully regained its former importance now that we are faced again with heavy international debts.

In fact, the possibility of "regulating," either by changes in exchange rates or by price changes, the balance of payments or the volume of employment greatly depends on the values of the elasticities referred to. The "classical" view that it is easy, for example, to restore equilibrium in the balance of payments by a change in exchange rates is valid only if these elasticities are high. With low elasticities quite different things may happen. The equilibrium in the balance of payments may become an indifferent or an unstable equilibrium; there may be a rather low limit beyond which no transfer is possible, etc.[3] The statistical measurements made in the Netherlands Central Statistical Office indicate the possibility that elasticities are rather low. The purpose of this paper is to give a brief account of these measurements, their results, and the problems they raise.

DEFINITIONS

We have to start with some definitions and an exposition of some simple mathematical con-

[1] I am indebted to Mr. W. H. Somermeyer for valuable assistance in the preparation of this paper; some theoretical questions will be treated by him in a forthcoming publication.

[2] R. Triffin, *Monopolistic Competition and General Equilibrium Theory* (Cambridge, Mass., 1940).

[3] Cf. J. Tinbergen, "Unstable and Indifferent Equilibria in Economic Systems," *Revue de l'Institut International de Statistique*, 9 (1941), p. 36. Also A. Lerner, *The Economics of Control* (New York, 1944), p. 378.

nections between the various concepts to be discussed.

Indicating by x_1 and x_2 the quantities demanded in two parts of a market and by p_1 and p_2 the corresponding prices, we shall, to begin with, assume that there is a functional relation between

$$X = \frac{x_1}{x_2} \text{ and } P = \frac{p_1}{p_2} : X = f(P) \text{ or } \frac{x_1}{x_2} = f\left(\frac{p_1}{p_2}\right) \tag{1}$$

The elasticity of $\frac{x_1}{x_2}$ with respect to $\frac{p_1}{p_2}$, i.e.,

$$\epsilon = \frac{P}{X} \cdot \frac{dX}{dP} \text{ or } \frac{\frac{p_1}{p_2}}{\frac{x_1}{x_2}} \cdot \frac{d\frac{x_1}{x_2}}{d\frac{p_1}{p_2}}$$

will be called *elasticity of substitution*. This definition coincides with that given by Allen and Hicks in the simplest case conceivable, i.e., the case where only two commodities are considered. The assumption from which it starts, i. e., the functional relationship (1), is rather special. It has the advantage of being one of the simplest ways of describing two competing markets or a "two-part" market. If the relation holds — albeit only approximately — it is a very easy way of describing some problems in such markets.

Instead of using the concepts X and P, it is, for some purposes, useful to replace them by slightly different concepts X' and P', which we may call *quotas*:

$$X' = \frac{x_1}{\alpha x_1 + (1-\alpha)x_2} \text{ and } P' = \frac{p_1}{\beta p_1 + (1-\beta)p_2} \tag{2}$$

where the denominators represent some sort of an average quantity and an average price; α, $1 - \alpha$, β, and $1 - \beta$ are weights in these averages. They are supposed to be constants during any (small) variations in x_1 and p_1.

It may easily be shown that the corresponding "quota elasticity," ϵ' depends on ϵ in the following simple way:

$$\epsilon' = \frac{(1-\alpha)X'P}{(1-\beta)XP'} \cdot \epsilon \tag{3}$$

If we propose to measure ϵ' and ϵ along a short interval of variation of x_1 and p_1, it will be logical to use weights α and β fulfilling the conditions:

$$\alpha : (1-\alpha) = P = p_1 : p_2$$
$$\beta : (1-\beta) = X = x_1 : x_2,$$

the usual weights in quantity and price indices. The use of these weights in (3) yields:

$$\epsilon' = \epsilon \tag{4}$$

i. e., the *elasticity of substitution equals the quota elasticity*.

From (3) it is also easily seen that, for arbitrary values of α, (4) holds good as soon as X and consequently also the quantity quota X' are small, for then $X' = \beta = 0$.

We conclude this theoretical section with the statement that under perfect competition between the two parts of the market the elasticities ϵ and ϵ' will be very high figures. This is the state of affairs which until recently was assumed, by most practical economists, to exist on most international markets. The common view held was that any country, by lowering its export prices only fractionally, could attract a considerable part of its competitors' customers. Small wage cuts would be sufficient to overcome a depression in exports. Small decreases in exchange rates would be sufficient to restore a disturbed equilibrium in the balance of payments.

We now turn to a description of the results obtained by some statistical measurements.

RESULTS OF STATISTICAL MEASUREMENTS

The general idea of these measurements was (i) to test whether a relation of type (1) fits the facts sufficiently well, (ii) if so, what elasticity of substitution is found, and (iii) whether any divergencies in elasticities can be explained.

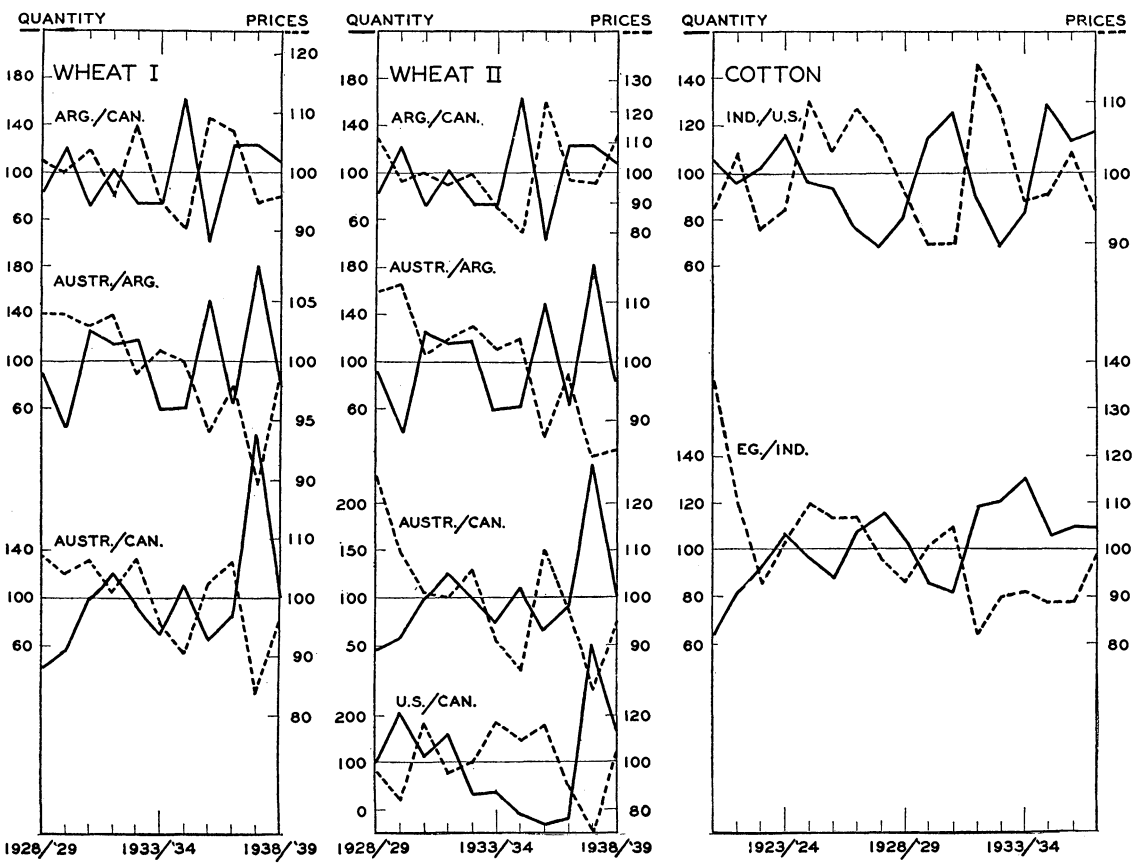
The material upon which these tests can be applied is abundant, since, in principle, all foreign trade statistics supply the figures needed. In principle, the imports into some given country by any two competing countries may be compared, or the imports by one country may be compared with total imports. Also, the substitution of imports by home production may be investigated. From a statistical viewpoint this is far more difficult since import statistics and

statistics of home production are not compared so easily as import figures from the same trade statistics. The procedure may, further, be applied to markets for separate commodities in the practical sense as well as for groups of commodities, which may even be the group of all commodities exported by a given country.

pose they seem the more accurate data. The second set (II) relates to the average export prices of the countries considered. Cotton prices are the prices of certain standard qualities at the Liverpool exchange.

This material shows clearly that there is a fairly high correlation between quantity ratios

CHART I. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS OF WHEAT AND COTTON, SELECTED COUNTRIES *



* Solid curves are quantity ratios; dashed curves are price ratios. Each pair of curves relates to two countries. Quantities are quantities exported by these two countries. Prices for wheat I are import prices in the United Kingdom for standard qualities from the countries concerned and for wheat II they are average export prices of these countries. Cotton prices are prices of certain standard qualities at the Liverpool exchange.

The first application to be discussed relates to two world staples, wheat and cotton. Each pair of curves in Chart I compares, for a given couple of countries, the quantity ratio and the price ratio.[4] Quantities are total exports in crop years. For wheat prices, two sets of data have been used. The first set (I) relates to import prices in the United Kingdom for standard qualities from the countries concerned; for our purand price ratios. In the case of cotton there seems to be a lag: quantities lag behind prices.

The elasticities for wheat have been calculated by using the first and second elementary regression equation (i. e., considering high prices and then quantities as independent variables), in order to obtain an idea of the range of error. The figures shown in Table I are rather high. The explanation seems to be the high degree of organization of the wheat market and the high degree of exchangeability of various sorts. It is interesting to note that, with one exception,

[4] Data for wheat are from Food Research Institute, Stanford University, *Wheat Studies*; data for cotton are from U. S. Department of Agriculture, *Agricultural Statistics*.

the figures for countries with the same crop season are higher than those for countries with a different season.

TABLE 1. — ELASTICITY OF SUBSTITUTION FOR WHEAT BASED ON (I) IMPORT PRICES FOR STANDARD QUALITIES IN THE U. K. AND (II) AVERAGE EXPORT PRICES IN THE COUNTRIES CONSIDERED; (1) FIRST AND (2) SECOND REGRESSION

Countries	I		II	
	1	2	1	2
Argentine/Canada	−3.6	−8.5	−1.8	−4.4
Australia/Argentine ...	−6.2	−15.0	−2.4	−7.7
Australia/Canada	−5.2	−9.4	−3.0	−5.9
U.S./Canada	...	...	−5.3	−11.3

In the case of cotton, elasticities were calculated with the aid of a multiple regression equation, "explaining" fluctuations in relative quantities by simultaneous fluctuations in relative prices and fluctuations in relative prices one year before. The results are presented in Table 2.

TABLE 2. — ELASTICITY OF SUBSTITUTION FOR COTTON DETERMINED BY LAGGED CORRELATION

Countries	Elasticity deduced from 1st regression	Correlation coefficient	Lag
India/U.S.A.	−2.9	0.89	6 months
Egypt/India	−1.1	0.69	3 months

In a twofold sense the competition between Indian and American cotton appears to be keener than that between Egyptian and Indian cotton: relative quantities react more accurately on price changes (higher correlation coefficient) and more intensively (higher elasticity). This corresponds to expectations. As to length of staple, American cotton takes the intermediate position. The elasticities found, however, are moderate.

A second application of the theory presented is given in Charts 2–6. Here the theory is applied not to single commodities but to groups of commodities of which *total exports of various countries* are composed. Quantity ratios are now calculated as the ratio of the "quantum index of exports" of the given country to the "quantum index of world trade" as published by the League of Nations.[5] Price ratios are calculated similarly. Although the correlations are lower, as would be expected, there are clear traces of correlation, except for India and Canada. For the other countries the elasticity of substitution may be calculated in one way or the other, as shown in Table 3.

[5] See *Review of World Trade*, 1938.

TABLE 3. — QUOTA ELASTICITIES FOR TOTAL EXPORTS

Country	Period	Method*	Elasticity determined from	
			1st regr.	2nd regr.
Argentine	1924–37	1	−0.5	−1.9
Australia	1924–37	1	−1.1	−1.5
Germany	1925–31	3	−2.3	−2.7
Hungary	1925–34	2	−2.5	−3.7
	1934–38	2	−0.7	−1.1
Japan	1924–38	1	−1.5	−1.7
South Africa	1924–37	1	−1.1	−1.9
United Kingdom	1924–32	1	−2.1	−2.5
	1924–32	4	−1.4	−2.3
United States	1924–38	4	−2.1	−3.9

* The methods used are indicated by the numbers 1 to 4.

Method 1. By simple correlation between X' and P' a regression coefficient a is found: $X' = aP'$, if X' and P' indicate deviations from average. The elasticity $\epsilon = a\frac{\overline{P'}}{\overline{X'}}$, where $\overline{P'}$ and $\overline{X'}$ are now average values.

Method 2. By simple correlation between the first differences $\Delta X'$ and $\Delta P'$ a regression coefficient a is found: $\Delta X' = a\Delta P'$, where $\Delta X'$ and $\Delta P'$ designate deviations from averages; again $\epsilon = a\frac{\overline{P'}}{\overline{X'}}$

Method 3. In order to account for a lag, a multiple correlation is established:

$$\Delta X' = a\Delta P' + b\Delta P'_{-1} \qquad (5)$$

where $\Delta X'$, $\Delta P'$ and $\Delta P'_{-1}$ are deviations from average and $\Delta P'_{-1}$ represents $\Delta P'$ lagged one year. The elasticity is taken as:

$$\epsilon = (a + b)\frac{\overline{P'}}{\overline{X'}}. \qquad (6)$$

Method 4. In order to account for a trend difference between X' and P', a multiple correlation is calculated:

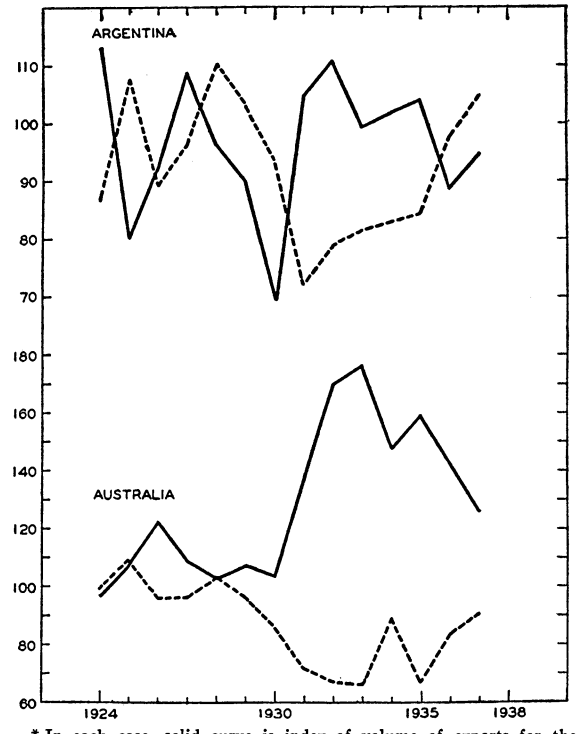
$$X' = aP' + bt \qquad (7)$$

where X' and P' are deviations from average and t represents time, counted from middle of period.

$$\epsilon = a\frac{P'}{X'}$$

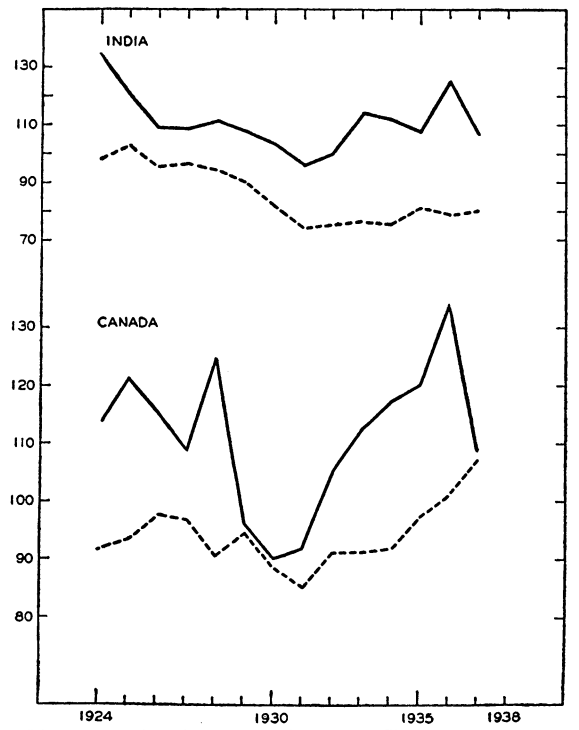
The highly interesting feature of the results of Table 3 is that they are centered around such a low figure as 2. This means that a 10 per cent reduction in the export price level only induces a 20 per cent increase in the volume of exports. This is far from the figures suggested by the theory of free competition. The results show, in other words, that for these problems some

CHART 2. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR ARGENTINA AND AUSTRALIA *



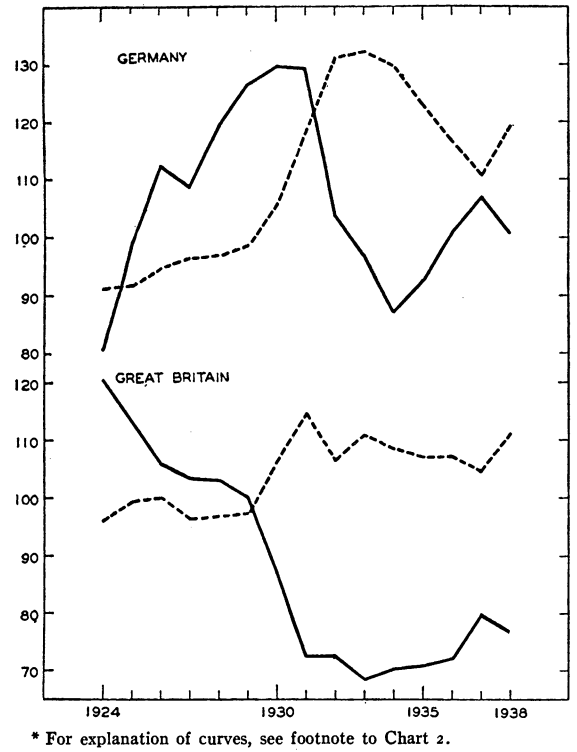
* In each case, solid curve is index of volume of exports for the country concerned divided by index of volume of world trade; dashed curve is index of export prices for the country concerned divided by index of world trade prices.

CHART 3. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR INDIA AND CANADA *



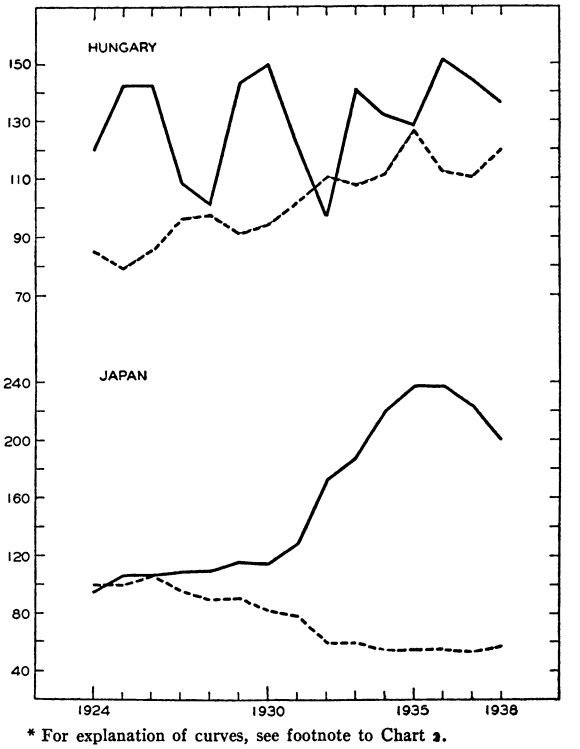
* For explanation of curves, see footnote to Chart 2.

CHART 4. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR GERMANY AND GREAT BRITAIN *



* For explanation of curves, see footnote to Chart 2.

CHART 5. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR HUNGARY AND JAPAN *

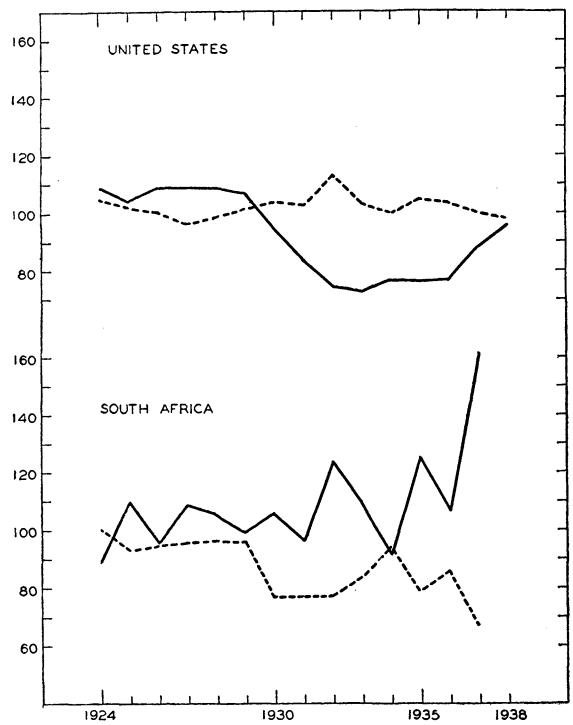


* For explanation of curves, see footnote to Chart 2.

theory of imperfect competition is needed to cover the facts. As already stated in the introduction, these figures are of great importance for the judgment of some practical problems of international financial and trade policy.

As a third application some results may be given concerning the *substitution of imports by home production.* These cases, as stated above, are less simple from the statistical viewpoint, since as a rule import statistics and home production statistics are less comparable than import statistics among each other. Chart 7 relates to the case of potatoes in the United Kingdom. This is one of the rare cases where a good correlation was obtained. The corresponding

CHART 6. — COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR UNITED STATES AND SOUTH AFRICA *



* For explanation of curves, see footnote to Chart 2.

elasticity of substitution appeared to be −1.7.

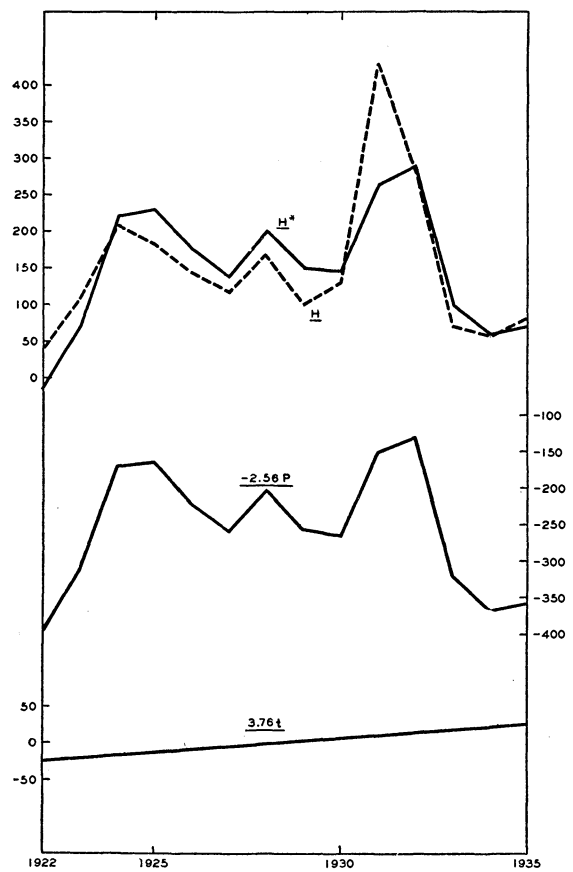
Charts 8–10 give some results obtained from a study of Swedish figures. Most of the cases did not yield a satisfactory correlation; the cases represented are the best ones. They relate to cast iron, metal plates, and plate and tin plate articles. The elasticities found center round a figure of about unity: −0.91, −1.21, and −0.97, respectively.

COMPARISON WITH SOME EARLIER RESULTS

A comparison of the results with some other investigations, published earlier, is of interest.

In a sense the demand studies made by Schultz [6] cover the same subject, as far as the prices of more than one commodity are included. The best-known cases are those of beef,

CHART 7. — ENGLAND: COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR POTATOES *



* H: ratio of quantities imported to quantities produced (observed).
H*: best approximation of H, calculated as a linear function of P, ratio of import price to home price, and t, trend.
Equation found: $H^* = -2.56\,P + 3.76\,t$, the terms of which are presented by the two lower curves. The corresponding elasticity of substitution is −1.70.

pork, and mutton, tea, coffee, and sugar, and barley, corn, hay, and oats. The relations Schultz fits are, however, of a more general character than ours. In his simplest case, where only two prices are considered, the relations studied are of the type:

$$\log x_1 = a_{11} \log p_1 + a_{12} \log p_2 \qquad (8)$$

[6] H. Schultz, *The Theory and Measurement of Demand* (Chicago, 1938), particularly Part III.

$$\log x_2 = a_{21} \log p_1 + a_{22} \log p_2, \qquad (9)$$

omitting further terms as income terms etc.

Subtracting, we obtain

$$\log \frac{x_1}{x_2} = (a_{11}-a_{21}) \log p_1 + (a_{12}-a_{22}) \log p_2 \qquad (10)$$

This can be written in the form:

$$\log \frac{x_1}{x_2} = A \log \frac{p_1}{p_2} \qquad (11)$$

only if $a_{11} - a_{21} = a_{22} - a_{12}$, which generally is not the case. For example, for beef and pork in the United States [7] Schultz finds $a_{11} = -0.49$, $a_{12} = +0.46$, $a_{21} = +0.35$, $a_{22} = -0.81$; hence $a_{11} - a_{21} = -0.84$ and $a_{22} - a_{12} = -1.27$. It is interesting, nevertheless, that these figures also are rather low and in the neighborhood of unity.

In a study by L. J. Cools [8] a relation is established between C, the ratio of butter and margarine consumption, and P, the price ratio for these commodities:

$$C = -1.64\,P + 0.025\,P^2 - 2.56\,T - 14.7$$

where T represents a trend series. For the average values of C and P the corresponding elasticity, which is the elasticity of substitution according to our definition, amounts to -2.4.

A. J. Brown, when studying the liquidity preference schedules of the London Clearing Banks,[9] assumed the relation between cash balance and interest rate to be of the type:

$$\log C/B = a_1 + b_1 \cdot \log \frac{y_B}{y_C} + c_1 \cdot \log (CB)$$

where C = cash and short-term claims; B = bills; y_C = average yield on C; and y_B = average yield on B.

Evidently b_1 is the elasticity of substitution. Brown found it to be -0.26, a very low figure indeed. In a similar relation explaining the fluctuations of log I/A, where I = investments and A = advances, he found an elasticity of substitution of -0.66.

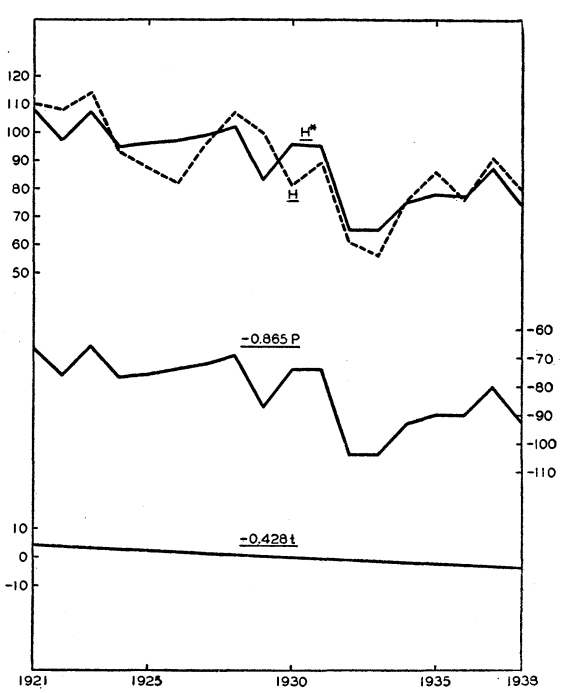
TABLE 4. — ELASTICITIES OF SUBSTITUTION OF A NUMBER OF DUTCH EXPORT PRODUCTS

Commodity	Market	Competing Countries Considered	Period	Elasticity
Butter *	England	Australia, New Zealand	1922–37	−2.0
Cheese	England	Canada, New Zealand	1927–37	−1.2
Cheese	Belgium	France	1927–37	−2.3
Bacon	England	All other countries	1926–37	−3.0
Coal	Belgium	" " "	1926–37	−2.0
Cotton manufacture	Netherland Indies	" " "	1927–37	−4.8 †
Shoes	Netherland Indies	" " "	1928–38	−2.8

* Danish and Dutch butter together.

† Average of figures obtained for separate periods.

CHART 8. — SWEDEN: COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR CAST IRON *



* For descriptions of curves, see footnote to Chart 7. Equation found: H* = −0.865 P −0.428 t, the terms of which are represented by the two lower curves. The corresponding elasticity of substitution is −0.914.

[7] *Ibid.*, pp. 582, 583.

[8] L. J. Cools, "Réactions réciproques des marchés du beurre, de la margarine et du saindoux en Belgique de 1920 à 1937," *Bulletin de l'Institut de Recherches Economiques*, August, 1938.

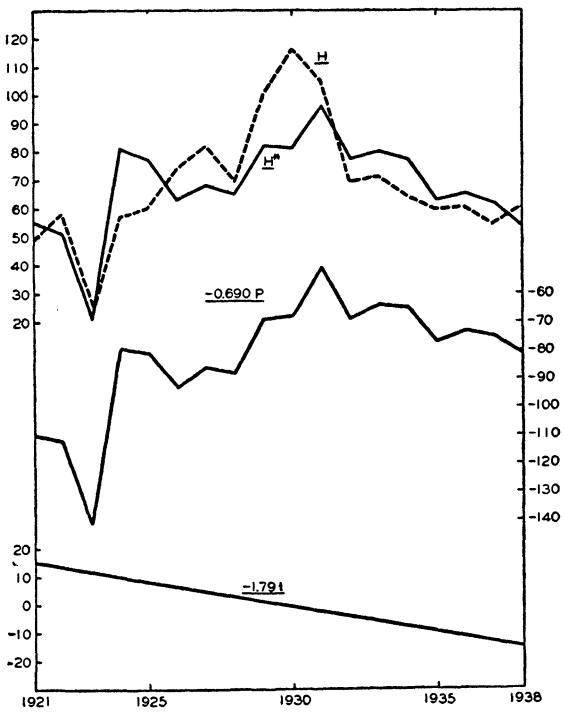
[9] A. J. Brown, *The Liquidity Preference Schedules of the London Clearing Banks*, Oxford Economic Papers 1 (1938), p. 49.

Derksen and Rombouts [10] published a number of figures on elasticities of substitution for Dutch export products at various markets. Their chief results may be summarized as in Table 4.

CONCLUSIONS

Two *conclusions* of the results discussed seem outstanding: 1. In a great number of cases it is impossible to obtain a good fit simply by assuming that the ratio of quantities is a function of the price ratio.

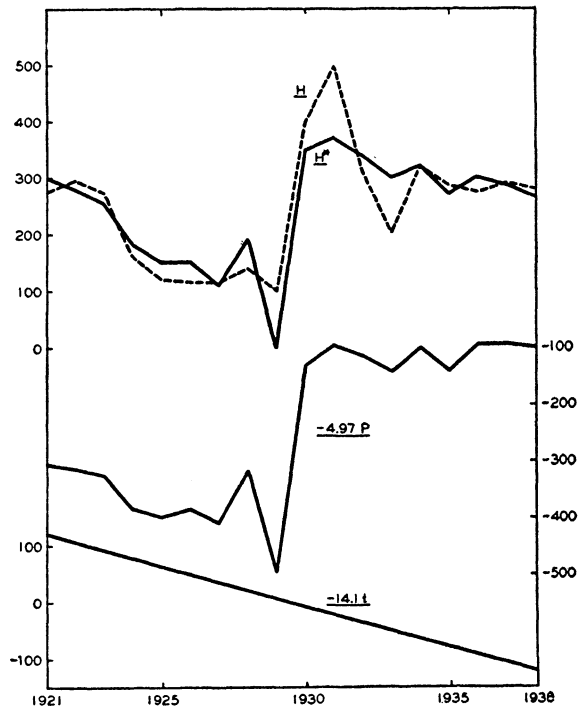
CHART 9. — SWEDEN: COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR METAL PLATES *



* For descriptions of curves, see footnote to Chart 7. Equation found: $H^* = -0.690\ P - 1.79\ t$, the terms of which are represented by the two lower curves. The corresponding elasticity of substitution is -1.21.

[10] "The Influence of Prices on Exports," *De Nederlandsche Conjunctuur*, Special Memorandum No. 1 (The Hague, 1939).

CHART 10. — SWEDEN: COMPARISON OF QUANTITY RATIOS AND PRICE RATIOS FOR PLATE AND TIN PLATE GOODS *



* For descriptions of curves see footnote to Chart 7. Equation found: $H^* = -4.97\ P - 14.1\ t$, the terms of which are represented by the two lower curves. The corresponding elasticity of substitution is -0.97.

2. Where a good or a fairly good fit is obtained, the corresponding elasticities of substitution are almost always rather low. They center around a value of -2. Considerably higher values are found for well-organized staple markets; for exports as a whole, and still more for imports as a whole, low figures are obtained, much lower than the theories of free competition would suggest.

The consequences for the theory of international trade and the balance of payments are very important. It would be of great use to the further development of these theories if the field were explored more intensively.

[Footnotes]

[3] **Unstable and Indifferent Equilibria in Economic Systems**

J. Tinbergen

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